Problem Set #5 Answer Key

1) Assume that there are two types of farmers in a region – those who grow cotton, and those who grow roses. Let W denote the applied water. The marginal benefit of water to the cotton growers (for one season) is \( MB_C = 100 - \frac{1}{5}W \). The marginal benefit of water to the rose growers is \( MB_R = 400 - 2W \). Let S denote the amount of water available for the season. S is measured in acre-feet. There are two states of nature in this region – high rain (flood) years, and low rain (drought) years. In a flood year, the supply of water is \( \bar{S} \), and in a drought year, the supply of water is \( S \).

Let \( \bar{S} = 600 \), and \( S = 400 \).

(i) In one or two paragraphs, describe the system of water rights in the United States. How and why did it develop? Discuss the economic efficiency of the system.

Water rights in the United States are on what is called a ‘queuing system’ in the detailed texts. This means that there is a system of assigning rights to certain quantities of water, and that there are regulations defining an order in which those rights can be used. The reason these systems were developed was to entice people to move to and develop new and unsettled land. Without rights to resources like land and water, homesteaders would have been unable to continue moving west. A similar analysis could be applied to the Homestead Act or the Railroad Act, both of which were instrumental in settling the Western United States. The Homestead Act gave land to any person who moved west and worked a piece of land for 5 years, while the Railroad Act gave parcels of land to the railroad companies in exchange for building railroad lines.

These rights take the form of either Riparian or Prior Appropriation rights. Riparian rights allow landowners to use surface water on land that is located adjacent to the water body (lakes or rivers for example). These water rights must be used on the land located by the water body. Therefore, these rights cannot be sold separately; they come with the land. Prior Appropriation water rights are based on history. Whoever is the first person to divert water from a river has rights to use that water. These rights are considered ‘senior’ to any person who later decides to use the water. While these rights are not attached to a particular piece of land (as with Riparian rights), there are often limitations on trade in these water rights. From an economic perspective, these systems are inefficient because water will go to those with historical rights, and not to those who use the water in the most beneficial manner.

For the following questions, be sure to show your work.

(ii) Draw the graph showing each MB curve, along with the aggregate MB, against the total supply. Be sure to label the graph clearly.
To calculate the aggregate MB is mathematically the same as working with heterogeneous producers (problem set #2).

\[ \text{MB}_C = 100 - \left(\frac{1}{5}\right)W_C \Rightarrow W_C = 500 - (5)\text{MB}_C \]

\[ \text{MB}_R = 400 - 2W_R \Rightarrow W_R = 200 - \left(\frac{1}{2}\right)\text{MB}_R \]

Adding these together to calculate aggregate demand:

\[ W = W_C + W_R = 700 - (11/2)\text{MB}_{agg} \]

Next, invert to find the aggregate MB, keeping in mind the kinks in the curve.

\[ \text{MB}_{agg} = 0 \text{ for } W > 700 \]

\[ \text{MB}_{agg} = \frac{1400}{11} - \left(\frac{2}{11}\right)W \text{ for } 150 < W < 700 \]

\[ \text{MB}_{agg} = 400 - 2W \text{ for } 0 < W < 150 \]

Graph: The graph is at the end of the document.

(iii) **No-trade regime.** Assume that trade in water is not allowed, and the cotton growers have senior rights relative to rose growers. For each state of nature, calculate the quantity of water used by each group, and the marginal benefit of water use to each group.

Here, since cotton growers have senior rights, they receive their full demand (up until their MB is zero) before the rose growers get any water. At \( W_C = 500 \), \( \text{MB}_C = 0 \), so cotton growers will demand up to 500 acre-feet of water. Anything after this goes to the rose growers. To calculate the MB of each, just plug the quantity received into the MB function of each group.

Drought: \( W_C = 400 \), \( W_R = 0 \), \( \text{MB}_C = 20 \), \( \text{MB}_R = 400 \).

Flood: \( W_C = 500 \), \( W_R = 100 \), \( \text{MB}_C = 0 \), \( \text{MB}_R = 200 \).

(iv) **Water market regime.** Now suppose that trade in water is allowed, and that there is no transaction or conveyance costs of water trading. Under each state of nature, calculate the quantity of water used by each group, and the marginal benefit (price) of water in the water market.

If there are no transaction costs, trade in the water market will occur until the MB of each group is equal. Mathematically, this is a similar problem to the heterogeneous producer problems we used earlier in the semester (see homework #2 for a reference to this type of problem). To find the equilibrium MB, we need to find the intersection of the \( \text{MB}_{agg} \) curve and the supply. In this case, the supply is vertical in a given season (there is no outside source of water available, or it is too expensive to be a viable substitute).

Drought: \( \text{MB} = \text{price} = 54.5 \), \( W_C = 227.3 \), \( W_R = 172.7 \).

Flood: \( \text{MB} = \text{price} = 18.2 \), \( W_C = 409.1 \), \( W_R = 190.0 \).

(v) **Third Party Effects.** These cotton growers use flood irrigation on their fields. Using flood irrigation results in only 60% of applied water going to the cotton crop. The other 40% of applied water is residual. This recharges the groundwater table, which is used by the nearby town of Scarcityville for consumptive purposes (drinking, cleaning, etc). In each state of nature, how much water did the town receive before trading was allowed? After trading was allowed? What potential solutions could compensate for the third party effects associated with water trading?
The calculation here is not very difficult, although it is meant to illustrate an important point. Since Scarcityville only gets residual water from the cotton growers (not the rose growers), if the cotton growers sell water and apply less water, they inadvertently hurt the town. Since no irrigation technique is 100% efficient, there will always be some water that is residual in agricultural production. To calculate the amount of water the town receives; you need to calculate 40% of the amount applied by the cotton growers in each situation. This shows that when cotton growers sell their water, the town receives less for consumptive uses.

No-Trade: Drought = .4(400) = 160, Flood = .4(500) = 200  
Trade: Drought = .4(227.3) = 90.9, Flood = .4(409.1) = 163.6

There are a number of potential solutions to this problem. One solution is to only allow cotton growers to sell the effective water they use (60% of their total), and require that the rest go to Scarcityville. Another option is to pay the town of Scarcityville enough money for them to find other sources of water – they could do this by building a dam somewhere with a canal, or having water supplied by another source.

(vi) **Conveyance Costs.** The residents of Scarcityville have gotten fed up with fighting for water and have all moved to Abundanceville (so you can now neglect the third party effects). However, there is now a cost of moving the water from the cotton region to the rose region. The cost is $50 per acre-foot of water. Under each state of nature, calculate the quantity of water used by each group, and the marginal benefit of water use to each group. In general, do you expect the conveyance cost of water to be small or large? How might this change the effectiveness of water markets?

With a conveyance cost, we will lose the equilibrium condition that \( MB_R = MB_C \). The reason that this condition made sense with no transaction costs was that if an additional unit of water is worth more to one group, the two groups can agree on a price between their respective values. In doing so, both groups will be better off. However, when there is a cost of conveyance, the rose growers will have to pay both the price to the cotton growers (\( MB_C \)) and the cost of conveyance ($50). So, the equilibrium condition is \( MB_R = MB_C + 50 \). This, along with the supply constraint is the necessary information to solve this problem.

\[
400-2W_R = 100 - \frac{1}{5}W_C + 50  
= 150 - \frac{1}{5}W_C
\]

Drought: \( W_R + W_C = 400 \)  
\( W_R = 150, W_C = 250 \)  
\( MB_R = 100, MB_C = 50 \)

Flood: \( W_R + W_C = 600 \)  
\( W_R = 168.2, W_C = 431.8 \)  
\( MB_R = 63.6, MB_C = 13.6 \)

In general, since water is heavy and difficult to move, we would expect the conveyance cost to be large relative to the value of the water. Because of this, markets might not function as well in
reality as they do in theory. This is because the benefit to a potential water buyer has to be larger than the cost of the water and the cost of moving it.

2) Suppose that Rip Van Winkle plants a single stand of trees on an empty plot. The volume of timber at time T is \( Q(T) = 20 T^2 - 2/3 T^3 \). T is measured in years, and \( Q(T) \) is measured in board-feet. The interest rate is 10%, and the price of wood in the market is $10/board-foot.

If Rip decides to go to sleep at time \( T=0 \), when does he have to wake up to maximize his profits from a one-time harvest (single rotation)? When does he have to wake up to maximize the average yield (maximum sustainable yield, or mean annual increment)?

One time harvest (profit max):
Here, the problem is to maximize the present value of the profits that Rip will receive when he cuts the trees. Let \( P = \) price of wood, \( T = \) time to harvest, and \( r = \) interest rate. Using continuous discounting, \( e^{-rT} \) is the discount factor (see a GSI if you don’t understand where this came from).

\[
\pi = \max_T P Q(T) e^{-rT}
\]

\[
\frac{\partial \pi}{\partial T} = P e^{-rT} Q'(T) - r P Q(T) = 0
\]

\[
Q'(T) / Q(T) = r
\]

Using this equilibrium condition, and plugging in \( Q(T) = 20 T^2 - 2/3 T^3 \), \( Q'(T) = 40T - 2T^2 \), and \( r = 10\% \) gives \( T = 12.7 \) years. You might have found two possible values of \( T \) by using the quadratic formula. Remember that when you solve first order conditions, you could be finding either a maximum or a minimum. If you plug the values into the profit function, one should give a much higher profit value than the other.

Maximum average yield (MSY, MAI):
The problem is to maximize the average amount of wood taken. Note that there is nothing about economics here – no price, no discount rate, no demand or supply. This is purely in terms of total quantity (the maximum average amount of wood).

\[
\max_T \frac{Q(T)}{T}
\]

\[
\frac{1}{T} (Q'(T) - \frac{Q(T)}{T}) = 0
\]

\[
\frac{Q'(T)}{Q(T)} = \frac{1}{T}
\]

As before, plugging in the values for \( Q'(T) \) and \( Q(T) \) gives \( T = 15 \) years.

Essay

Pretend that the year is 1960. Elvis is king of the jukebox, big cars rule the roads, and Floyd Dominy is king at the Bureau of Reclamation. The State of California is trying to convince residents to pass a referendum to develop the State Water Project – a giant water project that
uses water from the Upper Feather River area to provide water to users downstream. The project will cost approximately $1.75 billion and will provide water to agriculture in the San Joaquin Valley and to urban consumers throughout the state. Discuss the benefits and the costs of the project. Be sure to consider all of those affected – residents, agricultural producers, consumers, industry, and the environment.

In 1973, the United States government passed the Endangered Species Act. This Act protects the habitat of any endangered species. How does the Endangered Species Act value the preservation of species relative to other uses, such as agriculture or development? How could this Act affect the environmental costs and benefits discussed above?

First, this question is not based on a fictitious issue – California voters passed the State Water Project referendum in 1960. It allowed the state to spend $1.75 billion to build a large dam and reservoir (Lake Oroville), along with a hydroelectric plant. The water is moved via a system of canals and levees to the delta area, where it is sent south through the Central Valley, and over the Tehachapi Mountains to Los Angeles. In an analysis of the project, it is important to look at the expected benefits and costs over time.

There are many benefits of the project. At Lake Oroville, benefits include flood control and recreational benefits (in the reservoir). A hydroelectric plant will provide a cleaner source of electricity than using fuels like coal or natural gas. Downstream, urban residents and agricultural producers will benefit from a more stable supply of water. It will reduce reliance on the pumping of groundwater, which is limited. Having a reliable water supply could allow development of urban areas in Southern California. Also, increased agricultural production benefits the consumers as well as the farmers. This is because consumers will pay lower prices for agricultural products in the marketplace.

However, there are also significant costs to the project. First, $1.75 billion is not pocket change (especially in 1960). This is a lot of money for the residents of California if the benefits are not large. There are other costs as well – there could be serious environmental damage to the Feather River ecosystem by the creation of the dam. Areas that were fertile lands for plants and wildlife will now be at the bottom of a lake. This could destroy habitat that is essential for certain species. It will also ruin the river for recreation benefits like whitewater kayaking or canoeing, as well as for the fish populations. Also there are downstream water users of the river who had plenty of water supplied before the project, but will now find that some of the water they used to use is being diverted to the south. Another consequence of the project could be increased congestion in the state. As water becomes available, urban areas are allowed to grow, and more and more people will move to sunny California.

The enactment of the Endangered Species Act in 1973 changes the role of the environment in a benefit-cost analysis. This Act protects the habitat of endangered species at all costs. This Act places the water rights of fish species like salmon or Delta smelt above the rights of any person using the water. So, regardless of the benefit of developing an area, if it is a protected area, the development will not be allowed as originally planned. However, development could be changed still build the project but also protect the habitat. It might be possible to still allow the project to be built, but in a different location, or it could require the installation of fish ladders
(these allow the movement of salmon upstream). An example of this type of negotiation occurred in the choice of the site for the future UC-Merced campus. The site it was originally planned at is habitat for an endangered species so the site of the future campus was moved. Another example of the interaction of water policy and the Endangered Species Act could be seen last summer, when water to farmers in the Klamath Basin was cut off entirely because of the water needs of the suckerfish population.

There are lots of other things that you could have mentioned in this essay. The point was to think about the tradeoffs involved in any development, and the interaction of different policies.
Marginal Benefit

Quantity of Water

S=400

S=600

MB(rose)

MB(cotton)

MB(aggregate)