Solutions to Homework Set #3

1 Public Good

You find a very exciting summer internship job as an assistant to the director of a regional recreation office, and your first assignment is to give advice to the director about managing a beautiful wildlife reserve, which is called Diminishing Island. Below is the summary of information you have:

- The maintenance cost is $MC = 80A$ where $A$ is the acreage of land maintained in Diminishing Island.

- There are 100 citizens. But only two types of residents are attracted to Diminishing Island: 10 bird-watchers and 20 wild-flower lovers. The individual marginal benefits are given by $MB_b = 20 - 2A$ for each bird-watcher, and $MB_w = 20 - A$ for each wild-flower lover.

1. Aggregate demand and optimal level of public good

(a) What is the aggregate demand for land acreage in Diminishing Island?

**Solution:** The total marginal benefit $TMB$ is a vertical summation of marginal benefits of 10 bird-watchers ($TMB_b$) and 20 wild-flower lovers ($TMB_w$) such that

$$
TMB_b = MB_b * N_b = (20 - 2A) * 10 = 200 - 20A,
$$
$$
TMB_w = MB_w * N_w = (20 - A) * 20 = 400 - 20A,
$$

$$
TMB = \begin{cases} 
TMB_b + TMB_w & \text{if } 0 \leq A \leq 10 \\
TMB_w & \text{if } 10 < A \leq 20 
\end{cases}
$$

\[ TMB = \begin{cases} 
600 - 40A & \text{if } 0 \leq A \leq 10 \\
400 - 20A & \text{if } 10 < A \leq 20
\end{cases} \tag{1}

**Note:** The aggregate demand for public good is a vertical summation of all individual marginal benefits; and the aggregate demand for private good is a horizontal summation of all individual marginal benefits.

(b) What is the social optimal land acreage maintained in Diminishing Island?

**Solution:** The social optimal land acreage maintained is achieved when the total marginal benefit equals the total marginal cost. In this case, the total marginal benefit written by Equation (1) has two pieces.

if $TMB$ intercepts $TC$ at the range of $A$ such that $0 \leq A \leq 10$ $\rightarrow 600 - 40A = 80A \rightarrow A = 5,$

if $TMB$ intercepts $TC$ at the range of $A$ such that $10 < A \leq 20$ $\rightarrow 400 - 20A = 80A \rightarrow A = 4.$

Conclusion: the optimal acreage of land maintained is $A = 5$ \tag{2}

2. Management of public good

After two-week reading and talking with people, you have the four proposals on management of Diminishing Island.

(a) **Proposal one: the local government operates Diminishing Island using its tax revenue.** How much is the optimal maintained acreage in Diminishing Island? How much is tax needed
to be collected by the government? How much does each citizen need to pay in taxes for Diminishing Island? How much is total consumer surplus?

**Solution:** The optimal maintained acreage is given by \( A = 5 \). If the public good is maintained by government using tax revenue, government needs to collect tax to cover the total cost of providing the optimal level of public good.

\[
TC_G = \int_0^5 MC = \int_0^5 (80A)dA = 40A^2\bigg|_0^5 = 40 \times 5 \times 5 = $1000
\]  

Thus, government need to collect taxes up to $1000, and each citizen needs to pay for \( 1000/\text{100} = $10 \).

Consumer surplus \( CS_G \) is given by

\[
CS_G = \int_0^5 TMBdA - \int_0^5 MCdA = \int_0^5 (600 - 40A)dA - \int_0^5 8AdA = $1500.
\]

(b) **Proposal two:** the local government operates Diminishing Island by collecting entry fee.

How much is the optimal maintained acreage of Diminishing Island? How much is the entry fee? Who will pay the entry fee? How much is total consumer surplus?

**Solution:** The optimal maintained acreage is given by \( A = 5 \). Government needs to collect entry fee to cover the total costs, i.e., the total entry fee equals to the total costs. Therefore, the entry fee per person is given by

\[
e_g = TC_G/N = 1000/30 = 100/3.
\]

where \( e_g \) is greater than the individual benefit for bird-watcher and wild-flower lovers \( (TB_b = \int_0^5 (20 - 2A)dA = 75 \text{ and } TB_w = \int_0^5 (20 - A)dA = 175/2) \). Thus, all bird-watchers and wild-flower lovers will pay the entry fee. Total total consumer surplus is \( \int_0^5 (600 - 40A)dA - \int_0^5 8AdA = 1500 \).

(c) **Proposal three:** contracts a concession to a private firm to manage Diminishing Island.

This firm will charge admission fee and has the same maintenance cost \( MC = 80A \) (note: it is possible that a concessionaire has a lower marginal maintenance cost since a private firm possible manage public good more efficiently). What is the optimal maintained acreage of Diminishing Island? How much is the admission fee per person? How much is total consumer surplus and profit for this firm?

**Solution:** The optimal acreage is given by \( A = 5 \). Concessionaire will charge the entry fee such that the collection of entry fee will cover the management cost and also its producer surplus.

\[
E_c = (A^*\lambda)/N = 5 \times MC(5)/N = 5 \times (80 \times 5)/30 = 200/3
\]

where \( A^* \) is the optimal level of public good and \( \lambda \) is the shadow price of public good such that \( \lambda = MC(A^*) \). All bird-watchers and wild-flower lovers will pay the entry fee since \( 200/3 < TB_b = 75 \text{ and } 200/3 < TB_w = 175/2 \). Profit and total consumer surplus are given by

\[
\Pi_c = E_c \times N - TC_c = 200/3 \times 30 - \int_0^5 (80A)dA = 2000 - 1000 = $1000,
\]

\[
CS_c = \int_0^5 TMBdA - E_c \times N = \int_0^5 (600 - 40A)dA - 200/3 \times 30 = 2500 - 2000 = $500.
\]
Proposal four: give it to the director’s big campaign donor, Mr. Greedy, to run (which you read from the director’s mind).

How much is the acreage of land Mr. Greedy decides to maintain? How much is the admission fee per visitor? How much is total consumer surplus and profit?

**Solution:** He can either charge a high entry fee to attract only wild-flower lovers, or charge a low entry fee to attract bird-watchers and wild-flower lovers. Whether he choose the high or low entry fee depends on the corresponding profit.

If Mr. Greedy chooses to charge the high entry fee \( e_1 \), the optimal acreage of land maintained is achieved when \( TMB_w = MC \) such that \( TMB_w = 400 - 20A \) and \( MC = 80A \). Therefore, the optimal acreage is \( A = 4 \), and the entry and profit \( \pi_1 \) are given by

\[
e_1 = TB_w = \int_0^4 (20 - A)dA = 72, \tag{9}
\]

\[
\pi_1 = e_1 \times 20 - TC = 72 \times 20 - \int_0^4 80AdA = 1440 - 640 = $800. \tag{10}
\]

In this case, bird-watchers are not willing to pay since \( TB_b = \int_0^4 (20 - 2A)dA = 64 \) which is lower than the entry fee.

Consequently, total consumer surplus \( CS_1 \) is given by

\[
CS_1 = \int_0^4 TMB_w dA - e_1N_w = \int_0^4 (400 - 20A)dA - 72 \times 20 = 0. \tag{11}
\]

If Mr. Greedy chooses to charge the low entry fee \( e_2 \), the entry fee and profit \( \pi_2 \) are given by

\[
e_2 = TB_w = \int_0^{A_m} (20 - 2A)dA = 20A_m - A_m^2, \tag{12}
\]

\[
\pi_2 = e_2 \times (10 + 30) - TC = (20A_m - A_m^2) \times 30 - \int_0^{A_m} 80AdA = 600A_m - 70A_m^3. \tag{13}
\]

Mr. Greedy will maximize his profit by choosing the optimal \( A_m \), which is given by Equation (14). The optimal level of land acreage maintained is given by \( 600 - 140A_m = 0 \). Thus, \( A_m = 30/7 \).

\[
\max_{A_m} \pi_2 = \max_{A_m} (600A_m - 70A_m^2) \tag{14}
\]

Substituting \( A_m = 30/7 \) into Equations (12) and (13) yields the entry fee \( e_{m2} = 3300/49 \) and profit \( \pi_2 = 63000/49 \). Consumer surplus \( CS_2 \) is given by

\[
CS_2 = TMB(A_m) - e_2 \times N = \int_0^{30/7} (600 - 40A)dA - 3300/49 \times 30 = 9000/49. \tag{15}
\]

Since the profit from charging low entry fee is lower than charging high entry fee (\( \pi_2 = 63000/49 > \pi_1 = 800 \)), Mr. Greedy will charge a low entry fee to attract both types of consumers.

**Note:** In this case, we assume that monopoly cannot distinguish types of consumers, and thus Mr. Greedy can only charge a single entry fee. If monopoly can distinguish consumers, he can charge different entry fee to different consumers, i.e, he can charge the entry fee up to the total benefit for each consumer.
You want to summarize your proposals in the table below and send it to the director as a memo (you are also required to show your work for all questions).

<table>
<thead>
<tr>
<th>Table 1: Summary of Your Report</th>
</tr>
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<tbody>
<tr>
<td><strong>Manager</strong></td>
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<tr>
<td><strong>Funding</strong></td>
</tr>
<tr>
<td><strong>Optimal Maintained Acreage of Land</strong></td>
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<tr>
<td><strong>entry fee</strong></td>
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<tr>
<td><strong>total consumer surplus</strong></td>
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<tr>
<td><strong>profit</strong></td>
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</tbody>
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2 Technology Adoption

You take EEP101 and work as a part time consultant at a high-tech company in downtown Berkeley this semester. Your current assignment is to give advice about whether it is profitable for this company to adopt a new technology. The production function is given by $f(h_i(a_i)a_i) = \sqrt{h_i(a_i)a_i}$ where $a_i$ is the applied input and $h_i(a_i)$ is the input use efficiency rate for technology $i$. The other information of the current and new technology is summarized in the following table:

<table>
<thead>
<tr>
<th>Table 2: Technology Adoption</th>
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</thead>
<tbody>
<tr>
<td>Current Technology</td>
</tr>
<tr>
<td>fixed cost $K$</td>
</tr>
<tr>
<td>input use efficiency rate $h_i(a_i)$</td>
</tr>
<tr>
<td>output price $p$</td>
</tr>
<tr>
<td>input price $w$</td>
</tr>
</tbody>
</table>

1. Should this firm stick to the current technology or switch to the new technology?

**Solution:** Two steps to determine whether this firm should switch to the new technology:

(a) *Find out the profit-maximizing input and corresponding profit level of using the current or new technology;*

(b) *choose the technology which has the higher profit.*

In this case, if this company sticks to the current technology, its profit maximizing problem is given by Equation 16.

$$\pi_1 = \arg\max_{a_1} \left\{ 200\sqrt{0.4a_1} - 10a_1 - K_1 \right\}$$

**First Order Condition:**

$$\frac{200\sqrt{0.4}}{2\sqrt{a_1}} - 10 = 0 \rightarrow a_1 = 40$$

$$\pi_1 = 200 \times \sqrt{0.4 \times 40} - 10 \times 40 - 80 = 320$$

Similarly, if this company switches to the new technology, its profit maximizing problem is given by
Equation (17).

\[
\pi_2 = \underset{a_2}{\text{argmax}} \{200 \sqrt{0.8a_2} - 10a_2 - K_2\} \quad (17)
\]

First order condition:

\[
\frac{200 \sqrt{0.8}}{2 \cdot \sqrt{a_2}} - 10 = 0 \rightarrow a_2 = 80
\]

\[
\pi_2 = 200 \cdot \sqrt{0.8 \cdot 80} - 10 \cdot 80 - 400 = 400
\]

Thus, this firm shall switch to the new technology since it generates the higher profit ($\pi_1 = 320 < \pi_2 = 400$).

2. Now you obtain other pieces of information:

- The utilization of input will generate pollution, and the pollution discharge is given by $a(1 - h_i(a_i))$;
- Berkeley City Hall will impose pollution tax, $V = 50$, to reduce adverse impact on health.

Should this firm switch to the new technology?

Berkeley City Hall encourage this firm to adopt new technology since the new technology is cleaner by subsidy. What’s the minimum subsidy in order to make this firm switch to the new technology?

**Solution:** Given the pollution tax, the profit maximization for this firm are given by

\[
\pi_1 = \underset{a_1}{\text{argmax}} \{p \sqrt{0.4a_1} - wa_1 - va_1(1 - 0.4) - K_1\} \quad \text{if it sticks to the current technology} (18)
\]

\[
\pi_2 = \underset{a_2}{\text{argmax}} \{p \sqrt{0.8a_2} - w_{a_2} - va_2(1 - 0.8) - K_2\} \quad \text{if it switches to the new technology} (19)
\]

Solving these two profit maximization yields $a_1 = 2.5$ and $a_2 = 20$, and $\pi_1 = 25$ and $\pi_2 = 0$. Thus, this firm should not switch to the new technology if there is a pollution tax.

The minimum subsidy should the difference in profits of using the current and new technology. That is, at least government needs to provide subsidy which is equal to 25.

3 Essay Question: Public Good

You went back to school and take upper level environmental economics course this semester after you finished internship at the regional recreation office. Your first on-class presentation is about public good, and you want to use your experience of internship and give the presentation (see Section 1). Address the following issues in less than one page (it is serious requirement which implies that grader will only read the first one page if you have more than one).

1. Is wildlife reserve, for example, Diminishing Island, a pure public good?

2. Out of these four proposals, which proposal you prefer and why?

   (hint: you may talk about feasibility, efficiency, and welfare distribution)

Your presentation should include the following points:

1. **Wildlife reserve is a public good but not a pure public good.**
   A pure public good should be completely non-rival and non-excludable.

   Wildlife reserve has features as a public good. One person can benefit from wildlife reserve without
preventing others from benefiting (non-rivalry); and if a wildlife reserve is large enough and has no fence, it is non-excludable.

But the features of non-rivalry and non-excludibility for wildlife reserve are possibly not complete. For example, if there are too many people in the wildlife reserve, it may have congestion problem, i.e., individual consumer’s marginal benefit is declining with the size of consumers. Another example, if indeed the wildlife reserve has a fence and each visitor needs to pay entry fee, then it is not completely excludable.

2. **The design of optimal management mechanism needs to consider feasibility both economically and politically, efficiency, and welfare distribution.**

Basically, there are three common management mechanisms: (1) government operates the wildlife reserve either by tax revenues or charging entry fee. If it is funded by tax revenue, the potential problem is that not all citizens benefit from the wildlife reserve but all of them need to pay tax; (2) government gives a permit to a private firm to run the wildlife reserve. This private firm, a concessionaire, earns regulated profit by charging entry fee; (3) permit a private firm to run the wildlife reserve without any regulation.

Assume that $e_i$ is entry fee, $\pi_i$ is profit, and $cs_i$ is total consumer surplus for each management mechanisms ($i = 1$: run by government; $i = 2$: run by a concessionaire; $i = 3$: run by a monopoly). We have the following conclusions: $e_1 < e_2 < e_3$; $\pi_1 < \pi_2 < \pi_3$, and $cs_1 > cs_2 > cs_3$. As a social planner, it will choose the management mechanism which yields the highest social welfare. But practically, the choice of management mechanism also depends on other factors such as interest groups, management efficiency, etc.