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NOTES AND COMMENTS

A CLASS OF DECOMPOSABLE POVERTY MEASURES

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SEVERAL RECENT STUDIES OF POVERTY have demonstrated the usefulness of breaking down a population into subgroups defined along ethnic, geographical, or other lines [e.g. **1**, **20**]. Such an approach to poverty analysis places requirements on the poverty measure in addition to those proposed by Sen [**15**, **16**]. In particular, the question of how the measure relates subgroup poverty to total poverty is crucial to its applicability in this form of analysis. At the very least, one would expect that a decrease in the poverty level of one subgroup *ceteris paribus* should lead to less poverty for the population as a whole. At best, one might hope to obtain a quantitative estimate of the effect of a change in subgroup poverty on total poverty, or to give a subgroup's contribution to total poverty.

One way to satisfy the above criteria is to use a poverty measure that is additively decomposable in the sense that total poverty is a weighted average of the subgroup poverty levels.[2] However, the existing decomposable poverty measures are inadequate in that they violate one or more of the basic properties proposed by Sen.[3] Stated another way, of all the measures [**1**, **3**, **10**, **19**] that are acceptable by the Sen criteria, none is decomposable. In fact, the Sen measure and its variants that rely on rank-order weighting fail to satisfy the basic condition that an increase in subgroup poverty must increase total poverty (see footnote 6). This note is a first step towards resolving these inadequacies.

In what follows we present a simple, new poverty measure[4] that (i) is additively decomposable with population-share weights, (ii) satisfies the basic properties proposed by Sen, and (iii) is justified by a relative deprivation concept of poverty. The inequality measure associated with our poverty measure is shown to be the squared coefficient of variation and indeed the poverty measure may be expressed as a combination of this inequality measure, the headcount ratio, and the income-gap ratio in a fashion similar to Sen [**15**]. We generalize the new poverty measure to a parametric family of measures where the parameter can be interpreted as an indicator of "aversion to poverty." A brief empirical application demonstrates the usefulness of the decomposability property.

1. A DECOMPOSABLE POVERTY MEASURE

Let $y = (y_1, y_2, \ldots, y_n)$ be a vector of household incomes in increasing order, and suppose that $z > 0$ is the predetermined poverty line. Where $g_i = z - y_i$ is the income shortfall of the ith household, $q = q(y; z)$ is the number of poor households (having income no greater than z), and $n = n(y)$ is the total number of households, consider the

[1] We would like to thank the participants of the Cornell Development Seminar, Gary Fields, and the anonymous referees for helpful comments. In addition, we owe a special debt of gratitude to Amartya Sen for his thoughtful remarks and encouragement. This note is based on a longer working paper [**8**] and on dissertation research by J. Greer.

[2] See [**1**, **20**]. In contrast, decomposability as applied to inequality measures involves a "between-group" term to account for differences among subgroup mean incomes [**4**, **17**]. Here one poverty level is postulated to apply to all subgroups; hence a "between-group" poverty term would appear to be unnecessary.

[3] In their empirical work, Anand [**1**], Kakwani [**9**], and Van Ginneken [**20**] use decomposable measures that violate the transfer axioms.

[4] While revising the initial submitted version, we became aware of independent work by Kundu [**12**] which also gives P_2 and indicates some of its properties. However, Kundu's paper addresses quite different issues and, in particular, decomposability is not mentioned.

poverty measure P defined by

$$(1) \qquad P(y;z) = \frac{1}{nz^2} \sum_{i=1}^{q} g_i^2 .$$

Following Sen [15], poverty is a (normalized) weighted sum of the income shortfalls of the poor. In contrast to the Sen measure, which adopts a "rank order" weighting scheme, P takes the weights to be the shortfalls themselves; deprivation depends on the distance between a poor household's actual income and the poverty line, not the number of households that lie between a given household and the poverty line.

Despite this basic difference in weighting, several of the arguments advanced in support of the Sen measure also justify P. For instance, Sen has proposed that poorer households should have higher weights [15, Axioms E and M]. Clearly P satisfies this requirement. Further, Sen argues that the weights should be based on a notion of relative deprivation experienced by the poor households. In his seminal work on the subject, Runciman considers several different aspects of relative deprivation including the magnitude of relative deprivation, or "the extent of the difference between the desired situation and that of the person desiring it (as he sees it)" [14, p. 10]. Where the "desired situation" is to receive enough income to be able to "meet the accepted conventions of minimum needs" [16, p. 29] and the "existing situation" is given by the poor household's income, the magnitude of relative deprivation is precisely the income shortfall of that household. Clearly, the weighting scheme behind P is closely related to this aspect of relative deprivation.

Sen [15, 16] has formulated two axioms for a poverty measure to satisfy:

MONOTONICITY AXIOM: *Given other things, a reduction in the income of a poor household must increase the poverty measure.*

TRANSFER AXIOM: *Given other things, a pure transfer of income from a poor household to any other household that is richer must increase the poverty measure.*[5]

It can be shown that P satisfies these two axioms (see Proposition 1, below). Further, P is associated with a well-known inequality measure, the squared coefficient of variation. Let $H = q/n$ be the headcount ratio, $I = \sum_{i=1}^{q} g_i/(qz)$ be the income-gap ratio, and $C_p^2 = \sum_{i=1}^{q}(\bar{y}_p - y_i)^2/(q\,\bar{y}_p^2)$, where $\bar{y}_p = \sum_{i=1}^{q} y_i/q$. Then

$$(2) \qquad P(y;z) = H\left[I^2 + (1-I)^2 C_p^2\right],$$

as shown in [8]. Finally, the squared coefficient of variation C^2 is the measure of inequality "corresponding" to P in the sense that C^2 is obtained when n and $\bar{y}$ (the mean of y) are substituted for q and z in the definition of P [see 15, p. 224].

2. A CLASS OF DECOMPOSABLE MEASURES

It can be seen from (2) and the properties of C_p^2 (see [2]) that a given transfer has the same effect on P at low or high income levels. Kakwani has proposed a property that stresses transfers among the poorest poor:

[5] See also [19]. Kundu and Smith [13] question the desirability of the Transfer Axiom.

TRANSFER SENSITIVITY AXIOM: *If a transfer $t > 0$ of income takes place from a poor household with income y_i to a poor household with income $y_i + d$ ($d > 0$), then the magnitude of the increase in poverty must be smaller for larger y_i* [**10**, p. 439].

While P does not satisfy this axiom, it can be generalized to a class which contains poverty measures that do. For each $\alpha \geq 0$, let P_α be defined by

$$(3) \qquad P_\alpha(y; z) = \frac{1}{n} \sum_{i=1}^{q} \left(\frac{g_i}{z} \right)^\alpha .$$

The measure P_0 is simply the headcount ratio H, while P_1 is $H \cdot I$, a renormalization of the income-gap measure. The measure P is obtained by setting $\alpha = 2$. The parameter α can be viewed as a measure of poverty aversion: A larger α gives greater emphasis to the poorest poor. As α becomes very large P_α approaches a "Rawlsian" measure which considers only the position of the poorest household. The properties of this family of measures are summarized in the following proposition.

PROPOSITION 1: *The poverty measure P_α satisfies the Monotonicity Axiom for $\alpha > 0$, the Transfer Axiom for $\alpha > 1$, and the Transfer Sensitivity Axiom for $\alpha > 2$.*

PROOF: The Monotonicity Axiom holds for $\alpha > 0$ by the fact that g_i increases as y_i falls. To verify the Transfer Axiom, note that any transfer from a poor household to a richer one may be viewed as some combination of the following two types of "regressive" transfers: (i) from a poor household to another poor household that stays poor, or (ii) from a poor household to a household at or above the poverty line. The strict convexity of P_α in the vector of poor incomes for $\alpha > 1$ covers (i), while a transfer of the form (ii) increases P_α by inspection. That P_α satisfies the Transfer Sensitivity Axiom for $\alpha > 2$ follows from Kolm [**11**, p. 88].

3. DECOMPOSABILITY

Suppose that the population is divided into m collections of households $j = 1, \ldots, m$ with ordered income vectors $y^{(j)}$ and population sizes n_j. In analyzing poverty by population subgroups, the following axiom may be taken as a basic consistency requirement.

SUBGROUP MONOTONICITY AXIOM: *Let $\hat{y}$ be a vector of incomes obtained from y by changing the incomes in subgroup j from $y^{(j)}$ to $\hat{y}^{(j)}$, where n_j is unchanged. If $\hat{y}^{(j)}$ has more poverty than $y^{(j)}$, then $\hat{y}$ must also have a higher level of poverty than y.*

When incomes in a given subgroup change (the rest remaining fixed), this axiom requires subgroup and total poverty to move in the same direction. By this criterion the Sen measure and its variants [**1**, **9**, **10**, **18**, **19**] are not well-suited for poverty analysis by subgroup, since they violate this consistency requirement in certain cases.[6] On the other

[6] For instance, where $y = (1, 6, 6, 7, 8, 12)$, $\hat{y} = (3, 3, 6, 7, 8, 13)$, and $y^{(1)} = (1, 6, 12)$, $y^{(2)} = (6, 7, 8)$, $\hat{y}^{(1)} = (3, 3, 13)$, and $y^{(2)} = \hat{y}^{(2)}$, we have more poverty in $y^{(1)}$ than $\hat{y}^{(1)}$ by the Sen measure as long as $z \geq 13$ and yet $\hat{y}$ has more poverty than y using the same measure. Note that the mean incomes of $y^{(1)}$ and $\hat{y}^{(1)}$ are the same in this example. This example can be found in Cowell [**6**] who applied it to the Gini inequality measure.

hand it can be shown that P_α satisfies subgroup monotonicity, and an even stronger decomposability property:

PROPOSITION 2: *For any income vector y broken down into subgroup income vectors $y^{(1)}, \ldots, y^{(m)}$,*

$$P_\alpha(y; z) = \sum_{j=1}^{m} \frac{n_j}{n} P_\alpha(y^{(j)}; z). \tag{4}$$

P_α is additively decomposable with population share weights.

The decomposition in (4) allows a quantitative, as well as qualitative, assessment of the effect of changes in subgroup poverty on total poverty. In fact, increased poverty in a subgroup will increase total poverty at a rate given by the population share n_j/n; the larger the population share, the greater the impact. The quantity $T_j = (n_j/n)P_\alpha(y^{(j)}; z)$ may be interpreted as the total contribution of a subgroup to overall poverty while $100T_j/P_\alpha(y; z)$ is the percentage contribution of subgroup j.

4. AN ILLUSTRATIVE EXAMPLE

In this section, the poverty measure P_2 is applied to data from the 1970 Nairobi Household Survey to illustrate the usefulness of decomposability.

The Nairobi Household Survey was conducted by the Institute for Development Studies. A total of 1416 families were interviewed and information obtained regarding income, education, occupation, marital status, and other characteristics of all adult household members. To analyze poverty, a poverty line of 515 Kenya Shillings/year/adult was derived from the one calculated by Crawford and Thorbecke [7]. The overall results of the analysis contain few surprises. The very tentative estimates by previous

TABLE I

DECOMPOSITION OF POVERTY (P_2) BY NUMBER OF YEARS HOUSEHOLD HEAD HAS LIVED IN NAIROBI[a]

Years in Nairobi	Number of Individuals n_j	Level of Poverty $P_2(y^{(j)}; z)$[b]	Percentage Contribution to Total Poverty[c]	Average Income of Poor[d]	Proportion of Poor in Each Group
0	29	.4267	5.6	93.3	.55
.01–1	117	.1237	6.5	221.9	.30
2	116	.1264	6.6	140.0	.20
3–5	438	.0257	5.1	295.1	.09
6–10	793	.0343	12.1	273.8	.11
11–15	719	.0291	9.4	286.6	.10
16–20	565	.0260	6.6	329.3	.11
21–70	954	.0555	23.8	198.8	.12
Permanent Residents	116	.1659	8.7	203.3	.35
Don't Know	140	.2461	15.5	93.0	.34
Total	3987	.0558	99.9		

[a] The poverty line z is 515 Kenya Shillings per adult equivalent per year, or roughly \$72 (U.S.) per year (See [7]).
[b] $P_2(y^{(j)}; z) = 1/n_j z^2 \sum_{i=1}^{q_j} g_i^2$ where q_j is the number of individuals below the poverty line in group j.
[c] I.e., $100(n_j/n)(P_2(y^{(j)}; z)/P_2(y; z))$. (This column does not sum to 100 due to rounding errors.)
[d] In Kenya Shillings per adult equivalent per year.

authors [5, 7] that poverty is not a major problem in Nairobi were confirmed. Only 13 per cent of the survey sample live in poor households and P_2 is equal to 0.056 (see Table I).

The relation between poverty and certain specific household characteristics may be analyzed with the aid of a collection of tables called a poverty profile. Table I shows one such relation, describing poverty for subgroups differentiated by the number of years since the household head moved to Nairobi. The first column gives the total number of household members in the subgroup. The second gives the level of poverty for each subgroup as measured by P_2. The poverty level is then weighted by the population share to determine the contribution of the subgroup to total poverty, which is given as a percentage of total poverty in column 3. Complete elimination of poverty within a subgroup would lower total poverty precisely by this percentage.

As can be seen in Table I, of all those answering the question, poverty is worst among short-term residents (i.e., those in Nairobi less than two years). However, these recently arrived subgroups do not contribute much to total poverty due to their relatively small population. Rather, it is the group of households whose heads migrated to Nairobi twenty or more years ago which contributes most prominently to total poverty: 23.8 per cent of total poverty is accounted for by this subgroup. This cannot be seen from their level of poverty, which is far below the level for the recently arrived subgroups.

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[Footnotes]

[2] **Decomposable Income Inequality Measures**
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