Fall2014 **ARE211**

Final Exam
Wednesday, Dec 17, 2014

This is the final exam for ARE211. As announced earlier, this is an open-book exam. However, use of computers, calculators, Iphones, Ipads, cell phones, Blackberries and other non-human aids with on-off switches is forbidden. Read all questions carefully before starting the test. Allocate your 180 minutes in this exam wisely. The exam has 150 points, so aim for an *average* of roughly 1.2 minutes per point. However, some questions & parts are distinctly easier than others. Make sure that you first do all the easy parts, before you move onto the hard parts. Always bear in mind that if you leave a part-question completely blank, you cannot conceivably get any marks for that part. On the other hand, to give a 100% complete answer to all of the questions would almost certainly take you longer than the time available. In some problems, it may be efficient to simply write down the correct answer (if you know what it is) then come back and justify the answer later.

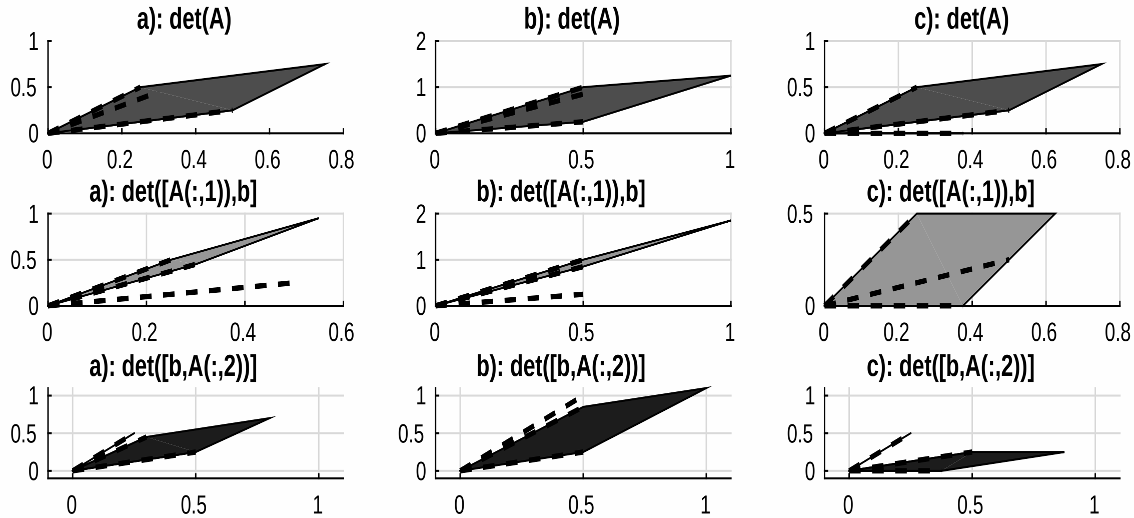

FIGURE 1. Figure for problem 2

Problem 1 (Analysis) [17 points]:

A) [12 points] Let $A = (0, 1) \subset \mathbb{R}$ be endowed with the Euclidean metric. Consider the set $\mathcal{O} = \{(\frac{1}{n}, 1) : n \in \mathbb{N}\}$. Does the open cover $\mathcal{O}$ of A have a finite open subcover? If so, provide it. If not, prove that one does not exist.

Problem 2 (Linear Algebra) [25 points]:

A) [10 points] Suppose that X is a vector space and that A is a subset of X. Prove that if A is a vector subspace then X/A is not a vector subspace.

B) [15 points] For each of panels a), b) and c) of figure 1, write down your best guess for the solution of the equation $A\boldsymbol{x} = \boldsymbol{b}$, where $A = [\boldsymbol{v_1}\boldsymbol{v_2}]$. (Note: full marks will be awarded if the maximum element-wise distance between your answer and the true answer is < 0.2. Otherwise the further your answer is from the correct answer, the fewer marks you will recieve. *For full marks use only the graphical approach to Cramer's rule (no algebra).* Hint: be careful about the sign of your solution.)

Problem 3 (Calculus) [25 points]:
Let $f(x) = x^3 + 2x^2 - 3x$.

A) [9 points] Use a first order Taylor expansion around $x = 0$ to estimate the change in the function when x is increased by 2, i.e. $dx = 2$. What is the estimated sign of the change?

B) [9 points] Now use a second order Taylor expansion around $x = 0$ to estimate the change in the function when x is increased by 2, i.e. $dx = 2$. What is the estimated sign of the change?

C) [7 points] Evaluate the original function at $x = 2$. Discuss how the actual sign of the change compares to your answers in the previous two parts.

Problem 4 (Constrained Optimization) [25 points]:
After working hard all semester you decide to take a vacation by the seaside. Each day you can either spend your time sunbaking, s, or jetboating, j. Consuming one unit of either takes a day. Your vacation is 15 days long. Additionally sunbaking costs \$1 per day and jetboating costs \$4 per day. You have allocated a budget of 24 dollars for your vacation. Your utility from sunbaking and jetboating is given by $f(s,j) = sj$.

A) [3 points] Set up your optimization problem so you have the best time possible on vacation (please ignore non-negativity constraints).
(Strictly speaking this problem should have non-negativity constraints on s and j. We omit them in this problem since it increases the number of cases unnecessarily. We will only consider $(s,j) \in \mathbb{R}_{++} \times \mathbb{R}_{++}$ to avoid the discontinuity of f at $(0,0)$.)

B) [3 points] Sketch the constraint set and draw a level set of the utility function.

C) [3 points] Write down the Lagrange and the KKT conditions.

D) [8 points] Find the solution to the problem. Show all your work for maximum credit.

E) [8 points] What will be the approximate increase in your utility if:
  (a) Your holiday is extended for a day?
  (b) You find a dollar on the beach?

Problem 5 (Comparative Statics) [25 points]:
Consider the following constrained maximization problem.

$$\max x + \alpha \text{ subject to } -x^3 - x \leq -\alpha$$

A) [5 points] In $\alpha \times x$ space, draw the feasible set and a level set of the objective function.

B) [10 points] Use the implicit function theorem to calculate $\frac{dx^*}{d\alpha}$.

C) [10 points] Use the envelope theorem to estimate the maximized value of the objective function when $\alpha = 1.9$.