Irrigation Economics

The use of irrigation water depends on:

- Economics (prices and costs)
- Crop Selection
- Land Quality and Environmental Conditions
- Irrigation Technology

Water Management Choices depend on:

- Type of Crops
- Irrigation Technology
- Level of Water Availability
Some Stylized Facts About Irrigation:

Irrigation water is measured in "acre-feet," AF, which is the amount of water needed to cover one acre of land to a one foot depth (before water is lost to percolation).

Irrigation Efficiency measures the percentage of water that is actually consumed by the crop.

Typical Water Use of Common Crops:

<table>
<thead>
<tr>
<th>Heavy water users:</th>
<th>Rice; Alfalfa:</th>
<th>5-7 AF/year</th>
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<tbody>
<tr>
<td>Medium water users:</td>
<td>Fruits:</td>
<td>2.5-4 AF/year</td>
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<tr>
<td></td>
<td>Cotton:</td>
<td>2.5-4 AF/year</td>
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<tr>
<td></td>
<td>Vegetables:</td>
<td>2-3.5 AF/year</td>
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<tr>
<td>Low water users:</td>
<td>Wheat:</td>
<td>1.8-2.5 AF/year</td>
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Irrigation efficiencies of several irrigation technologies:

<table>
<thead>
<tr>
<th>Gravitational:</th>
<th>Furrow</th>
<th>.65</th>
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<tr>
<td>Border</td>
<td>.65</td>
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<thead>
<tr>
<th>Sprinkler:</th>
<th>Manual move</th>
<th>.8</th>
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<tbody>
<tr>
<td>Center pivot</td>
<td>.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>With field crops</td>
<td></td>
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<tr>
<th>Low volume:</th>
<th>Drip</th>
<th>.95</th>
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<tr>
<td>LEPA</td>
<td>.9</td>
<td></td>
</tr>
<tr>
<td>Mini-sprinkler</td>
<td>.9</td>
<td></td>
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<tr>
<td></td>
<td>Used in field crops</td>
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</tr>
<tr>
<td></td>
<td>Not used with alfalfa, wheat</td>
<td></td>
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<td></td>
<td>Used with trees</td>
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Currently, the price of water is set administratively and is not the result of the maximizing behavior of economic agents.
How The Choice of Irrigation Technology Affects Output:

Water is applied to the surface, percolates through the soil and is taken up by the root system.
• If the soil is dry and a bucket of water is poured on it, most of it will fail to permeate the soil, but will instead exit the land in the form of runoff.
• If the soil is first moistened, then it much more readily absorbs water

**drip irrigation:**
• applies water slowly so that the crop can absorb it better
• Most of the water applied is absorbed by the plant.
• drip irrigation systems are very expensive to set up.

**sprinkler irrigation:**
• distributes water unevenly (through space and time)
• Less of the water applied is utilized by crops; greater evaporation and runoff
• sprinkler systems are relatively inexpensive to set up.

**gravitational systems such as flood or furrow:**
• pool water on a portion of land.
• less water efficient than sprinkler or drip (greater evaporation and surface runoff).
A simple model of irrigation technology choice:

Ag. production is a function of the effective water taken up by the crop. (called Effective Irrigation Water)

Effective water is the product of two components:
- **applied irrigation water** (the quantity of water applied)
- **irrigation efficiency** (fraction of applied water taken up by the crop)

Per-acre production function for an ag. product: 
\[ y = f(e) \]
- \( y \) = agricultural output per acre
- \( e \) = effective irrigation water per acre
  - \( fe > 0 \) and \( fee < 0 \)

Effective water equation:
\[ e_i = a * h(i, q, c) \]
- \( a \) = applied water per acre
- \( h(i, q, c) \) = irrigation efficiency

- \( i \) = irrigation technology, where we assume two possible irrigation technologies, labeled with an index variable \( i \):
  - Traditional technology: \( i = 1 \)
  - Modern technology: \( i = 2 \)
  - \( \frac{dh}{di} > 0 \): higher \( i \) results in higher irrigation efficiency

- \( q \) = land quality: water-holding capacity, soil quality and topographical conditions such as slope.
  - \( hq > 0 \), thus, an increase in land or water quality increases irrigation efficiency.

- \( c \) = climate variables (temperature, humidity, etc.)
  - \( hc \) is ambiguous
Model of irrigation technology choice (cont)

The farmer’s per-acre profit-maximization problem can be expressed as the following **discrete/continuous choice problem**:

\[
\max_{i,a} \Pi = Pf(e) - (w + z_i)a - k_i \\
\max_{i,a} \Pi = Pf(ah(i,q,c)) - (w + z_i)a - k_i
\]

\( \Pi = \) profit; \( P = \) output price \\
\( z_i = \) cost of water pumping and pressurization \\
\( k_i = \) fixed cost of technology \( i; \) \( w = \) price of water

**Assumptions:**

\( h(i=1) > h(i=0) \)  efficiency is higher with modern technologies. \\
\( k_1 > k_0 \)  Modern technology requires higher fixed cost. \\
\( z_1 > z_0 \)  Pumping cost is higher with modern technology.

For a given \( i \), optimal water use is determined by the F.O.C.:

\[
\frac{d\Pi}{da} = Pf_{e}e_{a} - (w + z_i) = Pf_{e}h(i,q,c) - (w + z_i) = 0
\]

where \( f_{e} \) and \( e_{a} \) are the appropriate partial derivatives.

Rearranging:

\[
\frac{w + z_i}{h(i,q,c)} = Pf_{e}
\]

**MC of effective water = MVP of effective water.**
The Technology Adoption Decision

At some level of land quality, all else being equal, switching technologies will maximize profits. This level of land quality is called the **switch point**.

- It is never profitable to farm land of quality \( q_m(i=1) \).
- High-quality land: either technology is profitable, although the traditional technology is more profitable.
- Moderate quality (between \( q_m(i=1) \) and \( q_s \)): modern technology makes profitable land that was previously not worth farming.

- The switch point is based on a comparison of Profit\((i=0)\) with Profit\((i=1)\).
Crop selection

- Crop selection depends on the levels of economic parameters such as land quality and water prices.

In general, farmers simultaneously choose crop type, irrigation technology and applied water use levels to maximize profits, based on the levels of economic parameters such as land quality.

Modern technologies are more likely to be adopted with:
- Moderate to low quality land; high value crops
- Low quality water; high price water.

What are some of the effects of technology adoption?

Recall that profit maximization requires:

\[ \frac{w + z_i}{h(i)} = P_{fe} \]

assume for simplicity that \( z_0 = z_1 \), (pumping costs are the same) and recall from page 2 that \( f_e > 0 \implies h(i=0) < h(i=1) \implies f_e(i=0) > f_e(i=1) \implies e_0 < e_1 \)

Modern technology increases the optimal level of effective water use. (does not imply higher applied water use)

In most cases, modern technology REDUCES the optimal level of applied water use, and is therefore water-saving.

If \( e_0 < e_1 \), then \( q_0 < q_1 \). Thus, modern technology increases crop output.
Crop Selection (cont)

If land quality is high, water quality is high and weather is mild:

\( h(\text{i}=1) \) and \( h(\text{i}=0) \) are not very different

The adoption of modern irrigation technology does not change the optimal levels of crop output or applied water use by much.

If land quality is low, water quality is low, or weather is hot:

Adoption of modern irrigation technology may affect optimal crop output and applied water use significantly.

When land quality is low and temperature is high, the effect of adopting new technology depends on water price:

<table>
<thead>
<tr>
<th>Water Price</th>
<th>Increase In Crop Output</th>
<th>Decrease In Applied Water Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low ($15/AF)</td>
<td>Minimal (0-5%)</td>
<td>High (30-40%)</td>
</tr>
<tr>
<td>Med ($15-$80/AF)</td>
<td>Medium (5-15%)</td>
<td>Medium (15-20%)</td>
</tr>
<tr>
<td>High ($80+/AF)</td>
<td>High (25-50%)</td>
<td>Low or neg. (&lt;5%)</td>
</tr>
</tbody>
</table>
An Example of Technology Choice Under Markets:

an individual farmer is growing a crop, Y

- Crop Production Function: \( Y = 30e - 0.2e^2 \)
- The price of \( y \) is: \( P = $80/\text{ton} \)
- The price of water is: \( V = $400 / \text{A-F} \)
- \( \pi = PY - Va - F \)
  - \( a \) = the input ‘applied water’, and \( F \) = fixed costs

two technologies:
- Sprinkler Irrigation is 50% efficient and costs $10,000 to install
- Drip Irrigation is 75% efficient and costs $20,000 to install

calculate profits under each system, then compare:

Under Sprinkler Irrigation:

\[
\text{Max. } \left\{ \pi^S = 80(30e - 0.2e^2) - 400a - $10,000 \right\} \\
\text{subject to: } e = 0.5a
\]

- substituting in the constraint
  \[
  \text{Max. } \left\{ \pi^S = 800a - 4a^2 - $10,000 \right\}
  \]

- Foc:
  \[
  \frac{d\pi^S}{da} = 800 - 8a = 0
  \]
  
  which yields: \( a^* = 100 \text{ AF} \).

- Substituting the value \( a^* \) into the profit expression we get:
  \[
  \pi^S = $800(100) - 4(100)^2 - 10,000 = $30,000
  \]
Example (cont)

Under Drip Irrigation:

\[
\begin{align*}
\max_a \{ \pi^D &= 80(30e - 0.2e^2) - 400a - 20,000 \} \\
\text{subject to: } e &= 0.75a
\end{align*}
\]

• substituting in the constraint:

\[
\begin{align*}
\max_a \{ \pi^D &= 1,400a - 9a^2 - 20,000 \}
\end{align*}
\]

• Foc:

\[
\frac{d\pi^D}{da} = 1,400 - 18a = 0
\]

which yields: \(a^* = 77.78\) A-F.

• Substituting the value \(a^*\) into the profit expression we get:

\[
\pi^D = 800(77.78) - 4(77.78)^2 - 20,000 = 34,444
\]

Since \(\pi^D > \pi^S\), the farmer is better off investing in a drip irrigation system

• Drip Irrigation Uses Less Applied Water: \((77.78 < 100)\)

• Drip Irrigation Uses More Effective Water: \(0.75(77.78) = 58.34 > 50 = 0.5(100)\)

• Output Per Acre is Higher Using Drip Irrigation: \(Y^D > Y^S\)
Irrigation Under Water Markets and Queuing Systems

Total water available for all acreage in a watershed is A
Total acreage of productive ag. land in the region is L

The production function is:

\[ y = f(e) \quad e = ah_i \]

\( e \) = effective water per acre; \( a \) = applied water per acre
\( y_m = f(e_m) \) = maximum output per acre
\( e = e_m \), effective water associated with maximum yield

Under a queuing system

- Senior rights owners use water until the VMP of water = 0, which is the level that will maximize yields.
  - applied water use is \( a_m = \frac{e_m}{h_0} \) per acre
  - Junior rights owners use whatever water is left.

Under a queuing system of water rights:
  - water price = 0
  - per acre fee for water use = \( \mu \)

Total acres under a water rights system: \( \frac{A}{a_m} = \frac{Ah_0}{e_m} < L \)

Total output = \( \frac{Ah_0 y_m}{e_m} \)

Output price = \( P \)

Producer surplus = \( P \frac{Ah_0 y_m}{e_m} - u \frac{Ah_0}{e_m} \).
Irrigation Under Water Markets and Queuing Systems (cont)

Is the queuing system efficient?
Junior rights owners do not get enough water if scarcity exists.
• A unit of water would provide positive MVP to junior owners
• The last unit of water on a senior owners land provides MVP =0.
  => The Queuing System is Inefficient. MVP_J + MVP_S

The queuing system leads to under-utilization and over-irrigation of land.

Under a Market System

efficient solution involves applying water uniformly across all land to equate the MVP.
• water per acre = \( \frac{A}{L} \)
• yield per acre = \( y_i = f\left(\frac{kiA}{L}\right) \)
• price of water = VMP of applied water = \( Pf_i h_i \)
• \( k_1 \) = the fixed cost of implementing the technology \((k_1 > k_0)\)

The producers' annual profits per acre are:
\[
\pi_i = Py_i - \frac{A}{L} Pf_i h_i - k_i - \mu - t
\]

so that:
\[
\pi_i = Py_i - \frac{A}{L} Pf_i h_i - k_i - \mu - t
\]
\[
\pi_0 = Py_0 - \frac{A}{L} Pf_o h_0 - k_0 - \mu - t
\]

\[
(\pi_i - \pi_0) = P(y_i - y_o) - \frac{A}{L} Pf_i (h_i - h_o) - (k_i - k_0)
\]
Irrigation Under Water Markets and Queuing Systems
(cont)

Technology 1 is selected if: \((\pi_1 - \pi_0) > 0\)

Both technologies require the same water per acre, because water is evenly distributed across all acres as a result of equating the MVP.

When the market price of water is taken as a constant in the problem, the choice of technology can be expressed as:

Select technology 1 when: \(P(y_1 - y_0) > k_1 - k_0\)

Both technologies result in the same water use per acre, but the modern technology increases the yield by raising the amount of effective water received by the crop.

If the market value of the increase in yield is greater than the extra capital costs involved with investing in the new technology, the farmer should invest.
Comparing Market and Queuing Outcomes

Assume that, under market conditions, technology \( i \) is optimal and adopted by all farmers. Under a market system all arable land is utilized. The transition to market will increase irrigated land from \( \frac{A}{e_m h_0} \) to \( L \).

Output will increase by: \( \Delta Y = Lf \left( \frac{h_i A}{L} \right) - \frac{A h_{i_m}}{e_m} f(e_m) > 0 \)

as water is shifted from low MP land to the high MP lands of junior rights holders.

- Output per acre of senior rights owners will decrease from \( f(e_m) \) to \( f(A/L e_i) \).
- Water per acre of all users will decrease from \( e_m h_0 \) to \( A/L \).

In the transition to the market, \( \left( e_m - h_i \frac{A}{L} \right) \frac{A}{h_0 e_m} \) units of water which were used to produce the output associated with area B in the figure are allocated to irrigate new lands.
Comparing Market and Queuing Outcomes (cont)

If the senior rights owners have to buy water under markets, they are losing from the transition. Under water markets, they now have lower yields, they now have to pay for water, and they also must pay to adopt the new technology, since doing so is now optimal. Their loss per acre is:

\[ P_f(e_m) - P_f\left(h_i\frac{A}{L}\right) + P_f h_i \frac{A}{L} + k_i + u. \]

But if the senior rights are given the property rights to the water, they may win. They still have lower output than under queuing, but the gain from selling excess water may overcome this output loss. Their income per acre will be:

\[ P_f\left(h_i\frac{A}{L}\right) + \left(e_m - h_i\frac{A}{L}\right)P_f h_i \frac{A}{L} - t - k_i \]

If the transaction costs are high, there is no incentive to switch to a system of water markets. Namely, if:

\[ t > \frac{P\Delta Y + k_i L - k_0 A / (h_0 \cdot e_m)}{L} \]

When transaction costs per acre exceed the per acre change in output plus the cost of adopting the optimal modern technology less the cost savings of senior owners not adopting the conventional technology, water markets may be inefficient.

Because markets for final products have negatively sloped demand, the transition from queuing to markets will also reduce the market price of agricultural commodities. Senior rights owners may thus lose, even if they sell water because of the price decline of their output. Producers as a whole may actually lose, but consumer surplus will increase.
### Numerical Example of Market vs. Queuing

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</tr>
<tr>
<td>900</td>
<td>1</td>
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</tr>
<tr>
<td>900</td>
<td>50</td>
<td></td>
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<tr>
<td>1050</td>
<td>1</td>
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<tr>
<td>1050</td>
<td>50</td>
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#### Queuing Outcomes

- **Output (10⁶ lbs.)**: 936, 936, 936, 936
- **Output Price ($/lb.)**: 0.75, 0.75, 0.75, 0.75
- **Irrigated Land (10³ acres)**: 720, 720, 720, 720
- **Producer Profits (10⁶ dollars)**: 342, 342, 342, 342

#### Market Outcomes: Adjustment Costs = $5/acre

- **Output (10⁶ lbs.)**: 1159, 1161, 1161, 1344
- **Output Price ($/lb.)**: 0.572, 0.746, 0.57, 0.744
- **Water Price ($/AF)**: 62.0, 73.75, 63.7, 118.4
- **Irrigated Land (10³ acres)**: 900, 900, 902, 1050
- **Technologies**: 2, 2, 3, 2, 3
- **Snr. Rights Net Profits ($10⁶)**: 5.3, 139, 0, 43.4
- **Snr. Rights Gross Profits ($10⁶)**: 191, 361, 191.1, 398.5
- **Percent Gain in Social Welfare**: 5.4%, 16.3%, 5.4%, 23.8%

#### Market Outcomes: Adjustment Costs = $50/acre

- **Output (10⁶ lbs.)**: 1150, 1160, 1150, 1342
- **Output Price ($/lb.)**: 0.579, 0.746, 0.579, 0.744
- **Water Price ($/AF)**: 53.5, 81.0, 53.5, 118.4
- **Irrigated Land (10³ acres)**: 890, 900, 890, 1050
- **Technologies**: 2, 2, 3, 2, 3
- **Snr. Rights Net Profits ($10⁶)**: 0, 107, 0, 10.9
- **Snr. Rights Gross Profits ($10⁶)**: 160.7, 328.2, 160.7, 366.1
- **Percent Gain in Social Welfare**: -0.5%, 4.6%, -0.5%, 10.3%
The demand functions for water can be derived according to the price of output and the available technology. The demand function has several steps and each is associated with a different technology.

when urban demand is small (D1), gains from markets are not spectacular, and traditional technology is optimal since the MVP of water in alternate uses is small.

when water demand is high (D2), markets lead to adoption of modern technologies and reduce ag. water use substantially.

**In general, there will be less objection to markets among users of scarce resources if existing users of the resource (senior rights holders or polluters) are given the right to sell permits.**
Queuing Vs. Markets: A Numerical Example

the farm production function is: \( y = 8e - 2e^2 \)

There are two technologies available:
- The traditional technology has a 50% water efficiency: \( h_0 = 0.5 \)
- The modern technology has a 60% water efficiency: \( h_1 = 0.6 \)

Suppose for two technologies: \( P = 125, \mu = 100, t = 50 \).

Total water stock is, \( A = 6000 \text{ AF} \),

Total land stock, \( L = 2000 \).

Queuing Outcome:

Technology 0 is chosen.

Senior rights holders max. yield: \( \text{Max}_{e} \{ y = 8e - 2e^2 \} \)

the FOC: \( \frac{\partial y}{\partial e} = 8 - 4e = 0 \Rightarrow e_m = 2, y_m = 8 \),

Water applied per acre is: \( a_m = \frac{e_m}{h_0} = \frac{2}{0.5} = 4 \).

Acreage utilized simply depends on how far the water flows:
\( A_Q = \frac{6000}{4} = 1500 \).

Aggregate output: \( Y_Q = y_m(A_Q) = 8(1500) = 12,000 \).

Income per acre: \( \pi_Q = 8(125) - 100 = 900 \).

Total Farm Income= (1,500 acres)($900/acre) = $1,375,000.
Queuing Vs. Markets: A Numerical Example (cont)

Market Outcome:

the case when only the traditional technology is available.

Declining MVP of water and homogeneous land quality implies it is optimal to distribute water evenly across all land

\[ a = \frac{6000}{2000} = 3 \]

Applied water per acre:

Effective water: \( e^* = 3(0.5) = 1.5 \)

Output per acre: \( y^* = 8(1.5) - 2(1.5)^2 = 7.5 \)

Aggregate Output: \( Y_M = 2000 \cdot 7.5 = 15000 \).

Total income: \( py = 2000 \cdot 7.5 \cdot 125 - 100 - 50 \cdot 1 = 157500 \)

Water price = VMP = \( p \frac{\partial f}{\partial e} h_0 = 125(8 - 4 \cdot 1.5 \cdot 0.5) = 125 \).

Senior rights owners are given the right to sell surplus water:

- Senior rights owner owns 4 AF of water, uses 3 AF under a system of water markets and sells the remaining AF for $125:

  \[ \pi^* = 125(7.5) - 100 - 50 + 1(125) = 912.50 \]

- Junior rights owners' profit per acre is:

  \[ \pi^* = 125(7.5) - 100 - 50 - 3(125) = 537.50 \]
the case when the modern technology is available

Effective water: \( a = \frac{e}{h_i} \Rightarrow 3 = \frac{e^*}{0.06} \Rightarrow e^* = 18 \)

Output per acre: \( y^* = 8(18) - 2(18)^2 = 7.92 \)

Aggregate output under market: \( Y^* = (2,000)(7.92) = 15,840 \)

\[
1500(7.92 \cdot 125 - 100 - 50 - 30) = 1620000
\]

Total income = \( py \mu \tau c \)

Water price = \( p \frac{\partial f}{\partial e} h_i = 125[8 - 4(1.8)].6 = 125 \cdot 8 \cdot 0.6 = 60. \)

Assuming, as before, that Senior rights owners are allowed to sell their surplus water:

• Senior rights owners' income =

\[
(7.92)125 - 100 - 50 - 30 + 60 = $870
\]

• Junior rights owners' income =

\[
7.92 \cdot 125 - 100 - 50 - 30 - 180 = 630.
\]
Implications:

- Establishing water markets is welfare improving.
- Adoption of modern technology is in the best interest of society.
  - Total farm income is higher when farmers adopt new technology.
- Adopting modern technology is in the interest of junior rights holders.
- Adopting modern technology is not in the interest of senior rights holders.
  - Senior rights owners profit from a market system, but only when modern technology is not adopted.

Now Say the Government Appropriates the Water Rights

If the government is given the water rights, then both senior and junior rights holders must purchase water. The water price is still $P = VMP$ in either case.

Profit per acre is now the same for the both senior and junior rights holders, which is what we already calculated for junior rights holders above. Therefore, it is now in the best interest of senior rights holders to invest in the new technology.

The allocation of property rights achieves the same result in terms of allocative efficiency: $P = MVP = \text{marginal profit per acre}$

Different incentives are created for technology investment.