# Problem Set #06
## First CompStat Problem Set
## Due date: Oct 14

(1) In problem 3 on your last problem set, you found the maximum and minimum distance from the origin to the ellipse $x_1^2+x_1x_2+x_2^2=3$. Generalize this problem to "minimize/maximize the distance from the origin to the ellipse $x_1^2+x_1x_2+\alpha x_2^2=3$" and use the Envelope theorem (starting from $\alpha=1$) to estimate the maximum and minimum distance from the origin to the following
ellipse,

$$x_1^2+x_1x_2+0.9x_2^2=3$$

.

(2) a) Prove that the expression $\alpha^2-\alpha x^3+x^5=17$ defines $x$ implicitly as a function of $\alpha$. in a neighborhood of $(\bar{\alpha},\bar{x})=(5,2)$
b) Estimate the $x$-value which corresponds to $\alpha=4.8$ using a first order approximation.

(3) (a) Consider the function $f(x,y,\gamma)=xy+\gamma y$ subject to the following constraints: $g(x,y,\gamma)\leq 1$, $x\geq 0$, $y\geq 0$, where $g(x,y)=x^2+\gamma y$.
(i) For $\gamma=1$, solve this maxmization problem using either the Lagrangian or KKT method.
(ii) Now, use the envelope theorem to estimate the maximized value of $f$ when $\gamma=1.2$
(b) Consider the problem $\max_x\ f(x;\alpha)\quad$ s.t. $\quad g(x;\alpha)\leq b$, where $x,\alpha,b\in\mathbb{R}$, $f$ and $g$ are twice continuously differentiable, $g_x(\cdot,\alpha)>0$. Let $x^*(\alpha)$ denote the solution to this problem, given $\alpha$. Use the implicit function theorem to identify sufficient conditions for $x^*(\cdot)$ to be everywhere strictly increasing in $\alpha$. Are the conditions you identified necessary as well? If so prove it. If not, provide a counter-example.

(4) Consider the equation $\alpha_1^3+3\alpha_2^2+4\alpha_1x^2-3x^2\alpha_2=1$. Does this equation define $x$ as an implicit function of $\alpha_1,\alpha_2$
a) in a neighborhood of $(\bar{\alpha_1},\bar{\alpha_2})=(1,1)$
b) in a neighborhood of $(\bar{\alpha_1},\bar{\alpha_2})=(1,0)$
c) in a neighborhood of $(\bar{\alpha_1},\bar{\alpha_2})=(0.5,0)$
If so, compute $\frac{\partial x}{\partial\alpha_1}$ and $\frac{\partial\delta x}{\partial\alpha_2}$ at this point.

(5) The economy of Iceland can be expressed by the following three variables:
x : hard-core liquor to survive dark cold winters
y : imported oil for heating in the cold winter
z : beaver pelts for people crazy enough to leave the house
The equilibrium is given by the two equations:

$$2xz+xy+z-2\sqrt{z}=11 \tag{1}$$

$$xyz=6 \tag{2}$$

Assume the economy finds itself at the initial equlibrium where $x=3,y=2,z=1$. The government fixes the number of allowances to hunt beaver exogenously.
a) If the prime minister raises $z$ to 1.1, calculate the change in $x$ and $y$.
b) The anti-drug alliance argues that too much alcohol is consumed in the country. They therefore argue to fix the amount of alcohol consumed exogenously by law at 2.95 and instead abolish the beaver hunting constraint. Can they use the implicit function theorem to estimate the changes in y and z?

(6) Assume you are given a competitive industry of identical firms with open entry. (i.e., two equations characterize the system: (1) producers are profit maximizers and the derivative of the price w.r.t a firm's own quantity is zero $\frac{\delta p(Q)}{\delta q_i} = 0$ and (2) new entry into the market occurs until profits are zero).

The government is thinking about imposing a lump-sum tax on firms. What would be the effects on a firm's individual output $q$, it's profit $\pi$, the total market output $Q$, the price $p(Q)$ and the number of firms in the industry $n$?

*This problem involves one of the many internal contradictions that economists make all the time. First you assume that* $\frac{\delta p(Q)}{\delta q_i} = 0$*, i.e., that individual firms face perfectly elastic demand curves. Then you asume that the* industry *faces a downward sloping demand curve, i.e., that* $\frac{dp(Q)}{dQ} < 0$*. Mathematically this is of course absurd, since* $Q = nq_i$*, but economists have been doing this shamelessly for centuries. There's a way of "fixing" this contradiction, but we're not going to get into it. So just do it and shudder quietly to yourselves.*