NOTES ON THE FARM-HOUSEHOLD MODEL

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Contents

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1. Introduction

The basic models economists use typically involve *consumers* and *firms*, and differences among different fields of economics often boil down to differences in the level of aggregation involved. For microeconomists, consumers are often aggregated up to level of households. There are two main reasons for choosing this level of aggregation. The first is practical: most data on things like expenditures and income that we're able to easily collect is on outcomes for households, not individuals. The second is a feature of most economic environments: there's a great deal of sharing of both income and consumption within many households, so that it may in fact be difficult to draw sharp distinctions among individuals.

In higher-income societies people's lives often seem to come close to approximating the economists' artificial division of economic phenomena between households and firms. Individuals make consumption decisions that depend on the preferences of those in their household; on the prices the household faces, and on the household's resources. Some individuals work for firms, which is where the locus of production is located; for many people there's a fairly clear separation between the productive activities they pursue at work and the lives they pursue within their households.

This separation, however, is often less clear-cut for agricultural households in lower-income countries. The household may be the locus for both consumption and production. Based on this observation, one of the basic building blocks used by development economists is the notion of a "farm-household".

Here our plan is to sketch the basic farm-household model, along the lines pursued by Singh et al. [1986]. That model sketched, we'll discuss circumstances in which the property of separation between consumption and production may obtain; and demonstrate other circumstances in which it will not.

We'll then turn our attention to ways in which we can think about matching up this simple model with the real world. We'll discuss how to estimate a demand system, and labor supply, all in an environment with no risk.

2. Elements of the Basic Model

2.1. **Household-level Decision-making.**

Of course, the "household" can't really make decisions; really what's being assumed here are three things:

- That household resources are pooled;

- That allocations within the household are efficient; and
- That the organization of production is efficient.

2.1.1. *Commodity Space.*

- Bardhan and Udry [1999] (following Singh et al. [1986]) assume that the household derives utility from the consumption and leisure of its members. In addition, land and labor are used in production of some *numeraire* good.
- In a more general formulation, we can think of some x as a "netput" vector of commodities.
- In either case, generally assume that the set of feasible allocations (both for consumption and production) is convex and compact.

Example: Problem for the Farm-Household

The farm-household maximizes a joint utility function

$$\max_{c_i, l_i, A, L} U(c_1, c_2, l_1, l_2)$$

subject to a budget constraint

$$p(c_1 + c_2) + w(l_1 + l_2) \leq [F(A, L) - rA - wL] + rE^A + w(E_1^L + E_2^L),$$

and subject to a collection of non-negativity constraints on consumptions, leisures, and farm inputs.

2.1.2. *Objective Function.*

- For Bardhan-Udry, objective is $U(c_1, c_2, l_1, l_2)$, where there are two household members $i = 1, 2$, and where (c_i, l_i) is the consumption-leisure pair for person i.
- For the moment, there's little of importance lost by assuming that the *household* utility can be additively decomposed,

$$U(c_1, c_2, l_1, l_2) = \sum_{i=1}^{n} U_i(c_i, l_i).$$

(though we'll return to this issue when we discuss issues related to intra-household allocation).
- Basically, we will typically want to assume that objective function is increasing, concave, and continuously differentiable. The first of these two assumptions correspond to important beliefs about preferences; the last is a mostly harmless assumption made for technical reasons.

2.2. **Feasible Set.**

- In the Bardhan-Udry formulation, the feasible set depends on:
  1. Total household land endowment E^A (note pooling of resources among household members).
  2. Total time available to each household member: E^L_i, $i = 1, 2$.
  3. Prices for consumption, labor, and land: (p, w, r).
- The household takes endowments (E^A, E^L_1, E^L_2) and prices (p, w, r) as given.
- Let $\Gamma(E^A, E^L_1, E^L_2, p, w, r)$ denote the feasible set for the household.

2.3. **Solution to the Household-Farm's Problem.**

2.3.1. *Solving the Problem.*

- With concave, increasing objective function and a non-empty, compact, convex feasible set, theory of the maximum implies a unique solution.
- Since (in addition) the objective function *doesn't* depend on any of the variables that determine the feasible set, the "separation property" is satisfied. Households can solve their problem in two separate steps:
  1. Maximize farm profits; and
  2. Given "total income" (including, but not limited to, farm profits), choose a consumption-leisure allocation to maximize utility.

2.3.2. *General Properties of the Solution.*

- Demands[1] for leisure and consumption should depend only on total income and prices; **not** on (E^A, E^L_1, E^L_2) (except to the exent these determine total income) and **not** on production decisions such as the choice of the allocation of land and labor to production (A, L).
- Operation of farm should **not** depend on household characteristics which influence only objective function.
- Since **other** farm-households presumably face the same prices, marginal products of labor and land should be equated across farms.

[1] That is, *Marshallian* demands will depend on total income and prices. However, the Marshallian demand system is not always the most convenient representation of demand. For example, Hicksian demands will instead depend on prices and the level of utility, while Frischian demands will depend on prices and marginal utility.

2.4. **Estimation and Inference.**

- Making additional progress on understanding behavior of farm households will require additional assumptions on either the feasible set (perhaps particularly the farm production function) or on the household utility function.
- Ideally, we'd like to use data to allow us to recover both U and Γ; then we'd have a complete model of the farm-household.
- In practice, this model may be too simple to capture important elements of the problem facing the farm-household. We can get at this by testing.

Example:

Estimating utility functions

- *If* separation property is satisfied, then we can ignore production side, and just look at demands for goods and leisure.
- Parameterize utility function; e.g.,

$$U(c_1, c_2, l_1, l_2; X_1, X_2) = \theta_1 \exp(\delta' X_1) \left[\alpha_1 \frac{(c_1 - \phi_1)^{1-\gamma} - 1}{1-\gamma} + (1-\alpha_1)\frac{(l_1)^{1-\gamma} - 1}{1-\gamma}\right] + \theta_2 \exp(\delta' X_2)\left[\alpha_2 \frac{(c_2 - \phi_2)^{1-\gamma} - 1}{1-\gamma} + (1-\alpha_2)\frac{(l_2)^{1-\gamma} - 1}{1-\gamma}\right]$$

2.4.1. *Features of Utility.* Example preferences feature:

- Linear Engel Curves (important for aggregation)
- "Subsistence" parameters ϕ_i
- Demands can depend on individual characteristics X_i.
- Demands depend on "disposable" total income $\bar{x} = y - p\sum_{i=1}^{i} \phi_i$.

Demands for consumption and leisure take the form

$$c_i(\bar{x}, p) = \left(\frac{\theta_i \alpha_i e^{\delta' X_i}}{p}\right)^{1/\gamma} \bar{x} + \phi_i$$

and

$$l_i(\bar{x}, w) = \left(\frac{\theta_i (1-\alpha_i) e^{\delta' X_i}}{w}\right)^{1/\gamma} \bar{x}$$

2.4.2. *Estimating Demands.* If we have data on c_i, l_i, prices (p, w), and disposable total income $\bar{x}$, then we can imagine using these to try and estimate these demand relationships. See Deaton [2008] for an overview of the applied literature on estimating demand.

For now, consider taking logs of the expression for $c_i(\bar{x}, p)$ and rearranging,

$$\log(c_i - \phi_i) = \frac{1}{\gamma}[\log(\theta_i) + \log(\alpha_i) + \delta' X_i - \log(p)] + \log(\bar{x}),$$

which is *almost* something we could use OLS to estimate. Notice that the coefficients associated with the logarithm of disposable income and with price don't vary across consumption and leisure. Except for the 'subsistence' parameters, this demand system features unitary income elasticities and a common price elasticity of $-1/\gamma$ across both consumption and leisure.

Alternatively, if prices aren't observed, rearrange again to get

$$\log \frac{p(c_i - \phi_i)}{\bar{x}} = (1 - \frac{1}{\gamma}) \log p + \frac{1}{\gamma}[\log(\theta_i) + \log(\alpha_i) + \delta' X_i],$$

which is an expression which would allow one to relate budget shares to household characteristics.

One might try estimating a 'reduced form' version of this demand system: something like

$$\log \frac{p(c_i - \phi_i)}{\bar{x}} = a_i + \beta' X_i,$$

where the coefficient $a_i = \frac{1}{\gamma}[\log \theta_i + \log \alpha_i] + (1 - \frac{1}{\gamma}) \log p$, and the coefficents $\beta = \delta/\gamma$.

2.4.3. *Testing.* If we can estimate demand system or Engel curves using the previously derived equations, we may be able to recover the utility function!

Question: How will we know if our estimates of preference parameters are adequate?

Answer: We won't. We can only know if they're *not* adequate.

Check the following, in this order:

(1) Are residuals from estimating equations independent of functions of prices and net total income? If not, suggests a problem with specification of utility function.
(2) Are residuals independent of production side characteristics? If not, suggests a problem with separation hypothesis.

Example:

Benjamin (1992)

The discussion above focuses on the independence of consumer demands from production characteristics of the household. In contrast, Benjamin [1992] provides a nice example of testing the independence of *production* decisions from exogenously determined household characteristics. In particular, Benjamin sets out to test whether household size and composition can help to explain the use of labor in agricultural production, using data from Java. Do households with many workers use more labor on their land?

To explore this question, we modify the basic model presented above just slightly, allowing (i) utility to depend on a vector of exogenous household characteristics (e.g., number of children) X; and letting (ii) the total time endowment of the household also depend on household characteristics, and denoting this endowment by $E^L(X)$.

This gives the maximization problem

$$\max_{c_i, l_i, A, L} U(c_1, c_2, l_1, l_2; X)$$

subject to a budget constraint

$$p(c_1 + c_2) + w(l_1 + l_2) \leq [F(A, L) - rA - wL] + rE^A + wE^L(X)$$

and subject also to a collection of non-negativity constraints on consumptions, leisures, and farm inputs.

As before, the solution to this problem involves a separation between the consumer and producer roles played by the household. In particular, given the agricultural production function F, input demands for agricultural land and labor will depend only on input prices (w, r), and *not* on household characteristics.

Benjamin [1992] adopts a Cobb-Douglas parameterization of the agricultural technology, with

$$F(A, L) = \frac{\beta e^{-\alpha/\beta}}{1+\beta} L^{1+1/\beta} A^{\gamma/\beta} + M, \tag{1}$$

with $(\alpha, \beta, \gamma, M)$ all parameters governing the behavior of the function. Though this parameterization looks slightly crazy, there's method behind the madness. First, the marginal product of labor takes the slightly saner form

$$\frac{\partial F}{\partial L}(A, L) = e^{\alpha/\beta} L^{1/\beta} A^{\gamma/\beta},$$

while the first order conditions equating this marginal product to the wage w can be arranged to give the following "partial" demand[2] for labor:

$$\log L = \alpha + \beta \log w + \gamma \log A. \tag{2}$$

The key to Benjamin's test is to note that household characteristics X *do not* appear in the input demand for L. Some alternative models (e.g., the "Peasant" models of Chayanov [1966]) suggest instead that the household labor endowment $E^L(X)$ ought to be a key determinant of the labor employed on the farm.

Accordingly, Benjamin uses a cross-sectional dataset with data on farm operations in a region of Java. These data are clustered, with clusters corresponding roughly to villages, so index clusters by $v = 1, \dots, V$. Within cluster (or village) v, there are N_v households, indexed by j. So consider estimating the regression

$$\log L_v^j = \alpha + \beta \log w_v + \gamma \log A_v^j + \delta X_v^j + \epsilon_v^j,$$

with X_v^j some set of household characteristics (e.g., size), and ϵ_v^j a disturbance term.

Benjamin poses the test of a Chayanovian alternative against the null hypothesis of separation as a test of whether or not the coefficient $\delta = 0$. He fails to reject this null, and so one may infer that the farm-household model featuring separation may be adequate, at least for making predictions regarding agricultural labor demand in Java.

3. When Separation Fails

When the property of separation fails, our job as economists becomes much more difficult! In general consumption may depend not just on

[2]We call this the "partial" demand because it depends on the price w but on the *quantity* A; the fully worked-out input demand would substitute the input demand for A into this expression, showing how L depends on the price of land r instead of its quantity A. Notice that this points out a possible weakness in the analysis—though it's natural to assume that households take prices as given, it's not so natural to assume that they take landholdings as given! And if A is an endogenous quantity, then including it on the right-hand-side of a regression will tend to create problems. On the other hand, Benjamin might argue that at least in the short-run households take A to be fixed. In evaluating this possible argument we should wonder about how easily and quickly households can change their operated land-holdings by leasing land in or out.

prices of consumption goods and summary measures of resources, but on the details of what productive assets are held, etc. And as the Benjamin exercise suggests, production decisions may similarly depend on what would otherwise be irrelevant details—household composition, for example.

To make progress in the face of this additional complexity we really need a structured alternative to the farm-household model with separation. Deciding on *what* structured alternative is appropriate becomes an important problem, and making an intelligent choice is likely to depend on what the *reason* for the failure of separation is.

3.1. **Possible reasons for failure of separation.** "Shallow" reasons for failure of separation:

- Mis-specified utility function.
- Utility depends directly on production arrangements (e.g., prefer to work on own land).
- Production depends directly on consumption side (e.g., marginal product of labor depends on how well-fed workers are).
- "Transaction" costs.

"Deep" reasons for possible failure of separation involve so-called *missing markets*, though this begs the question of why markets may be missing.

3.2. **Farm-Households and Missing Contingent-Claims Markets.**

3.2.1. *Feasible Set.* Suppose that there's some randomness to production, so that output is given by $F(A, L, \epsilon)$, where ϵ is a random variable, instead of simply $F(A, L)$.

Let's suppose that $\epsilon \in \Omega$, and that Ω has a finite number of elements.

Now, the farm household's constraints still have to be satisfied, as before, but *now* they have to be satisfied for every value of ϵ which may be realized. This gives us additional constraints (a constraint for every possible value of ϵ). We write the new set as

$$\Gamma(E^A, E^L_1, E^L_2, \epsilon, p, w, r).$$

Decisions about how land and time are allocated have to be made before ϵ is observed. So what's the household's new problem? To maximize *expected* utility subject to this new constraint set:

$$\max_{c_i(\epsilon), l_i, A, L} \sum_{\epsilon} \Pr(\epsilon) U(c_1(\epsilon), c_2(\epsilon), l_1, l_2)$$

such that

$$p(c_1(\epsilon)+c_2(\epsilon))+w(l_1+l_2) \leq [F(A,L,\epsilon)-rA-wL]+rE^A+w(E_1^L+E_2^L)$$

for all ϵ. Associate a multiplier $\Pr(\epsilon)\lambda(\epsilon)$ with the budget constraints. Solution must also satisfy a collection of non-negativity constraints.

Notice that consumption depends on ϵ!

3.2.2. *First-order conditions for the consumption side.*

$$c_i(\epsilon): \quad \Pr(\epsilon)\frac{\partial U}{\partial c_i} = p\Pr(\epsilon)\lambda(\epsilon)$$

$$l_i: \quad \mathrm{E}\frac{\partial U}{\partial l_i} = w\mathrm{E}\lambda(\epsilon)$$

Summing the FOC w.r.t. $c_i(\epsilon)$ over ϵ, we obtain the optimality condition

$$\frac{\mathrm{E}\partial U/\partial c_i}{\mathrm{E}\partial U/\partial l_i} = \frac{p}{w}.$$

Contrast with case of no risk; effect on consumption and leisure depends on curvature of utility function.

3.2.3. *First-order conditions for the production side.*

$$A: \quad \mathrm{E}\left[\lambda(\epsilon)\frac{\partial F}{\partial A}\right] = r\mathrm{E}\lambda(\epsilon)$$

$$L: \quad \mathrm{E}\left[\lambda(\epsilon)\frac{\partial F}{\partial L}\right] = w\mathrm{E}\lambda(\epsilon)$$

The thing to note is that the first order conditions for production can't in general be disentangled from the multiplier $\lambda(\epsilon)$. As a consequence, the choice of productive inputs will depend on the probability distribution of the marginal utility of income for the household.

Example:

Here's a particular example. Use the preferences described above, with $\gamma = 1$ and $\phi_i = 0$ (i.e., logarithmic utility) and a Cobb-Douglas production function

$$y = A^\beta L^{1-\beta} e^\epsilon,$$

where $\mathrm{E}e^\epsilon = 1$.

Now, optimality on the consumption side implies

$$\frac{\alpha_i}{1-\alpha_i}\mathrm{E}\frac{l_i}{c_i(\epsilon)} = \frac{p}{w}.$$

Since the left-hand side of this is a convex function of a random variable, the expected marginal rate of substitution consumption must be greater than in the case of no risk (Jensen's inequality). Accordingly, in this case the farm household will work more than it would if separation held [see Kochar, 1999, for evidence on this point].

3.2.4. *Key points from example.*

- Failure of separation
- "Precautionary labor" (but direction of distortion depends on preferences and technology)
- Farm inputs and consumption demands now become a complicated function of just about everything in the environment. The approach to estimation taken above won't work (in the sense that the demands are mis-specified, so we can't expect to use this as the basis for a (consistent) estimator).

Exercises

(1) Suppose that the property of separation holds, so that the production of farm-households can be treated as though they simply operated profit-maximizing firms. Let the agricultural production function F take the form (1) assumed by Benjamin [1992]. Also, you may find it convenient to take the price at which the household can market its output to be some q (instead of letting this be a *numeraire* good as above).

a) Write the profit-maximization problem facing a firm operating this production function.

b) What parameter restrictions are necessary to guarantee that this F is concave and increasing? Under what conditions will F display constant returns to scale?

c) Assuming these parameter restrictions, find the input demand functions for A and L that solve the firm's problem, taking as given prices (p, q, w, r).

d) How will the production of the farm-household change as each of these prices change? (Compute an expression for price elasticities.) Describe in particular the supply function (output as a function of output price q).

(2) One stylized fact about developing country agriculture is the so-called "inverse productivity puzzle"—data on agricultural productivity for smallholders often suggests that productivity

(output conditional on inputs) is larger when operated plots are smaller [see e.g., Lamb, 2003]. One simple way of introducing this into the farm-household model is to modify the agricultural production technology assumed above, taking

$$F(A,L) = \frac{\beta e^{-\alpha/\beta}}{1+\beta} L^{1+1/\beta}(A+1)^{\gamma/\beta} + M.$$

a) Following the steps outlined in the previous problem, compare the farm-household supply function with 'inverse-productivity' to the case with a constant returns version of F.
b) What is the total or 'full' income of the farm-household, taking into account not only farm profits but also returns to land owned and the value of time endowments?
c) Now suppose that there are two farm-households. These face the same prices and are otherwise identical in every way, except that one owns twice as much land as the other. Explain how these differences in endowments will affect farm profits.

(3) Consider the basic farm-household model of Section 2, with the household having a utility function defined over consumptions (c_1, c_2) and leisures (l_1, l_2) for each of two people taking the form

$$U(c_1, c_2) = \frac{1}{2}\log c_1 + \psi \log c_2 + \frac{1}{2}\log l_1 + (1-\psi)\log l_2.$$

Take $E_1^L = E_2^L = 1$, and assume that $F(A,L) = A^{1/2}L^{1/2}$.

a) What are farm profits? [Note that the production function is a special case of one you solved above.]
b) Solve the (Marshallian) demand system for the household; that is, express quantities of consumptions and leisures as a function of prices, total (or 'full') income, and the preference parameter ψ.
c) Interpret the resulting allocation within the household. What's the cost of laziness?

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