An Economic Model of Positive Externalities:

Consider a fertilizer manufacturer who uses animal waste as an input and generates a positive externality by removing the waste from the environment. Let:

- $X =$ the amount of waste used by fertilizer manufacturers.
- $D(P) =$ the fertilizer manufacturers’ demand for $X$
- $PB(X) =$ the fertilizer manufacturers’ private benefit from output $X$ (i.e., the area under the demand curve).
- $EB(X) =$ environmental benefit of removed waste $X$.
- $SB(X) =$ social benefit of $X = PB(X) + EB(X)$.
- $C(X) =$ cost of obtaining $X$.
- $SW(X) =$ social welfare of using $X = PB(X) + EB(X) - C(X)$

Now Consider the Market for Animal Waste

Social optimization problem:

$$\max_X \{SW(X) = PB(X) + EB(X) - C(X)\}$$

First-Order Condition: $PB_X + EB_X - C_X = 0$, or $MPB + MEB = MC$.

Hence, the socially optimal solution is to use $X^*$ animal waste, such that:

$$MSB(X^*) = MC(X^*)$$
Positive Externalities (cont.)

\[ Q^* = \text{optimal output} \]
\[ P^*_c = \text{optimal consumer price} \]
\[ P^*_p = (P^*_c + S^*) = \text{optimal producer price} \]
\[ Q_c = \text{competitive output} \]
\[ P_c = \text{competitive price} \]
\[ S^* = P^*_p - P^*_c = \text{MEB} = \text{optimal subsidy} \]

In the Figure:
- social optimum, where MSB=MC at point A
- competitive solution is MPB = MC at point B.
  -- under-utilization of X

A subsidy \( S^* = \text{MEB}(X^*) \) will achieve the optimal solution.

- consumer gain \( = P^*_c P^*_c BC \)
- producers gain \( = AB P_c P^*_p \)
- environment gain \( = \text{MBCA} \)
- subsidy cost \( = P^*_c CA P^*_p \)
- net social gain \( = \text{BAM} \).
Polluter Heterogeneity and Markets for Pollution

• Firms are heterogeneous and differ in their ability to abate pollution.

• Efficient solution may require all firms to abate the same amount, or perhaps that only one of many firms should abate.

• Pollution markets utilize economic incentives to allocate pollution abatement between firms in the most cost-effective manner.

• Assume there are $i$ groups of polluters (different industries, firms, etc.) emitting pollution into a common medium. Let

$$X_i = \text{pollution generated by polluter } i.$$  

$$B^i(X_i) = \text{the monetary benefit of polluter } i \text{ derived from pollution (avoided abatement costs)}$$

Total pollution $= X = X_1 + X_2 + X_3, \ldots, X_I = \sum_{i=1}^{I} x_i$

$$SC(X) = \text{social cost of pollution (depends on total pollution).}$$
Heterogeneity (cont.)

The social optimization problem is:

\[
\text{max } \sum_{i=1}^{I} B^i(X_i) - SC(X) \quad \text{subject to } X = \sum_{i=1}^{I} X_i.
\]

Using Lagrange multiplier techniques, this problem becomes

\[
\text{max } L = \sum_{i=1}^{I} B^i(X_i) - SC(X) + \lambda \left[ X - \sum_{i=1}^{I} X_i \right]
\]

where \( \lambda = \) shadow price of pollution = marginal cost to society from an added unit of pollution.

FOC:

\[
L_{X_i} = \frac{\partial L}{\partial X_i} = \frac{\partial B^i}{\partial X_i} - \lambda = 0 \quad \text{for } i = 1, I
\]

\[
L_X = \frac{\partial L}{\partial X} = -\frac{\partial SC}{\partial X} + \lambda = 0
\]

where: \( \frac{\partial B^i}{\partial X_i} = B^i_{X_i} = MB_i = MB \) from polluting for firm \( i \)

and \( \frac{\partial SC}{\partial X} = SC_x = MSC \) of pollution.

At the optimal solution, \( MB_i = MSC = \lambda \), for all \( i \).

Gov’t policies

• a unit pollution tax, \( t^* = MSC(X^*) \)
• tradeable permits, with total pollution restricted to \( X^* \).

-- if competitive, price of a pollution permit will be \( \lambda \)
Heterogeneity: The Case of Two Polluters (I = 2)

- ABC = horizontal sum of MB1 and MB2 = aggregate demand for pollution
- \( \frac{\partial SC}{\partial X} = SC_X = MSC \) of pollution
- \( x_1^*, x_2^*, X^* \) = optimal levels of pollution
- \( x_2^0, x_1^0, x^0 \) = initial unregulated levels of pollution
The Two Polluter Case (cont.)

To achieve $X^*$ using a pollution tax:
-- set a unit tax on pollution = MSC at $X^* = \lambda$.

With tradable pollution permits:

• each polluter receives $X^*/2$ pollution coupons
• they are traded at an equilibrium price of $\lambda$
• Polluter 1 buys $x_1^* - x_1^*/2$ from polluter 2
  -- there are gains from trade

Note: Welfare is smaller if each polluter is restricted to $X^*/2$ pollution units and trade is disallowed.

• Seller Gains:
  revenue received - total benefits of sold permits

• Buyer Gains:
  total benefits of gained pollution - cost of permits

Recalling the previous Figure:

gains from trade for polluter 1:
$$KNT = KTX_1^* \frac{X^*}{2} - NTX_1^* \frac{X^*}{2}$$

gains from trade of polluter 2:
$$MNL = NMX_2^* \frac{X^*}{2} - LMX_2^* \frac{X^*}{2}$$
Problems Associated with Enactment of Pollution Permit Markets

**Measurement and monitoring:**

Pollution may not be easily observable or measurable.

   technological standards, such as mandated pollution containment and control equipment may be preferred to pollution taxes.

Cost of pollution regulation depends on monitoring and containment costs. Development of new technologies may lead to change in policy tools.

**Multitude of pollutants:**

When small numbers of activities generate multitudes of pollutants, it may be easier to regulate polluting activities, rather than attempting to regulate pollution itself.

**Number and variability of polluters:**

Permit markets are more effective when the number of participants is larger and highly varied. Yet, if pollution impacts differ between locations, should you have many small markets recognizing regional variations or a few large markets that cover heterogeneous regions?
Polluter Heterogeneity and the Choice of Pollution Taxes or Standards

Assume that firms have a fixed-proportions production technology; that is, each firm has a fixed labor/output ratio and a fixed pollution/output ratio.

(Source: Hochman and Zilberman)

P = output price,
x = labor used per unit of output
w = the wage rate for labor
z = pollution produced per unit of output
v = a pollution tax
\bar{z} = a quota/standard on pollution per unit of output.
Polluter Heterogeneity and the Choice of Pollution Taxes or Standards (cont.)

In the figure:
- labor-efficient firms are toward the bottom of the graph
- pollution-efficient firms are toward the left of the graph.
- a firm will produce whenever \( P \geq wx + vz \)

the pre-regulation case:
- there is no pollution tax, \( v = 0 \), and firms will operate provided that \( P/w \geq x \)
  -- line AF delineates this “survival region”
- labor-inefficient firms (those above line AF) shut down

the pollution standard/quota case: \( \ddot{z} \) per unit of output
- with no pollution tax, firms with \( z \leq \ddot{z} \) and \( P/w \geq x \) will survive. The survival region is the area OABD.
- pollution standard eliminates the highly polluting firms in region DBFE.

the pollution tax case: \( v \)
- firms with \( P \geq wx + vz \) will continue to operate
- the line AE is the border line of "survival region"
- a pollution tax eliminates the highly labor intensive, highly polluting firms in region AFE.
Thus, a given level of pollution can be achieved with either a pollution tax or standard; however, the types of firms which shut down may differ:

(1) A pollution tax achieves a given level of pollution by eliminating highly polluting producers, but some of the remaining low-cost producers may be highly polluting (producers in area CED). Critics may charge the policy maker with “letting big polluters off the hook”.

(2) A pollution standard achieves a given level of pollution by eliminating highly polluting firms, but some of the eliminated producers may be low-cost firms (producers in area CED). Additionally, highly labor-inefficient firms continue to produce (producers in area ABC). Critics may charge the policy maker with “shutting down the most efficient businesses.”

Although standards achieve the same environmental targets less efficiently (at higher cost), there is another reason they are often used in practice: standards achieve a given level of pollution with a smaller impact on prices. It may be important to policy-makers to moderate the effects of environmental regulation on output, because output is closely related to employment and employment is a sensitive political issue. Similarly, it may be important to policy-makers to moderate the effects of environmental
regulations on prices, because consumers can be quite sensitive to significant price changes (especially in poor countries). We will show this result in the example below: standards achieve the same pollution target at a higher cost, but also at higher output levels than pollution taxes.
Polluter Heterogeneity and the Choice of Pollution Taxes or Standards (cont.)

- a given level of pollution can be achieved by a pollution tax or standard

- the types of firms which shut down may differ

  (1) A pollution tax achieves a given level of pollution by eliminating highly polluting producers, but some of the remaining low-cost producers may be highly polluting (producers in area CED). Critics may charge the policy maker with “letting big polluters off the hook”.

  (2) A pollution standard achieves a given level of pollution by eliminating highly polluting firms, but some of the eliminated producers may be low-cost firms (producers in area CED). Additionally, highly labor-inefficient firms continue to produce (producers in area ABC). Critics may charge the policy maker with “shutting down the most efficient businesses.”

- standards are less efficient than taxes (higher costs)

- standards have smaller impacts on prices
Pollution Taxes vs. Standards (cont.)

Consider three groups of firms in the figure:

- Group I will survive under either a pollution tax or a pollution standard (firms in the area OACD).
- Group II will survive only under a standard (firms in the area ABC)
- Group III will survive only under a tax (firms in the area CED)

For Group i, let: where i=I, II III

- \(z(i)\) = pollution per unit of output of group i
- \(Q(i)\) = output of group i
- \(Z(i)\) = pollution of group i \[i.e., Z(i) = z(i)Q(i)\]

Comparing the outcomes of taxes and subsidies:

- Under a pollution tax: Total pollution = \(Z(I) + Z(III)\)
- Under a pollution standard: Total pollution = \(Z(I) + Z(II)\).
  -- equal if \(Z(II) = Z(III)\)

Note that, by definition:
- \(Z(II) = z(II)Q(II)\), and
- \(Z(III) = z(III)Q(III)\).

Since \(z(III) > z(II)\), it must be the case that \(Q(II) > Q(III)\)

Now, since
- \(Q(\text{standard}) = Q(I) + Q(II)\), and
- \(Q(\text{tax}) = Q(I) + Q(III)\),

it must be the case that \(Q(\text{tax}) < Q(\text{standard})\)
Pollution Taxes vs. Standards (cont.)

Summary

- Taxes achieve environmental targets at the least cost (highest efficiency).

- Standards achieve environmental targets at a lower level of economic efficiency, but with less impact on output and employment.

- Taxes cause the least-efficient plants to close, but some highly polluting firms may remain open.

- Standards cause the most highly polluting plants to close, but may allow some inefficient plants to remain open.
Pollution Caused by an Output

Let:
\[ Y = \text{output}; \quad P = \text{output price} \]
\[ X = \text{a single input}; \quad W = \text{the per unit cost of } X \]
\[ M = \text{a second input}; \quad V = \text{the per unit cost of } M \]
\[ Z = \text{pollution}. \]

- The production function for each firm is: \( Y = f(X, M) \).
- Pollution damage per firm is \( h(Z) \); we assume:
  \[ h_Z > 0, \quad h_{ZZ} > 0. \]

**Pollution Z depends on output Y:** \( Z = g(Y) \).

- The optimal policy can be determined by solving:

\[
\max_Y \left\{ W(Y) = PY - C(Y, W, V) - h(g(Y)) \right\}
\]

FOC: \[ P - C_Y - h_Z g_Y = 0. \]

- Price = marginal cost per unit (\( C_Y \)) + marginal external cost per unit (\( h_Z \cdot g_Y \)),
- \( h_Z \cdot g_Y \) = (marginal damage per unit of pollution)(marginal pollution generated per unit of output)

**Policies to obtain the optimal solution:**
- Tax pollution by \( h_Z \) (i.e., a per unit tax on pollution), or
- Tax output by \( h_Z \cdot g_Y \) (i.e., a per unit tax on output)
Pollution Caused by an Input

Let:
\[ Y = \text{output}; \ P = \text{output price} \]
\[ X = \text{a single input}; \ W = \text{the per unit cost of } X \]
\[ M = \text{a second input}; \ V = \text{the per unit cost of } M \]
\[ Z = \text{pollution}. \]

Pollution \( Z \) depends on the input \( X \): \( Z = g(X) \).

• The social optimization problem is now:
\[
\text{Max. } \{ W(X, M) = Pf(X, M) - VM - WX - h(g(X)) \}
\]

• So that we now have two optimality conditions:
\[
(1) \ W_M = Pf_M - V = 0, \\
\text{--the marginal product of } M \text{ is equal to its wage.}
\]
\[
(2) \ W_X = Pf_X - W - h_Z \cdot g_X = 0, \\
\text{--the marginal product of } X \text{ is equal to its wage plus the marginal environmental cost of output } X
\]

Policies to obtain the optimal solution:

• Tax of \( h_Z \) on pollution, (a pollution tax)

• Tax of \( h_Z g_X \) on \( X \) (a unit tax on the polluting input).
Policy Tools to Control Environmental Externalities

**Education:**
A preventive policy used to instill environmental values (i.e., do not litter) about new environmental issues (i.e., global warming) and about pollution control technologies.

**Clear property rights definition:**
To enable private parties to efficiently use the legal system to resolve externality problems.

**Direct control policies:**
Often take the form of standards/quotas and technology standard on production or on pollution.

**Taxes:**
Applied directly to pollution and/or to pollution-generating outputs, inputs, or production activities.

**Subsidies:**
Given to firms for pollution reduction and abatement activities, or for the development of substitute technologies to eliminate pollution-generating activities.

**Trading in pollution rights:**
Gives firms the right to collect and trade permits for pollution-generating activities.

**Support for research and development (R&D):**
Government assistance to develop better technologies to reduce, mitigate or monitor pollution.
Technology diffusion

Technology diffusion: A gradual process in which new technology spreads through the economy.

Diffusion curve: Denotes number of adopters of new technology as a function of time. Diffusion curves tend to be S-shaped.

The government can accelerate diffusion of pollution-reducing policies by:
- Engaging in extension and education activities
- Subsidizing new technology
- Regulating technology adoption (setting a timetable for diffusion).

Example: Catalytic converters
Drip irrigation
Scrubbers in Coal-Fired Electric Plants

If gov’t promotes a new technology, the technology will be adopted more readily, and the diffusion curve will shift to the left.