Managing migration from the traditional to modern sector in developing countries[*][†]

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Abstract

We model the process of migration from a traditional to a modern sector. Migrants from the traditional sector experience a period of unemployment before finding modern sector jobs. Because of congestion in the process that matches the unemployed with jobs, an increase in the amount of unemployment increases the expected duration of unemployment for the representative migrant. Skilled workers can provide education and other services that decrease the expected duration of unemployment, but the competitive market under-provides these services. Congestion in the search process and the under-provision of migration services are two market failures, requiring two types of government policies. We explain how the analytic model can be studied using numerical methods in order to evaluate government policy.

Keywords: transitional economies, unemployment, congestion

1 Introduction

In many developing countries, there are too many workers leaving the traditional agricultural sector, and too few of these migrants getting jobs in the

modern sector at any point in time. Transferring workers from the traditional to the modern sector requires resources, including skilled workers and capital that would otherwise be employed in the modern sector. Skilled workers provide training, converting the unskilled agricultural migrants into semi-skilled workers ready for modern-sector jobs. Capital is needed in order to provide the infrastructure requirements (transportation systems, facilities for markets) arising from migration. This paper examines two sources of inefficiency that result in excessively fast migration out of the traditional sector, and slow movement into the modern sector, leading to high unemployment rates and large adjustment costs for society.

First, agricultural migrants create a negative externality when they attempt to enter the modern sector. Friction in the labor market can cause congestion, which migrants fail to internalize. It takes time to match job openings and job seekers in the modern sector. At the beginning of a period firms decide on the number of semi-skilled workers they would like to hire, and the number of job seekers (unemployed migrants) is given. Job seekers and firms search for matches; this search process results in an increase in employment, but not all agents find matches. An increase in the number of job seekers typically increases the number of matches, simply because there are more agents on one side of the market. However, if the increase in the expected number of matches is less than proportional to the increase in job seekers, the probability that other job seekers find employment is lower. In this case, there is congestion in the labor market.

For example, suppose that there are initially 100 unemployed workers, and the search process is such that in expectation 60 of these obtain jobs in a period. Now consider an exogenous increase of 10 unemployed migrants. If this exogenous change results in a 10% increase in the expected number of matches, the preexisting unemployed have the same chance of getting a job as before the arrival of the new migrants. However, if their arrival results in a 5% increase in the expected number of jobs, the probability of getting a job falls from 0.6 to $\frac{63}{110} = 0.57$.

The friction, and the resulting congestion, is not a market failure. Friction is simply a type of adjustment costs, and congestion is analogous to increasing marginal adjustment costs. To some extent, these costs are determined by technology and institutions that we take as given for the purpose of this model. The market failure is that migrants do not internalize the congestion. In deciding whether to look for a job in the modern sector, a potential migrant takes into account the probability of getting a job, but

not the effect of his migration decision on the likelihood that others obtain jobs. (We ignore other kinds of externalities, such as stress on the urban infrastructure, that migrants might create.)

The amount of friction in the labor market depends in part on the quantity of modern-sector resources allocated to this market. The free market does not efficiently allocate modern-sector resources to this activity; this is the second distortion in our model. Migrants need education and training in order to move from traditional to modern-sector jobs. For a variety of reasons (e.g. credit constraints, asymmetric information) migrants may be unable or unwilling to pay for the full value of educational services. Firms are reluctant to pay for this training, because job mobility makes it impossible for them to capture all of the benefits that arises from it. There may also be external economies of scale in the educational sector, leading to inefficiency in the market for the educational services needed to convert the unskilled agricultural migrants into semi-skilled workers for the modern sector. Much of the additional infrastructure required by migration is similar to a public good, the provision of which cannot be left to the market.

Even in OECD countries, where we expect markets to function more efficiently than in developing countries, the government is active in labor markets. For example, within OECD countries, public expenditures on "active labor-market programs", i.e. those which assist workers in getting jobs (including by retraining), as distinct from merely providing unemployment insurance, range from 0.16% of GDP in the US to nearly 2% of GDP in Denmark (The Economist, page 33, January 20, 2007). In developing countries, where even basic education can be viewed as an "active labor-market program", there is likely much greater need for government involvement in assisting factors to move out of the traditional and into the modern sector.

There are a variety of factors that can be used either for production in the modern sector or to smooth the transition of migrants from the traditional to the modern sector. For simplicity of exposition, we hereafter refer to this combination of factors as skilled labor. This skilled labor provides many kinds of services that help migrants move into the modern sector. We refer to this skilled labor as "teachers", since training migrants to become semi-skilled workers and assisting them in finding jobs in the modern sector are among their important functions. We refer to the sector in which teachers work as either the educational, or the transport sector (since migrants are being "transported" into the modern sector, a process that requires education). The opportunity cost of hiring teachers is their wage as skilled workers in the

modern sector.

Due to the two types of market failure, policymakers need two instruments to achieve an efficient outcome. The government needs to subsidize modern-sector factors to enter education, implicitly allocating skilled workers to the education sector. A larger number of teachers makes it possible to train the migrants more quickly, decreasing the duration of unemployment. This policy intervention helps the unemployed move into modern sector jobs. However, the number of migrants is endogenous. Increasing the number of teachers not only increases the speed of exit from the unemployment pool into the pool of semi-skilled employed, but it also changes the size of the unemployed pool.

Unskilled agricultural workers decide to enter this pool based on their expectation of the semi-skilled wage and of the duration of unemployment. Inducing skilled workers to become teachers reduces the stock of skilled workers in the modern sector, likely decreasing the value of marginal product of semi-skilled workers there. The resulting fall in the semi-skilled wage (the "wage effect") makes migration less attractive. However, the increased number of teachers and the resulting fall in the expected duration of unemployment (the "duration effect") makes migration more attractive. The negative externality created by an unemployed worker means that the number of migrants is inefficiently large. Hiring more teachers can either exacerbate or ameliorate this inefficiency, depending on whether the duration effect or the wage effect is more powerful.

In either case, the government needs a second policy instrument to correct the externality arising from unemployment. A variety of policies could slow migration. The requirement that people moving to cities hold a residence permit has been used in China and other planned economies. Subsidizing the traditional sector, increasing the wage there, also diminishes the incentive to migrate. This market-based approach avoids the need to monitor compliance with residency rules, and it increases the income of the poorest.

The next section provides a more detailed but still informal description of the model. The following section formally describes the model. The subsequent section sketches a calibration and explains how numerical methods can be used for policy analysis.

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2 An informal sketch of the model

We model the migration dynamics and study the role of policy using a three-sector model with an endogenous employment probability. There are two traded goods sectors: the modern sector uses skilled and semi-skilled labor and sector-specific factors; the traditional sector uses unskilled labor and a sector-specific factor. The third (nontradable) sector, "transport", provides education and other migration services. Using skilled labor, this sector transforms unemployed migrants from the traditional sector into semi-skilled workers who (after some delay) obtain jobs in the modern sector.

The total number of skilled workers is fixed; government policy and endogenous labor market conditions determine the allocation of these workers between the modern and transport sectors. The number of semi-skilled workers with jobs in the modern sector is a *predetermined* endogenous variable: its level is given at a point in time, but it changes endogenously over time as unemployed workers get jobs. The number of unemployed workers is endogenous. Traditional agricultural workers flow freely into the pool of unemployed to satisfy an equilibrium condition. The traditional sector shrinks over time, as some workers there move into the pool of unemployed.

At a point in time the equilibrium semi-skilled wage depends on the number of skilled and semi-skilled workers in the modern sector. This wage is endogenous (unlike in the Harris-Todaro model where it is treated as exogenous). The wage in the traditional sector is either fixed or endogenous, depending on whether the marginal productivity of workers in that sector is constant or decreasing. Government policy might affect the traditional wage, e.g. by means of a wage subsidy or an output subsidy that increases labor's value of marginal product.

The probability, per unit of time, that an unemployed worker gets a job in the modern sector is the hazard rate. An increase in the number of teachers (our shorthand for the modern-sector factors devoted to the transport sector) increases the hazard rate; with more teachers, the unskilled migrants are trained more rapidly and given greater assistance in finding a job. An increase in the number of unemployed workers increases congestion, lowering the hazard rate. At each point in time traditional workers decide whether to remain in the traditional sector or to enter the pool of unemployed in the hope of obtaining a job in the modern sector. In equilibrium, these workers are indifferent between the two options.

The opportunity cost of being unemployed, per unit of time, is the tra-

ditional wage for that unit of time. The benefit of being unemployed is the hazard rate times the present discounted value of having a semi-skilled job rather than a traditional job. This discounted value depends on the how we model workers' expectations. Under rational point expectations, workers are able to predict future wages and employment probabilities. (There is no aggregate uncertainty in this model, although individual migrants are unsure of when they will get a job.) Under myopic expectations, workers act as if current wages and employment probabilities will persist indefinitely.

We assume that workers have myopic expectations. This assumption is probably more realistic (compared to rational point expectations). It also leads to a simpler (lower dimensional) optimization problem. In addition, it means that instead of specifying a time horizon and a trajectory of discount factors, we can evaluate the present discounted value of the wage differential using a single parameter. Finally, the assumption avoids the problem of the time-inconsistency of second best policies. Consequently, we do not have to take a position regarding whether policymakers have the ability to commit today to actions that they will take in the future.

3 The model

The economy consists of three sectors. The two tradable goods sectors are Agriculture (A) and Modern Industry (M). The third sector, Transport (T), uses skilled labor to produce migration services; these services convert unskilled unemployed migrants to semi-skilled labor and assist with their entry into the modern sector. Industry uses skilled and semi-skilled labor and a sector-specific factor. Agriculture uses unskilled labor and a sector-specific factor. In order to move from agriculture to industry, a migrant passes through a pool of unemployed.

3.1 Wages in the modern sector

Over time, education converts unskilled labor to semiskilled labor. In our stationary model, the amount of both skilled labor and the sum of semiskilled plus unskilled plus unemployed labor are constants. By choice of units we normalize both of these constants to 1. Let S equal the fraction of skilled labor in the Transport sector, so the amount of skilled labor in the modern sector is $1 - S$. Let L equal the fraction of unskilled+semiskilled+unemployed

labor in the modern sector. The relative price of output in the modern sector is p (or p is a TFP parameter); $pF(1 - S, L)$ is the sector's value of output. There are decreasing returns to scale due to the presence of fixed factor in modern sector:

$$F(1 - S, L) = \left((1 - S)^{1-\rho} (L)^\rho\right)^\theta$$
$$= \left((1 - S) \left(\tfrac{L}{1-S}\right)^\rho\right)^\theta .$$

The wages of skilled and semiskilled workers in modern sector, v and ω, solve the share equations

$$\text{skilled:} \quad \frac{v(1-S)}{p\left((1-S)\left(\frac{L}{1-S}\right)^\rho\right)^\theta} = (1 - \rho)\,\theta$$

$$\text{semi-skilled:} \quad \frac{\omega L}{p\left((1-S)\left(\frac{L}{1-S}\right)^\rho\right)^\theta} = \rho\theta.$$

These equations imply

$$\text{skilled:} \quad v = (1 - \rho)\,\theta p \left((1 - S) \left(\tfrac{L}{1-S}\right)^\rho\right)^\theta \tfrac{1}{1-S}$$

$$\text{semi-skilled:} \quad \omega = \rho\theta p \left((1 - S) \left(\tfrac{L}{1-S}\right)^\rho\right)^\theta \tfrac{1}{L}.$$

(1)

3.2 Unemployment and entry into the modern sector

Let U be the pool of unemployed. In this model, skilled workers are always fully employed. By "unemployment rate" we always mean the percentage of unskilled and semi-skilled workers who are unemployed. Our normalization that the size of the unskilled/semi-skilled population is 1 means that U is the unemployment rate. The probability per unit of time dt that an individual worker gets a job is $f(U, S)dt + o(dt)$; f is a "hazard rate". Congestion means that $f_U < 0$. An increase in skilled workers in Transport increases the flow out of the pool of unemployed: $f_S > 0$. Skilled workers in Transport are paid by the government at their equilibrium wage, v. The flow of workers into the modern sector is

$$\frac{dL}{dt} = U_t f(U_t, S_t).$$

(2)

For our numerical experiments we specify the hazard rate as

$$f(U, S) = (b + S)^\beta (c + U)^{-\sigma},$$

(3)

where the parameters b, c, β and σ are positive. With this model

$$\frac{\partial f}{\partial U} = -\sigma (b + S)^\beta (c + U)^{-\sigma - 1} < 0$$

$$\frac{\partial f}{\partial S} = \beta (b + S)^{\beta - 1} (c + U)^{-\sigma} > 0 \qquad (4)$$

$$\frac{\partial^2 f}{\partial U \partial S} = -\sigma \beta (b + S)^{\beta - 1} (c + U)^{-\sigma - 1} < 0.$$

The first line of equation (4) shows that an increase in U decreases the hazard rate, i.e. it increases the duration of unemployment. The parameter σ determines the importance of congestion; congestion vanishes as $\sigma \to 0$. The second line shows that an increase in S increases the hazard rate; this effect depends on the magnitude of β, so this parameter determines the importance of S. The cross-partial in the third line shows that an increase in S increases the congestion effect: a larger value of S means that an additional unemployed worker has a greater effect on increasing the expected unemployment duration for other unemployed workers.

The sign of the cross-effect means that the two distortions tend to offset each other. For example, suppose that the government has a particular second-best policy menu that enables it to increase S, but it is not able to discourage agricultural workers from leaving the sector. Since the competitive equilibrium leads to too low a level of S, the government wants to increase this value. However, in doing so it encourages more workers to enter the pool of the unemployed (because $\frac{\partial f}{\partial S} > 0$); the larger S also exacerbates the congestion problem (because $\frac{\partial^2 f}{\partial U \partial S} < 0$). Thus, the second-best level of S is likely smaller than the first best. This example also illustrates that the use of one policy instrument (here, a subsidy on S) increases the importance of dealing with the other distortion. Policies to promote entry of modern sector resources into the transport sector increase the need to adopt other policies that discourage too rapid an exit by workers from the agricultural sector. In this sense, policies that target the two distortions are complementary.

The parameters b and c determine the steady state of the model, and the steady state wage differential between agricultural workers and semi-skilled workers in the modern sector. In a steady state, there is no entry into the modern sector, so $U = 0$. In the steady state, there is no role for the transport sector, so $S = 0$. Thus, in the steady state, if one additional worker were to decide to leave the agricultural sector and look for a job in the modern sector, his expected duration of unemployment is $f^{-1}(0, 0) = (b)^{-\beta} (c)^\sigma > 0$. Due to the cost of moving sectors, even when unemployment is 0, there is a steady state wage gap between agricultural and semi-skilled modern workers.

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3.3 The agricultural wage

At the sectoral level there are decreasing returns to unskilled labor in agriculture due to the presence of a fixed factor or to the presence of underemployment/job sharing in that sector. The amount of labor in agriculture is $A = 1 - L - U$ and the value of output of agriculture is aA^α, where a is a TFP parameter and $0 < \alpha < 1$. Each worker in the sector obtains (by assumption) the value of average product[1], so the wage in the sector is $w_t = aA_t^{\alpha-1}$.

3.4 The equilibrium amount of unemployment

The wage differential (between a semiskilled worker in the modern sector and an unskilled agricultural worker) at time t is

$$
\begin{aligned}
m_t &\equiv \omega(S_t, L_t; p) - a\left(1 - L_t - U_t\right)^{\alpha-1} \\
&= \rho\theta p \left((1-S)\left(\tfrac{L}{1-S}\right)^\rho\right)^\theta \tfrac{1}{L} - a\left(1 - L - U\right)^{\alpha-1}.
\end{aligned}
\tag{5}
$$

When there is no ambiguity we suppress time subscripts. If a worker in the agricultural sector discounts the future wage differential at the rate $r(s)$ and has a time horizon of τ, his PDV of having a job in the modern rather than the agricultural sector (under rational point expectations), denoted q_t, is

$$
q_t \equiv \int_0^\tau e^{-\int_0^{s'} r(s)ds} m_{t+s'} ds'.
$$

If workers are myopic (so that they expect current wage differential to persist), as we hereafter assume, then

$$
q_t = \frac{m_t}{r},
\tag{6}
$$

where

$$
\int_0^\tau e^{-\int_0^{s'} r(s)ds} ds' = \frac{1}{r}.
$$

The assumption of myopic expectations means that we can model the worker's problem as if he has an infinite horizon with a constant discount rate r; we refer to r as the "pseudo-discount rate"

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[1]This assumption means that agricultural workers obtain an equal share of the returns to the fixed factor. The alternative assumption that ag workers obtain their value of marginal product would also be easy to model.

When $U > 0$ (i.e. migration is positive) the equilibrium condition is

$$a\left(1 - L_t - U_t\right)^{\alpha-1} = q_t f\left(U_t, S_t\right). \tag{7}$$

The left side is the Agricultural wage, which is the opportunity cost of leaving the traditional sector. (The worker has to leave the sector in order to receive training and look for a job in modern sector.) The right side of equation (7) is the expected benefit of looking for a job (the benefit of having a job times the probability of finding one). We can use equation (6) to eliminate q, to write the equilibrium condition as

$$a\left(1 - L_t - U_t\right)^{\alpha-1} = \frac{m_t}{r} f\left(U_t, S_t\right). \tag{8}$$

Using equations (5) and (3) we can write this equilibrium condition as

$$G\left(U, L, S\right) \equiv ra\left(1 - L - U\right)^{\alpha-1} -$$
$$\left(\rho\theta p\left((1-S)\left(\tfrac{L}{1-S}\right)^\rho\right)^\theta \tfrac{1}{L} - a\left(1 - L - U\right)^{\alpha-1}\right)\left(b + S\right)^\beta\left(c + U\right)^{-\sigma} = 0. \tag{9}$$

This equation holds of $U \geq 0$. Our model describes the decline of the traditional sector and the growth of the modern sector, so $U > 0$ during the transformation, and $U = 0$ once the transformation is complete (i.e. in a steady state). It is not plausible that the processes of growth and decline of the modern sector are governed by the same equations; the decline of the modern sector is not interesting in this context.

In this model, the exogenous parameters rather than policies determine the steady state. In the steady state $U = 0 = S$; using these values and equation (9), the steady state level of L is the smallest positive root of

$$ra - \frac{L^{\rho\theta-1}}{(1-L)^\alpha}c^{-\delta}b^\beta\rho\theta p + \frac{c^{-\delta}b^\beta\rho\theta p L^{\rho\theta}}{(1-L)^\alpha} + c^{-\delta}b^\beta a = 0. \tag{10}$$

The equilibrium condition, equation (7), holds for non-negative values of L less than or equal to this root.

3.5 Welfare

The flow of national income is

$$Y_{t+s} \equiv pF_{t+s}\left(1 - S_{t+s}, L_{t+s}\right) + aA_{t+s}^\alpha, \tag{11}$$

equal to the value of output in the tradable sectors. The sum of income to semi-skilled and unskilled workers is

$$\omega_{t+s} L_{t+s} + a \left(1 - L_{t+s} - U_{t+s}\right)^{\alpha}.$$

We assume that social welfare, W, is a convex combination of national income and income to low wage workers, with weights λ and $1 - \lambda$:

$$
\begin{aligned}
W\left(S_t, U_t, L_t\right) \equiv \lambda \left(pF_{t+s} + aA_{t+s}^{\alpha}\right) + \\
(1 - \lambda)\left(\omega_{t+s} L_{t+s} + a\left(1 - L_{t+s} - U_{t+s}\right)^{\alpha}\right)),
\end{aligned}
\tag{12}
$$

The unskilled and semi-skilled wage bill is a component of national income, so this wage bill receives the weight $\lambda + 1 - \lambda = 1$. The remaining component of national income consists of the return to specific factors and the skilled wage bill in the modern sector; these values receive the weight λ. This formulation means that society is willing to sacrifice $\frac{1}{\lambda}$ units of income from skilled labor and modern sector specific factors in order to transfer 1 unit of income to semi-skilled and unskilled workers.

The flow of adjustment costs equals the difference between maximized national income

$$pF_{t+s}\left(1, L_{t+s}\right) + a\left(1 - L_{t+s}\right)^{\alpha},$$

obtained by setting $S = 0 = U$, and the equilibrium level of national income, given by equation (11).

Given a constant discount rate δ, the social planner's objective is to maximize

$$\int_0^{\infty} e^{-\delta s} W\left(S_t, U_t, L_t\right) ds.
\tag{13}$$

If agricultural workers have myopic expectations, so that equation (6) is valid, then we have a standard optimal control problem with one state variable, L, with the equation of motion (2).

4   The numerical problem

There are a number of ways to study this problem, depending on whether we are interested in the competitive equilibrium (absent government intervention), or the first best or a second best outcome. To study the competitive equilibrium, we need only solve a pair of differential equations (without optimization). In order to study the first best outcome, we proceed in two

steps. First, we maximize the government's objective, under the assumption that the government directly chooses S and U, and then find policies that support these choices as a competitive outcome, respecting the equilibrium condition, equation (9). In order to study a second best outcome, we select the limited menu of policy options, such as a subsidy to teachers in the transport sector, or a subsidy to agricultural workers, impose the equilibrium condition, equation (9), and solve the resulting control problem.

Time limitations preclude a full analysis of these scenarios. We report some preliminary calibration results and then outline the plan for subsequent analysis. This material indicates how the model can be numerically implemented and used for policy analysis.

4.1   Calibration of model

The unit of time is one year. Most of the model parameters have an obvious interpretation, enabling us to select plausible values to fit the economy that we wish to describe. Table 1 lists provisional parameter values.

parameter	definition	value
θ	skilled + unskilled labor share in modern sector	0.8
ρ	unskilled labor share of total labor share in modern sector	0.5
p	relative price of modern sector (or TFP parameter)	1
a	TFP parameter in agriculture	0.5
α	unskilled labor share in ag output	0.9
r	"pseudo-discount rate" for ag migrants	0.2
b, c, β, σ	parameters of the hazard function	see text
λ	inverse of value of one unit of income for low paid workers	1
δ	social planner's pure rate of time preference	0.05

Table 1: Parameters of model

The parameters of the hazard function are not so familiar, so it is convenient to choose their values by using the fact that when the arguments of the hazard rate are constant, the inverse of the hazard rate is the expected duration of unemployment. That is, we choose constant values (S_i, U_i) and a corresponding expected duration of unemployment, d_i and obtain the parameters of the hazard function by solving

$$d_i = f^{-1} = (b + S_i)^{-\beta} (c + U_i)^{\sigma}. \tag{14}$$

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Although there are four parameters in the hazard function, the system of calibration equations (14) contains one degree of freedom, so we can select one parameter arbitrarily. We calibrate the system by setting $c = 0.12$ and requiring that

$$(S_i, U_i, d_i) \in \{(0, 0, 0.5), , (0, 0.1, 1), (0.05, 0.1, 0.5)\}.$$

For example, the first calibration assumption (where $S = 0 = U$) means that in a steady state, an agricultural worker who decides to look for a job in the modern sector (requiring entry into the "empty" unemployment pool) has an expected duration of unemployment of half a year. The fact that the expected duration of unemployment of a migrant is positive, even when there are no unemployed workers, means that in the steady state the semi-skilled wage in the modern sector is higher than the agricultural wage. The last calibration assumption means that if 5% of skilled workers are employed in the transport sector, and the unemployment rate is 10%, the expected duration of unemployment is also half a year. The calibration assumptions, together with $c = 0.12$, imply that $b = 0.8696705076$, $\beta = 12.39953799$, and $\sigma = 1.142550961$.

Figure 1 shows the expected duration of unemployment, as a function of the unemployment rate, for three values of S. For example, when the unemployment rate is 30%, increasing S from 0% to 5% of skilled workers reduces the expected duration of unemployment from over 2 years to about 1 year.

The equilibrium condition $G(U, L, S) = 0$ from equation (9) implicitly gives U as a function of L, S. Denote this function as $U = \hat{g}(L, S)$, i.e. $G(\hat{g}(U, S), L, S) = 0$. We cannot obtain an analytic expression for $\hat{g}$, but we need its approximation in order to simulate the dynamics, or to solve one of the optimization problems. Equation (9) holds for $U \geq 0$ (because this model describes the growth but not the decline of the modern sector). Therefore, the equation of interest is $U = g(L; S) = \max\{\hat{g}(L; S), 0\}$. There is no analytic expression for the function g, but we can obtain a good numerical approximation. Figure 2 shows the graphs of third order Chebyshev approximations of $g(L, S)$ for three values of S.

Recall that L is the fraction of labor (excluding skilled labor) working in the modern sector, and $1 - L - U$ is the fraction of this labor in the agricultural sector. For values of L close to 0, the modern semi-skilled wage is very large and the agricultural wage is low (using equation (5) and the
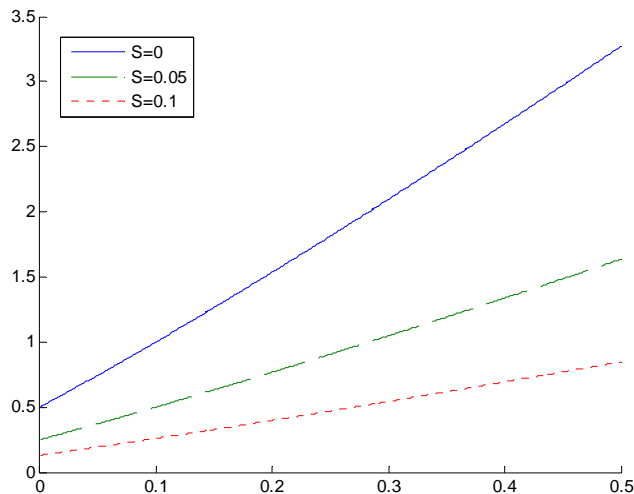
Figure 1: Expected unemployment duration (in years) as a function of unemployment rate for different levels of S

agricultural wage $w = a\left(1 - L - U\right)^{\alpha - 1}$). Thus, for small values of L, the unemployment rate approaches 100%, since most agricultural workers are willing to risk a long spell of unemployment in order to obtain a high wage in the modern sector. This model is useful only for values of L bounded away from 0. The steady state, $L_\infty\left(S\right)$, is the smallest value of L at which $g\left(L; S\right) = 0$. Of course, in an efficient steady state, S must be 0, since by definition there is no unemployment in the steady state in this model.

For our calibration, and for $0\% \leq S \leq 10\%$, an increase in S always increases the level of unemployment. An increase in S lowers number of skilled workers in the modern sector, lowering the marginal product (and the wage) of semi-skilled workers in that sector, thus lowering the incentive to migrate. However, an increase in S decreases the expected duration of unemployment, increasing the incentive to migrate. For our parameterization, the second effect dominates.
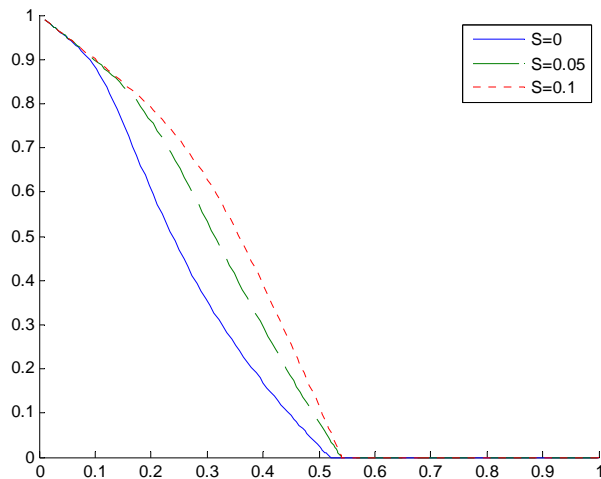
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Figure 2: Equilibrium level of unemployment as a function of L for different levels of (constant) S

4.2 Plan for simulation and optimization

This section describes the plan to simulate the competitive equilibrium (where $S \equiv 0$) and to solve for the equilibria with policy interventions.

4.2.1 The competitive equilibrium

In this setting $S = 0$ and the steady state, L_∞, is the smallest solution to $\hat{g}(L, 0) = 0$. Given our approximation to the function $\hat{g}$, and thus to g, we can numerically solve the ODE (2), rewritten here as

$$\frac{dL}{dt} = g(L; S) f(g(L; S), S), \quad L(0) = L_0, \text{ given, with } S = 0. \tag{15}$$

We are also interested in social welfare. For a given value of L, the present discounted value of social welfare is

$$V(L) = \int_0^\infty e^{-\delta s} W(S, g(L_t; S), L_t) \, ds \text{ evaluated at } S = 0. \tag{16}$$

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At the steady state, L_∞, we have

$$V(L_\infty) = \frac{W(0, 0, L_\infty)}{\delta}. \tag{17}$$

The function $V(L)$ obeys the ODE

$$g(L; S) f(g(L; S), S) \frac{dV}{dL} = \delta V(L) - W(0, g(L; 0), L_t) \tag{18}$$

with the boundary condition, equation (17). To obtain the competitive equilibrium trajectory and social welfare we need to numerically solve the pair of ODEs (15) and (18), together with their boundary conditions.

4.2.2 The first best outcome

The first best policy chooses U and S directly. The assumption that the government has enough instruments to achieve the first best means that we can solve the maximization problem ignoring the equilibrium condition (9). Once we have the solution (the optimal trajectories of L, U, S) we can calculate government policies that support this trajectory. For example, we can use the wage equation (1) to calculate the equilibrium trajectory of skilled wage $\{v_t\}$ for skilled transport workers; this wage induces the optimal number of skilled workers to enter that sector. We can also use equation (9) to calculate an equilibrium policy trajectory for $\{a_t\}$ (TFP in agriculture) or for some other policy variable, that induces the optimal amount of migration.

We again designate the value function – here, the maximized present discounted value of the stream of welfare – as $V(L)$. (This is an abuse of notation because V represents both the value function under the competitive equilibrium and in the first best outcome. We can use superscripts for clarity, but we omit them here in order not to encumber the notation.) The Bellman equation is

$$\delta V(L) = \max_{U,S} \{W(S, U, L) + V'(L) U f(U, S)\}. \tag{19}$$

In a steady state, $S = U = 0$, so the steady state is $L_\infty(0)$, obtained in section 4.2.1 and the boundary condition is equation (17).

The solution to the optimization problem leads to control rules $S = h(L)$ *and* $U = k(L)$. Using these control rules we can solve the ODE

$$\frac{dL}{dt} = k(L) f(k(L), h(L)) \qquad L(0) = L_0, \text{ given,}$$

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to find L as a function of time. With that function we can use $S_t = h(L_t)$ and $U_t = k(L_t)$ to find the controls as a function of time. We can graph these in order to compare the evolution of L and U under the competitive equilibrium and the first best. We also want to compare $V(L)$ at the initial condition, under the competitive and first best equilibria, in order to determine the welfare effect of optimal intervention.

4.2.3 Second best equilibria

There are several possible types of second best equilibria, depending on the planner's policy menu. For example, the planner may be able to subsidize the agricultural sector or tax the modern sector, taking as given a non-optimal relation between S and L; alternatively, the planner may be able to choose S, taking as given a non-optimal tax or subsidy for the tradable sectors. We can compare the trajectories of L and U in these scenarios, and the associated levels of welfare, with the trajectories and welfare levels in both the first best and the competitive equilibrium trajectories.

5 Conclusion

We described a model of migration of unskilled workers from the agricultural to the modern sector. In order to enter the modern sector, migrants pass through a pool of unemployed workers. Their expected duration of unemployment depends on the number of unemployed workers and on the number of skilled workers, primarily teachers, who provide training and other adjustment services.

There are two sources of market failure in a competitive equilibrium. First, for a variety of reasons including migrants' credit constraints and imperfect information, the market provides a suboptimal amount of migration services. The government has a role in subsidizing skilled workers to provide these services. Second, because of congestion in the process by which unemployed workers are matched with jobs, the entry of one more migrant into the pool of unemployed increases the expected duration of unemployment of other workers in the pool. The migrant does not take this negative externality into account when deciding to look for job in the modern sector. The government has a role in slowing migration.

The government needs two instruments to deal with the two market im-

perfections. A subsidy to the agricultural sector, which makes it less attractive to leave the sector, targets the congestion. A subsidy to skilled workers providing migration services targets the under-provision of these services. The subsidy to workers providing migration services has an ambiguous effect on the level of congestion. As more of these workers provide the services, the number of skilled workers in the modern sector decreases, which tends to decrease the value of marginal productivity of modern sector semiskilled workers, reducing their wage and thereby reducing the incentive to migrate. However, an increase the supply of migration services reduces the expected duration of unemployment, thereby increasing the incentive to migrate. We found that for a reasonable calibration, the second effect dominates, so an increase in skilled workers providing migration services increases the amount of unemployment, thereby increasing the amount of congestion at a point in time.

When the second effect dominates, a subsidy to migration services increases the incentive to subsidize the agricultural sector (or tax the modern sector). Subsidizing migration services increases the number of unemployed (for a given level of employment in the modern sector). Since the exit from agriculture is too rapid in the competitive equilibrium even in the absence of the additional migration services, it becomes even more important to slow migration once the additional services are provided. In contrast, the use of an agricultural subsidy reduces the level of unemployment (for a given level of employment in the modern sector), reducing the incentive to subsidize migration services.

In order to understand the magnitude of the levels of optimal policies, and their relation to the parameters of the model, we need to use numerical methods. We sketched a calibration of the model and provided suggestions for further numerical analysis.