Journal of Development Economics 106 (2014) 227–238

Contents lists available at ScienceDirect

Journal of Development Economics

journal homepage: www.elsevier.com/locate/devec

The demand for insurance against common shocks[☆]

A. de Janvry [a], V. Dequiedt [b,*], E. Sadoulet [a]

[a] University of California at Berkeley, USA
[b] CERDI, Université d'Auvergne, France

ARTICLE INFO

Article history:
Received 19 March 2012
Received in revised form 23 September 2013
Accepted 2 October 2013

JEL classification:
D14
D81
G22

Keywords:
Producer groups
Weather insurance
Group insurance

ABSTRACT

In recent years, index-based insurance has been offered to smallholder farmers in the developing world to protect against common shocks such as weather shocks. Despite their attractive properties, these products have met with low demand. We consider the frequent situation where farmers are members of groups with common interests. We show that this creates strategic interactions among group members in deciding to insure that reduce the demand for insurance for two reasons. One is free riding due to positive externalities on other group members when a member chooses to insure. The other is potential coordination failure because it may not be profitable for a risk-averse member to insure if the other members do not. As a consequence, we argue that the demand for insurance against common shocks could increase if the insurance policy were sold to groups rather than to individuals.

1. Introduction

Uninsured weather shocks remain a major challenge in increasing productivity and reducing poverty among smallholder farmers in developing countries. In anticipation of shocks, they need to manage risk by diversifying their incomes toward low productivity activities (Dercon and Krishnan, 1996) and they may fail to adopt higher-yielding but higher-risk varieties (Eswaran and Kotwal, 1990). After the occurrence of a shock, they need to cope with risk by reducing consumption expenditures, often with irreversible health consequences, eventually taking children out of school (Jacoby and Skoufias, 1997), and selling productive assets that will subsequently be difficult to accumulate again (Rosenzweig and Wolpin, 1993).

The revenue of agricultural households in a community is subject to different kinds of shocks. A typology of shocks is possible according to the statistical properties of their distribution. The two polar types are idiosyncratic and common shocks. Idiosyncratic shocks are independently distributed in the community. As a first approximation, health shocks are an example of idiosyncratic shocks. Common shocks affect everyone in the community. Price fluctuations and weather shocks are, again as a first approximation, examples of common shocks. Insurance against idiosyncratic shocks is possible via mutualization. In a celebrated empirical study, Townsend (1994) found that some village communities in India manage to achieve an outcome close to full insurance against idiosyncratic shocks via risk-sharing. Insurance against common shocks is more difficult to achieve. If one abstracts from credit or storage capacities that allow for intertemporal smoothing, insurance usually requires the intervention of third-parties like insurance companies and/or financial markets. For instance, insurance against price fluctuations is possible on derivative financial markets (Moschini and Lapan, 1995). Insurance against weather shocks requires the availability of suitable weather derivatives or the existence of re-insurance companies willing to diversify their portfolio. Naturally, the idiosyncratic or common nature of the shock depends on the geographical configuration of the community considered.

In this paper, we analyze the demand for insurance against common shocks such as weather shocks, when farmers belong to communities with shared interests. We highlight two distinct characteristics of insurance against common shocks in such communities or groups. The first is that insurance decisions taken by one individual may exert a positive externality on other group members. Therefore the demand for insurance may be plagued by a free-rider problem and it is plausible that the sum of the individual willingness to pay for insurance is lower than the group willingness to pay. The second characteristic is that the value of insurance against common shocks can be positive or negative for an individual, depending on the insurance decisions of the other group members. The game played by community members when they

☆ This paper was partly written while the second author was visiting the University of California at Berkeley. Financial support from FERDI and from the Programme d'Investissements d'Avenir of the French government (ANR-10-LABX-14-01) is gratefully acknowledged.

* Corresponding author. Tel.: +33 4 73 17 75 13; fax: +33 4 73 17 74 28.
E-mail address: Vianney.Dequiedt@udamail.fr (V. Dequiedt).

0304-3878/$ – see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.jdeveco.2013.10.001

choose whether or not to take insurance is, in some circumstances, a coordination game. Group members may fail to coordinate on the Pareto dominant outcome in which they all choose to take insurance.

As a consequence of these two characteristics, the demand for insurance against common shocks can increase if policies are offered at the group rather than at the individual level. This is what we analyze in this paper.

For most of the analysis, we rely on the following specification for individual preferences: the utility of individual j in a group of N members depends on his own wealth and on the aggregate wealth of the group. If the group of individuals that we consider, for example, produces a local public good, then under some conditions that are standard in the literature, the equilibrium utilities of individuals depend on these two variables. Therefore we believe that our specification is particularly suitable to study the demand for weather insurance in agricultural marketing cooperatives or other producer organizations.

A free-riding problem may occur because the decision by one individual to take insurance reduces the risk associated with aggregate wealth in the sense of second-order stochastic dominance. This risk reduction will be valued by other group members. As a consequence, the sum of the individual inverse demands for insurance, i.e., the sum of the individual risk premiums, may be lower than what the group as a whole would be ready to pay, i.e., the group risk premium.

When the two variables, own wealth and aggregate wealth, that enter individual utility functions are complements, a risk averse individual may prefer to stay uninsured if other group members do not take insurance. This occurs because individuals prefer to be rich when the group as a whole is rich and poor when the group as a whole is poor rather than poor when the group is rich and rich when the group is poor. In this case, coordination of group members is necessary for uptake and coordination may be achieved simply by offering the insurance policy to the group rather than to the individuals.

It is worth emphasizing that the coordination problem does not appear if shocks are idiosyncratic. Such a problem is distinctive of insurance against common shocks. Similarly, while free-riding is not impossible when insurance covers idiosyncratic shocks, it is exacerbated when shocks are common.

In recent years, there has been a growing interest in designing weather index insurance products for farm households in developing countries (see Barnett and Mahul, 2007; World Bank, 2009).[1] Since 2003, there have been experiments in Malawi (World Bank CRMG, 2009), Morocco (Stoppa and Hess, 2003), Peru and Vietnam (Skees et al., 2007), India (Manuamorn, 2007), and several other developing countries. Yet, individual uptake for these insurance products has been disappointingly low (see for instance Giné and Yang, 2009; Carter et al., 2010). India has achieved the most success in bringing provision of index insurance for small farmers to scale, with a number of private weather insurance schemes that together reached 150,000 farmers in 2009, while the public AIC program reached more than 1 millions farmers that year (Hazell et al., 2010).[2] In terms of contractual arrangements, almost all products are individual, even though they may be delivered through a variety of different channels, including small community-based schemes, NGOs, or micro-finance institutions. There are a few cases where the policy holder is the group itself: an oxen insurance that successfully worked for 26 years in Burkina Faso (Roth and Cord, 2008) and Fondos in Mexico (Ibarra and Mahul, 2004), for example. The main purposes of these group insurance schemes are to lower management costs and reduce basis risk, as the group can provide some loss adjustment among its members.

Several explanations have been proposed for the low individual demand for weather insurance. For instance, it has been argued that insurance products are complex and that farm households with little financial education face difficulties in understanding their logic (see Cole et al., 2009). It has also been argued that agricultural households already use a wide array of risk management strategies such as credit and savings and that weather insurance does not add much to them (see Gollier, 1994). Others consider that index-based insurance policies are poor products that leave too much basis risk uninsured (see Clarke, 2011a). Finally, some authors have argued that interlinked transactions must be taken into account in order to understand the demand for weather insurance (see Attanasio and Rios-Rull, 2000). In particular, it is plausible that, in village communities, formal weather insurance interacts with informal risk sharing (see Mobarak and Rosenzweig, 2012).

Our model scrutinizes this interlinked transactions argument and illustrates the potential interest of group contracts. Group contracts were also advocated by Clarke (2011b) on the basis of a costly state verification model. In his model, group contracts are useful because they allow crowding in mutual insurance and reduce the overall verification cost. In contrast, our model is closer to the functioning of index-based policies and does not incorporate a verification of the losses.

Our work can also be related to Arnott and Stiglitz's (1991) model of insurance which stresses the crowding out of formal insurance by informal risk-sharing. In that model of insurance in a moral hazard setting, informal risk-sharing decreases equilibrium welfare unless peer-monitoring can be used to discipline group members. Our model abstracts from moral hazard because we are interested in modeling index-based policies for which misbehavior is not an issue. Neither do we consider explicitly informal risk-sharing. However, our specification of indirect utilities that takes into account interactions among group members can encompass cases of informal risk-sharing and is used to explain low demand for, or crowding out of, formal insurance. By contrast to Arnott and Stiglitz's analysis, interactions among group members are productive and increase welfare even if they are not peer-monitored.

The paper is organized as follows. In Section 2 we present the model and relate our specification to the functioning of farmer cooperatives in the developing world. In Section 3 we provide two illustrative examples. In the first, we compare individual and group risk premiums and show that the sum of individual premiums can be lower than the group premium. In the second, we show that insurance against common shocks can have a negative value. In Section 4 we relate these illustrative examples to general properties of correlated stochastic variables and offer a more systematic analysis. Section 5 presents concluding remarks. When proofs do not immediately follow the Lemmas and Propositions, they can be found in an Appendix.

2. Social preferences in producer cooperatives

2.1. Social preferences

The community we consider is a group of N individuals. Each individual $j \in \{1,\ldots,N\}$ in the group is endowed with wealth w_j. We denote by $W = \sum_{j=1}^{N} w_j$ the aggregate wealth in the group, and $W_{-j} = \sum_{l \neq j} w_l$ the sum of the wealth of individuals other than j. To take into account the fact that group members interact, we assume

[1] Most products use a meteorological index such as temperature or cumulative rainfall at a given geographical location, strongly correlated with farm losses, and condition payments to the insured on the realized value of the index. Some other products use indices based on estimates of area yields (e.g. AIC in India), local level livestock mortality rate (Mongolia), or, more recently, satellite-based measures of available vegetative cover (livestock insurance in Kenya). Because the index measures a purely exogenous event, offering these policies is not subject to the moral hazard issues that typically plague insurance markets. Moreover, because the risk is observable by the insurer, there is no reason to suspect that adverse selection may be a concern. Finally, the recent availability of cheaper and more reliable automated meteorological stations has decreased the fixed costs associated with such products. Index-based products thus seem to be the right approach to insure the rural poor (Skees et al., 2006).

[2] As mentioned before, price shocks are another example of common shocks. In an interesting experimental study in Ghana, Karlan et al. (2011) found that offering price insured loans does not increase significantly the demand for loans.

that each individual j has preferences given by a von Neumann-Morgenstern utility function that may depend on the whole vector of wealths:

$$u_j = u_j(w_1, \ldots, w_N). \quad (1)$$

Such preferences may be the result of having group members involved in several interlinked transactions with each others. Therefore, everyone in the group cares not only about his own wealth but also about the wealth of the others in the group. A particular form of Eq. (1) to which we devote much attention in the following is

$$u_j = u_j\left(w_j, W\right). \quad (2)$$

With Eq. (2), each individual cares not only about his own wealth but also about the aggregate wealth of the group. This simple kind of social preference is, as we explain below, very plausible when the group considered is a cooperative or any other group of producers.

2.2. Producer cooperatives

In this subsection, we detail examples of the functioning of producer cooperatives that highlight the public good aspect of some of their activities. In these examples, preferences of cooperative members can be represented by indirect utility functions of the form given in Eqs. (1) and (2). They are inspired by our own work on coffee cooperatives in Guatemala, but are expected to apply more generally.

2.2.1. Cost-sharing with economies of scale

Some cooperatives are organized to exploit economies of scale and share the burden of collective production costs. For example, coffee cooperatives typically own processing equipment and machines to wash, sort, and dry coffee beans before they are sold. Cooperative members share capital purchase and maintenance, processing costs, administrative costs, and marketing costs. Some of these costs are recurrent and need to be covered by regular contributions of cooperative members. Usually, however, cooperatives do not charge an annual fee, but meet recurrent costs through a rebate (a tax) on the unit price they pay to producers for the coffee they brought to be processed at the cooperative. It is therefore reasonable to assume that the unit price $p(\cdot)$ a cooperative pays to producers is an increasing function of $Q = \sum_l q_l$, the total production of cooperative members.

Suppose that individual j obtains revenue $\widetilde{w}_j$ in a first period that comes from the sale of his production to the cooperative. At this period, he spends a fixed amount $\overline{c}_j$ to meet his current consumption needs and invests the remaining amount $\widetilde{w}_j - \overline{c}_j$ for next year's production.[3] This investment captures decisions concerning the seedlings to be bought, the fertilizer to be applied, etc. For simplicity, we assume that next year's production $q_j(\cdot)$ will be proportional to investment and that the proportion is the same for all cooperative members. Therefore, $q_j(\widetilde{w}_j - \overline{c}_j) = b(\widetilde{w}_j - \overline{c}_j)$, and the value of this production is given by $p(\sum_l q_l(\widetilde{w}_l - \overline{c}_l)) q_j(\widetilde{w}_j - \overline{c}_j)$. When individual j evaluates future revenue according to an increasing function $v(\cdot)$, and by denoting $w_j = \widetilde{w}_j - \overline{c}_j$ for all j, $W = \sum_j w_j$, we can represent its preferences with an indirect utility function of the form given in Eq. (2):

$$u_j\left(w_j, W\right) = v\left(p(bW) b w_j\right).$$

[3] Here we consider for simplicity that consumption is constrained and does not result explicitly from the maximization of some utility function. In the more complex case where individuals are free to adjust current consumption, the model would be similar to models of voluntary contributions to an impure public good such as studied by Bardhan et al. (2007) because individual utility would be a function of consumption c_j, own contribution $\widetilde{w}_j - c_j$, and total contribution $\sum_l \widetilde{w}_l - c_l$. In such models, equilibrium utilities are a function of the whole vector of wealths $\widetilde{w}_l$ and would take the form given in Eq. (1).

2.2.2. Collective asset

Another important function of many cooperatives is to manage a collectively owned asset such as a financial contract or a sales contract. In these situations, the purpose of the cooperative can be to give access to the market, bypassing intermediaries. This is possible because the cooperative can contract on volumes that an individual producer is unable to guarantee. The purpose can also be to give access to formal credit. Again this is possible because the cooperative can provide some collateral that an individual producer could not. The management of this collective asset necessitates time and money to be fully profitable, and, most importantly, cooperative members must contribute to fulfilling the contract for its durability. In these situations, the collective asset can be seen as a public good for cooperative members. It benefits everyone in the cooperative and generates free-riding problems. Most cooperatives, for example, count on their members to deliver all of their production, and trade coffee with large foreign importers on contracts signed long before they have the coffee under control. This raises the well-known problem of enforcing deliveries from members when prices vary over the course of the year inducing the temptation to side-sell. In that case, implicit monetary contributions include what members lose by giving up other deals. On the credit side, the cooperative can only pay back its loan and maintain its reputation and access to credit if all the members contribute by paying back their loans.

To elaborate on this public good aspect and to derive closed-form expressions for indirect utility, we assume that the collective asset can be considered as pure public goods in the sense that it benefits every cooperative member to the same extent, whatever the individual contribution of that member. Therefore, each individual cooperative member j can use his wealth w_j to buy a quantity c_j of the private goods and contribute G_j to the functioning of the cooperative. We normalize the price of both goods to be 1 so that the individual's budget constraint is $w_j = c_j + G_j$. We also denote by $G_j = \sum_j G_j$ the total amount contributed to the cooperative functioning. The utility of individual j depends on his consumption of the private goods and on the cooperative functioning. We assume that it is given by the function

$$U_j\left(c_j, G\right) = \left(a_j c_j^{\gamma_j} + b_j G^{\gamma_j}\right)^{\frac{1}{\lambda_j}}, \quad (3)$$

where $0 \le \gamma_j \le \lambda_j$, and $0 < a_j$, $0 < b_j$.

Whether contribution to the collective asset of the cooperative is voluntary or mandatory depends on the degree of institutionalization of collective decision processes in the cooperative. Mandatory contributions are more likely in cooperatives that rely on more formal transactions.

Consider first that contributions to the collective asset are mandatory. These contributions result from a collective choice rule adopted by the group, as studied for instance in Epple and Romano (2003). For simplicity let us assume that public goods provision is financed through a proportional tax enforced at the group level. In that case, if we denote by T the tax rate, it is straight forward to establish that

$$U_j\left(c_j, G\right) = U_j\left((1-T)w_j, T\sum_l w_l\right) = u_j\left(w_j, W\right).$$

With Eq. (3), we obtain up to an increasing and linear transformation

$$u_j\left(w_j, W\right) = \left(\alpha_j w_j^{\gamma_j} + \beta_j W^{\gamma_j}\right)^{\frac{1}{\lambda_j}}, \quad \alpha_j, \beta_j > 0. \quad (4)$$

When $\gamma_j = \lambda_j = 0$, we get the Cobb–Douglas function

$$u_j\left(w_j, W\right) = w_j^{\alpha_j} W^{\beta_j}. \quad (5)$$

Moreover it can be shown (see Appendix A) that in the latter case, the preferred tax rate of individual j does not depend on the level of his wealth w_j. This property ensures that any collective choice procedure based on individual preferences will select a tax rate that does not depend on the distribution of wealth in the group.

Consider now that individuals contribute voluntarily to the cooperative. We assume that the public goods provision game is a strategic form game in which individuals decide simultaneously and non-cooperatively how much to contribute. When preferences are given by Eq. (3), this game has a unique Nash equilibrium (see Bergstrom et al. (1986) or Cornes and Hartley (2007)). Let us assume that in equilibrium every individual contributes a positive amount to the public goods, which occurs when individuals are not too asymmetric. When preferences are given by Eq. (3), the equilibrium utilities are given up to an increasing and linear transformation by

$$u_j(W) = W^{\frac{\gamma_j}{\lambda_j}}. \tag{6}$$

Similarly, when $\gamma_j = \lambda_j = 0$,

$$u_j\left(w_j, W\right) = W^{\alpha_j+\beta_j}. \tag{7}$$

See Appendix B. In the equations above, the indirect utility depends only on aggregate wealth in the community and not on an individuals' own wealth. This is a direct consequence of the well-known fact that private provision of a public good is independent of the distribution of income (Warr, 1983).[4] In a sense, here, the public goods contribution mechanism achieves an indirect mutualization of wealth. A similar indirect utility would also arise in the case of a productive association that pulls all members' proceeds together before sharing income.

3. Free-riding and coordination: illustrative examples

Now, we elaborate on the indirect utilities described in Eq. (2) to study the demand for insurance by individuals in the group.

In order to study insurance, we introduce risk in the environment. We assume that the initial wealth profile $w = (w_1,\ldots,w_N)$ is a stochastic variable that takes values in the interval $[\underline{w}, \overline{w}]^N$. The wealth of individuals is subject to shocks that can be idiosyncratic and/or common.

Risk-sharing inside the group, i.e. mutual insurance, can be used to provide insurance against idiosyncratic shocks. Mutual insurance consists in redistributing wealth among individuals without changing the aggregate wealth of the group. It will be valued by all individuals in the group provided that their indirect utilities exhibit risk aversion with respect to own wealth and individuals are not too different concerning their risk exposure, i.e. the shocks they face are not too asymmetric. This last point is easy to understand: if individual 1 is risk averse and individual 1's wealth is not subject to shocks while individual 2's wealth is, then individual 1 will not agree to share risk with individual 2. On the other hand, if the two levels of wealth are given by i.i.d. stochastic variables w_1 and w_2 and the two individuals are risk averse, then it is clear that they both benefit from replacing their stochastic wealth w_1 by $(w_1 + w_2)/2$, i.e. they both benefit from risk-sharing.

Notice that when indirect utility is given by Eqs. (6) and (7), mutual insurance is unnecessary. In particular, when indirect utility depends only on aggregate wealth because it comes from a public goods voluntary contribution game, individuals are already insured against idiosyncratic shocks by the public good contribution game.

Below we focus on risks that cannot be dealt with using mutual insurance, i.e. we focus on shocks that affect the aggregate wealth in the group such as weather shocks. For this reason, we simplify the problem by assuming that individual wealths w_j are subject to the same common shock, i.e. for all j, $w_j = w$ where w is a positive and non-degenerate real-valued random variable distributed according to g. The notation E_g and $\hat{w}$ are used for the expectation operator and expected value of w, respectively. For simplicity, we also consider a group of N identical individuals whose indirect utility functions are given as before by $u(w,W)$.

In such a setting, we provide formal examples to illustrate the existence of free-riding and coordination problems in insurance uptake decisions.

A more general treatment is postponed to Section 4.

3.1. Free-riding

When preferences of individuals depend on their own wealth and on the aggregate wealth in the group, it is likely that insurance decisions create externalities. Indeed, when one individual takes insurance, this decision modifies the distribution of his own wealth and, by a composition effect, modifies also the distribution of the aggregate wealth. Therefore the insurance of one individual may change the welfare of all individuals in the group. In what follows we provide an example where these externalities generate free-riding in the sense that the group is ready to pay more for insurance than the sum of what each individual is ready to pay.

To model the insurance decisions, we assume that an insurance company proposes to fully insure the risk, i.e. to replace one individual wealth w by its expected value $\hat{w}$, for a positive price.[5]

Assumption 1. The indirect utility function $u(w, W)$ is increasing in w, strictly increasing in W, and concave.[6]

Suppose that insurance is offered to the group and that its price is shared equally among group members. Let us denote by c the per member price of insurance. Individuals in the group will unanimously decide to buy insurance whenever

$$E_g u(w, Nw) \leq u(\hat{w}-c, N(\hat{w}-c)).$$

We now define c^g the group risk premium, i.e. the highest price group members are ready to pay when insurance is offered at the group level. This risk premium is given by

$$E_g u(w, Nw) = u\left(\hat{w}-c^g, N\left(\hat{w}-c^g\right)\right). \tag{8}$$

It exists and is positive and unique under Assumption 1.

Suppose now that insurance is offered to the individuals. Each individual j is offered insurance at a price c_j. When he anticipates that all other individuals will buy insurance at their offered price, an individual j will buy insurance whenever

$$E_g u\left(w, w + (N-1)\hat{w} - \sum_{l\neq j} c_l\right) \leq u\left(\hat{w}-c_j, N\hat{w} - \sum_{l\neq j} c_l - c_j\right).$$

Lemma 1. Under Assumption 1, and if the insurance company wants to sell insurance to all group members, the maximal price it can charge for insurance is the same for all individuals in the group. It is given by c^i which solves

$$E_g u\left(w, w + (N-1)\left(\hat{w}-c^i\right)\right) = u\left(\hat{w}-c^i, N\left(\hat{w}-c^i\right)\right). \tag{9}$$

[4] To be precise, this result requires that the set of contributors does not change when one changes the distribution of wealth.

[5] Full insurance is an extreme assumption that we adopt here in order to facilitate interpretation of the results. The free-riding effect that we want to highlight in this Section would still hold with partial insurance but would be more difficult to isolate.

[6] Since the function u has two arguments, concavity means that $u(\lambda w_1 = (1-\lambda) w_2, \lambda W_1 + (1-\lambda)W_2) \geq \lambda u(w_1, W_1) + (1-\lambda)u(w_2, W_2)$ for any (w_1, W_1) and (w_2, W_2) and any $\lambda \in [0,1]$.

Proof. Appendix C.

This lemma shows that when individuals are symmetric, the equilibrium of the insurance market is necessarily symmetric: the risk premium is necessarily identical for all individuals. It is equal to the premium agent j is ready to pay when the other agents take full insurance and pay a premium.

We now provide conditions under which $c^g > c^i$. We focus on the case where the individual indirect utility functions are identical for all individuals in the group, depend on aggregate wealth only, and exhibit constant relative risk aversion:

$$u(w,W) = u(W) = \begin{cases} \dfrac{W^{1-\sigma}}{1-\sigma} & \text{with} \quad \sigma \neq 1, \\ \log(W) & \text{with} \quad \sigma = 1. \end{cases} \tag{10}$$

Suppose first, that insurance is offered at the group level and that the total cost is shared equally among agents. Applying Eq. (8), each agent can be charged an amount up to c^g with

$$E_g u(Nw) = u(N(\hat{w} - c^g)). \tag{11}$$

Suppose now that insurance is offered at the individual level. Applying Eq. (9), agent j can be charged an amount up to c^i with

$$E_g u\left(w + W_{-j}\right) = u\left(\hat{w} - c^i + W_{-j}\right),$$

where $W_{-j} = (N-1)\left(\hat{w} - c^i\right) > 0$ is the constant aggregate level of wealth obtained by the other agents taking the full insurance.

Proposition 1. Suppose that individual utility functions are given by Eq. (10), then $c^i < c^g$.

Proof. Appendix C.

Here, when an agent decides to buy insurance coverage, he creates a positive externality on the welfare of others. This occurs because, as we demonstrate formally in Section 4, a reduction in the risk of any w_j reduces the variability of the sum W, holding all else equal. When insurance is offered at the individual level, nobody internalizes these externalities and the equilibrium results in underprovision, i.e. in our setting, $c^i < c^g$. The discrepancy between the individual and the group risk premia is non-negligible as the next numerical example shows.

Example. To evaluate the difference between c^i and c^g, let us consider the following numerical application. Assume that the utility of individuals is only a function of aggregate wealth (or coffee production) $u(W) = \log W$. Individual production w is equal to 2 in good years, and falls to 1 whenever there is catastrophic excess rainfall. The probability of this extreme rainfall event is 1/7. The group risk premium c^g solves

$$\frac{6}{7}\log 2 = \log\left(\frac{13}{7} - c^g\right),$$

which gives

$$c^g \approx 0.0457.$$

The individual risk premium c^i solves

$$\frac{1}{7}\log\left(1 + (N-1)\left(\frac{13}{7} - c^i\right)\right) + \frac{6}{7}\log\left(2 + (N-1)\left(\frac{13}{7} - c^i\right)\right) = \log\left(N\left(\frac{13}{7} - c^i\right)\right).$$

For $N = 2$, this gives

$$c^i \approx 0.0133.$$

In a group of two individuals, the sum of individual premia is less than one third of the group premium.

For $N = 15$, this gives

$$c^i \approx 0.0022,$$

which implies that, in a group of 15 individuals, the sum of the individual premia is 20 times less than the group premium.

To solve the free-riding problem, it is possible to offer the insurance at the group level. When group members have to choose between insurance for everybody in the group and insurance for nobody in the group they will be ready to pay the higher premium c^g. In order to extract this higher premium, it is important that individuals in the group are not offered the opportunity of buying individual insurance instead.[7]

3.2. Coordination

In the preceding subsection we analyzed the individual incentives to take insurance under the hypothesis that the other group members purchase insurance. Whatever individual j decided, i.e. to take or not to take insurance, it was assumed that the other group members took insurance. As will become clear in this section, fixing the other group member decisions was necessary because individual incentives depend on the other's insurance decisions. In fact, the externalities we highlighted in the preceding subsection imply that purchasing or not insurance is a strategic decision. To gain a better understanding of this issue, it is useful to recast the insurance decision problem in terms of a strategic form game in which each group member must decide simultaneously to take or not to take insurance. In this game, the payoffs are the ex ante expected utilities (computed before the realization of the shock on the wealth distribution). We show below that this game can be a coordination game with multiple equilibria. In particular, we provide an example where insurance has a negative value: even if insurance is costless, nobody in the group wants to buy insurance if he (correctly) anticipates that the others in the group will not buy insurance. It illustrates the fact that the demand for insurance against common shocks can give rise to multiple equilibria with either all agents or none being insured.

In this subsection it is assumed that insurance is perfect and sold at an actuarially fair price. This extreme hypothesis is considered in order to give the best chances to the insurance policy. With imperfect or costly insurance, the negative value result would only be more likely to hold.

We assume that indirect utilities are given by Eq. (5), i.e.:

$$u(w,W) = w^{\alpha}W^{\beta}, \qquad \alpha > 0, \beta > 0, \alpha + \beta < 1. \tag{12}$$

Suppose that agents can obtain full insurance for free, i.e. they can choose to exchange their stochastic wealth w for a certain wealth equal to the expected value $\hat{w}$. As mentioned above, we are interested in the strategic form game in which individuals simultaneously choose

[7] In a sense, offering the insurance at the group level helps the group deal with free-riding on insurance purchases. It would be unnecessary if the group were able to achieve costless discipline among its members.

to take the insurance or not to take the insurance. The payoffs in this strategic form game are as follows. If k other individuals choose to take the insurance, individual j gets

$$E_g u(\hat{w}, (k+1)\hat{w} + (N-k-1)w) = \hat{w}^\alpha E_g((k+1)\hat{w} + (N-k-1)w)^\beta,$$

if he takes insurance and

$$E_g u(w, k\hat{w} + (N-k)w) = E_g w^\alpha (k\hat{w} + (N-k)w)^\beta,$$

if he does not.

If all agents except agent j take the insurance, it is in the interest of that agent to take the insurance as well because, in this case, his utility is given by $w^\alpha(w+(N-1)\hat{w})^\beta$ which is strictly concave with respect to w and therefore

$$E_g w^\alpha (w + (N-1)\hat{w})^\beta < \hat{w}^\alpha (N\hat{w})^\beta.$$

The insurance game in which the agents simultaneously choose to take or not the actuarially fair insurance always possesses an equilibrium in which all agents take the insurance. But it may not be the only equilibrium of that game. To see this, suppose that no other individual takes the insurance. If individual j does not take the insurance his payoff is

$$E_g w^\alpha (Nw)^\beta,$$

while, if he takes the insurance, it is

$$\hat{w}^\alpha E_g(\hat{w} + (N-1)w)^\beta.$$

Proposition 2. Suppose indirect utilities are given by Eq. (12) and w is distributed on $\{0, \overline{w}\}$ with probabilities $\{p, 1-p\}$.

- For any value of N, there are admissible values for α, β and p such that there is an equilibrium of the insurance game in which nobody takes the insurance.
- For any admissible values of α, β and p, there are values of N such that there is an equilibrium of the insurance game in which nobody takes the insurance.

Proof. Appendix C.

In that case, the insurance game has two Nash equilibria: one with full insurance, i.e. insurance taken by all agents, the other with no insurance, i.e. insurance taken by no agent. Of course, the full insurance equilibrium Pareto dominates the no-insurance equilibrium, nevertheless there is a priori no guarantee that agents will manage to coordinate on the full insurance equilibrium.[8] In order to illustrate Proposition 2, we provide a numerical example below.

Example. When $u(w, W) = w^{\frac{1}{3}} W^{\frac{1}{3}}$, $p = \frac{1}{7}$, $N = 7$ and $\overline{w} = 1$,

$$E_g u(w, Nw) \approx 1.639,$$

$$E_g u(\hat{w}, \hat{w} + (N-1)w) = 1.621,$$

and

$$E_g u(w, Nw) > E_g u(\hat{w}, \hat{w} + (N-1)w).$$

If nobody else takes insurance, individual j prefers not to take insurance.

Group insurance can solve the coordination problem because it would let the group choose between the two equilibrium outcomes: full insurance or no insurance. In this case, there would be unanimous agreement on the full insurance outcome since strict concavity of u guarantees that $E_g u(w, Nw) < u(\hat{w}, N\hat{w})$.

4. Free-riding and coordination: more general results

In this section we go beyond the illustrative examples of the preceding section and scrutinize the conditions under which free-riding and coordination problems appear in insurance purchase settings. To do so, we consider settings in which shocks on individual wealth can be idiosyncratic or common, i.e. the individual wealths can be independently distributed or not. We also consider settings in which the indirect utility functions may depend on own wealth only, on own wealth and aggregate wealth or on the whole vector of individual wealth in an arbitrary way. The section is organized as follows. In Section 4.1, we introduce the notion of mean-preserving spread for multidimensional settings which characterizes risk-reduction. In Section 4.2, we establish several lemmas that relate individual insurance decisions to multidimensional risk-reduction. In Section 4.3, we exploit these lemmas to scrutinize when free-riding and coordination problems occur.

4.1. Mean-preserving spread and multidimensional risk

To understand the value of insurance for risk averse individuals, it is convenient to use the notion of a mean-preserving spread. This notion captures the fact that a lottery, i.e. a stochastic variable, is more risky than another with the same mean, or equivalently that any risk averse decision maker prefers the former to the latter. When two stochastic variables X and Y take values in $\mathbb{R}$, the following well-known and simple characterization is due to Rothschild and Stiglitz (1970).

Definition 1. When the stochastic variables X and Y take values in $\mathbb{R}$, Y is a mean-preserving spread of X if and only if (i) and (ii) hold

(i) The two stochastic variables have the same mean, $E(Y) = E(X)$;
(ii) For all $x \in \mathbb{R}$,

$$\int_{-\infty}^{x} F_Y(v)\partial v \geq \int_{-\infty}^{x} F_X(v)\partial v,$$

where F_Y (resp. F_X) denotes the cumulative distribution of Y (resp. X).

When individuals in a group have indirect social preferences as in our model, their welfare depends not only on the distribution of their own wealth but on the distribution of the wealth profile in the group. To study the behavior of individual j it is not sufficient to scrutinize the properties of the one-dimensional stochastic variable w_j that describes his own wealth. It is necessary to scrutinize the properties of the multidimensional stochastic variable $w = (w_1, w_2, \ldots, w_N)$ that takes value in $\mathbb{R}^N$ and describes the wealth profile in the group. We therefore present the definition of stochastic dominance for multidimensional variables, and analyze in Lemmas 2 to 5 some specific cases of wealth distributions where stochastic dominance prevails. In what follows, X and Y are two multidimensional stochastic variables that take values in $\mathbb{R}^N$.

Definition 2. The multidimensional stochastic variable Y is a mean-preserving spread of the multidimensional stochastic variable X when the equivalent statements (i) or (ii) hold

(i) For all continuous and concave functions f, $E(f(Y)) \leq E(f(X))$;
(ii) There exists a stochastic variable Z such that Y has the same distribution as Z and (X,Z) is a martingale, i.e. $E(Z|X) = X$.

Equivalence between the two statements is established by the Blackwell–Sherman–Stein theorem (see, for instance, chapter 7 in

[8] The game theory literature has repeatedly stressed the fact that in coordination games there is a priori no reason to focus exclusively on the Pareto dominant equilibrium. See for instance Harsanyi and Selten (1988) and Carlsson and van Damme (1993).

Shaked and Shantikumar (2007)). It follows that (*ii*) is a suitable definition of increasing risk because all risk averse decision makers, those whose preferences are represented by a concave function, prefer X to Y, according to (*i*).

4.2. Insurance and mean-preserving spread

In what follows, the wealth of individual j is given by a stochastic variable w_j. We denote by g_j the marginal distribution of w_j, E_{gj} the expectation operator with respect to that distribution, and $\hat{w}_j = E_{g_j} w_j$ the mean value or expectation of w_j. In this setting, what we call insurance is the possibility for an individual to replace his own stochastic wealth w_j by its mean value $\hat{w}_j$. The stochastic variables w_j may be independently distributed. In that case, the density of $(w_{1,\ldots,}w_N)$ is the product of the marginal distributions $g = \prod_{j=1}^{N} g_j$. The stochastic variables w_j may also be correlated and $g \neq \prod_{j=1}^{N} g_j$. In what follows, we consider both possibilities, unless specified otherwise.

Lemma 2. Suppose that individual wealths w_j are independently distributed. The stochastic variable $(w_1,\ldots w_{j-1}, w_j, w_{j+1}\ldots w_N)$ is a mean-preserving spread of the stochastic variable $(w_1, \ldots, w_{j-1}, \hat{w}_j, w_{j+1}, \ldots, w_N)$.

Proof. It is straightforward to verify statement (*ii*) in Definition 2 with $Y = Z = (w_1, \ldots, w_N)$ and $X = (w_1, \ldots, \hat{w}_j, \ldots, w_N)$.

This lemma shows that when shocks on wealth are idiosyncratic, i.e. the random variables w_j, $j = \{1,\ldots,N\}$ are independently distributed, replacing the stochastic wealth of any individual j by its expected value $\hat{w}_j$ induces a reduction in risk. Therefore, any risk averse individual will find insurance against idiosyncratic shocks profitable even if his preferences depend on the distribution of the whole wealth profile of the group.

The next lemma shows that even if shocks on wealth are not purely idiosyncratic, individual insurance is also desirable for any risk averse individual when all other group members are already insured.

Lemma 3. Suppose that individual wealths are given by the stochastic variables w_j with mean $\hat{w}_j$. The stochastic variable $(\hat{w}_1, \ldots, \hat{w}_{j-1}, w_j, \hat{w}_{j+1}, \ldots, \hat{w}_N)$ is a mean-preserving spread of the (degenerate) stochastic variable $(w_1, \ldots, \hat{w}_j, \ldots, \hat{w}_N)$.

Proof. It is straightforward to verify statement (*ii*) in Definition 2 with $Y = Z = (\hat{w}, \ldots, \hat{w}, w, \hat{w}, \ldots, \hat{w}))$ and $X = (\hat{w}, \ldots, \hat{w})$.

If one recasts the insurance decision problem as a strategic form game played by all group members in which they must simultaneously decide whether or not to take insurance, as we did in Section 3.2, Lemma 3 establishes that it is always a Nash equilibrium for everyone to take insurance, and Lemma 2 establishes that it is the only equilibrium if wealths are independently distributed. In the latter case, it is actually a dominant strategy equilibrium.

In contrast, when shocks on wealth are common then the random variables w_j, $j = \{1, \ldots, N\}$ are perfectly correlated. In this case the result established in Lemma 2 no longer holds, and we have the following property:

Lemma 4. Suppose that individual wealths are given by the same (non degenerate) stochastic variable w with mean $\hat{w}$. The stochastic variable $(w, \ldots, w)$ is not a mean-preserving spread of the stochastic variable $(w, \ldots, \hat{w}, \ldots, w)$.

Proof. Consider two stochastic variables X and Z such that $X = (w, \ldots, \hat{w}, \ldots, w)$ and $E(Z|X) = X$. If Z puts positive weights only on diagonal elements of the form $(z, z, \ldots, z)$, then $E(Z|X)$ can only be a diagonal element. This is in contradiction with the equality $E(Z|X) = X$ and the fact that X does not put positive weights only on diagonal elements because w is non degenerate (i.e. $w = \hat{w}$ is not always true). Therefore Z necessarily puts positive weights on elements outside the diagonal and cannot possess the same distribution as $Y = (w, \ldots, w)$.

The fact that one individual insures himself against his own wealth variations does not imply a reduction in risk concerning the distribution of the whole wealth profile. As a consequence, when the utility of an individual depends not only on his own wealth but on the whole wealth profile, and even if this utility function is concave because the individual is risk-averse, insurance may not be valuable. Actually, as shown in Proposition 2 it is sufficient that the utility depends on own wealth and aggregate wealth for this to happen, i.e. for insurance to decrease one's utility in some circumstances.

It can be inferred from Lemma 3 and Lemma 4 that the problem is one of coordination because the value of being insured may depend on the decisions of others. If all other group members are insured, insurance is necessarily valuable. If other group members are not insured, the value of insurance can be positive or negative.

In our initial model specified in Eq. (2), individual preferences depend on the vector of wealths in a specific way: indirect utilities are functions of own wealth and aggregate wealth. To go a little further, we can also scrutinize the behavior of aggregate wealth when one introduces insurance.

Lemma 5. Suppose that individual wealths are given by the same stochastic variable w with mean $\hat{w}$. For $0 \le k < N$, define the aggregate wealth when k group members take insurance as $W_k = k\hat{w} + (N-k)w$. The real valued stochastic variable W_k is a mean-preserving spread of the real valued stochastic variable W_{k+1}.

Suppose that the individual wealths are given by i.i.d. stochastic variables w_j with mean $\hat{w}$. Define W_k as above. The real valued stochastic variable W_k is a mean-preserving spread of W_{k+1}.

Proof. First, it is clear that in both cases $E(W_k) = E(W_{k+1})$.

Consider the first case, i.e. of a purely common shock. The aggregate wealth W_k is less than a given threshold x whenever

$$(N-k)w + k\hat{w} \le x,$$

or equivalently

$$w \le \hat{w} + \frac{x - N\hat{w}}{N-k}.$$

The right-hand side of the above inequality is increasing (resp. decreasing) in k when x is above $N\hat{w}$ (resp. below $N\hat{w}$). Therefore, the cumulative density F_{W_k} of W_k is increasing with k (resp. decreasing with k) above $N\hat{w}$ (resp. below $N\hat{w}$). As a consequence,

$$\int_{-\infty}^{x} F_{W_k}(y) - F_{W_{k+1}}(y)dy \ge 0, \forall x \in \mathbb{R},$$

and, according to Definition 1, W_k is a mean-preserving spread of W_{k+1}.

Consider now the second case, i.e. idiosyncratic shocks. In that case, Lemma 2 ensures that the vector of individual wealths when k individuals get insurance is a mean-preserving spread of the vector of wealths when $k+1$ individuals get insurance. Consider then any concave function f defined over $\mathbb{R}$ and the linear function Σ which is the sum of N real valued variables. The function Σ maps $\mathbb{R}^N$ on $\mathbb{R}$. It is straightforward to see that the function $f \circ \Sigma$ is a concave function defined over $hbbR^N$. Therefore, by applying Definition 2, we know that

$$E(f(W_k)) \le E(f(W_{k+1})),$$

which according to Definition 2 again ensures that W_k is a mean-preserving spread of W_{k+1}.

This lemma shows that when shocks are common and one individual takes insurance, it decreases the risk associated with the distribution of aggregate wealth. Such a property holds also when shocks are idiosyncratic, i.e. when individual wealths w_j are independently distributed. However, such a property does not hold without restrictions on the distribution of shocks. It is easy to see that if $N = 2$ and shocks are negatively correlated such that $w_1 = -w_2$, aggregate wealths as defined in Lemma 5 are such that $W_0 = W_2 = 0$, while W_1 is a non-degenerate stochastic variable and is not a mean-preserving contraction of W_0.

As a consequence of Lemma 5, if indirect utilities depend only on the aggregate wealth of group members and shocks are either common, i.e. individual wealths are given by the same stochastic variable, or idiosyncratic, i.e. individual wealths are given by i.i.d. stochastic variables, then we obtain that insurance cannot have a negative value and that insurance decisions create positive externalities, i.e. a risk-averse individual is better off when someone else in the group takes insurance.

4.3. Externalities and the value of insurance

In this subsection, we detail the conditions under which free-ridingand coordination problems may occur with respect to the demand for insurance. We study settings in which the utility functions may depend on the individual's own wealth, the aggregate wealth, or both individual and aggregate wealths. The shocks affecting wealth may be idiosyncratic or common, i.e. stochastic wealths may be independently distributed or not.

By adopting this broad perspective, we highlight the fact that the common nature of the shocks and the fact that individuals have social preferences, i.e. that their indirect utility functions depend not only on their own wealth but also on the other group member's wealth, are crucial for our results. They exacerbate the free-riding problem and open the possibility for a coordination problem.

We start with a simple observation.

Observation. Suppose that the utility of individuals depends only on one's own wealth and is given by concave functions $u_j(w_j)$ then insurance is always valued positively by individuals, whatever the nature of the shocks, i.e. idiosyncratic or common. Moreover, the decision by one individual to be insured does not modify the incentives of any other group member to get insured.

When individuals only care about their own wealth, no matter whether shocks are idiosyncratic or common, the free-riding and coordination problems emphasized in Section 3 cannot occur. In this case, the insurance decision problem is separable and can be studied in isolation for each individual.

Proposition 3. Suppose that the utility of individuals depends on aggregate wealth and is given by concave functions $u_j(W)$.

- If individual wealths are given by independent stochastic variables w_j then insurance by one individual is always positively valued by all individuals.
- If individual wealths are given by the same stochastic variable w, then insurance by one individual is always positively valued by all individuals.
- For fixed and identical marginal distributions of individual wealths, the group's willingness to pay for insurance is higher when shocks on wealth are purely common, i.e. when individual wealths are given by the same stochastic variable, than when shocks on wealths are idiosyncratic, i.e. when individual wealths are given by i.i.d. stochastic variables.

Proof. The first two items are direct consequences of Lemma 5 which establishes the fact that when one individual takes insurance, this induces a mean-preserving contraction of aggregate wealth. This in turn is positively valued by any risk-averse individual whose utility depends on aggregate wealth only.

To prove the third item, let us remind the definition of the group willingness to pay for insurance c^g, which we introduced in Section 3.1 for the case of purely common shocks, as the unique solution to

$$E_g u(Nw) = u\left(N\left(\hat{w} - c^g\right)\right).$$

If shocks are idiosyncratic, i.e. the w_j are i.i.d. with the same marginal distributions as w and mean $\hat{w}$, the group willingness to pay for insurance, $\tilde{c}^g$ is given by

$$E_g u\left(\sum_{j=1}^{N} w_j\right) = u\left(N\left(\hat{w} - \tilde{c}^g\right)\right),$$

or equivalently by

$$E_g u\left(N\left(\frac{1}{N}\sum_{j=1}^{N} w_j\right)\right) = u\left(N\left(\hat{w} - \tilde{c}^g\right)\right).$$

In order to establish that $\tilde{c}^g \leq c^g$, it is sufficient to establish that w is a mean-preserving spread of $\frac{1}{N}\sum_{j=1}^{N} w_j$. This in turn comes from the fact that w and the w_j have the same marginal distributions and for any concave function f,

$$E_g f(w) = \frac{1}{N}\sum_{j=1}^{N} E_g f\left(w_j\right) \leq E_g f\left(\frac{1}{N}\sum_{j=1}^{N} w_j\right).$$

The interpretation of Proposition 3 in terms of free-riding behavior and a coordination problem is the following. When indirect utility depends on aggregate wealth only and shocks on group members' wealths are either idiosyncratic or purely common, it is always profitable for one individual to take an actuarially fair insurance. Therefore, the coordination problem that was emphasized in Section 3.2 cannot appear.

In addition and still when shocks on group members' wealths are either idiosyncratic or purely common, Proposition 3 establishes that when one individual takes insurance, this creates positive externalities on other group members: insurance by one individual increases the welfare of all the others in the group. This property is key for the free-riding behavior highlighted in Section 3.1.

Therefore, free-riding behavior can appear when shocks on wealths are idiosyncratic or purely common. The last item in the proposition establishes that in a sense, the free-riding problem is more acute when shocks on wealths are common than when they are idiosyncratic. Indeed, for fixed marginal distributions of shocks, it is easy to see that the individual willingness to pay for insurance provided everyone else takes insurance is the same whatever the correlation among individual shocks. Therefore, the last item establishes that when individual willingness to pay for insurance is lower than the group's willingness to pay, it is more so if shocks are common. The following example illustrates this point.

Example. Let us come back to the numerical application developed in Section 3.1. Assume that the utility of individuals is only function of aggregate wealth $u(W) = \log W$. Individual wealth w is equal to 2 with probability 6/7 and is equal to 1 with probability 1/7. We consider below the case $N = 2$. As we already know, when shocks on wealth are purely common, the group risk premium is

$$c^g \approx 0.0457.$$

Now if shocks on wealths are idiosyncratic so that individual wealths are given by i.i.d. stochastic variables with the same marginal distributions as w, the group risk premium $\tilde{c}^g$ solves

$$\frac{36}{49}\log 4+\frac{12}{49}\log 3+\frac{1}{49}\log 2=\log\left(2\left(\frac{13}{7}-\tilde{c}^g\right)\right),$$

which gives

$$\tilde{c}^g\approx 0.0194.$$

In the latter case, the group's willingness to pay for insurance is still higher than the individual's willingness to pay, $c^i\approx 0.0133$, but less so than when shocks are purely common.

Proposition 4. Suppose the utility of individuals is given by concave functions $u_j(w_j,W)$. If individual wealths are independent stochastic variables, i.e. shocks are purely idiosyncratic, then insurance by one individual is always positively valued by all individuals.

Proof. Consider any concave function $u_j(w_j,W)$ defining the indirect utility of individual j. Define the function $U_j(w_1,\ldots,w_N)=u_j\,(w_j,W)$. Such a function U_j inherits the concavity properties of u_j. When one individual, say individual l, decides to get insured, he replaces his stochastic wealth w_1 by its mean value $\hat{w}_l$ and, according to Lemma 2, this induces a mean-preserving contraction of the stochastic vector of wealths, whatever the insurance decisions of other group members. Because U_j is concave, such an insurance decision increases the expected utility of individual j.

As a consequence of Proposition 4, when shocks on wealth are idiosyncratic, free-riding problems may occur but coordination problems cannot.

To understand why an actuarially fair insurance can have a negative value as in the case of Proposition 2, we must realize that individual insurance decisions have two effects. The first effect is a reduction of the risk associated with variation in own wealth. This is certainly positively valued by risk averse individuals. The second effect is a modification of the joint distribution of (w_j,W). In particular, if shocks are highly correlated among individuals and individual j is the only one taking insurance, this may result in a lower correlation between the two variables w_j and W. In some circumstances detailed below, individuals prefer to have the two variables that enter their utility function highly correlated. This second effect can therefore be negatively valued and can also dominate the first effect. This is what happened for Proposition 2 to hold.

As is now more clear, complementarities between individual wealth and aggregate wealth of the other group members are key to explain the negative value of insurance against common shocks at the individual level. Because of these complementarities, it is preferable for individual i that his own wealth be subject to the same shocks as the wealth of other group members, rather than being insured against these shocks. Beyond the example provided in the preceding section, we now present sufficient conditions on the indirect utility functions that guarantee that insurance against common shocks can have a negative value.

Assumption 2. For each j, the indirect utility function $u_j\,(w,W)$ is a real valued function defined on $[0;+\infty[^2$, increasing, concave, increasing in the second argument, differentiable, and such that $u_j(0,0)\neq-\infty$ and for all w,

$$\lim_{W\to+\infty}\frac{\partial u_j}{\partial w}(w,W)=+\infty.$$

The last part of Assumption 2 is not equivalent to the hypothesis of constant sign of the cross-partial derivative $\frac{\partial^2 u_j}{\partial w\partial W}$, it neither implies or is implied by single-crossing. But it is linked to it. It is in particular weaker than the hypothesis of a strictly positive and bounded away from zero cross-derivative. It is another way of capturing some elements of complementarity between the two variables. Assumption 2 is satisfied by the preferences given in Eq. (12).

Proposition 5. Suppose that the indirect utility functions of individuals satisfy Assumption 2, then insurance against a common shock can have a negative value for any individual if no one else is insured.

Proof. Appendix C.

For example, when indirect utilities are given by Eq. (4), Assumption 2 is satisfied, and insurance against common shocks can have a negative value at the individual level.

To summarize this subsection, we established that free-riding and coordination problems are the more likely to plague the demand for insurance when group members are interdependent and shocks are common. Coordination problems appear only when shocks are common (Proposition 4). As a consequence, the demand for weather index-based insurance among agricultural smallholders is certainly the most likely to be affected by free-riding and coordination problems.

5. Final remarks

The model we developed in this paper sheds light on two important phenomena that may arise when insurance is offered to individuals who are members of groups with shared interests. First, in that context, insurance decisions may create positive externalities and induce free-riding if insurance policies are offered directly to individuals. While these externalities can exist when insurance covers an idiosyncratic shock, they are exacerbated when it covers a common shock. Second, and contrary to what occurs with independent shocks, insurance against a common shock may have a negative value: if an individual anticipates that the others in the group will not take insurance, he may find insurance for himself unprofitable. Both phenomena contribute to explaining the low uptake of index-based weather insurance among farmers with interlinked interests. They also suggest that the demand for index-based weather insurance could increase if contracts are sold to intermediaries such as producers cooperatives as opposed to individual farmers.

Practitioners in the weather insurance sector are aware of the potential interest of dealing directly with cooperatives. As E. Meherette, Nyala Insurance S.C.'s deputy CEO, explains:

> "Nyala has found that farmers' unions serve as effective delivery channels for the weather insurance products. By working with cooperative unions, Nyala insures all farmers who belong to the cooperative under the same contract. The cooperative is responsible for both paying the premium and distributing potential payouts to each insured farmer, reducing transaction costs for Nyala" (Meherette, 2009).

From the point of view of insurance companies, group policies certainly decrease transaction costs. They also contribute to the scaling up necessary to recover fixed costs. Beyond these advantages, we showed that group policies can also increase the demand for insurance.

In the microfinance sector, group contracts have been extensively used for credit. Group loans with joint liability have been an important contributor to the success of micro-credit institutions. Theoretical arguments have been proposed to explain their superiority over individual loans in terms of overcoming adverse selection and moral hazard problems (see Armendariz de Aghion and Morduch (2005) for a synthesis of the different arguments). The case for group insurance, as developed in this paper, relies on largely distinct arguments. In particular, group insurance must be targeted at individuals that share a common interest. What occurs after the insurance contract is signed is not changed by the fact that it is a group contract. What is changed is mainly what occurs in interlinked transactions that affect the uptake decision.

Appendix A. Mandatory contribution to the public good

Consider the case where preferences are given by Eq. (5). With a tax rate T, the log-linearized utility of agent i is

$$\alpha_i \log((1-T)\omega_i) + \beta_i \log\left(T\sum_{j=1}^{N}\omega_j\right).$$

This function is concave in T and the most preferred tax rate of agent i is

$$\overline{T}_i = \frac{\beta_i}{\alpha_i + \beta_i},$$

which does not depend on the wealth of individual i but only on his preference parameters α_i and β_i.

Appendix B. Voluntary contribution to the public good

We derive the unique equilibrium of the voluntary contribution game by using the technique suggested by Cornes and Hartley (2007). Consider the case where individual preferences are given by Eq. (5). The marginal rate of substitution of individual j is given by

$$MRS_j\left(c_j, G\right) = \frac{\alpha_j}{\beta_j}\frac{G}{c_j}.$$

For an aggregate level of public good provision equal to G, individual j maximizes his payoff if and only if his contribution Gj satisfies

$$G_j = r_j(G) = \max\left\{0, w_j - \frac{\alpha_j}{\beta_j}G\right\},$$

because either his contribution equals zero and the marginal rate of substitution is less than 1 (corner solution) or the marginal rate of substitution is equal to 1 (interior solution). By summing over j we obtain:

$$\sum_{j=1}^{N} r_j(G) = \sum_{j=1}^{N} \max\left\{0, w_j - \frac{\alpha_j}{\beta_j}G\right\}.$$

Let us denote

$$I(G) = \left\{j \in N : w_j - \frac{\alpha_j}{\beta_j}G > 0\right\}.$$

When the players are not too asymmetrical, we will have $I(G^*) = N$ i.e. all the players contribute a strictly positive amount to the public good. In that particular case, equilibrium conditions give:

$$G^* = \frac{\sum_{j=1}^{N} w_j}{1 + \sum_{j=1}^{N}\frac{\alpha_j}{\beta_j}},$$

$$c_j^* = w_j - G_j^* = \frac{\alpha_j}{\beta_j}\frac{\sum_{i=1}^{N} w_i}{1 + \sum_{i=1}^{N}\frac{\alpha_i}{\beta_i}}.$$

So that the equilibrium utility of a particular player j is:

$$u_j\left(w_j, W\right) = W^{\alpha_j + \beta_j}.$$

Suppose now that the utility of individuals is given by Eq. (3). In this case, the marginal rate of substitution of individual j is given by

$$MRS_j\left(c_j, G\right) = \frac{a_j}{b_j}\left(\frac{G}{c_i}\right)^{1-\gamma_j}.$$

In an interior equilibrium of the voluntary contribution game, there is a linear relation between c_j and G. The same argument as developed above establishes that, in an interior equilibrium, i.e. when everyone contributes, the public good quantity G is proportional to the aggregate wealth W and the indirect utility function of individuals is of the form

$$u_j(W) = W^{\frac{\gamma_j}{\kappa_j}}.$$

Appendix C. Proofs

C.1. Proof of Lemma 1

We prove by contradiction that individuals cannot differ in the maximal amount they are ready to pay for insurance, given that all the others pay their maximal amount and get insurance. Let us denote c_j^i the maximal amount individual j is ready to pay and assume that there exist k and l such that $c_k^i > c_l^i$. We know that

$$E_g u\left(w, w + (N-1)\hat{w} - \sum_{j\neq k} c_j^i\right) = u\left(\hat{w} - c_k^i, N\hat{w} - \sum_j c_j^i\right).$$

The fact that u is increasing in w_i ensures that

$$E_g u\left(w, w + (N-1)\hat{w} - \sum_{j\neq k} c_j^i\right) \leq u\left(\hat{w} - c_l^i, N\hat{w} - \sum_j c_j^i\right),$$

or

$$E_g u\left(w, w + (N-1)\hat{w} - \sum_{j\neq k} c_j^i\right) \leq E_g u\left(w, w + (N-1)\hat{w} - \sum_{j\neq l} c_j^i\right).$$

When u is strictly increasing in its second argument, this last equation is in contradiction with $c_k^i > c_l^i$. Therefore, the individual risk premium is necessarily the same for all agents and solves Eq. (9).

C.2. Proof of Proposition 1

From the definition of the individual risk premium c^i, and by denoting $W_{-i} = (N-1)\hat{w} - \sum_{j\neq i} c^i$, we know that

$$E_g u(w + W_{-i}) = u\left(\hat{w} - c^i + W_{-i}\right).$$

In words, c^i is the risk premium an individual with utility given by $u(\cdot + W_{-i})$ would be ready to pay. Now we use the fact that when $u(\cdot)$ exhibits constant relative risk aversion, then it exhibits strictly decreasing absolute risk aversion. So that an individual with preferences given by $u(\cdot + W_{-i})$ is strictly less risk averse than an individual with utility given by $u(\cdot)$. A standard result in risk theory (see Pratt (1964) or Gollier (2001) for a synthetic presentation) tells us that for any non-degenerate lottery, the risk premium of the latter individual is larger than the risk premium of the former. According to Eq. (11), c^g is the risk premium of the latter individual and we deduce that $c^i < c^g$.

C.3. Proof of Proposition 2

We have to compare the payoff of individual i without insurance

$$(1-p)N^{\beta}\overline{w}^{\alpha+\beta},$$

to the payoff with insurance

$$(1-p)^{\alpha}\overline{w}^{\alpha}\left(p((1-p)\overline{w})^{\beta}+(1-p)((1-p)\overline{w}+(N-1)\overline{w})^{\beta}\right).$$

After simple manipulations, we obtain that individual i prefers no insurance whenever

$$g(N,p)=(1-p)^{\alpha+\beta-1}\left[\frac{p}{N^{\beta}}+(1-p)\left(\frac{N-p}{N(1-p)}\right)^{\beta}\right]<1.$$

Notice that

$$\lim_{N\to+\infty} g(N,p)=(1-p)^{\alpha}<1,$$

which proves the second assertion.

To establish the first assertion, we study the properties of $g(N,p)$ when p is close to zero. It is straightforward to establish that

$$g(N,0)=1,$$

and

$$\frac{\partial g}{\partial p}(N,0)=\frac{1}{N^{\beta}}-\alpha-\frac{\beta}{N}.$$

When $N=2$, this partial derivative is negative when $\beta=\frac{1}{2}$ and $\alpha>\frac{2\sqrt{2}-1}{4}$. And because $\frac{\partial g}{\partial p}(N,0)$ is decreasing with N, for $N>2$, it is also possible to find values for α and β such that $\frac{\partial g}{\partial p}(N,0)$ is negative.

We conclude that for these values of α and β, $g(N,p)$ is smaller than 1 for sufficiently small values of p.

C.4. Proof of Proposition 5

We assume that the shock on individual wealths is common and given by the random variable w which takes value in $\{0,\overline{w}\}$, with $\overline{w}>0$. The distribution of w is such that $w=0$ with probability 1/2 and $w=\overline{w}$ with probability 1/2. Consider individual i and suppose the others do not take insurance. His expected payoff if he does not take insurance is

$$\frac{1}{2}u_i(\overline{w},N\overline{w})+\frac{1}{2}u_i(0,0),$$

while if he takes insurance he gets

$$\frac{1}{2}u_i\left(\frac{\overline{w}}{2},\frac{\overline{w}}{2}+(N-1)\overline{w}\right)+\frac{1}{2}u_i\left(\frac{\overline{w}}{2},\frac{\overline{w}}{2}\right).$$

The agent strictly prefers not to take insurance whenever

$$u_i(\overline{w},N\overline{w})-u_i\left(\frac{\overline{w}}{2},\left(N-\frac{1}{2}\right)\overline{w}\right)>u_i\left(\frac{\overline{w}}{2},\frac{\overline{w}}{2}\right)-u_i(0,0),$$

where $u_i(0,0)$ is distinct from $-\infty$. Because the function u_i is increasing in its second argument and differentiable, we know that

$$u_i(\overline{w},N\overline{w})-u_i\left(\frac{\overline{w}}{2},\left(N-\frac{1}{2}\right)\overline{w}\right)\geq u_i(\overline{w},N\overline{w})-u_i\left(\frac{\overline{w}}{2},N\overline{w}\right)=\int_{\frac{\overline{w}}{2}}^{\overline{w}}\frac{\partial u_i}{\partial w_i}(x,N\overline{w})\partial x.$$

Because u_i is concave we obtain

$$u_i(\overline{w},N\overline{w})-u_i\left(\frac{\overline{w}}{2},\left(N-\frac{1}{2}\right)\overline{w}\right)\geq\frac{\overline{w}}{2}\frac{\partial u_i}{\partial w_i}(\overline{w},N\overline{w}).$$

Under Assumption 2, the right-hand side of the last equation goes to $+\infty$ as N goes to $+\infty$. Suppose now that starting from an initial group of size N_0 with possible heterogeneous individuals, we replicate this economy by creating k avatars of each initial individual-type. As k goes to $+\infty$, the size of the replicated group, $N=kN_0$, goes to infinity while keeping the number of individual-type fixed. Therefore it is possible to find k sufficiently high such that for all individual i in the group

$$u_i(\overline{w},N\overline{w})-u_i\left(\frac{\overline{w}}{2},\left(N-\frac{1}{2}\right)\overline{w}\right)>u_i\left(\frac{\overline{w}}{2},\frac{\overline{w}}{2}\right)-u_i(0,0).$$

References

Armendariz de Aghion, B., Morduch, J., 2005. The Economics of Microfinance. MIT Press.
Arnott, R., Stiglitz, J., 1991. Moral hazard and nonmarket institutions: dysfunctional crowding out or peer monitoring? Am. Econ. Rev. 81 (1), 179–190.
Attanasio, O., Rios-Rull, J., 2000. Consumption smoothing in island economies: can public insurance reduce welfare? Eur. Econ. Rev. 44 (7), 1125–1158.
Bardhan, P., Ghatak, M., Karaivanov, A., 2007. Wealth inequality and collective action. J. Public Econ. 91, 1863–1874.
Barnett, B., Mahul, O., 2007. Weather index insurance for agriculture and rural areas in lower income countries. Am. J. Agric. Econ. 89 (5), 1241–1247.
Bergstrom, T., Blume, L., Varian, H., 1986. On the private provision of public goods. J. Public Econ. 29, 25–49.
Carlsson, H., van Damme, E., 1993. Global games and equilibrium selection. Econometrica 61, 989–1018.
Carter, M., Sarris, A., Cheng, L., 2010. The economics of interlinking credit and insurance: the demand side. Working Paper. University of California at Davis.
Clarke, D., 2011a. A theory of rational demand for index insurance. Oxford Dept. of Economics Discussion Paper 572.
Clarke, D., 2011b. Reinsuring the poor: group microinsurance design and costly state verification. Oxford Dept. of Economics Discussion Paper 573.
Cole, S., Giné, X., Tobacman, J., Topalova, P., Townsend, R., Vickery, J., 2009. Barriers to household risk management: evidence from India. Federal Reserve Bank of New York Staff Report.
Cornes, R., Hartley, R., 2007. Aggregative public good games. J. Pub. Econ. Theory 9 (2), 201–219.
Dercon, S., Krishnan, P., 1996. Income portfolios in rural Ethiopia and Tanzania: choices and constraints. J. Dev. Stud. 32 (6), 850–875.
Epple, D., Romano, R., 2003. Collective choice and voluntary provision of public goods. Int. Econ. Rev. 44 (2), 545–572.
Eswaran, M., Kotwal, A., 1990. Implications of credit market constraints for risk behaviour in less developed economies. Oxf. Econ. Pap. 42 (2), 473–482.
Giné, X., Yang, D., 2009. Insurance, credit and technology adoption: field experimental evidence from Malawi. J. Dev. Econ. 89 (1), 1–11.
Gollier, C., 1994. Insurance and precautionary saving in a continuous-time model. J. Risk Insur. 61 (1), 78–95.
Gollier, C., 2001. The Economics of Risk and Time. MIT Press, Cambridge.
Harsanyi, J., Selten, R., 1988. A General Theory of Equilibrium Selection in Games. MIT Press, Cambridge.
Hazell, P., Anderson, J., Balzer, N., Hastrup Clemmensen, A., Hess, U., Rispoli, F., 2010. The Potential for Scale and Sustainability in Weather Index Insurance. IFAD, Rome.
Ibarra, H., Mahul, O., 2004. Self insurance funds as agriculture insurance providers: the case of Fondos in Mexico. World Bank Mimeo.
Jacoby, H., Skoufias, E., 1997. Risk, financial markets and human capital in a developing country. Rev. Econ. Stud. 64 (3), 311–345.
Karlan, D., Kutsoati, E., McMillan, M., Udry, C., 2011. Crop price indemnified loans for farmers: a pilot experiment in rural Ghana. J. Risk Insur. 78 (1), 37–55.
Manuamorn, O., 2007. "Scaling Up Microinsurance: The Case of Weather Insurance for Smallholders in India", Discussion Paper. World Bank.
Meherette, E., 2009. Providing weather index and indemnity insurance in Ethiopia. Innovations in Insuring the Poor. IFPRI Focus.
Mobarak, M., Rosenzweig, M., 2012. Selling formal insurance to the informally insured. Center for Microfinance, Working Paper.
Moschini, G., Lapan, H., 1995. The hedging role of options and futures under joint price, basis and production risk. Int. Econ. Rev. 36 (4), 1025–1049.
Pratt, J., 1964. Risk aversion in the small and in the large. Econometrica 32 (1–2), 122–136.
Rosenzweig, M., Wolpin, K., 1993. Credit market constraints, consumption smoothing and the accumulation of durable production assets in low-income countries: investment in bullock in India. J. Polit. Econ. 101 (2), 223–244.
Roth, J., Cord, M.J., 2008. Agricultural Microinsurance. The Micro Insurance Center.
Rothschild, M., Stiglitz, J., 1970. Increasing risk: I, a definition. J. Econ. Theory 2 (3), 225–243.
Shaked, M., Shantikumar, J., 2007. Stochastic Orders. Springer, New York.

Skees, J., Hartell, J., Yao, J., 2006. Weather and index-based insurance for developing countries: experience and possibilities. In: Sarris, A., Hallam, D. (Eds.), Agricultural Commodity Markets and Trade: New Approaches to Analyzing Market Structure and Instability. Edward Elgar Publishing.
Skees, J., Hartell, J., Goes, A., 2007. Using index-based risk transfer products to facilitate micro-lending in Peru and Vietnam. Working paper.
Stoppa, A., Hess, U., 2003. Design and use of weather derivatives in agricultural policy: the case of rainfall index insurance in Morocco. Working Paper.
Townsend, R., 1994. Risk and insurance in village India. Econometrica 62 (3), 539–591.
Warr, P., 1983. The private provision of public goods is independent of the distribution of income. Econ. Lett. 13, 207–211.
World Bank, 2009. Catastrophe Risk Financing in Developing Countries: Principles for Public Intervention, Washington.
World Bank CRMG, 2009. Managing drought risk for food security in Africa: an innovative solution in Malawi. Note 44.