B Appendix for "The Political Economy of Environmental Policy with Overlapping Generations"

This appendix collects supplementary information alluded to in the journal publication.

B1 Exogenous Productivity Growth

In the context of most environmental problems, the natural resource is degrading on the 0-tax trajectory. In our model of constant productivity and capital, the world becomes poorer and future generations have lower welfare on that trajectory. This Appendix introduces exogenous productivity growth in both sectors. Let $a \geq 0$ be the growth rate of total factor productivity in manufacturing and $b \geq 0$ the growth rate of efficiency in output per unit flow of the resource. Sectoral output is

$$M_t = e^{at}(1 - L_t)^{\beta} \quad \text{and} \quad F_t = e^{bt} L_t \gamma x_t.$$

The inequality $a > 0$ can also be interpreted as exogenous growth in the stock of capital. The extraction of the resource is still $L_t \gamma x_t$ (not $e^{bt} L_t \gamma x_t$). This model of resource productivity growth implies that each extracted unit of the resource increases the supply of the resource-intensive commodity. If we think of the resource as being energy, the assumption means that the economy becomes less energy intensive. The assumption of exponential productivity growth simplifies the discussion, but the next proposition also holds if the productivity parameters a and b decrease over time. The exponential productivity growth implies a growth factor of $e^{(a-b)}\left(1 + \bar{r}(T_t, x_t)\right)$ for the price level and of e^{a} for all other variables (w_t, R_t, and π_t). For the following proposition we assume that $\chi \in (0, 1)$ is constant and that there is no transfer between generations, i.e. $\xi = 0$.

Proposition 7 *A larger value of $a - b$ increases the stringency of the necessary and sufficient condition under which a small constant tax increases the welfare of the young.*

Proof. Using a derivation parallel to that contained in the proof of proposition 4, we have

$$\left.\frac{dU_0^y}{d\varepsilon}\right|_{\varepsilon=0} > 0 \Leftrightarrow (1-\chi)\left(\frac{e^{-(a-b)\alpha}\left(1+\bar{r}(0,x_0)\right)^{\alpha}}{1+\rho}-1\right)\bar{T}_0 > 0.$$

The second inequality is equivalent to

$$\left(\frac{1+\bar{r}(0,x_0)}{e^{(a-b)}}\right)^{\alpha} > 1+\rho. \tag{B.1}$$

The left side of inequality (B.1) is decreasing in $a-b$, so an increase in $a-b$ decreases the set of parameter values and initial conditions under which the inequality is satisfied, i.e. the circumstances under which the young benefit from the tax. ■

Under proportional growth ($a=b$), the condition for the young to benefit from the tax is the same as when $a=b=0$. The welfare effect of the tax, for the young, depends on the change in the price level. A *ceteris paribus* increase in $a-b$ increases the next period relative supply of the manufacturing good, thereby increasing the future relative price of the resource-intensive good, P_{t+1}. The higher price lowers the marginal utility of next period income, making it "less likely" that the young are willing to forgo income today in order to have higher income in the next period. For $a>b$, the young would require a higher transfer from the old in order to agree to the tax. If, however, the productivity in the resource sector grows much faster than in the manufacturing sector ($b>>a$), the young might support a tax even when the resource is shrinking on the 0-tax trajectory, and in the absence of a transfer.

B2 Future generations

Merely in order to avoid uninteresting complications, we assume that for future generations the tax is constant: $\bar{T}_0=\bar{T}_1=\bar{T}_2$.... The life-time welfare of the next young generation is

$$U_1^y(\varepsilon)=\mu p(\bar{T}_1\varepsilon,x_1)^{-\alpha}\left(w(\bar{T}_1\varepsilon)+\chi R(\bar{T}_1\varepsilon)+\frac{\left(1+\bar{r}(\bar{T}_1\varepsilon,x_1)\right)^{\alpha}}{1+\rho}(1-\chi)R(\bar{T}_2\varepsilon)\right).$$

Differentiating this expression with respect to ε gives

$$\frac{dU_1^y}{d\varepsilon} = \frac{d}{d\varepsilon}\mu P_1^{-\alpha}(w(\bar{T}_1\varepsilon)+\chi R(\bar{T}_1\varepsilon))+\frac{d}{d\varepsilon}\left[\mu P_1^{-\alpha}\frac{\left(1+\bar{r}(\bar{T}_1\varepsilon,x_1)\right)^{\alpha}}{1+\rho}(1-\chi)R(\bar{T}_2\varepsilon)\right]$$

Using the simplifications found in the proof of Proposition 4, (including $R(0)=0$ and $\frac{\partial P_1^{-\alpha}}{\partial x_1}=\alpha P_1^{-\alpha}x_1^{-1}$) the expression simplifies to

$$\begin{aligned}\left.\frac{dU_1^y}{d\varepsilon}\right|_{\varepsilon=0} > 0 \;&\Leftrightarrow\; \bar{T}_1 P_1^{-\alpha}(1-\chi)\left(\frac{(1+\bar{r}(0,x_1))^{\alpha}}{1+\rho}-1\right)\left.\frac{dR}{d\varepsilon}\right|_{\varepsilon=0} > -\bar{T}_0 w(0)\frac{\partial P_1^{-\alpha}}{\partial x_1}\frac{\partial x_1}{\partial T_0}\\ &\Leftrightarrow\; (1-\chi)\left(\frac{(1+\bar{r}(0,x_1))^{\alpha}}{1+\rho}-1\right)\bar{T}_0 > -\left(w(0)\alpha x_1^{-1}\frac{\partial x_1}{\partial T_0}\right)\left(\frac{1}{dR/d\varepsilon|_{\varepsilon=0}}\right)\bar{T}_0\end{aligned}$$

Comparing this condition to inequality (A.27), we see that when the stock is degrading (i.e. $\bar{r}(0,x_0)<0$), a small tax is more likely to benefit the next period's young generation compared to today's, which always loses in the absence of transfers. The difference arises for two reasons: A lower stock increases the BAU growth rate, $\frac{d\bar{r}(0,x_t)}{dx_t}=-r<0$, so that the left side is less negative. The right side of the inequality above is negative. Therefore, the condition here is weaker than the condition in inequality (A.27). In fact, it is satisfied for any initial stock value in the calibration used in Section 5.

B3 Nash Bargaining

Using equations (5) - (7), we define the lifetime welfare of the two agents

$$\begin{gathered}U^o\left(x_t,\Upsilon\left(x_t\right),\chi_t\right)=P^{-\alpha}e^o\\ = p^{-\alpha}(x_t,\Upsilon\left(x_t\right))\left[\pi(\Upsilon\left(x_t\right))+(1-\chi_t)R(\Upsilon\left(x_t\right))\right]+\bar{\sigma}(x_t,\Upsilon\left(x_t\right)).\\ U^y\left(x_t,\Upsilon\left(x_t\right),\chi_t\right)=\\ = p^{-\alpha}(x_t,\Upsilon\left(x_t\right))\left[w(\Upsilon\left(x_t\right))+\chi_t R(\Upsilon\left(x_t\right))\right]\\ +\tfrac{1}{1+\rho}p^{-\alpha}(x_{t+1},\Upsilon\left(x_{t+1}\right))\left(1-\chi_{t+1}\right)R(\Upsilon\left(x_{t+1}\right).\end{gathered}$$

Denote

$$\tilde{U}^o\left(x_t\right)=U^o\left(x_t,0,\chi\right) \text{ and } \tilde{U}^y\left(x_t\right)=U^y\left(x_t,0,\chi\right),$$

the value of lifetime utility of the two agents when they impose a zero tax in the current period. Under a 0 tax, $R=0$, so the current value of χ

does not affect the agents' lifetime welfare. Therefore $\tilde{U}^o(x_t)$ and $\tilde{U}^y(x_t)$ do not depend on χ. They do, however, depend on the current value of x and of course they depend on the decision rules used in the future; they are functionals. The pair $\left(\tilde{U}^o(x_t), \tilde{U}^y(x_t)\right)$ is the threat-point in the Nash bargaining game. Total surplus equals

$$S(x_t, T_t, \chi_t) \equiv U^o(x_t, T_t, \chi_t) + U^y(x_t, T_t, \chi_t) - \left(\tilde{U}^o(x_t) + \tilde{U}^y(x_t)\right).$$

The Nash bargaining solution maximizes the Nash product,

$$\left(U^o(x_t, T_t, \chi_t) - \tilde{U}^o(x_t)\right)\left(U^y(x_t, T_t, \chi_t) - \tilde{U}^y(x_t)\right).$$

It is well known that when there are lump sum transfers (χ is unconstrained) the bargaining solution maximizes surplus, which is equivalent to maximizing aggregate lifetime welfare, the maximand in equation (14). That maximand does not involve χ_t. The choice of χ enables decisionmakers to make a lump sum transfer between generations. In this case, the transfer is chosen to split the surplus evenly between the two generations, implying:

$$\begin{aligned} U^o(x_t, T_t, \chi_t) - \tilde{U}^o(x_t) &= U^y(x_t, T_t, \chi_t) - \tilde{U}^y(x_t) \Longrightarrow \\ U^o(x_t, T_t, \chi_t) - U^y(x_t, T_t, \chi_t) &= \tilde{U}^o(x_t) - \tilde{U}^y(x_t) \end{aligned}$$

Using the formulae for equilibrium and disagreement payoffs and solving the last equation for χ_t gives

$$\chi_t = \Xi(x_t) = \frac{1}{2} + \frac{\pi(\Upsilon(x_t)) - w(\Upsilon(x_t))}{2R(\Upsilon(x_t))} + \frac{1}{2p^{-\alpha}(x_t, \Upsilon(x_t))R(\Upsilon(x_t))}C \tag{B.2}$$

with the definition

$$C \equiv \bar{\sigma}(x_t, \Upsilon(x_t)) - \frac{1}{1+\rho}p^{-\alpha}(x_{t+1}, \Upsilon(x_{t+1}))(1 - \Xi(x_{t+1}))R(\Upsilon(x_{t+1})) - \left(\tilde{U}^o(x_t) - \tilde{U}^y(x_t)\right).$$

The function Ξ appears in the definition of C, both explicitly and implicitly via the definition of $\tilde{U}^y(x_t)$. Therefore, equation (B.2) is a functional in Ξ. We numerically approximate the fixed point to this equation, and to the functional equation that determines Υ. If χ is constrained, and the constraint is binding, then it is no longer the case that aggregate surplus is split equally between the two generations. In that case, the equilibrium T_t that maximizes the Nash product, does not maximize aggregate surplus.

B4 Numerics

We approximate $\Upsilon(x_{t+1})$ and $\bar{\sigma}(x_{t+1}, \Upsilon(x_{t+1})) \equiv \Phi(x_{t+1})$ as polynomials in x_{t+1}, and find coefficients of those polynomials so that the solution to

$$\max_{T_t} P^{-\alpha}(x_t, T_t) Y(T_t) + \\ \left\{\Phi(x_{t+1}) + \tfrac{1}{1+\rho} P^{-\alpha}(x_{t+1}, \Upsilon(x_{t+1}))\left[\pi(\Upsilon(x_{t+1})) + (1 - \Xi(x_{t+1})) R(\Upsilon(x_{t+1}))\right]\right\} \\ \text{subject to } x_{t+1} = (1 + \bar{r}_t(x_t, T_t)) x_t \quad \text{with } x_0 \text{ given.}$$

approximately equals $\Upsilon(x_t)$. Appendix B3 explains the functional equation used to approximate Ξ in the Nash bargaining case. In the probabilistic voting model, Ξ is a known constant. We use 13-degree Chebyshev polynomials evaluated at 13 Chebyshev nodes on the $[0.1, 0.9]$ interval. At each node the recursion defining $\bar{\sigma}(x_t, \Upsilon(x_t))$,

$$\Phi(x_t) = \frac{1}{1+\rho}\left\{p^{-\alpha}(x_{t+1}, \Upsilon(x_{t+1}))\pi(\Upsilon(x_{t+1})) + \Phi(x_{t+1})\right\} \tag{B.3}$$

and the optimality condition

$$\frac{d}{dT_t}\left[P^{-\alpha}(x_t, T_t) Y(T_t) + \tfrac{1}{1+\rho}\Omega\right] = 0$$
$$\text{with} \quad \Omega \equiv \left\{\Phi(x_{t+1}) + \tfrac{1}{1+\rho} P^{-\alpha}(x_{t+1}, \Upsilon(x_{t+1}))\left[\pi(\Upsilon(x_{t+1})) + (1 - \Xi(x_{t+1})) R(\Upsilon(x_{t+1}))\right]\right\} \tag{B.4}$$

subject to $x_{t+1} = (1 + \bar{r}_t(x_t, T_t)) x_t$ and $T_t = \Upsilon(x_t)$ must be satisfied. If χ is endogenous, we additionally require that $\chi_t = \tilde{\Xi}(x_t) = \Xi(x_t)$ with $\tilde{\Xi}(x_t)$ as explained in Appendix B3.

Starting with an initial guess for the coefficients of the approximations of $\Phi(\cdot)$ and $\Upsilon(\cdot)$ and, possibly, $\Xi(\cdot)$, we evaluate the right side of equation (B.3) for at each node. Using these function values, we obtain new coefficient values for the approximation of $\Phi(\cdot)$. We then use the optimality condition (B.4) to find the values of $\Upsilon(\cdot)$ at the nodes; we use those values to update the coefficients for the approximation of $\Upsilon(\cdot)$. For endogenous χ, the new coefficients for $\Phi(\cdot)$ and $\Upsilon(\cdot)$ also allow the updating of the coefficients for the approximation of $\Xi(\cdot)$. We repeat this iteration until the coefficients' relative difference between iterations falls below 10^{-6}. See chapter 6 of Miranda and Fackler (2002) for details.

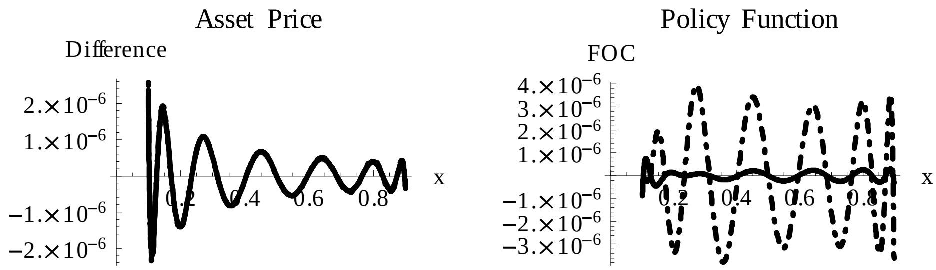

Figure 5: Deviation of asset price (left) and policy function (right) approximation from true value outside of approximation nodes for the efficient bargaining (solid) and the social planner's (dot-dashed) problems

Figures 5 graph the differences (the "residuals") between the right and left sides of equations (B.3) and (B.4), respectively. These residuals equal 0 at the nodes, because we set both the degree of the polynomial and the number of nodes equal to n. We choose $n = 13$ to ensure that residuals are at least 5 orders of magnitudes below the solution values on the $[0.1, 0.9]$ interval.

B5 Robustness checks

We computed two variations as a further robustness check. In the first variation, young agents select the current tax and receive all of the surplus, but have to compensate the old generation to ensure that the latter's welfare does not fall below a default level. This default level equals their welfare under the tax chosen in the previous period. The rationale for this model is that inertia favors the existing tax, and that young agents have to compensate the now-old agents to persuade them to change the tax that the latter chose when they were young. For this experiment we set $\chi = 1$. We find that this variation results in a tax policy very close to, but slightly lower than the policy under the previous formulation with $\chi = 1$. We conclude that our results are not sensitive to changes in χ or to moderate changes in the structure of the political economy model.

In the second variation, motivated by Proposition 5 and the comments following it, $\chi = 0$. Here, the old in the first period to propose a transfer rate ξ. Conditional on this choice, the old and the young each propose a constant tax. Due to inertia, society chooses the smaller of these two taxes. We then confirmed numerically that this tax is time consistent. Future young generations would like to lower the tax and future old generations would like to increase it, but the welfare gain that either achieves is insufficient to compensate the other. Therefore, no proposed change achieves consensus. The belief in the initial period that the tax will be constant is therefore confirmed by the equilibrium. The steady state stock is about 2% higher than in the political economy framework (with $\chi = 0$) and 10% lower than under the social planner.