ARE211, Fall2015

NPP3: TUE, SEP 22, 2015 PRINTED: AUGUST 25, 2015 (LEC# 9)

Contents

3. Nonlinear Programming Problems and the Kuhn Tucker conditions (cont)

Key points:

(1) Interpretation of the Lagrangian multiplier: λ_j is the rate at which the maximized value of the objective increases as the j'th constraint is relaxed.
   (a) multipliers increases with the length of the gradient of the objective at the solution
   (b) the j'th multiplier *decreases* with the length of the gradient of the j'th constraint at the solution

(2) Lots of practice at computation

3.4. **A worked solution to an NPP: S&B #18.18 (on the problem set)**

The example: S&B qu 18.18

$$\min 2x^2 + 2y^2 - 2xy - 9y \quad \text{s.t.}$$
$$\begin{aligned} 4x + 3y &\leq 10 \\ y - 4x^2 &\geq -2 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

Flip the signs so it's a max problem, respecify nonnegativity constraints as inequalities

$$\max f(x,y) \quad = \quad 2xy + 9y - 2x^2 - 2y^2 \quad \text{s.t.}$$
$$\begin{aligned} 4x + 3y &\leq 10 \\ 4x^2 - y &\leq 2 \\ -x &\leq 0 \\ -y &\leq 0 \end{aligned}$$

Now set up Lagrangian:

$$L(x, y, \boldsymbol{\lambda}) = (2xy + 9y - 2x^2 - 2y^2) + \lambda_1(10 - 4x - 3y) + \lambda_2(2 - 4x^2 + y) + \lambda_3 x + \lambda_4 y$$

Recall from (**??**), that the first order conditions were

$$\partial L(\bar{\mathbf{x}}, \bar{\boldsymbol{\lambda}})/\partial x_i = 0; \;\; \partial L(\bar{\mathbf{x}}, \bar{\boldsymbol{\lambda}})/\partial \lambda_j \geq 0; \;\; \bar{\lambda}_j \partial L(\bar{\mathbf{x}}, \bar{\boldsymbol{\lambda}})/\partial \lambda_j = 0. \tag{1}$$

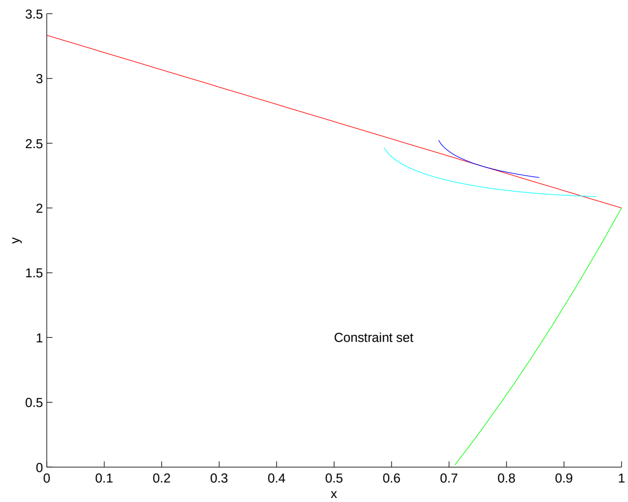

FIGURE 1. The feasible set for problem 18.18

In this particular problem, these conditions imply

$$
\begin{array}{rcl}
L_x & = & 2y - 4x \quad - 4\lambda_1 - 8x\lambda_2 + \lambda_3 \quad = \quad 0 \\
L_y & = & 2x - 4y + 9 - 3\lambda_1 + \lambda_2 + \lambda_4 \quad = \quad 0 \\
L_{\lambda_1} & = & 10 - 4x - 3y \quad \geq \quad 0 \\
L_{\lambda_2} & = & 2 - 4x^2 + y \quad \geq \quad 0 \\
L_{\lambda_3} & = & x \quad \geq \quad 0 \\
L_{\lambda_4} & = & y \quad \geq \quad 0
\end{array}
$$

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3) Assume at least one nonnegativity constraint satisfied with equality:

   (a) *First assume we're at the origin.* $x = y = 0$;
      - $L_{\lambda_1} > 0$; $L_{\lambda_2} > 0$
      - $\lambda_1 = \lambda_2 = 0$
      - $L_x = 0 - 0 - 0 - 0 + \lambda_3 = 0 \Longrightarrow \lambda_3 = 0$;
      - $L_y = 0 - 0 + 9 - 0 + 0 + \lambda_4 > 0.$ $\otimes$

   (b) *Now assume just x is positive* $x > y = 0$;
      - $\lambda_3 = 0$;
      - $L_{\lambda_2} = 2 - 4x^2 \geq 0 \Longrightarrow x \leq \sqrt{0.5}$
      - $L_{\lambda_3} > 10 - 4\sqrt{0.5} > 0 \Longrightarrow \lambda_1 = 0$
      - $L_y = 2x + 9 + \lambda_2 + \lambda_4 > 0.$ $\otimes$

   (c) *Now assume just y is positive* $y > x = 0$;
      - $\lambda_4 = 0$;
      - $L_{\lambda_2} = 2 + y > 0$
      - $\lambda_2 = 0$;
      - $L_x = 2y + \lambda_3 - 4\lambda_1 = 0 \Longrightarrow \lambda_1 > 0 \Longrightarrow y = 10/3$;
      - $L_y = -40/3 + 9 - 3\lambda_1 < 0$; $\otimes$

4) Conclude that $x > 0$; $y > 0$; $\lambda_3 = \lambda_4 = 0$.

5) Assume both x and y are positive:

   (a) $L_{\lambda_1} = L_{\lambda_2} = 0$
      - $\lambda_3 = \lambda_4 = 0$ (because both x and y are positive).
      - $L_{\lambda_1} = 10 - 4x - 3y = 0 \Longrightarrow y = (10 - 4x)/3$
      - $L_{\lambda_2} = 2 - 4x^2 + (10 - 4x)/3 = 0 \Longrightarrow 3x^2 + x - 4 = 0.$
      - i.e., $(3x + 4) * (x - 1) = 0 \Longrightarrow x = 1 \Longrightarrow y = 2.$
      - $L_y = 4 - 8 + 9 > 0$; $\otimes$

   (b) $L_{\lambda_1} > 0; L_{\lambda_2} = 0$ (only the quadratic constraint satisfied with equality)
      - $\lambda_1 = \lambda_3 = \lambda_4 = 0$
      - $L_{\lambda_2} = 2 - 4x^2 + y = 0 \Longrightarrow y = 4x^2 - 2$;
      - $L_x = 0 \Longrightarrow y \geq 2x$ (otherwise $L_x < 0$).

- $y = 4x^2 - 2$ and $y \geq 2x \Longrightarrow x \geq 1 \Longrightarrow y \geq 2$.
- $x \geq 1, y \geq 2 \Longrightarrow L_{\lambda_2} = 10 - 4x - 3y \leq 0$; $\otimes$

(c) $L_{\lambda_1} > 0; L_{\lambda_2} > 0$ (solution in the interior of constraint set)

- $\boldsymbol{\lambda} = 0$; (i.e. the whole vector zero)
- $L_x = 0 \Longrightarrow y = 2x$;
- $L_y = 0 \Longrightarrow 6x = 9 \Longrightarrow x = 1.5$;
- $y = 3$;
- $L_{\lambda_1} = 10 - 4 * 1.5 - 3 * 3 = -5$; $\otimes$

6) Computing the solution: $L_{\lambda_1} = 0; L_{\lambda_2} > 0$ (only the linear constraint satisfied with equality)

- $\lambda_2 = \lambda_3 = \lambda_4 = 0$
- $L_{\lambda_1} = 10 - 4x - 3y = 0 \Longrightarrow y = (10 - 4x)/3$
- $L_x = 2(10 - 4x)/3 - 4x - 4\lambda_1 = 0$ or $20 - 20x - 12\lambda_1 = 0$
- $L_y = 2x - 4(10 - 4x)/3 + 9 - 3\lambda_1 = 0$ or $22x - 13 - 9\lambda_1 = 0$

Two equations in two unknowns, i.e., $\begin{bmatrix} 20 & 12 \\ 22 & -9 \end{bmatrix} \begin{bmatrix} x \\ \lambda_1 \end{bmatrix} = \begin{bmatrix} 20 \\ 13 \end{bmatrix}$ so $x = 0.7568$; $\lambda_1 = 0.4054$.

From the constraint that's satisfied with equality, i.e., $10 - 4x - 3y = 0$, we have $y = (10 - 4 * 0.7568)/3 = 2.3242$. Now let's check that our answer satisfies the mantra:

$$\begin{aligned} \bigtriangledown f &= [2y - 4x \quad 2x - 4y + 9] \\ &= [1.6216 \quad 1.2162] \end{aligned}$$

For the mantra to be satisfied, $\bigtriangledown f$ must be collinear with $\bigtriangledown g = [4 \quad 3]$, or, equivalently, $f_1/f_2 = g_1/g_2$. And $1.6216/12162$ indeed equals $4/3$, so the mantra is satisfied.

3.5. **Computing a solution to an NPP: a simple worked example**

How do you actually solve an NPP? Answer is: a process of elimination. You check all the possibilities to see if you can find a point that satisfies the KT conditions, and then you eliminate anything that fails this test. Here you are using the fact that the KT conditions are *necessary* for a solution, i.e., if they fail this test, they *can't* be a maximum. Once you've found something that does satisfy the KT conditions, then you have to go back and check that the second order conditions are satisfied.

The example:

$$\begin{aligned} \max f(\mathbf{x}) &= (x_1+2)(x_2-2) \text{ s.t.} \\ p_1x_1+p_2x_2 &\leq y; \\ x_i &\geq 0; \end{aligned}$$

In this case, g is the matrix above, i.e.,

$$\begin{bmatrix} p_1 & p_2 \\ -1 & 0 \\ 0 & -1 \end{bmatrix};$$

Check the nonvanishing gradient condition: $\triangledown f(\mathbf{x}) = 0$ iff $x_1 = -2$, $x_2 = 2$. Clearly this point is outside the constraint set, so we know the gradient is nonvanishing on the constraint set.

Set up Lagrangian:

$$L(\mathbf{x}, \lambda) = (x_1+2)(x_2-2) + \lambda_0(y - p_1x_1 - p_2x_2) + \lambda_1x_1 + \lambda_2x_2$$

Recall from (**??**), that the first order conditions were

$$\partial L(\bar{\mathbf{x}}, \bar{\boldsymbol{\lambda}})/\partial x_i = 0; \;\; \partial L(\bar{\mathbf{x}}, \bar{\boldsymbol{\lambda}})/\partial \lambda_j \geq 0; \;\; \bar{\lambda}_j \partial L(\bar{\mathbf{x}}, \bar{\boldsymbol{\lambda}})/\partial \lambda_j = 0.$$

In this particular problem, these conditions imply

$$\begin{aligned}
L_{x_1} &= x_2 - 2 - \lambda_0 p_1 + \lambda_1 = 0 \\
L_{x_2} &= x_1 + 2 - \lambda_0 p_2 + \lambda_2 = 0 \\
L_{\lambda_0} &= y - p_1 x_1 - p_2 x_2 \geq 0 \\
L_{\lambda_1} &= x_1 \geq 0 \\
L_{\lambda_2} &= x_2 \geq 0.
\end{aligned}$$

Observe that the last three FOC give you back precisely the constraint conditions.

We will set $p_1 = p_2 = y$ and solve explicitly for a solution. Under this condition, the solution will be at a corner. *NOTE WELL: this solution depends on the particular specification of parameters. In general, you could get a solution on the face of the budget line.*

Go through the interior, faces and vertices of the constraint set in turn. (Emphasize that while I can tell by inspection the solution to this problem, so I don't have to go thru all this hassle, in general I don't know the answer in advance, so don't have a clue about which corner, face, etc. to start with.)

(1) Try none of the constraints binding; KT says $\bar{\lambda}_0 = \bar{\lambda}_1 = \bar{\lambda}_2 = 0$. which implies $x = (-2, 2)$. Contradiction. Assumed that $\bar{x}$ was nonnegative; found that if there were a $\bar{x}$ that satisfied the KT conditions under these assumptions, then $\bar{x}_1$ would be negative. Note that we couldn't have had a point satisfying this condition anyway, because of the non-vanishing gradient property, we checked above

(2) Try $\bar{x}_1 > 0$, $\bar{x}_2 > 0$ and $p \cdot \bar{\mathbf{x}} = y$. KT conditions say that $\bar{\lambda}_1 = \bar{\lambda}_2 = 0$; $\bar{\lambda}_0 \geq 0$. Plugging these is gives

$$\begin{aligned}
x_2 - 2 &= \lambda_0 p_1 \\
x_1 + 2 &= \lambda_0 p_2
\end{aligned}$$

But when $p_1 = p_2$, this means that $x_2 - x_1 = 4$. On the other hand, since $p_1 = p_2 = y$, the budget constraint implies that $x_1 + x_2 = 1$. Substituting yields $2x_2 = 5$, which implies x_1 MUST be negative, contradicting our initial condition that x_1 must be nonnegative.

(3) Try $\bar{x}_1 > 0$, $\bar{x}_2 = 0$ and $p \cdot \bar{\mathbf{x}} = y$. KT conditions say that $\bar{\lambda}_0, \bar{\lambda}_2 \geq 0$; $\bar{\lambda}_1 = 0$. Plugging these is gives

$$\begin{aligned} L_{x_1} &= x_2 - 2 - \lambda_0 p_1 + \lambda_1 \\ &= -2 - \lambda_0 p_1 = 0; \end{aligned}$$

which implies $\lambda_0 = -2/p_1$, which is a contradiction.

(4) Try $\bar{x}_1 = 0$, $\bar{x}_2 > 0$ and $p \cdot \bar{\mathbf{x}} = y$. KT conditions say that $\bar{\lambda}_0, \bar{\lambda}_1 \geq 0$; $\bar{\lambda}_2 = 0$. Plugging these in gives

$$L_{\lambda_0} = y - p_2 x_2 = 0$$

which implies $x_2 = y/p_2 > 0$. Also,

$$\begin{aligned} L_{x_2} &= x_1 + 2 - \lambda_0 p_2 \\ &= 2 - \lambda_0 p_2 = 0 \end{aligned}$$

which implies $\lambda_0 = 2/p_2$. Now consider L_{x_1}, i.e.,

$$\begin{aligned} L_{x_1} &= x_2 - 2 - \lambda_0 p_1 + \lambda_1 \\ &= y/p_2 - 2 - 2 + \lambda_1 \text{ (since } \lambda_0 = 2/p_1) \\ &= 1 - 4 + \lambda_1 \end{aligned}$$

which implies that $\lambda_1 = 3$. So we have a solution, i.e., $(0, y/p_2)$ with $\lambda_0 = 2/p_2$, $\lambda_1 = 3$.

3.6. Computed solution to a NPP: ARE problem set example.

This example was a homework problem for Econ 201A, 1999:

<u>The example:</u>

$$\begin{aligned} \max u_i(\mathbf{x}_i) &= x_{1i}(4 - x_{2i}) \text{ s.t.} \\ p_1 x_{1i} + p_2 x_{2i} &= p_1\omega_{1i} + p_2\omega_{2i}; \\ x_i &\geq 0; \end{aligned}$$

where $\omega_1 = (4,3)$, $\omega_2 = (1,0)$. The problem that 201 students faced was to solve for the $\mathbf{x}_i$'s for $i = 1, 2$, and for the equilibrium prices. What I'll do in these notes is to solve for the demand functions for good #1, *and* to derive some equilibrium properties of the price vector.

For this problem, g is the matrix above, i.e.,

$$\begin{bmatrix} p_1 & p_2 \\ -1 & 0 \\ 0 & -1 \end{bmatrix};$$

while b is

$$\begin{bmatrix} p_1\omega_{1i} + p_2\omega_{2i} \\ 0 \\ 0 \end{bmatrix};$$

We'll normalize by setting $p_1 = 1$ and let $\xi^i : \mathbb{R}^2 \to \mathbb{R}^2$ denote the demand function for agent i, i.e., the demand function is now a function only of relative prices. We know from the answer sheet that

$$\begin{aligned} \xi^1(p_2) &= \begin{cases} \left(\frac{4-p_2}{2}, \frac{4+7p_2}{2p_2}\right) & \text{if } p_2 \leq -4/7 \\ (4+3p_2, 0) & \text{otherwise} \end{cases} \\ \xi^2(p_2) &= \begin{cases} \left(\frac{1-4p_2}{2}, \frac{1+4p_2}{2p_2}\right) & \text{if } p_2 \leq -1/4 \\ (1, 0) & \text{otherwise} \end{cases} \end{aligned}$$

We'll now derive the demand functions for agent #1, and check that our answers agree with ξ^1. You should check as an exercise that you can repeat the same steps for agent #2, and arrive at ξ^2.

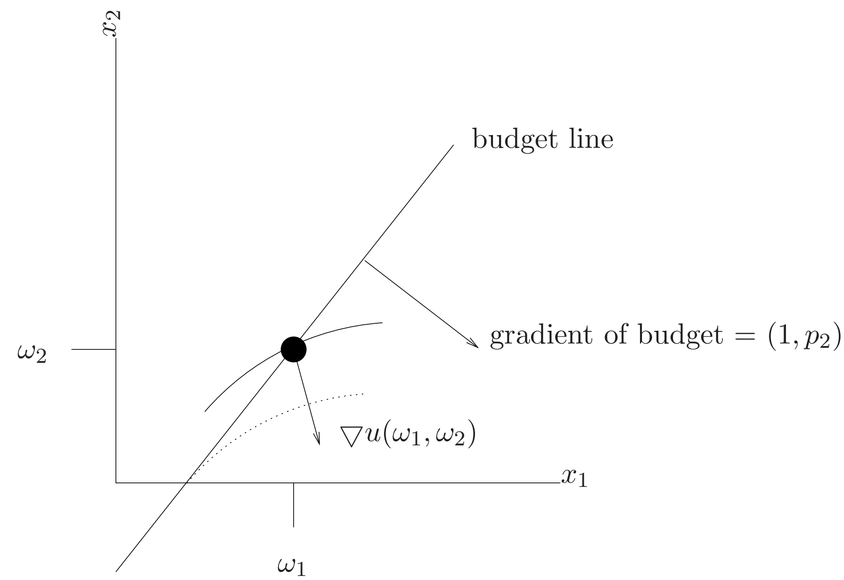

FIGURE 2. The problem facing agent #1

Before embarking on this problem, we'll get some intuition for the solution. Note first from the definition of the utility function that

(1) provided that a positive quantity of good 1 is consumed, good 2 is a "bad";
(2) as x_{2i} increases above 4, then good 1 becomes a bad also, and the gradient of utility is a negative vector.
(3) it may not be *immediately* obvious, but when $x_{2i} > 4$, $u_i(\cdot)$ is a strictly quasi-*convex* function.
(4) now assume that both prices are both positive:
    (a) if $x_{2i} < 4$, you cannot get an interior (i.e., strictly positive) solution to the KT conditions because the gradient of the constraint is a strictly positive vector while the gradient of the utility function has one positive and one negative component.
    (b) if $x_{2i} > 4$, you *can* get an interior solution to the KT conditions because the graident of the utility function is strictly negative. In this solution, the *non-typical* constraint will be binding, i.e., instead of wanting to move NE in the positive quadrant you want to move SW. *However*, in this case the utility function is quasi-convex not quasi-concave.

When you solve for an interior solution to the KT conditions, you'll have found a *minimum* on the constraint set, not a maximum.

(5) conclude from this that you cannot obtain an interior maximum to this problem if both prices are positive.

Rather than exploring all the possibilities exhaustively, we'll henceforth assume that $p_2 < 0$, i.e., good 2 is a bad. Now we'll draw the picture. The gradient of the budget constraint points down and to the right, i.e., SE., and the budget line is a positively sloped line through endowment point. Fig. 2 indicates the budget line with a relatively small negative price p_2; The optimum for this player is obviously a corner solution. Clearly, in order to get an interior solution to #1's optimization problem, you have to flatten the budget line, i.e., *lower* p_2.

From now on, I'm going to dump all the i subscripts since we're only dealing with $i = 1$.

Set up the Lagrangian, setting $p_1 = 1$ and $y(p_2) = \omega_1 + p_2\omega_2$, i.e., $y_1(p_2) = 4 + 3p_2$ and $y_2(p_2) = 1$.

$$L(\mathbf{x}, \lambda) = x_1(4 - x_2) + \lambda_1 x_1 + \lambda_2 x_2 + \lambda_3(y(p_2) - x_1 - p_2 x_2) + \lambda_4(x_1 + p_2 x_2 - y(p_2))$$

Recall from (**??**), that the first order conditions were

$$\partial L(\bar{\mathbf{x}}, \bar{\boldsymbol{\lambda}})/\partial x_i = 0; \quad \partial L(\bar{\mathbf{x}}, \bar{\boldsymbol{\lambda}})/\partial \lambda_j \geq 0; \quad \bar{\lambda}_j \partial L(\bar{\mathbf{x}}, \bar{\boldsymbol{\lambda}})/\partial \lambda_j = 0. \qquad (\textbf{??})$$

In this particular problem, these conditions imply

$$\begin{aligned}
L_{x_1} &= (4 - x_2) + \lambda_1 - (\lambda_3 - \lambda_4) = 0 \\
L_{x_2} &= -x_1 + \lambda_2 - (\lambda_3 - \lambda_4)p_2 = 0 \\
L_{\lambda_1} &= x_1 \geq 0 \\
L_{\lambda_2} &= x_2 \geq 0 \\
L_{\lambda_3} &= y(p_2) - (x_1 + p_2 x_2) \geq 0 \\
L_{\lambda_4} &= x_1 + p_2 x_2 - y(p_2) \geq 0
\end{aligned}$$

Note that $L_{\lambda_3} > 0$ implies $L_{\lambda_4} < 0$ while $L_{\lambda_4} > 0$ implies $L_{\lambda_3} < 0$. Conclude that $L_{\lambda_3} = L_{\lambda_4} = 0$ leaving open the possibility that *either* λ_3 *or* λ_4 could be positive. Accordingly, it will be convenient to define $\lambda_0 = (\lambda_3 - \lambda_4)$, which can be either positive negative or zero. Thus for agent 1:

$$\begin{aligned}
L_{x_1} &= (4 - x_2) + \lambda_1 - \lambda_0 = 0 \\
L_{x_2} &= -x_1 + \lambda_2 - \lambda_0 p_2 = 0 \\
L_{\lambda_0} &= x_1 + p_2 x_2 - (4 + 3p_2) = 0 \\
L_{\lambda_1} &= x_1 \geq 0 \\
L_{\lambda_2} &= x_2 \geq 0
\end{aligned}$$

By inspection of the figure we can see that there are really two possibilities:

(A) $p_2 < 0$; the budget line alone is binding (if p_2 is large in abs value, i.e., budget line relatively flat). In this case,

$$\lambda_1 = \lambda_2 = 0, \lambda_0 = \lambda_3 > 0$$

(B) $p_2 < 0$; the budget line and the nonneg constraint on good 2 are both binding (if p_2 is small in abs value, i.e., budget line relatively steep). In this case,

$$\lambda_1 = 0, \lambda_2 > 0, \lambda_0 = \lambda_3 > 0$$

On the other hand, Fig. 2 suggests that there are several possibilities that we can *exclude* based on the Lagrangian conditions. We'll focus on one of them, just for practice, but there are *many* more that we won't check.

(C) $p_2 > 0$ and $x_i > 0$, $i = 1, 2$, i.e., the budget line is the only constraint satisfied with equality.

We'll begin with (C), write down the Lagrangian system and show that all of the requirements *cannot simultaneously* be satisfied. From the mantra, we know the reason: the gradient of the budget line and the gradient of the objective have to be co-linear, but they can't be, because the objective's gradient points NE, while the budget's gradient points SE. Our task now is to show this using the Lagrangian. The conditions are:

$$
\begin{aligned}
L_{x_1} &= (4 - x_2) - \lambda_0 = 0 \\
L_{x_2} &= -x_1 - \lambda_0 p_2 = 0 \\
L_{\lambda_0} &= x_1 + p_2 x_2 - (4 + 3p_2) = 0 \\
\lambda_1 &= 0; \qquad \lambda_2 = 0;
\end{aligned}
$$

From L_{x_1} we have that

From L_{x_2} and $p_2 > 0$, we have that

$$\lambda_0 = 4 - x_2 \lambda_0 = \qquad -x_1/p_2 < 0$$

so that, substituting into L_{x_1}

$$(4 - x_2) + \frac{x_1}{p_2} = 0, \tag{2}$$

or, since $x_1 > 0$,

$$(x_2 - 4) = \frac{x_1}{p_2} > 0$$

But from L_{λ_0}, we have that

$$x_1 + p_2x_2 \quad = \quad 4 + 3p_2 \quad > \quad 4$$

or

$$4 - x_2 \quad > \quad \frac{x_1}{p_2} \quad > \quad 0 \tag{3}$$

But, obviously, (2) and (3) cannot simultaneously be satisfied, so we've established that the set of conditions listed in (C) cannot hold. Note, moreover, that we obtained the contradiction by showing that if $p_2 > 0$, then the combination of L_{x_1} and L_{x_2} would then be inconsistent with L_{λ_0}.

Now let's consider the possibilities which from the figure, we know *are* possible. We will take each of possibilities (A) and (B) in turn, and see their implications for the Lagrangian system;

(A) the budget line alone is binding:

$$\begin{aligned} L_{x_1} &= (4 - x_2) - \lambda_0 = 0 \\ L_{x_2} &= -x_1 - \lambda_0 p_2 = 0 \\ L_{\lambda_0} &= x_1 + p_2x_2 - (4 + 3p_2) = 0 \end{aligned}$$

Solving this in the usual way:

$$\begin{aligned}
(a) \qquad 0 &= (4 - x_2) + \frac{x_1}{p_2} \qquad \text{(from } L_{x_1}\text{)} \\
(b) \qquad 0 &= p_2(4 - x_2) + x_1 \qquad \text{(rearranging (a))} \\
&= x_1 - p_2 x_2 + 4p_2 \\
(c) \qquad (4 + 3p_2) &= x_1 + p_2 x_2 \qquad \text{(from } L_{x_2}\text{)} \\
(d) \qquad (4 + 3p_2) &= 2p_2 x_2 - 4p_2 \qquad \text{(subtracting (b) from (c))} \\
(e) \qquad x_2 &= \frac{4 + 7p_2}{2p_2} \qquad \text{(rearranging (d))} \\
(f) \qquad x_1 &= \frac{4 - p_2}{2} \qquad \text{(subst (3) into (c))}
\end{aligned}$$

Note that $x_2 \geq 0$ iff $|p_2| \leq 4/7$. Summarizing, (e) and (f) give us agent #1's demand function for $p_2 \in (-\infty, -4/7]$, i.e.,

$\xi^1(p_2) = (\frac{4+7p_2}{2p_2}, \frac{4-p_2}{2})$

(B) both budget line and nonneg constraint on 2 are binding:

$$\begin{aligned}
L_{x_1} &= 4 - \lambda_0 = 0 \\
L_{x_2} &= -x_1 + \lambda_2 - \lambda_0 p_2 = 0 \\
L_{\lambda_0} &= x_1 - (4 + 3p_2) = 0
\end{aligned}$$

From $L_{x_1} = 0$, $\lambda_0 = 4$. From $L_{\lambda_{01}} = 0$, $x_1 = (4 + 3p_2)$. Plugging both values into $L_{x_2} = 0$,

$$\begin{aligned}
L_{x_2} &= -(4 + 3p_2) + \lambda_2 - 4p_2 \\
&= -4 - 7p_2 + \lambda_2 = 0;
\end{aligned}$$

Now L_{x_2} can be zero with $\lambda_2 \geq 0$ iff $-7p_2 - 4 \leq 0$, i.e., if $|p_2| \leq 4/7$. Therefore, we have now computed agent #1's demand function for $p_2 \in (-4/7, 0]$, i.e.,

$\xi^1(p_2) = (4 + 3p_2, 0)$