Problem Set #08

First Calculus Problem Set

Due date: Nov 11

Please note: There are parts of one question in this assignment to which you may want to reply: “I beg your pardon?” or something possibly ruder than this. That kind of response is entirely appropriate, but you should add a sentence explaining why you are led to make such a remark.

(1) Consider the function $f(x) = 20x^3 - 120x^2 - 5x + 36$

a) What is the derivative of this function?

b) What is the derivative of this function evaluated at $x_0 = 4$?

c) What is the differential of this function at $x_0 = 4$?

d) Approximate the change in the function when moving from $x_0 = 4$ to $x_1 = 5$.

e) What is the actual change in the function when moving from $x_0 = 4$ to $x_1 = 5$?

(2) Multivariate calculus drill: Simon and Blume, question 14.1, parts (a), (d) and (f);

(3) Gradients: Simon and Blume, question 14.19


(5) Still more differential approximations: Simon and Blume, question 14.8b.

(6) And one more: Simon and Blume, question 14.10

(7) Consider the function $f(x) = x_1x_2$. Let $x_0 = (3, 2)$ and $h = (3, 4)$.

a) Directly compute the directional derivative of $f$ at $x_0$ in the direction $h$ by the method used in class and in the lecture notes.

b) Indirectly compute the directional derivative of $f$ at $x_0$ in the direction $h$ using the differential

c) Do both computations give the same answer. If not, why not?
(8) Consider the function \( f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2 + y^2}} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases} \)

(a) Using the chain rule, compute \( f_x(x, y) \), for \((x, y) \neq (0, 0)\).

(b) Compute \( f_x(0, 0) \), using the following definition: \( f_x(x, y) = \lim_{t \to 0} \frac{f((x,y) + (t,0)) - f(x,y)}{t} \).

(c) Compute the directional derivative of \( f \) at \((0, 0)\) in the direction \((3,3)\), using the method presented in class (and the lecture notes).

(d) Write down \( f_y(0, 0) \) and evaluate the expression \( \nabla f(0, 0) \cdot (3,3) \).

(e) For \( x_0 = y_0 = 0 \) and \( h = (3,3) \), what can be said about the expression
\[
\lim_{|k| \to \infty} \frac{(f((x_0,y_0)+h/k)-f(x_0,y_0)) - \nabla f(x_0,y_0) \cdot h/k}{||h||/k}
\]

(f) What can you conclude about the differentiability or otherwise of \( f \)?

(g) With \( x \) on the horizontal axis, plot \( f_x(x,x) \), for \( x \in [-1,1] \).

(i) From your graph, what can you say about the continuity or otherwise of \( f_x(\cdot,\cdot) \) at \((0,0)\).

(ii) From your graph, what can you say about the continuity or otherwise of \( f_x(0,0) \).

(iii) Explain why your answers to 8(g)i and 8(g)ii are different.

(iv) What is the relationship between your answers to 8(g)i and 8(g)ii and your answer to 8f?