Aggregate demand is kinked:

1. \( MB = 50 - 3X \) \( \quad 0 \leq X \leq 10 \)

2. \( MB = 40 - 2X \) \( \quad 10 \leq X \leq 20 \)

Government's Problem:
The government chooses an \( X^* \) such that \( MB = MC \).

Since demand is kinked, we must look at both segments of the demand curve separately. One segment will provide an INCONSISTENT answer, while the other will provide the correct \( X^* \).

1. \( MB = 50 - 3X \quad 0 \leq X \leq 10 \)  
   \( MB = MC \Rightarrow \quad 50 - 3X = 5X \)  
   \( 8X = 50 \)  
   \( X^* = 6.25 \)  
   0 \leq 6.25 \leq 10  
   \[ X^* = 6.25 \] is correct

2. \( MB = 40 - 2X \quad 10 \leq X \leq 20 \)  
   \( MB = MC \Rightarrow \quad 40 - 2X = 5X \)  
   \( 7X = 40 \)  
   \( X^* = 5.71 \)  
   but 5.71 is not between 10 and 20

The government will charge an entry fee that just covers costs, \( E_G \), i.e.,

\[
E_G = \frac{TC(X^*)}{3} \quad \text{Recall: } MC = 5X
\]

\[
TC = \int 5X = \frac{5}{2} X^2
\]

\[
E_G = \frac{5(X^*)^2}{3(2)} = \frac{5(6.25)^2}{2(3)} = \$32.55: \quad \boxed{E_G = 32.55}. \]

Now let's see if the two consumers are willing to pay this amount:

Rich person's MB = 20 - X and TB = \( \int MB \Rightarrow \)

\[
TB = \int_{0}^{6.25} 20 - X = 20X - \frac{1}{2} X^2 \bigg|_{0}^{6.25} = 105.47 > E_G.
\]

The rich person will enter.

Poor person's MB = 10 - X and TB = \( \int MB \Rightarrow \)

\[
TB = \int_{0}^{6.25} 10 - X = 10X - \frac{1}{2} X^2 \bigg|_{0}^{6.25} = 42.97 > E_G.
\]

The poor person will also enter.
Concessionaire's Problem:

She also provides $X^*$: i.e., $X_C = X^*$

$$\Rightarrow \left[ X_C = 6.25 \right]$$

$X_C = 6.25 \Rightarrow$ a shadow price $\lambda^* = 50 - 3 \times 6.25 = 31.25$.

Concessionaire's entry fee, $E_C$

$$E_C = \frac{X^* \lambda^*}{3} = \frac{31.25(6.25)}{3} = \frac{31.25}{3} = \frac{192.5}{3} = 65.10$$

Recall the rich person's benefit from $X = 6.25$.

$p = 105.45 > E_C$: rich person will enter.

Recall the poor person's benefit from $X = 6.25$.

$p = 42.97 < E_C$: poor person will not enter.

$\therefore$ Concessionaire's provision is inefficient because it is never economically efficient to exclude an individual from consuming a public good.

Monopolist's Problem:

The monopolist knows that the poor consumer cannot afford to enter so he provides the level $X_m$ of the public good where the marginal benefit of the rich consumers equal the MC: $MB_{1,2} = MC$.

$$MB_{1,2} = 40 - 2X = MC = 5X \Rightarrow 40 - 2X = 5X$$

$$7X = 40$$

$$\left[ X_m = 5.71 \right].$$

Note: $X_m < X_c = X^*$: the monopolist underprovides the public good.
The monopolist will charge each of the rich consumers their total benefit from consuming $X_m$:

$$E_m = \int MB = \int_{0}^{5.71} 20 - X = 20X - \frac{1}{2}X^2 \bigg|_{0}^{5.71} = 97.90 \quad \left( E_m = 97.90 \right).$$

Rich person's benefit

$$TB = \int_{0}^{5.71} 20 - X = 20X - \frac{1}{2}X^2 \bigg|_{0}^{5.71} = 97.90$$

The rich person will enter.

Poor person's benefit:

$$TB = \int_{0}^{5.71} 10 - X = 10X - \frac{1}{2}X^2 \bigg|_{0}^{5.71} = 40.80 < E_m$$

The poor person will not enter.

\[ \therefore \text{Monopoly provision is inefficient for two reasons:} \]

1. Exclusion from public good is never efficient.

2. Monopoly underprovides the public good,

$$X_m < X^*.$$ 

This happens because the monopolist knows that its output affects price and, therefore, restricts its provision.
New Case:  
"Low MC"

Aggregate demand is kinked:

(1) \( MB = 50 - 3X \) \( 0 \leq X \leq 10 \)

(2) \( MB = 40 - 2X \) \( 10 \leq X \leq 20 \)
Government's Problem:

The government chooses an $X^*$ such that $MB = MC$.

Again, we must examine both segments of the demand curve.

(1) $MB = 50 - 3X$ \quad 0 \leq X \leq 10 \\
$MB = MC \Rightarrow$ \\
$50 - 3X = X$ \\
$4X = 50$ \\
$X^* = 12.25$

(2) $MB = 40 - 2X$ \quad 10 \leq X \leq 20 \\
$MB = MC \Rightarrow$ \\
$40 - 2X = X$ \\
$3X = 40$ \\
$X^* = 13.33$

$MB = MC \Rightarrow$ \\
$50 - 3X = X$ \\
$3X = 40$ \\
$X^* = 12.25 \quad \text{INCONSISTENT}$ \\
$12.5 \text{ is not between 0 and 10}$ \\
$X^* = 13.33 \quad \boxed{\text{is correct}}$

The government will charge an entry fee that just covers costs, $E_G$, i.e.,

\[
E_G = \frac{TC(X^*)}{3} \\
\text{Recall: } MC = X \\
TC = \int X = \sqrt{2}X^2 \\
E_G = \frac{(X^*)^2}{2(3)} = \frac{(13.33)^2}{6} = 29.61: \boxed{E_G = 29.61}.
\]

Now let's see if the two consumers are willing to pay this amount:

Rich person's $MB = 20 - X$ and $TB = \int MB \Rightarrow$
\[
TB = \int_{0}^{13.33} (20 - X) = 20X - \frac{1}{2}X^2 \bigg|_{0}^{13.33} = 177.75 > E_G.
\]

The rich person will enter.

Poor person's $MB = 10 - X$ and $TB = \int MB \Rightarrow$
\[
TB = \int_{0}^{13.33} (10 - X) = 10X - \frac{1}{2}X^2 \bigg|_{0}^{13.33} = 44.55 > E_G.
\]

The poor person will also enter.
Concessionaire's Problem:

She also provides $X^*$: i.e., $X_C = X^*$

$$\Rightarrow \left| X_C = 13.33 \right|$$

$X_C = 13.33 \Rightarrow$ a shadow price $\lambda^* = 40 - 2(13.33) = 13.34$.

Concessionaire's entry fee, $E_C$

$$E_C = \frac{X^* \lambda^*}{3} = \frac{(13.33)(13.34)}{3} = 59.27: \left| E_C = 59.27 \right|.$$  

Recall the rich person's benefit from $X = 13.33$.

$177.75 > E_C : \text{ rich person will enter.}$

Recall the poor person's benefit from $X = 13.33$.

$44.55 < E_C : \text{ poor person will not enter.}$

$\therefore$ Concessionaire's provision is inefficient because the poor person is excluded from consumption.

Monopolist's Problem:

Again, the monopolist knows that the poor consumer cannot afford to enter. He provides the level $X_m$ of the public good where the marginal benefit of the rich consumers equals the MC: $MB_{1,2} = MC$.

$$MB_{1,2} = 40 - 2X = MC = X \Rightarrow 40 - 2X = X$$

$$3X = 40$$

$$\left| X_m = 13.33 \right|.$$  

Note: In this case, the monopoly provides the socially optimal amount of the public good.
The monopolist will charge each of the rich consumers their total benefit from consuming $X_m$:

$$E_m = \int MB = \int_{0}^{13.33} \left(20 - X = 20X - \frac{1}{2}X^2\right) \bigg|_{0}^{13.33} = 177.75 \quad \left[E_m = 177.75\right].$$

Rich person's benefit

$$TB = \int_{0}^{13.33} \left(20 - X = 20X - \frac{1}{2}X^2\right) \bigg|_{0}^{13.33} = 177.75$$

The rich person will enter.

Poor person's benefit:

$$TB = \int_{0}^{13.33} \left(10 - X = 10X - \frac{1}{2}X^2\right) \bigg|_{0}^{13.33} = 44.46 < E_m$$

The poor person will not enter.

∴ Monopoly provision is inefficient because the poor person is excluded from consumption.

Note: In the case of "low MC," all three providers provide the optimal amount of the public good,

$$X^* - X_c = X_m.$$  

Of course, the concessionaire and monopolist are still inefficient because they exclude the poor consumer.