Heterogeneity Revisited
Clarification for EEP101/Econ125

The following handout is intended to clarify the concept of heterogeneity among producers and the effect of particular policy interventions in this context. I will be using the same example that I attempted to present in section last week.

Setup
The story goes as follows: There are three firms, each with different demands for pollution. This is a strange way of looking at the world, but it might be helpful to think of pollution as an input to the production process. In this case, the marginal benefit for each firm to produce one more unit of pollution is like the extra amount of profit that they earn by imposing one more unit of pollution on society.

If you consider figure 1 (attached, since I can’t figure out how to draw it on the computer), you’ll see a three firm example. Note that firm 1 produces less pollution at any given price. This means that he (probably she) is more efficient. The big MB is just the horizontal sum of all of the firms marginal benefits curves, and the area under it represents the total benefits (profits) to the industry from producing at point Q.

As always, the socially optimal point is found be setting marginal benefits equal to marginal social cost. We can reach this point in one of several ways: by taxing, subsidizing, a uniform standard, or through tradable permits. We will go through these cases, but first, let’s set up the math so that we can go back and forth between algebra and graphs.

The three marginal benefits are given by:
\[ \text{mb}_1 = 75 - (5/4)q_1 \]
\[ \text{mb}_2 = 75 - q_2 \]
\[ \text{mb}_3 = 75 - (3/4)q_3 \]

Remember, q represents the amount of pollution here. To figure out MB, we need to sum these things horizontally. This requires taking the inverse of these functions, i.e., solving q in terms of mb.

The inverses are:
\[ q_1 = 60 - (4/5)\text{mb}_1 \]
\[ q_2 = 75 - \text{mb}_2 \]
\[ q_3 = 100 - (4/3)\text{mb}_3 \]

If you sum them up, you get the big Q of pollution for the whole industry.
\[ Q = 235 - (47/15)\text{MB} \]

Since we like to solve things by setting marginal benefit equal to other stuff, we need to take the inverse of this: \[ \text{MB} = 75 - 0.32Q. \]

The marginal social cost in this example is given by: \[ \text{MSC} = 30 + Q. \]
The socially optimal level of production, $Q^*$, is where $MSC = MB$, or $Q^* = 34.1$ and $MB^* = 64.1$. This is represented by the point $A$. Note that here, all of the firm’s marginal benefits are equal; they are not, however, producing the same amount of pollution. Au contraire. **Firm one is producing 8.7, firm two 10.9, and firm 3, 14.5.** You figure out these numbers by plugging in $mb = 64.1$ into each firm’s marginal benefits equation and solving for $q$. They will also be useful in calculating how much firms might trade, so don’t forget about them.

**Going Pareto: some policy options**

**The tax/subsidy**

We can figure out the optimal tax/subsidy both graphically and with math. The level of the two will be the same: the difference between marginal social cost and marginal private cost (zero in this case) at the optimal quantity. Here that distance is $AQ^*$. They will, however, be at different costs to the government.

Math:
The unit tax or subsidy here is just the difference between $MSC$ and $MPC$ at $Q^*$, which we already know is 64.1, since $MPC = 0$. The cost to the government, however, is quite different. The tax is charged on each unit up to the optimal $Q$, so the government revenue $= tQ^* = 64.1(34.1) = 2185.81$. The subsidy goes the other way; the government pays the unit subsidy for each unit not produced by polluters. Therefore, the cost to the government is $(235 - Q^*)64.1 = (235 - 34.1)64.1 = 12,877.69$. Obviously, the subsidy is an expensive option here.

**Uniform Standard**

Consider the case where the policymaker chooses a uniform standard to achieve the optimal pollution level. Let’s say that he divides up the optimal pollution level in equal parts: $Q^*/3 = 11.4$ units of pollution per person. He also decides that there will be no trading allowed. In this case, the polluters will be producing at points $D$, $E$, and $F$.

Note: their marginal benefits are not equal here. This is bad because it is wasteful. Firms 1 and 2 are emitting more than they would with a more efficient policy and firm 3 is emitting less. If you insert 11.4 into their marginal benefits equations, you will find 60.8, 63.6 and 66.5 for firms 1, 2, and 3. It is inefficient because, as a group, the producers are getting less surplus in this case than they would be if they emitted the level of pollution that equalized their marginal benefits. To see this, we need to compare the welfare under the efficient level and under the uniform standard level.

Because the total externality cost will be the same for both the socially optimal solution and the uniform standard, we can see the difference in welfare by comparing the producer’s surpluses and the two solutions. At the optimal level, producer surplus is about 2372. Summing up the three surpluses at 11.4 gives us about 2370. Obviously,
this isn’t the ideal solution to show the great efficiency gain from using a tax as opposed to a uniform standard. It is, however, interesting to see that uniforms standards might not be as horribly wasteful as sometimes is claimed.

**Tradeable Permits**

In the case above, if the firms could trade, they would do so until their marginal benefits were equal. “Gains from trade” refers to the welfare gained by being allowed to move to the point where everyone’s marginal benefits are equal. In this case, it’s only about 2, the difference between \( SW^* = 2372 \) and \( SW^{\text{permi}} = 2370 \). You could also figure this out by calculating the gains for each firm, which are just the triangles shaded in the figure with cross-hatching. These are the difference between what they earn on (or pay for) the permit and the benefits they derive from it, minus the externality cost associated with the emissions.

It was mentioned in lecture that if you allocate permits “historically”, as has been done repeatedly in the United States, then sometimes the most inefficient polluter wins. “Historically” means that the permits are divided up according to the percentage of pollution created by each firm when they are producing at their own optima. In this case, firm 1 produces 60, or 25.5% of the pollution, firm 2 31.9%, and firm 3 42.6%. The allocation of the permits to achieve the optimal level of pollution is then:

- Firm 1: \( .255(34.1) = 8.69 \)
- Firm 2: \( .319(34.1) = 10.8 \)
- Firm 3: \( .426(34.1) = 14.52 \)

When we compare this with our efficient levels of emissions, we see that they are the same. *Therefore, this method of allocation in this case provides no great advantage to the big polluter.* However, this will not always be true. This is a result of the chosen functional forms and not the general case.

Just to give an example of where this won’t be true, consider a two firm example where

\[
mb_1 = 100 - q_1 \\
mb_2 = 75 - \frac{1}{2}q_2
\]

The marginal externality costs are \( MEC = \frac{1}{2} Q \). Recalling that the MPC in these sorts of examples is zero, this equation is equivalent to MSC.

I’m not going to draw the graph for this, but you can certainly do it as an exercise. If we take inverses and sum the demands for pollution horizontally in the same way we did above, we find that \( Q = 250 - 3MB \). The inverse of this yields the equation we’ll be working with most: \( MB = 250/3 - Q/3 \).

Equating MB and MSC, we find the optimal level of pollution to be 100 units. This occurs where the marginal benefit is equal to 50 for both firms.
Historically, firm 1 was emitting 100 units of pollution, and firm 2 150 (you should be able to figure this out). If we allocate permits using their historical percentages, we will give 40%, or 40 units, to firm 1, and 60 to firm 2. This allocation mechanism will give firm one a marginal benefit of 60 and firm two a marginal benefit of 45. This distribution implies that it is worthwhile for firm 2 to sell some of its permits to firm 1. The fact that it can make money off this situation by selling some of its rights to firm 1 is what allows firm 2 to “win” in this case.

I realize that this second example was rather quickly described, however, you have all the information here to go through a more complete derivation should you be interested.