B Online Appendix for "Regulation with anticipated learning..."

This Supplementary Appendix, referred to as "Appendix B" in "Regulation with anticipated learning about environmental damages", consists of three sections. The first section shows that the subjective distribution for the unknown damage parameter G^* collapses to the true parameter value as $t \to \infty$. The second part describes the calibration outlined in section 5.1 and lists the computer packages that we used to solve the numerical problem. The third section shows that including an explicit inequality constraint on emissions would have no effect on our quantitative results.

B.1 Convergence of the distribution

The difference at the beginning of period t between the subjective expectation of g^* and its true value, $g_t - g^*$, depends on the realization of the sequence of random variables, $\Omega_t \equiv \{\omega_0, \omega_1, ...\omega_{t-1}\}$. Some straightforward but tedious calculations confirm that the expectation and variance at time 0 (with respect to the random sequence Ω_t) of this difference is

$$
\begin{aligned}
E_{\Omega_t}\left(g_t - g^*\right) &= \frac{\left(g_0 - g^*\right)\sigma_\omega^2}{\sigma_\omega^2 + t\sigma_{g,0}^2} \to 0 \text{ as } t \to \infty \\
Var_{\Omega_t}\left(g_t - g^*\right) &= \frac{\sigma_{g,0}^4}{\left(\sigma_\omega^2 + t\sigma_{g,0}^2\right)^2} t\sigma_\omega^2 \to 0 \text{ as } t \to \infty.
\end{aligned}
$$

The mean and the variance of the random variable $g_t - g^*$ asymptotically approach 0. The mean decreases monotonically. The variance might initially increase (if $\sigma_{g,0}^2 < \sigma_\omega^2$) but has a single turning point and thereafter monotonically decreases. From equation (20), $\sigma_{g,t}^2 \to 0$ as $t \to \infty$. These facts and equation (17) imply that the subjective distribution of G converges to the true parameter value G^*.

B.2 Model calibration and numerical methods

We set the length of a period equal to 10 years, using a ten-year discount factor of $\beta = 0.7408$. This discount factor implies an annual discount rate of 3%, a value used in previous studies (Kelly and Kolstad 1999) (Kolstad 1996b) (Nordhaus 1994b). Both costs and damages are measured in billions of 1998 US dollars.

CO_2 emissions and stock. The CO_2 atmospheric stock S_t is measured in billions of tons of carbon equivalent (GtC). The pre-industrial atmospheric stock is about 590GtC as estimated by Neftel, Friedli, Moor, and Lötscher and H. Oeschger and U. Siegenthaler and B. Stauffer (1999) and used in Kelly and Kolstad (1999) and Pizer (1999). We take this level to be the steady state stock given a low level of economic activity. Let e_t be total anthropogenic CO_2 emissions in period t. Approximately 64% of these emissions contribute directly to the atmospheric stock (Kolstad 1996b), (Nordhaus 1994b). Remaining emissions are absorbed by oceanic uptake, other terrestrial sinks, and the carbon cycle (Intergovernmental Panel on Climate Change 1996). The linear approximation of the evolution of atmospheric stocks is

$$S_{t+1} - 590 = \Delta\left(S_t - 590\right) + 0.64e_t.$$

We take $x_t \equiv 0.64e_t$, the anthropogenic fluxes of CO_2 into the atmosphere, as the control variable and rewrite the above equation as

$$S_{t+1} = \Delta S_t + (1 - \Delta)\,590 + x_t. \tag{37}$$

The estimate of the stock persistence is $\Delta = 0.9204$ (an annual decay rate of 0.0083 and a half-life of 83 years) (Kelly and Kolstad 1999) (Kolstad 1996b) (Nordhaus 1994b).

Equation (37), unlike equation (5), includes the constant, $\alpha \equiv (1 - \Delta)\,590$. In order to apply the formulae in Lemma 1 we define $s_t \equiv S_t - \frac{\alpha}{1-\Delta}$ and replace equation (37) with $s_t = \Delta s_{t-1} + x_t$. We then need to write damages as a function of s rather than S. Expected damages equal $\frac{G}{2}\left(s - \bar{s}\right)^2$, with $\bar{s} \equiv \bar{S} - \frac{\alpha}{1-\Delta} = 0$.

Environmental damage. Perhaps the most controversial issue concerns the relation between carbon stocks and environmental damages. Calibration of the damage function requires three parameters, $\overline{S}$ (the stock at which damages are 0), g^*, and σ^2_ω. In addition, we need two state variables, the initial mean and variance g_1 and $\sigma^2_{g,1}$. We set $\overline{S}$ equal to the pre-industrial level of stocks. The choice of the other four variables is less obvious.

As noted in the text, we describe our calibration in terms of the parameter ϕ, defined as the expected percentage reduction of Gross World Product (GWP) due to a doubling of stocks from their pre-industrial level. Nordhaus (1994a) surveys opinions of damages associated with an estimated 3°C warming, a temperature change associated with a doubling of CO_2 stocks. The opinions about ϕ range from 0 to 21 percent of GWP with mean 3.6 and coefficient of variation 1.6 (Table 2 in Roughgarden and Schneider (1999)). Thus, for a point estimate of $\phi = 3.6$, a 95% confidence interval includes damages of approximately 0% to 15% of GWP –

a substantial variation. In order to make our model consistent with this survey, we assume that the coefficient of variation of damages is 1.6.

We use the following formulae for expected damages and the coefficient of variation of damages, which are calculated using the formulae provided in Section 4:

$$E\left[D(S_t,\omega_t;g)|\Omega_t\right]=\frac{1}{2}\exp(g_t+\frac{1}{2}\sigma_{g,t}^2)\left(S_t-\bar{S}\right)^2=\frac{G_t}{2}\left(S_t-\bar{S}\right)^2, \tag{38}$$

$$CV\left[D(S_t,\omega_t;g)|\Omega_t\right]=\left[\exp(\sigma_{g,t}^2+\sigma_\omega^2)-1\right]^{\frac{1}{2}}. \tag{39}$$

There is a simple relation between ϕ and the parameters of our model. The 1998 estimate of GWP is 29,185 billion dollars (International Monetary Fund 1999), for a 10 year estimate of GWP of 291,850. The estimated damages due to doubling of CO_2 stocks during this period is 291,850$\frac{\phi}{100}$. Equating this value to the expected damages given by equation (38) gives us one calibration equation:

$$\begin{array}{c}291,850\phi\frac{1}{100}=\frac{1}{2}\exp(g_1+\frac{1}{2}\sigma_{g,1}^2)\,(590)^2\Longrightarrow\\ 1.676\,8\times10^{-2}\phi=\exp(g_1+\frac{1}{2}\sigma_{g,1}^2)=G_1.\end{array} \tag{40}$$

(We have set the time index $t=1$.) For example, if the regulator's expectation of ϕ is 1.33, we have $1.676\,8\times10^{-2}\,(1.33)=2.230\,1\times10^{-2}=G_1$

We obtain our second calibration equation using the coefficient of variation of damages in Nordhaus' survey and equation (39)

$$CV\,(Damages)=1.6=\left[\exp(\sigma_{1,t}^2+\sigma_\omega^2)-1\right]^{\frac{1}{2}}\Rightarrow3.56=\exp(\sigma_{g,1}^2+\sigma_\omega^2). \tag{41}$$

We need one more assumption to identify the model parameters. We assume that the regulator begins with diffuse priors $\left(\sigma_{g,0}^2=\infty\right)$ and has made one observation, so his posterior variance (using equation (20) is $\sigma_{g,1}^2=\sigma_\omega^2$. Using this equation, we can solve equation (41) to obtain $\sigma_{g,1}^2=\sigma_\omega^2=.634\,88$.

Using this value we can rewrite equation (40) as $g_1=-.317\,44+\ln\left(1.676\,8\times10^{-2}\phi\right)$. Thus, the value of g_1 corresponding to the belief that $\phi=1.33$ and the level of uncertainty $\sigma_{g,1}^2=.634\,88$ is

$$g_1=-.317\,44+\ln\left(1.676\,8\times10^{-2}(1.33)\right)=-4.120\,5.$$

Abatement cost. In order to use a stationary model, we assume that the *expected* BAU level of emissions is equal to the constant $\overline{x}$. We choose the constant $\bar{x}$ so that our model predicts a BAU level of CO_2 stocks of 1500 GtC in 2100, consistent with the IPCC IS92a scenario ((Intergovernmental Panel on Climate Change 1996), page 23). Given the current atmospheric CO_2 concentration $S_0 = 781$GtC ((Keeling and Whorf 1999)), using equation (37) the expected future BAU atmospheric CO_2 concentration is

$$S_t = \Delta^t S_0 + \frac{1-\Delta^t}{1-\Delta}\left[(1-\Delta)\,590 + \overline{x}\right].$$

We choose $\overline{x} = 116.73$ GtC so that the model predicts CO_2 stocks of 1500 GtC in 2100.

We want to choose the slope of abatement costs, b, so that abatement costs in our model are similar to those in Nordhaus (1994a). Nordhaus (1994a) sets abatement costs equal to $A = 0.0686u^{2.887} \times 291,850$, where u is the fractional reduction in CO_2 emissions, relative to the BAU level. We draw 1000 realizations of u from a uniform distribution with support $[0, 0.75]$ (the same support that Nordhaus (1991) used) and calculate A using this formula; we treat the pairs (u, A) as psuedo-observations for a regression. Each value of u implies a level of abatement, $\overline{x} - x = u\overline{x}$, with $\overline{x} = 116.73$.

When $\theta = 0$, our quadratic benefit-of-emissions function is equivalent to a quadratic abatement cost function

$$A = \frac{b}{2}\left(\bar{x} - x_t\right)^2 = \frac{b}{2}\left(u\overline{x}\right)^2.$$

We treat this equation as a regression and we use our pseudo-observations to estimate the parameter b, the slope of marginal benefits The estimated value is $b = 1.9212$ (billion \$/GtC^2). The corresponding estimate of the intercept is $a = b\bar{x} = 224.26$ (billion \$/GtC). The R^2 for this regression is 0.9762, implying that the quadratic function and the function in Nordhaus' formula are very similar, for reductions between 0 and 75% of emissions.

Cost uncertainty. We model cost uncertainty by allowing the actual BAU level of emissions to equal the constant $\overline{x}$ plus a mean-zero random variable $\tilde{\theta}_t = \frac{\theta_t}{b}$. The actual marginal abatement costs are then $b(\overline{x} + \tilde{\theta}_t - x_t)$. That is, the intercept but not the slope of marginal costs are random. We use 13 observations of historical emissions, at ten-year intervals, to estimate a detrended model of emissions, leading to an estimate of σ_θ^2. This parameter is needed to evaluate the magnitude (but not the sign) of the difference in value functions under taxes and quotas. (The difference in value functions is proportional to σ_θ^2). This parameter does not effect the relation between anticipated learning and abatement.

In our model, the cost uncertainty is linearly related to the BAU level of emissions. We used data on actual emissions, e_t, to estimate the variance and autocorrelation of the cost shock.

Using maximum likelihood and data from Marland, Boden, Andres, Brenkert, and Johnston (1999) (total global carbon emissions over every 10 years during the period 1867-1996) we estimated the following model:

$$e_t = e_0 + \kappa t + \varepsilon_t, \qquad \varepsilon_t = \rho \varepsilon_{t-1} + \nu_t, \qquad \nu_t \sim iid\, N\left(0, \sigma_\nu^2\right).$$

(Since we have only 13 observations, we view this procedure as merely a means of calibration.) The estimates are $\rho = 0.96$ and $\sigma_\nu = 4.55$ GtC. We convert the emission uncertainty σ_ν into cost uncertainty σ_θ by multiplying it by 0.64 (because $x_t \equiv 0.64 e_t$), and then by the slope of marginal abatement cost $b = 1.9212$ (because $\theta_t \equiv b\tilde{\theta}_t$). The result is $\sigma_\theta = 4.55 \times 0.64 \times 1.9212 = 5.5945\$/$(ton of carbon).

Numerical methods. We approximate ρ_∞ and μ_∞ as functions of $\left(g, \sigma_g^2\right)$ by solving the fixed point problems in equations (27) and (28) recursively using the collocation method, described in Miranda and Fackler (2002). We apply a third-order (cubic) piecewise polynomial spline to grids on the $\left(g, \sigma_g^2\right)$ plane with 10x10 collocation nodes. The approximation is twice continuously differentiable. We obtain the approximation using the following procedures from the toolbox that accompanies Miranda and Fackler (2002): FUNDEFN, FUNNODE, FUNFITXY, and FUNEVAL. Applying the collocation method using equation (36), we approximate the value function of payoff differences under taxes and quotas

B.3 The inequality constraint

Here we show that the probability that it would ever be optimal to set $x \leq 0$ is negligible, for all reasonable values of the damage parameter. Thus, there is essentially no loss in generality in ignoring the constraint $x \geq 0$, even if we believe that this constraint is reasonable. (For example, we may think that the possibility of sequestration of carbon could never be great enough to offset carbon emissions.)

We use figure 5 to explain how we obtain an upper bound on the probability that it is optimal to set $x \leq 0$. The solid curve labelled $C(0)$ shows the boundary in S, g space at which it is optimal to set $x = 0$ *when there is certainty about the parameter* g^* (i.e., when $\sigma_g^2 = 0$). As noted in the text, the optimal level of emissions (x or z) is a decreasing function of both the stock, S_t, and the current point expectation, G_t (equivalently, g_t). Consequently, the boundary $C(0)$ has a negative slope. Under certainty about g^*, it is optimal to set $x_t > 0$ if and only if (S_t, g_t) lies below the boundary $C(0)$. Our simulations (reported in the text) show that uncertainty about g^* increases the optimal level of emissions. The dashed curve, labelled $C(t)$ shows the boundary in (S, g) space on which it is optimal to set $x_t = 0$ for a given level of uncertainty. The precise location of this boundary depends on the level of uncertainty. However, for our purposes, all that matters is that the boundary $C(t)$ lies above the boundary $C(0)$.

Suppose that we begin at a point (S_0, g_0) shown in figure 5, where it is optimal to have positive emissions if there is no uncertainty about g^*. (The point (S_0, g_0) lies below $C(0)$.) Assume that $S_0 < S_\infty^{BAU}$, the BAU steady state. Assume also that the point $\left(S_\infty^{BAU}, g_0\right)$ (not shown) lies below the boundary $C(0)$. This assumption is true in our model even for values of g well outside the range of current opinions; we return to this point below.

Pick an arbitrary future time $t \leq \infty$; hold this time fixed for the following experiment. In our model, it is never optimal to set emissions above the BAU level. Denote $\bar{S}_t$ as the level of the stock at time t if emissions are set at the BAU level from the current time to time t. Because of the structure of the model, we know that $\bar{S}_t \leq S_\infty^{BAU}$, with strict inequality for $t < \infty$. Given the assumptions in the previous paragraph, the point $\left(\bar{S}_t, g_0\right)$ lies below the boundary $C(0)$, as shown.

When there is uncertainty about g^*, the value of g_t changes over time. In view of the previous comments, a *sufficient* condition for the optimal level of emissions to be positive at time t is that $g_t \leq \bar{g}$, defined as the value of g on the curve $C(0)$, associated with $S = \bar{S}_t$. (See figure 5.) Of course, $\bar{g}$ depends on the initial stock level and the time t (since $\bar{S}_t$ depends on those variables) but it does not depend on the uncertainty parameters. We do not need to use Monte Carlo methods to calculate $\bar{g}$. In order to obtain an upper bound on the probability that it would be optimal to set $x < 0$ we merely need to calculate (using Monte Carlo methods) the probability that $g_t > \bar{g}$.

We now describe the results of our Monte Carlo simulations, expressed in terms of the parameter ϕ rather than g. Recall that ϕ is defined as the percentage reduction in GWP due to a doubling of GHG, and ϕ_t is the subjective belief about this parameter. ϕ_t and G_t are positively linearly related, and thus ϕ_t is a monotonic function of g_t.

For the following experiment, we hold fixed the initial value of the stock at the baseline level, and we vary t. Different values of t imply different levels of $\bar{S}_t$, and thus different values of $\bar{g}$. For each of these values we calculate the corresponding value of ϕ, which we denote as $\bar{\phi}_t$. (These are the values at which it is optimal to set emissions equal to 0.) Figure 6 shows

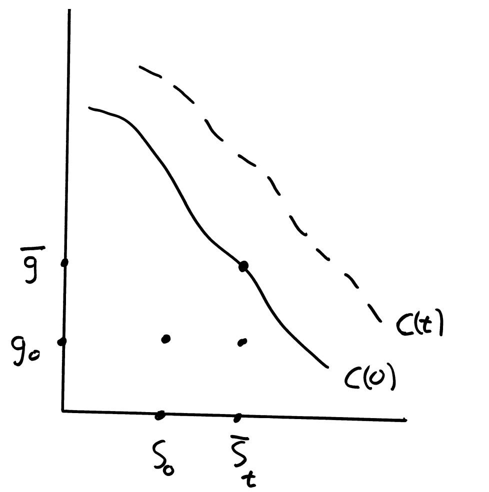

Figure 5: Critical region where emissions are positive

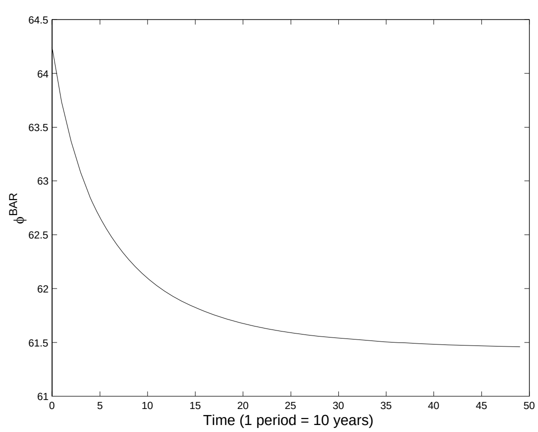

Figure 6: The graph of critical boundary for baseline parameters.

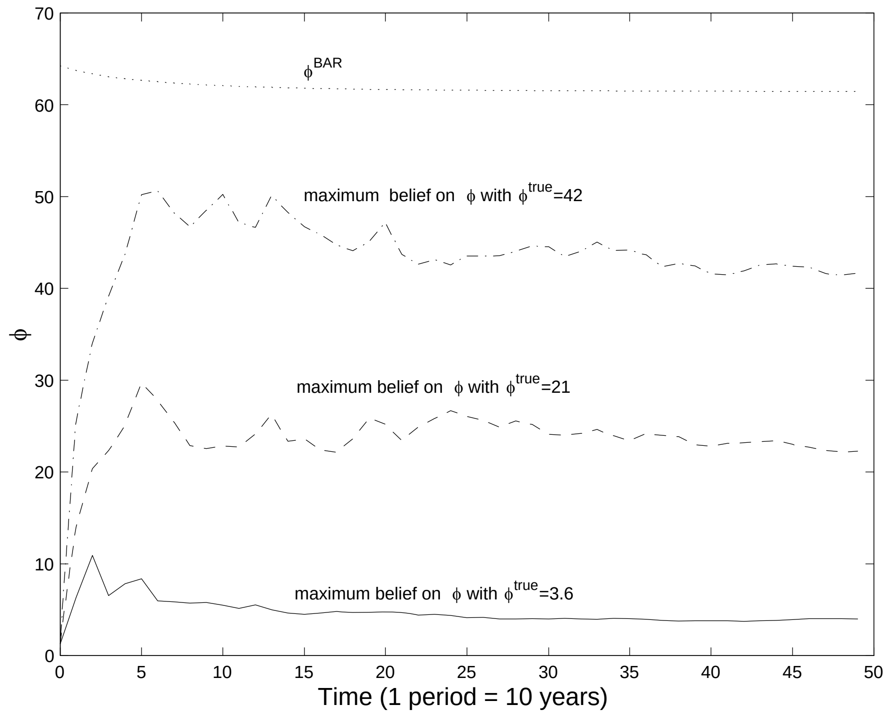

Figure 7: Simulation results

the trajectory of $\bar{\phi}_t$ under our parameterization.

Since the BAU stock asymptotes to a finite level, the graph in Figure 6 is also asymptotic to a finite level. The important point is that this level is in excess of 61. This value is nearly three times the most pessimistic guesstimate of ϕ (equal to 21 in Nordhaus's 1994a survey).

Of course, it is still possible that ϕ_t could exceed $\bar{\phi}_t$. To test this possibility, we ran 1000 simulations, each consisting of 50 periods (500 years). In each of these the initial belief is $\phi_0 = 1.33$. For each set of simulations we chose a different value of the true parameter ϕ^*. For each set of simulations we stored the largest value of ϕ_t in each of the 50 periods. Figure 7 plots these largest values for the three cases $\phi^* = 3.6$, $\phi^* = 21$, $\phi^* = 42$. Even for the extremely unlikely case where $\phi^* = 42$, we have no cases where $\phi_t \geq \bar{\phi}_t$.

We are able to find cases where $\phi_t \geq \bar{\phi}_t$ and thus the constraint $x \geq 0$ *might* be violated, but these cases are wildly outside the range of plausibility, given current evidence. For example, if the true value is $\phi^* = 42$ *and the initial belief is also* $\phi_0 = 42$ (double the most pessimistic opinion), there is only a 7% chance that $\phi_t \geq \bar{\phi}_t$