The following is a take-home exam covering selected material from the first half of ARE 251/Econ 270A. Your exam should be put in Ethan Ligon’s mailbox in Giannini 207 before 5:00 PM on Monday, October 19th. You should answer each question concisely, returning no more than five pages (written on one side) in total. Further, any single page with more than 3000 characters or symbols on it may be discarded.

Consider a village of \( n \) people, each belonging to one of \( m < n \) households; the number of people in the \( i \)th household is given by \( n_i \). You have data on basic demographic characteristics of all of these \( n \) people, and a panel of data collected over \( T > 1 \) years on income (\( y_{it} \)) and expenditures (\( c_{it} \)) collected at the level of the \( m \) households.

Unless stated otherwise, assume that the people who live in the village are all risk-averse, with time-separable preferences, and a CES momentary utility function with coefficient of risk aversion equal to two.

(1) Explain in detail how you would go about using these data to describe the extent of inequality within the village.

(2) Another researcher using the same data asserts that inequality has increased over time—the Gini coefficient in year 1 is much smaller than the Gini coefficient in year \( T \). But an Atkinson measure of inequality with an “inequality aversion” coefficient of two actually falls slightly.
   a) Without any further information, what can you say about how the Lorenz curve must have changed between the first and last periods?
   b) What can you say about the possible consequences of the increase in the Gini coefficient for (utilitarian) social welfare in this village?

(3) Explain in detail how you would go about using these data to test whether or not there was full risk-sharing in the village.

(4) A colleague of yours, using the same original data, is interested in knowing how variation in household wealth and in the price of rice (the major staple in the village) influence rice consumption, and hence nutrition. He gets data on household rice consumption (in kilograms) \( \{x_{it}\} \). The time series on rice prices \( \{p_t\} \) (which a Marshallian demand system would suggest) don’t seem trustworthy, however, so he decides to simply assume that rice prices are common to the village with a given year, and so substitutes a village-level time effect \( \eta_t \) and estimates the regression

\[
\log x_{it} = \alpha_i + \eta_t + \gamma \log y_{it} + \epsilon_{it},
\]

where \( \{\alpha_i\} \) is a set of household fixed effects, and \( \{\epsilon_{it}\} \) a collection of disturbance terms. However, he obtains an estimate of \( \gamma \) which isn’t significantly different from zero, and so your colleague concludes that increases in household income wouldn’t be effective at improving nutrition.

Date: October 16, 2009.
a) Suppose that each household’s utility function takes the form

\[ U(x_1, \ldots, x_m) = \sum_{j=1}^{m} \alpha_j \frac{x_j^{1-\gamma} - 1}{1 - \gamma} \]

where the \( x_j \) are different goods. If each household takes its income and prices as given, derive the corresponding system of Marshallian demands.

b) Show how to use the demand system you’ve derived to motivate a regression which takes the form estimated by your colleague.

c) In light of the preceding, is there any puzzle about his results? Explain.