I. Theory
1. Consider the standard static household model, where the household utility \( U(c, L - l; z^s) = c + v(L - l; z^s) \) is a function of consumption \( c \) and leisure \( L - l \), and its production \( q(L; z^s) \) is a function of labor \( L \). The household is endowed with \( L \) units of labor. The variables \( z^s \) and \( z^o \) are shifters in utility and in production, and \( l \) the household labor supply. Define \( h = L - l \) as net hired labor, with positive values indicating hired labor and negative values that the household is working off-farm. Assume that there are competitive labor and product markets, with prices \( w \) for labor. To simplify notation, the prices of \( q \) and \( c \) are normalized to 1. Write the optimization problem of the household and show that the household is maximizing profit.

2. Under what conditions does the household remain self-sufficient in labor? What is the optimal level of labor use? Under what conditions does the household hire in labor, or hire out some of its own labor?

3. Assume now that there are large transaction costs on the labor market, so that the effective cost of hiring one unit of labor is \( w^h \), while the household only gets \( w^o \) when it works off-farm, with \( w^o < w^h \). Write the household optimizing problem. Write the (Kuhn-Tucker) first-order conditions. [One way to simplify notations is to define \( h^+ = \max(0, h) \geq 0 \) for hired in labor and \( h^- = -\min(0, h) \geq 0 \) for hired out labor]

4. Derive the conditions under which the household chooses to not participate in the labor market. What is its optimal level of labor use in production? What is its corresponding marginal utility of leisure?

5. Similarly, under what conditions does the household choose to hire in some labor? Under what conditions does it choose to hire out some of its own labor?

6. Give the reduced form expression for labor use on farm, \( L \), in each of the 3 regimes of labor market participation defined above.

II. Empirical project: Labor intensity in farm production.

In this exercise, you will perform a simplified version of the analysis in Sadoulet, de Janvry, and Benjamin (1998), “Household Behavior with Imperfect Labor Market”, Industrial Relations, Vol. 37, No. 1, pp. 85-108. The data set posted on the website has been extracted from a large survey of the Mexican ejido sector that we conducted in collaboration with the ministry of Agrarian reform, in 1994. The ejido sector is the land reform sector established by the land reform of 1917.

Household labor has been divided in skilled and unskilled labor, depending on its education. Skilled labor is assumed not to participate to the agricultural labor market. All hired labor is assumed unskilled. While the observed monetary wage is absolutely identical across observations in our sample, one can argue that there are (monetary or non-monetary) transaction costs that make the de-facto take home benefit of off-farm work lower than the cost of employing a worker. We study the household participation to the unskilled labor market, and its behavior in corn production.

1. Give a few descriptive statistics on land use and labor allocation that set the stage for the analysis. What proportion of the households are net buyer, seller, or self-sufficient in unskilled labor? Contrast their endowment and behavior, with tables or graphs. You will see that a small percentage of the household both hire in and work off-farm. As this behavior cannot be accommodated in this simple model of labor market participation, we will discard those observations. (Alternately, you could choose a different empirical approach).
2. An empirical model of the labor market participation developed in the theory above could be written as:

\[
\begin{align*}
g(L, z', z'', z''') + \varepsilon & < a \quad \Rightarrow \quad h < 0 \\
a & \leq g(L, z', z'', z''') + \varepsilon \leq b \quad \Rightarrow \quad h = 0 \\
b & < g(L, z', z'', z''') + \varepsilon \quad \Rightarrow \quad h > 0
\end{align*}
\]

(1)

where \( \varepsilon \) represent unobserved individual effect. The term \( z''' \) has been added to represent potential fixed transactions costs in access to the labor market, which affect the choice of regime but not marginal decision on how much to hire in or out. And then production decisions under different regimes are written as:

\[
\begin{align*}
X = f^1(z'') + \mu & \quad \text{if} \quad h < 0 \\
X = k(z'', L, z''') + \mu & \quad \text{if} \quad h = 0 \\
X = f^3(z') + \mu & \quad \text{if} \quad h > 0
\end{align*}
\]

(2)

You will analyze labor use in corn production, an activity performed by all households.

- Fixed transaction costs in access to labor market, \( (z'') \), are characterized by two regional indices of infrastructure and density of labor market.
- Unskilled labor endowment \( (L) \) is characterized by unskilled men and women.
- Shifters in utility \( (z') \) include assets that represent sources of income, such as total owned irrigated and rainfed land, skilled labor force, and migration assets. In the ejido sector, households had usufruct of land, but could neither buy nor sell. So total land is a fixed asset.
- Other factors that could pertain either to \( (z'') \) or \( (z') \) are land quality (characterized by 2 indices of regional productivity), household age, common pasture area (because of the interaction between corn and livestock production, and the use of common property in this integrated system).

Finally, as we study the labor per ha of corn, we add the land planted in corn as a control variable to account for economies of scale, even though it is endogenous.

3. Estimate the three equations (2) by simple OLS, and test for the separation in production and consumption decisions for the households that participate to the labor market. Discuss the issue of selection bias.

4. Estimate the labor market participation equation (1) with an ordered probit.

5. Using a Heckman procedure, what equations should you estimate to correct for the selection bias? (If necessary, check in your textbook for the standard Heckman 2 step procedure). When the selection model is an ordered probit as it is here, the Inverse Mills ratios (IMRs) are given by:

\[
\begin{align*}
\lambda = -\frac{\varphi(a)}{\Phi(a)} & \quad \text{if} \quad h < 0 \\
\lambda = -\frac{\varphi(b) - \varphi(a)}{\Phi(b) - \Phi(a)} & \quad \text{if} \quad h = 0 \\
\lambda = \frac{\varphi(b)}{1 - \Phi(b)} & \quad \text{if} \quad h > 0
\end{align*}
\]

with \( \varphi \) and \( \Phi \) the density and distribution functions of the standard Normal distribution. Using the results from the selection model, compute these IMRs and estimate the labor intensity equations in (2) correcting for selectivity bias. Test for the separation of production and consumption decisions. Discuss your results.