Conservation capital and sustainable economic growth

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Abstract

An endogenous growth model, which links pollution to ineffective input-use, is developed to examine the potential for achieving balanced growth while preserving the environment through investment in conservation capital. We derive conditions under which individual preferences for environmental quality and private incentives for investment in conservation capital can lead to non-decreasing environmental quality with balanced growth even in the absence of environmental regulations. Additionally, conditions under which investment in conservation capital can enable an environmentally regulated economy to achieve a higher rate of sustainable balanced growth than otherwise are analyzed.

1. Introduction

Growing recognition of the need for sustainable development, defined by some as achieving growth while keeping the stock of natural environmental assets constant (Pearce et al., 1990), has provoked debates about the role of environmental policy and its impact on economic growth. While one view maintains that environmental regulations are needed to protect the environment, others suggest that technological change induced by market forces can protect the environment even in the absence of regulation (Green and Morton 2000). Additionally, while some argue that environmental regulations impede economic growth, proponents of environmental regulation stress that it can stimulate technological innovation which increases input productivity and the efficiency of resource use and can lead to higher rates of economic growth (Porter, 1990; Gardiner and Portney, 1994).

Does sustainable growth inevitably imply lower long run economic growth? Can conservation-oriented technological innovation lead to improvements in environmental quality in the absence of regulations? If not, can regulation designed to preserve environmental quality stimulate technological change without adversely impacting long run growth?

This paper addresses these issues by developing an endogenous growth model that recognizes various ways in which environmental quality and economic activity interact with each other and by analyzing the linkages between environmental policy and the endogenously determined long-run rate of economic growth. This framework recognizes that the environment provides two types of benefits: first, it provides an amenity value which arises because environmental quality affects utility, and second, the stock of environmental quality determines its capacity to assimilate pollution and to regenerate itself. Economic activity also affects the environment in two ways. Ineffective use of inputs increases pollution that degrades the environment. Investment in capital goods augments the effectiveness of input use and, thus, input-productivity and lowers the pollution intensity of inputs; thereby enabling the prevention of

environmental degradation associated with economic activity. Underlying this framework is the premise that pollution is caused because inputs are not used effectively in the process of production, that is, inputs are not converted completely (embodied) into useful output. The less effective the use of inputs the greater the waste which is released into the environment and causes degradation[1].

We refer to capital that can increase the effectiveness (or productivity) of input-use and reduce pollution per unit of input and per unit of output as *conservation capital*. This capital plays a dual role in the production process. It is both *resource-conserving* (input-productivity enhancing) and *pollution-reducing* (input-waste reducing). The notion that some technologies can increase productivity and reduce environmental degradation arises from the observed increase over the last century in the productivity with which resources, such as fossil fuel energy, water and materials, are used and in our ability to recycle and reuse waste, leading to lower carbon intensity per unit of energy used and lower material intensity of production[2] (see Ausubel, 1996; Khanna and Zilberman, 1997). Conservation capital is differentiated from other capital, called *production capital*, which does not affect the pollution intensity of inputs[3]. We investigate whether the productivity-enhancing features of conservation capital create strong enough private incentives for investment and can prevent environmental degradation even without regulation.

Additionally, we examine the impact of environmental regulations on the growth rate.

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[1] For example, ineffective uptake of chemical fertilizers and irrigation water by plants contributes to contaminated run-off from agricultural fields and water quality degradation. Similarly, incomplete conversion (embodiment) of toxic chemical inputs into final manufactured goods leads to polluted waste water streams. For other such examples see Khanna and Zilberman (1997).

[2] For example, technological change has reduced tin and aluminum requirements for manufacturing tinplate and cans and the silver content of film rolls and led to miniaturization of equipment. There are also several technologies for recycling and recovering inputs such as aluminum, mercury and sulfur for re-use as inputs in production, further increasing their productivity and reducing pollution.

[3] The role of environmentally friendly (human) capital that increases the productivity of polluting inputs is also recognized by Bovenberg and Smulders (1995). However they do not distinguish between its dual roles (input productivity-enhancing and pollution-reducing) since they define pollution to be identical to input-use. They also consider human capital to be a pure public good that would not be provided in a pure market economy. In contrast, environmentally friendly (conservation) capital is considered a rival input in this paper that provides both private benefits by augmenting input productivity and public benefits by reducing pollution per unit input use. The decision to invest in it is driven partly by private incentives and partly needs to be induced by environmental regulations.

Environmental regulations can impact growth, in our framework, in two ways. First, they can impact the productivity of production capital by affecting the use of the other inputs such as the polluting input and conservation capital. While the use of the polluting input is likely to decrease, the use of conservation capital and thus effective use of the polluting input is likely to increase. Second, it has the potential to crowd out investment in production capital by affecting the amount of savings and diverting available savings towards conservation capital. The direction of the productivity-effect and crowding-out effects of regulation on growth is ambiguous apriori.

Several studies have incorporated the economy-environment linkage in an endogenous growth framework (see survey by Smulders, 1995a). The framework developed here differs from these studies in several ways. First, in contrast to the common premise that pollution is a direct input to production, this study recognizes that the origins of pollution lie in the waste generated when inputs are not used effectively. A distinction between inputs and pollution allows input-use to increase while pollution may remain constant at sustainable levels[4]. Second, unlike Smulders (1995b) and Bovenberg and Smulders (1995), we consider conservation capital to be rival and excludable in its productive use, embodied in equipment or human skills and a private good (like production capital) acquired through investment by producers[5]. Thus, conservation capital does not

[4] Previous work model pollution to be directly dependent on the level of capital (Huang and Cai, 1994, Smulders and Gradus, 1996), input-use (Bovenberg and de Mooij, 1994, Bovenberg and Smulders, 1995, Smulders, 1995b) or consumption (Hung, Chang and Blackburn, 1993, Verdier, 1993) and that pollution control would require a reduction in these levels (Smulders 1995b, Michel and Rotillon, 1992) or a redirection of resources from productive activities towards clean up through investment in abatement capital (Smulders and Gradus, 1996, Huang and Cai, 1994, Den Butter and Hofkes, 1995). Keeler et al. (1971) and Brock (1977) treated pollution both as an inevitable by-product of production and as an input that contributes positively to production. While Brock does not consider abatement possibilities, Keeler et al. (1971) allow for expenditure on pollution control and show that capital stock and output are higher when there is no expenditure on pollution control. van der Ploeg and Withagen (1991) also relate pollution to output and consider both the negative effects of the stock of pollution and the flow of pollution on utility in the Ramsey model with no technological progress. They show that an emissions tax lowers consumption, capital and output in the steady state but do not examine the impact of environmental regulations on growth.

[5] The rival nature of conservation capital implies that acquisition of conservation capital is consistent with the assumption of perfectly competitive firms operating under constant returns to scale production technology and with real-world observations of private investment in such technologies (see Porter and van der Linde, 1995).

have to be publicly provided and its input-productivity enhancing effect implies that firms have some incentives to invest in it voluntarily even in the absence of environmental policy. Use of conservation capital, however, also reduces pollution that provides non-excludable benefits. This positive externality generated by conservation capital leads to a market failure and a deviation between the privately optimal and the socially optimal level of investment in it.

Third, the mechanism through which environmental regulations impact growth rates in this paper differs from that in Smulders (1995b) and Bovenberg and Smulders (1995) where the stock of environmental quality is included as an input in production. These previous works focus only on the productivity effect of environmental regulation and demonstrate that while a reduction in pollution implies a reduction in input-use and thus, in the productivity of capital and in the growth rate, it also improves the regenerative capacity of the environment and raises the stock of environmental quality which boosts production. They find that the net impact of a pollution control policy on growth can be positive only if its positive impact on environmental quality and regenerative capacity is large and positive. In this paper, the positive impact on productivity arises not from environmental quality improvement, but more directly from an increase in investment in conservation capital.

Further, this paper focuses not only on the productivity effect, but also considers the crowding out effect of environmental regulation. Unlike Rebelo (1991) and Gradus and Smulders (1993) where regulations invariably reduce growth because investment in abatement technologies crowds out resources from more productive uses, we show that this crowding out effect can be mitigated by freeing up resources expended on the polluting input. This is made possible by the productivity-enhancing effect of conservation capital that allows a reduction in the use of the polluting input, allowing for channeling of resources to investment activities.

Our analysis demonstrates that an economy can achieve non-decreasing environmental quality together with balanced growth even in the absence of environmental regulation if there

exists a high output elasticity of conservation capital, a high price of the variable polluting input, a high regeneration rate for environmental quality and relatively non-degraded environmental quality at steady state. However, if these conditions are not met, then a pollution tax or a tax on the polluting input and a subsidy on conservation capital are needed to achieve sustainable balanced growth. A pollution tax can improve economic growth if the preferences for environmental quality relative to those for consumption are strong enough. Strong preferences for environmental quality lower the share of present consumption in output and lower the need for a high tax rate to achieve sustainable balanced growth. This can reduce the negative impact on the polluting input and thus on productivity of capital and mitigate the crowding out effect on investment in production capital.

The next section describes the technology and preference structure of the underlying model. Section 3 discusses socially optimal choices and compares them to those in an unregulated economy. It also derives the policy instruments to achieve sustainable balanced growth. Section 4 investigates the impact of these instruments on growth rates. Section 5 concludes the paper.

2. Theoretical model

2.1. Production and preference structure in the economy

We consider an economy that produces a final output, Y, using a variable input, X, physical capital, K, and conservation capital, H. The effectiveness with which X is used in production (measured by the productivity coefficient of X, $\alpha(H)$) depends upon the stock of conservation capital in the economy. Define $E=\alpha(H)X$ as the "effective" variable input, with $\alpha_H>0$ and $\alpha_{HH}<0$. The production function is specified as:

$$Y=F(\alpha(H)X, K) \text{ ; } F_X>0,\ F_K>0 \text{ and } F_{XX}<0,\ F_{KK}<0 \qquad (1)$$

We assume that both physical capital and conservation capital can be produced one for one

from forgone consumption (as in Smulders, 1995b). An investment, M, has to be undertaken to increase the stock of conservation capital; thus, $\dot{H} = M$ (2)

Investment in production capital $\dot{K}$ is obtained after deducting expenditures on consumption and variable polluting input and on investment on conservation capital from total income,

$$\dot{K} = Y - C - wX - M \tag{3}$$

The variable input X is produced using renewable resources available to households. We assume that X can be produced at a constant marginal cost of w and that the supply of X is infinitely elastic[6].

As the stock of conservation capital increases, and the effective use of input X increases, input-waste per unit input is assumed to decrease. The pollution coefficient of the variable input (pollution generated per unit input), $\gamma(H)$, is an inverse function of the stock of conservation capital. The flow of pollution, P, at any point of time is:

$$P = \gamma(H)X \text{ with } \gamma_H<0 \text{ and } \gamma_{HH}>0. \tag{4}$$

The flow of pollution accumulates over time and decreases the stock of environmental quality, N. Environmental quality is considered to be a renewable resource with a fixed rate of regeneration, R_N, at any instant. The net change in the stock of environmental quality increases with the rate of regeneration, and with the existing level of environmental quality, but falls with pollution level (as in Musu, 1994, Smulders and Gradus, 1996; Huang and Cai, 1994; Xepapadeas, 1994):

$$\dot{N} = R_N N - \gamma(H)X \tag{5}$$

Infinitely lived, identical individuals derive utility from consumption goods, C, and environmental quality, N. A representative agent's time invariant utility function is $U(C, N)$ with $U_c>0$, $U_{cc}<0$, $U_N>0$ and $U_{NN}<0$. <u>For reasons explained below, we assume a Cobb-Doublas utility function which is separable in consumption and environmental quality and with a constant weight</u> ϕ_N attached to environmental quality:

[6] Examples include chemical inputs (fertilizers and industrial chemicals), pulp, irrigation using surface water.

$$U(C,N) = \ln C + \phi_N \ln N \tag{6}$$

With ρ representing the rate of time preference, social welfare is represented by:

$$W = \int_0^\infty e^{-\rho t} U(C,N)\, dt \tag{7}$$

Balanced growth is defined as one where all economic variables grow at a constant rate of growth [redacted] This implies that the allocative variables $s_M=M/Y$, $s_C=C/Y$, and $s_X = wX/Y$ are constant. *Sustainable growth*, on the other hand, is defined as one where the level of environmental quality is maintained fixed at all points in time, implying $g_N=0$ and [redacted]. *Sustainable balanced growth*, therefore, has: $g_N=0$, $\dot{g}_N = 0$ and $g_J = g \; \forall J$.

2.2 Feasibility of sustainable balanced growth

We now examine the conditions under which sustainable balanced growth is feasible. Changes in the rates of growth of production and conservation capital can be specified as:

$$\dot{g}_H / g_H = [g_M - g_H] \tag{8}$$

and

$$\dot{g}_K / g_K = \varepsilon_X g_X + \psi \varepsilon_E g_H + (\varepsilon_K - 1) g_K \tag{9}$$

Balanced growth requires [redacted] and that $g_X=g_H=g_K=g_M$, which implies that

[redacted] where $\varepsilon_H = \psi \varepsilon_E$ (10)

Thus, as in most endogenous growth models, balanced growth is feasible if the production function displays constant returns to scale in X, H and K and the output elasticities ε_X, ε_H and ε_K are constant over time. Balanced growth in the stocks of production and conservation capital and in the use of variable input X then leads to growth in output at the same rate.

For sustainable balanced growth we require g_N to be equal to zero at the sustainable level. This implies that: $\gamma(H)X/N=R_N$ and that $g_N=0$ when $N=\gamma(H)X/R_N$ and $g_X=g_H$ (11)

Differentiating both sides of the expression for N in (11) with respect to time we obtain:

$$\dot{N} = \frac{\gamma_H \dot{H} X}{R_N} + \frac{\gamma \dot{X}}{R_N} = -\xi g_H N + g_X N \text{ where } \xi = -\gamma_H H / \gamma > 0 \tag{12}$$

Thus $g_N=0$ if $-\xi g_H + g_X = 0$, which on a balanced growth path requires $\xi = 1$. (13)

2.3. Optimality of sustainable balanced growth

In addition to these restrictions on the production and pollution functions, restrictions on the preferences of social welfare maximizing economic agents are needed to ensure that sustainable balanced growth is welfare maximizing or socially optimal. Socially optimal choices of C, N, X, H, and K are determined by maximizing welfare in equation (7) subject to the three constraints which define the change in stock of production capital (3), conservation capital (2), and environmental quality (5), with θ_1, θ_2 and θ_3 denoting the multipliers for the three constraints respectively.

Consumers maximize intertemporal utility by choosing a consumption path such that the return on an additional unit of postponed consumption equals the marginal utility of current consumption. This is an extension of the well-known Ramsey rule.

$$\sigma \frac{\dot{C}}{C} - \sigma_{cN} \frac{\dot{N}}{N} + \rho = r \tag{14}$$

The right-hand side, r, denotes the social rate of return on a unit of output invested in capital (in terms of consumption forgone), is the inter-temporal elasticity of substitution of future for present consumption and is the elasticity of marginal utility of consumption with respect to N. Under balanced growth, with C growing and N becoming scarce, the price of C tends to fall relative to that of N creating a substitution effect towards C. But an elasticity of substitution of one between C and N ensures that the substitution towards C from N is exactly offset by the income-effect created by the increase in output under balanced growth, making consistent with balanced growth. For simplicity we assume that $\sigma = 1$ and that $\sigma_{CN} = 0$. The latter ensures that balanced growth is optimal even in the absence of a non-constant g_N. Sustainable balanced growth is therefore optimal when $r = g + \rho$.

The socially optimal level of input-use is determined by equating the marginal product of X to its per unit social cost, which includes its private cost, w, and its pollution costs given by the third

term in (15): (15)

where $\varepsilon_X = F_X \alpha X / Y$ is the output elasticity of the variable input X and s_X is the share of expenditures on X in output and given by wX/Y . The per unit pollution costs of X depend upon its pollution coefficient, γ, and the shadow price ($q_3=\theta_3/\theta_1$) of environmental quality relative to that of consumption goods. The smaller the stock of conservation capital, the greater the pollution coefficient of X and the higher its social costs. In a regulated economy, q_3, represents the tax on pollution that achieves the socially optimal level of environmental quality.

The optimal dynamic allocation of investment between K and H is determined such that the rate of return from both are equal to r as follows:

$$r = \varepsilon_K \frac{Y}{K} = \varepsilon_E \psi \frac{Y}{H} + q_3 \xi \gamma \frac{X}{H} \tag{16}$$

where is the output elasticity of production capital. The net social return from investing in conservation capital arises from two sources. A marginal increase in the stock of conservation capital increases the productivity of X and K. This is captured by which is a measure of the conservation-effect of investment in conservation capital and ε_E the elasticity of output with respect to the effectively used input E. The last term in (16) shows that investment in H generates social returns by reducing γ and lowering the social costs of X (given by $q_3\gamma$).

The desired level of environmental quality is chosen such that the marginal benefits from improving environmental quality normalized by its price plus the absorption capacity and rate of growth of its price, are equal to the rate of return on capital as follows.

$$U_N / q_3 U_C + R_N + \frac{\dot{q}_3}{q_3} = r \tag{17}$$

Socially optimal balanced growth requires while sustainability requires that $\xi=1$. Together these imply that is constant and equal to g (since differentiating (15) with

respect to time, we get: $0 = \dot{q}_3 \gamma X / Y - g q_3 \xi \gamma X / Y \Rightarrow \dot{q}_3 / q_3 = \xi g$) We substitute g for $\boxed{}$ and ρ+g for r (obtained above) in (17) to obtain:

$$q_3 = \frac{U_N}{[\rho - R_N]U_C} \tag{18}$$

For this optimal level of q_3 to grow at the constant rate g, while being consistent with a constant N and with C growing at rate g, it is necessary that U_N/U_c increases at rate g when N is constant and that $U_N N/U_c C = \phi_N$ remain constant under balanced growth preferences. Bovenberg and Smulders (1995) and King, Plosser and Rebelo (1988) show that ϕ_N remains constant under balanced growth if the intra-temporal elasticity of substitution between C and N is unity.

We, therefore, assume the following to ensure that sustainable balanced growth is feasible:

$$\boxed{}; Y = (H^{(1-\mu)} X^{\mu})^{\chi} K^{1-\chi}; \quad \gamma(H) = \frac{1}{H}; \; U(C,N) = \ln \mathrm{C} + \phi_N \ln N \tag{19}$$

The first three functions indicate that an increase in the stock of conservation capital increases the productivity of input X and lowers its pollution coefficient and that there are constant returns to scale between effective input-use and production capital. In the last expression, ϕ_N is constant and represents the weight attached to environmental quality in utility and the intra-temporal elasticity of substitution between C and N is unity. These assumptions imply that:

$$\varepsilon_K = (1-\chi); \varepsilon_X = \mu\chi; \psi = (1-\mu); \varepsilon_E = \chi; \; \xi = 1; \gamma = \frac{1}{\mathrm{H}}; \sigma = 1; \frac{U_N N}{U_c C} = \phi_N; \sigma_{cN} = 0 \tag{20}$$

which satisfy the conditions derived above for sustainable balanced growth to be feasible and optimal. We now use the specifications assumed in (20) to derive the socially optimal choices and compare them to those made in an unregulated economy.

2.4 Socially optimal solution

The socially optimal solution is obtained by maximizing:

$$W = \int_0^{\infty} e^{-\rho t} \ln \mathrm{C} + \phi_N \ln N \, dt \tag{21}$$

subject to: $\dot{K} = (H^{(1-\mu)} X^{\mu})^{\chi} K^{1-\chi} - wX - M - C$; $\dot{H} = M$; and $\dot{N} = \frac{X}{H} - R_N N$ (22)

The first order conditions for social optimality are derived in (A.1)-(A.6) in Appendix A. Using _ to denote $q_3 P/Y$, where $P=X/H$ represents pollution, these conditions reduce to the following 8 equations in 8 unknowns in the sustainable balanced growth steady state (as shown in A.7-A.11). The unknowns are C/Y, Y/K, H/Y N, P, g, r, and _.

$$r = g + \rho \tag{23}$$

$$\frac{wPH}{Y} = \mu\chi - \tau \tag{24}$$

$$r = (1-\chi)\frac{Y}{K} \tag{25}$$

$$r = \frac{\phi_N (C/Y)P}{\tau N} + R_N + g \tag{26}$$

$$r = [(1-\mu)\chi + \tau]\frac{Y}{H} \tag{27}$$

$$\frac{Y}{K} = \left[P^{\mu} \frac{H}{Y} \right]^{\chi/1-\chi} \tag{28}$$

$$g = [1 - \frac{C}{Y} - g\frac{H}{Y} - wP\frac{H}{Y}]\frac{Y}{K} \tag{29}$$

$$P = R_N N \tag{30}$$

Condition (23) implies that the rate of interest must be equivalent to the social discount rate plus the growth rate. For efficient production, it is optimal to choose K and H such that their marginal products are equalized and both are equal to the rate of return r (as shown by (25) and (27)). Further, the rate of return, r also governs the optimal choice of environmental quality (26); the latter depends on the elasticity of the marginal utility of consumption with respect to environmental quality (ϕ_N), the rate of regeneration, the shadow value of environmental quality and the economic growth rate. The pollution tax plays a dual role in this model. It penalizes the use of the pollution-generating input X and raises the social cost of using X (24). It also enhances the social marginal benefit of investing in H due to its pollution-reducing effect and thus induces greater investment in

H than would occur in the absence of the tax (27). The last two conditions (29) and (30) are obtained directly from the first and the third constraint in (22). The equation in (29) represents the goods market equilibrium conditions, which ensures that investment in production capital is equal to the residual savings after investment in *H*, expenditures on consumption and on *X*. The equation in (30) follows from the assumption of sustainable growth, which restricts pollution to be equal to the regenerative capacity of the environment.

We use these expressions to obtain socially optimal shares of C, *X*, and *M* in total output as a function of _ and to solve for *Y/K* as a function of _. We thereby seek to obtain an expression for the sustainable balanced growth rate *g* in (29) as a function of _. We can write $g = [1 - s_C - s_M - s_X]\frac{Y}{K}$ where $s_c = C/Y$, $s_M = gH/Y$, and $s_X = wX/Y$. We can express *Y/K* as a function of _ as follows (as shown by (A.15) to (A.17) in Appendix A):

$$\frac{Y}{K} = \left(\frac{\mu\chi - \tau}{w}\right)^{\frac{\mu\chi}{1-\mu\chi}} \left((1-\mu)\chi + \tau\right)^{\frac{\chi(1-\mu)}{1-\mu\chi}} (1-\chi)^{\frac{(\mu\chi-\chi)}{1-\mu\chi}} \tag{31}$$

The share of consumption to output, $s_C = C/Y$, is obtained from (2.26) after substituting for *P* from (30) and for *r* from (23). We then obtain: $s_C = \frac{\tau(\rho - R_N)}{\phi_N R_N}$ (32)

which implies that $(\rho - R_N)$ has to be positive to ensure that the output share of consumption is positive. Differentiating this with respect to _ we obtain: $\frac{ds_C}{d\tau} = \frac{(\rho - R_N)}{\phi_N R_N} > 0$ (33)

This implies that an increase in _, which increases the marginal social cost of pollution per unit output relative to the price of consumption, leads to an increase in the share of consumption in output. This occurs because the marginal benefits of *N* fall as τ and, therefore, *N* increase, hence, consumption must increase to ensure that the marginal returns from improving environmental

quality are equal to the rate of return from capital which is fixed along a balanced growth path (as shown by A.6 in Appendix A). The extent to which this occurs is inversely related to ϕ_N.

Using (27) and substituting for r from (25) we obtain the following expression for the output share of conservation capital investment, as shown by (A.19)-(A.21) in Appendix A.

$$s_M = [(1-\mu)\chi + \tau)]\frac{g}{r} = [(1-\mu)\chi + \tau)] - \{\rho[(1-\mu)\chi + \tau)]^{\frac{(1-\chi)}{1-\mu\chi}} w^{\frac{\mu\chi}{1-\mu\chi}} (\mu\chi - \tau)^{\frac{-\mu\chi}{1-\mu\chi}} (1-\chi)^{\frac{-(1-\chi)}{1-\mu\chi}} \} \qquad (34)$$

Differentiating (34) with respect to _ we find that s_M increases as _ increases (shown in A.22):

$$\frac{ds_M}{d\tau} = 1 + \rho[(1-\mu)\chi + \tau)]^{\frac{(\mu\chi-\chi)}{1-\mu\chi}} w^{\frac{\mu\chi}{1-\mu\chi}} \{\chi(\mu\chi - \tau) + \tau\}(1-\chi)^{-\frac{(1-\chi)}{1-\mu\chi}} (1-\mu\chi)^{-1} > 0 \qquad (35)$$

The share of X to total output, s_X, is directly obtained from (24) as

$$s_X = \mu\chi - \tau \qquad (36)$$

The output share of variable input is increases with $\mu\chi$ and falls with τ, since $ds_x/d_ = -1$.

2.5 Privately optimal solution

In a decentralized economy, a representative consumer maximizes utility over an infinite horizon (as given in (21)) by choosing consumption subject to an inter-temporal budget constraint. This would yield $\frac{\dot{C}}{C} = g = r - \rho$ where r is the market interest rate. This is similar to the condition that determines socially optimal consumption decisions as given by (23).

Each representative firm maximizes profits by choosing the level of production capital (K) and conservation capital (H) and the level of the variable input (X) that it uses. In the absence of any regulations, firms disregard the social costs of using X and the pollution that they generate as well as the social benefits of using H. The firm's privately optimal solution is obtained by maximizing profits, _, assuming that output price is normalized to 1.

$$Max\ \Pi = \left(H^{(1-\mu)} X^{\mu}\right)^{\chi} K^{1-\chi} - wX - M$$

$$\text{subject to: } \dot{K} = (H^{(1-\mu)} X^{\mu})^{\chi} K^{1-\chi} - wX - M - C;\ \dot{H} = M\ . \qquad (35)$$

This yields the following conditions:

$$\frac{wPH}{Y} = \mu\chi \tag{36}$$

$$r = (1-\chi)\tfrac{Y}{K} \tag{37}$$

$$r = [(1-\mu)\chi]\tfrac{Y}{H} \tag{38}$$

$$\frac{Y}{K} = \left[P^{\mu}\frac{H}{Y}\right]^{\chi/1-\chi} = (\frac{\mu}{w})^{\frac{\mu\chi}{1-\mu\chi}}(1-\mu)^{\frac{\chi(1-\mu)}{1-\mu\chi}}\chi^{\frac{\chi}{1-\mu\chi}}(1-\chi)^{\frac{(\mu\chi-\chi)}{1-\mu\chi}} \tag{39}$$

$$g = [1-\frac{C}{Y}-g\frac{H}{Y}-wP\frac{H}{Y}]\frac{Y}{K} \tag{40}$$

The marginal condition (37) determining the level of K to employ is the same as in the socially optimal solution (25). The output capital ratio in (39) is derived from (31) by setting _=*0*. The marginal conditions determining the privately optimal and socially optimal levels of X and H are different. The privately optimal share of X in total output is now s_X = __ (41)

It increases as the output elasticity of X increases. The privately optimal share of M is obtained by setting _=*0* in the socially optimal s_M in (33):

$$s_M = [(1-\mu)\chi]\frac{g}{r} = [(1-\mu)\chi]\left\{1-\frac{\rho w^{\mu\chi}\mu\chi^{\mu\chi}(1-\chi)^{(1-\chi)}}{[(1-\mu)\chi)]^{\chi(1-\mu)}}\right\}^{\frac{1}{1-\mu\chi}} \tag{42}$$

It increases as the output elasticity of H (given by *(1-_)χ))* increases and decreases as _ and w increase. As _ increases, incentives for present consumption instead of investment in either K or H increase while an increase in w reduces incentives to use X and therefore to use conservation capital which increases the productivity of X. Since s_M has been shown to increase with _, it implies that it will be higher in the socially optimal solution.

The share of consumption in output is derived in (A.21) in Appendix A as follows:

$$s_C = \frac{\rho(1-\mu\chi)}{(\frac{\mu\chi}{w})^{\frac{\mu\chi}{1-\mu\chi}}((1-\mu)\chi)^{\frac{\chi(1-\mu)}{1-\mu\chi}}(1-\chi)^{\frac{(1-\chi)}{1-\mu\chi}}} \tag{43}$$

This increases as _ increases and households become more impatient. The term in the denominator is simply equal to r as shown by (A.22) in Appendix A, implying that s_C falls as r increases. The term in the denominator is also an increasing function of the output elasticities of the inputs, X, H and K. As these elasticities increase, implying that output produced with given inputs increases, and as (1-_χ) in the numerator increases, (implying that the combined output elasticity of H and K increases relative to that of X), the share of consumption in output decreases and output is likely to be diverted towards investment in capital.

The change in environmental quality in this unregulated economy with balanced growth in the steady state is derived as follows (see A.23):

$$\dot{N} = R_N N - \frac{X}{H} = R_N N - \frac{\mu r}{(1-\mu)w} = R_N N - \left(\frac{\mu\chi(1-\chi)^{1-\chi}}{((1-\mu)\chi)^{1-\chi} w} \right)^{\frac{1}{1-\mu\chi}} \tag{44}$$

This implies that:

$$\dot{N} \geq 0 \text{ if } \mathrm{N} \geq \frac{\left(\frac{\mu\chi(1-\chi)^{1-\chi}}{((1-\mu)\chi)^{1-\chi} w} \right)^{\frac{1}{1-\mu\chi}}}{R_N} \tag{45}$$

This expression shows that environmental quality is more likely to be preserved or even improved in the steady state under an unregulated regime if $\mu\chi$ and $(1-\chi)$ are low while R_N, w and N are high. A low output elasticity of the polluting input ($\mu\chi$) and high w both discourage use of input X, while a low output elasticity of production capital ($1-\chi$) discourages use of K. A high $(1-_)\chi$ lowers pollution by encouraging usage of H, increasing the likelihood that environmental quality is increasing. Additionally, the conditions that the natural rate of regeneration and the steady state level of N must be high imply that in the absence of regulation, private choices of firms can ensure environmental quality improvement only if the environment is not highly degraded. If the conditions in (45) do not ensure that N is preserved at the socially optimal level in the balanced growth steady state then the regulator would need environmental policy instruments to achieve

sustainable balanced growth.

2.6 Stability analysis

The stability of the equilibrium is derived by linearizing the first-order conditions (23-30) and reducing them to the following two differential equations using methods described in Smulders (1995b)[7]:

$$\dot{\tilde{N}} = R_N \tilde{N} - \frac{P}{N}\tilde{P} \tag{46}$$

$$\dot{\tilde{P}} = \frac{Z}{\Psi}\tilde{P} \tag{47}$$

where $Z = (1-\chi)(\mu\chi - \tau)(r-\rho)\left(\frac{\mu\chi w((1-\mu)\chi+\tau) - (1-\mu\chi+\tau)}{1-\mu\chi+\tau}\right)$ and

$$\Psi = \frac{r\mu^2\chi^2 w^2((1-\mu)\chi+\tau)(1-\mu\chi+\tau) - w(1-\chi)(\mu\chi-\tau)^2[\mu\chi w((1-\mu)\chi+\tau) - (1-\mu\chi+\tau)]}{((1-\mu)\chi+\tau)w(1-\mu\chi+\tau)(\mu\chi-\tau)}.$$

We find that $\psi > 0$ and Z<0 if $\frac{(1-\mu\chi+\tau)}{((1-\mu)\chi+\tau)} > w\mu\chi$. This is always the case since the left hand side is $\frac{(1-\mu\chi+\tau)}{(\chi-\mu\chi+\tau)} > 1$ and if $w_\chi < 1$. A sufficient case for the latter is $w \leq 1$, since _ and χ are each less than 1. The determinant of the Jacobian formed by the differential equations (46) and (47) and given by $R_N \frac{Z}{\Psi}$ is, therefore, negative. This implies that a saddle point exists when w is low. This is the case even when $\tau=0$. For the rest of the analysis we focus on the case where w is less than one. This is also the more interesting case to examine, since a high w is more likely to preclude the need for any environmental regulation.

3. Policy instruments to achieve sustainable balanced growth

We examine alternative policy instruments that could achieve the socially optimal levels of X, H, K and P and sustainable balanced growth. The first one is a pollution tax with the tax rate q_3 (normalized by the price of output) set at the level determined from the social planner's problem to

achieve sustainable balanced growth with $P=R_N N$. The firm's maximization problem is now:

$$Max\ \Pi = \left(H^{(1-\mu)}X^{\mu}\right)^{\chi} K^{1-\chi} - wX - M - q_3 \tfrac{X}{H}$$

$$\text{subject to: } \dot{K} = (H^{(1-\mu)}X^{\mu})^{\chi} K^{1-\chi} - wX - M - C\,;\ \dot{H} = M\,. \tag{48}$$

The marginal conditions determining K would be the same as in (37). The tax would result in the following marginal conditions for choosing X and H, similar to those under social optimality.

$$\chi\mu \tfrac{Y}{X} - w - q_3 \tfrac{1}{H} = 0 \qquad \Rightarrow \qquad \frac{wPH}{Y} = \mu\chi - \tau \tag{49}$$

$$(1-\mu)\tfrac{Y}{H}\chi - q_3 \tfrac{X}{H^2} = \rho\theta_2 - \dot{\theta}_2 \ \Rightarrow \quad r = \left[(1-\mu)\chi + \tau\right]\tfrac{Y}{H} \tag{50}$$

To examine the factors determining the level of τ we derive the following expression for τ as an implicit function of the production and preference parameters as shown in (B.1-B.3).

$$\frac{\tau(\mu\chi-\tau)^{\frac{\mu\chi}{1-\mu\chi}}\left((1-\mu)\chi+\tau\right)^{\frac{\chi(1-\mu)}{1-\mu\chi}}}{1-\mu\chi+\tau} = \frac{\rho\phi_N R_N w^{\frac{\mu\chi}{1-\mu\chi}}}{(\rho-R_N)(1-\chi)^{\frac{(1-\chi)}{1-\mu\chi}}} \tag{51}$$

We do comparative statics to examine the effects of parameters such as ϕ_N, _, R_N, and w and evaluate them at $\tau=0$, as shown in (B.5-B.8) in Appendix B. These comparative statics show that the optimal _ increases with the weight attached to environmental quality, with the natural rate of regeneration, with the private marginal cost of the polluting input, w and falls with the social rate of discount. As the natural rate of regeneration increases, the marginal benefit from improving environmental quality (see (17)) increases. This implies that socially optimal levels of H should be high while those of X should be low; both of which can be achieved by a higher _. A high private marginal cost of the polluting input implies that the private optimal level of X is relatively low even without regulation. For regulation to reduce X further below this low level and increase investment in H, the optimal _ must be higher than in the case where w is relatively low. Further, when the social discount rate is high, the social planner is more myopic and concerned about present

[7] The details of the linearized model and the stability analysis are available from the authors upon request.

consumption than about investment in K and H and also chooses a lower N. The optimal tax rate in this case will be lower.

A comparison of (24) and (36) and (27) and (38) also indicate that in a decentralized economy, the firm disregards the effect of its input use on pollution. In particular it ignores the negative externality generated by the use of X and the positive externality generated by the use of H. We therefore examine if a two-part policy composed of a per-unit input tax and a per-unit subsidy on conservation capital investment would achieve social optimality. The firm's maximization problem can now be written as:

$$Max\ \Pi = \left(H^{(1-\mu)}X^{\mu}\right)^{\chi} K^{1-\chi} - wX - M - \tau_X X + s_H H$$

subject to: $\dot{K} = (H^{(1-\mu)}X^{\mu})^{\chi} K^{1-\chi} - wX - M - C$; $\dot{H} = M$. (52)

This would lead to the following marginal conditions determining the levels of X and H:

$$\chi\mu \tfrac{Y}{X} - w - \tau_X = 0 \tag{53}$$

$$(1-\mu)\tfrac{Y}{H}\chi + s_H = \rho\theta_2 - \dot{\theta}_2 \tag{54}$$

These conditions can be made equivalent to those in (36) and (38) by setting the input tax rate and the conservation capital subsidy rate as follows:

$$\tau_X X = s_H H = \tau Y \tag{55}$$

where X and H are set at the socially optimal levels. This will ensure that private decisions to use X (and H) will now incorporate its pollution-generating (-reducing) effect and the firm does not only equate the private marginal benefits to the private marginal costs of using X (and H) but instead equates it with the private *plus* social marginal cost (benefit) of using that input. These subsidy and tax rates also ensure a balanced budget because total tax revenues will be equal to total subsidy payments. Thus, this two-instrument scheme confers a zero net benefit to the government, unlike a pollution tax policy that would generate revenues for the government.

4. Impact of environmental policy on sustainable growth

We now examine whether it is possible for government intervention, through environmental regulations, to lead to a higher rate of economic growth than would have been possible in the absence of regulation. The goods market equilibrium condition in (29) implies that the growth rate is determined by *Y/K* and by the output shares of *X*, *M* and *C* as follows:

$$g = \frac{\dot{K}}{K} = [1 - \frac{C}{Y} - \frac{M}{Y} - \frac{wX}{Y}]\frac{Y}{K} = [1 - s_c - s_M - s_x]\frac{Y}{K} \tag{56}$$

where s_C, s_M and s_X are as defined in (32, 33 and 34) respectively and *Y/K* is as defined in (31). In a balanced growth steady state each of these is constant. To examine the impact of an increase in τ on the steady state growth rate we take the partial derivative of (56) with respect to τ and examine the conditions under which *g* increases with environmental stringency. In other words, we are comparing two steady state balanced growth rates when one state has a marginally higher _. The impact of a more stringent environmental policy on growth can be expressed as a function of several components. It consists of a productivity effect, a consumption effect, an input use effect and a conservation effect. We refer to the sum of the last three effects as the crowding out effect since they influence the amount of savings left for investment in production capital.

$$\frac{dg}{d\tau} = \overbrace{(1 - s_X - s_C - s_M)\frac{\partial Y/K}{\partial \tau}}^{Productivity-effect} - \overbrace{\frac{Y}{K}\left\{ \underbrace{\left(\frac{\partial s_C}{\partial \tau}\right)}_{Consumption-effect} + \underbrace{\left(\frac{\partial s_X}{\partial \tau}\right)}_{Input-effect} + \underbrace{\left(\frac{\partial s_M}{\partial \tau}\right)}_{Conservation-effect} \right\}}^{Crowding-out-effect} \tag{57}$$

The productivity effect due to the imposition of a tax could arise for two reasons as can be seen by expressing the output-capital ratio, *Y/K*, as a function of pollution and of the share of output invested in conservation capital (as shown in (A.17) in Appendix A). The pollution tax has a

negative impact on pollution and the use of the polluting input and this lowers the productivity of production capital. However, the tax raises the share of output invested in conservation capital which increases effective input use and thus the productivity of production capital. By substituting in the expressions for *P* and *H/Y* in terms of _ we obtain the expression for *Y/K* as a function of _ in (31). We differentiate that with respect to _ to obtain the following:

$$\frac{\partial \frac{Y}{K}}{\partial \tau} = \frac{(Y/K)\chi[\mu\{(1-\mu)\chi+\tau)\}(1-w)-\tau]}{(1-\mu\chi)(\mu\chi-\tau)((1-\mu)\chi+\tau)} \underset{<}{\geq} 0 \quad \text{if} \quad \tau \underset{>}{\leq} \frac{\mu\chi(1-\mu)(1-w)}{1-\mu(1-w)} \qquad (58)$$

The net impact of a pollution tax on *Y/K* (that is the productivity effect) is ambiguous and depends on the output share of the tax, the cost of the polluting input, *w*, as well as the on the productivity of the effective input, χ. While an increase in _ would reduce the use of *X* it increases investment in *H* which is also a productive input. Hence, as long as _ is low enough to ensure that the negative impact on *X* is low, the productivity effect is positive. The extent to which _ needs to be low varies directly with *w*, the cost of the polluting input, and inversely with the productivity of *X*. For the case where *w<1* and a saddle point exists (as shown in Section 2.6) we find that the lower *w* is, the higher _ can be and still result in a positive productivity effect. As *w* increases, the _ that can be accompanied by a positive productivity effect is lower. This is reasonable, because *w* and _ operate in the same direction in their effect on *X*. If both are high, it would reduce the use of the polluting excessively and this would have a negative effect on the productivity of *K*.

This result is in contrast to that obtained by Bovenberg and Smulders (1995) who find that environmental regulations would always lower the productivity of capital (and the rate of growth). While they model pollution as a direct input to production, they do not include the productivity enhancing effect of *H*. Additionally, they only focus only on the productivity effect of regulation on growth, which constitutes only part of the effect of the tax on growth rate, as given by the first term in (57). We also need to examine the crowding out impact of the tax on savings and on firm's

decision to invest in production capital. As shown above an increase in _ increases consumption and investment in H, and this will have a negative impact on the growth rate because it reduces savings available for investing in K. This crowding out effect may be offset to some extent by the reduction in the expenditures on X that occurs in response to an increase in _. To analyze the net effect of the tax on growth after taking into account both the productivity and the crowding out effects, we express g as an explicit function of _ after substituting in the expressions in (31) for Y/K and in (32) for s_C, and the sum of s_M and s_X from (34) and (36):

$$g = \left[1 - \frac{\tau(\rho - R_N)}{\phi_N R_N} - \chi + \frac{\rho[(1-\mu)\chi + \tau)]^{\frac{(1-\chi)}{1-\mu\chi}}}{\left(\frac{\mu\chi - \tau}{w}\right)^{\frac{\mu\chi}{1-\mu\chi}} (1-\chi)^{\frac{(1-\chi)}{1-\mu\chi}}} \right] \left(\frac{\mu\chi - \tau}{w}\right)^{\frac{\mu\chi}{1-\mu\chi}} [(1-\mu)\chi + \tau]^{\frac{\chi(1-\mu)}{1-\mu\chi}} (1-\chi)^{\frac{(\mu\chi-\chi)}{1-\mu\chi}} \tag{59}$$

Differentiating g with respect to τ and evaluating the derivative at _=0 we obtain:

$$\left.\frac{dg}{d\tau}\right|_{\tau=0} \gtreqless 0 \text{ if } \phi_N \gtreqless \frac{(\rho - R_N)(1-\mu\chi)(\mu\chi / w)^{\frac{\mu\chi}{1-\mu\chi}} ((1-\mu)\chi)^{\frac{\chi(1-\mu)}{1-\mu\chi}} (1-\chi)^{\frac{(1-\chi)}{1-\mu\chi}}}{R_N(1-w)(1-\chi) + \rho(1-\mu\chi)} \tag{60}$$

The expression in (60) shows that a marginal increase in _, starting from an unregulated equilibrium has a positive effect on the growth rate if ϕ_N is large. The extent to which ϕ_N needs to be large varies directly with w and _ and inversely with R_N. These results suggest that a pollution tax can have a positive impact on growth if the weight attached to environmental quality in the utility function is high enough, so that the effect of a tax on the share of output spent on consumption is small and if the discount rate is low enough to make a larger investement in H optimal. Additionally, the effect of small tax would be positive if the regenerative rate of environmental quality is high because that would allow the sustainable level of pollution to be high.

The main implications of these results are two-fold: First, even with the availability of conservation capital, which is both productivity-enhancing and pollution reducing, environmental regulations can lower the productivity of production capital because they reduce the use of the

polluting input. Second, despite the lower use of the polluting input which frees up some resources, environmental regulations can crowd out investment in production capital and lower the growth rate. Conditions under which environmental regulations have a positive impact on the sustainable balanced growth rate include a high weight attached to environmental quality, a low discount rate, a low cost of polluting input and a high regenerative rate of the environment.

5. Conclusions

This paper develops an endogenous growth model, in which consumers derive utility from private consumer goods and a stock of environmental quality that is modeled as a renewable resource. The production process is defined to link pollution to ineffective input-use, which can be reduced through conservation capital investment. Conservation capital is considered to be a private, rival good that is productivity-enhancing and also provides public pollution-reducing benefits. Since conservation capital is rival and productivity enhancing, there exist private incentives to invest in it. However, since producers are likely to disregard the public benefits of investing in conservation capital, they will under invest in it relative to the socially optimal level. Private incentives for investment in conservation capital can allow an unregulated economy to achieve sustainable balanced growth if the cost of the polluting input is high and the output elasticity of conservation capital is high while that of production capital and the polluting input are low.

In the absence of strong private incentives for achieving a sustainable level of pollution, environmental regulations such as a pollution tax, with the price of pollution growing at the same rate as the other economic variables in the long run, is needed to achieve sustainable balanced growth. The emissions tax is also shown to be equivalent to a two-instrument scheme composed of a tax on polluting input and a subsidy on conservation capital investment. The key question that is addressed here is the impact of environmental regulation on the long run growth rate.

We show that environmental regulations have a productivity effect and a crowding out effect

on the growth rate. The productivity effect arises because environmental regulations increase the share of the output invested in conservation capital but reduce the share of the polluting input in output. While the former tends to increase the productivity of production capital, the latter lowers it. Additionally, environmental regulations have a crowding out effect by increasing the share of consumption goods in output and diverting savings to conservation capital instead of to production capital. However, since environmental regulations lower the share of output spent on the polluting input X, this may reduce, or even offset, the crowding out effect. We find that the net impact of environmental regulations on the sustainable balanced growth rate could be positive under restrictive conditions which include a low tax rate, strong preferences for environmental quality, a high output elasticity of production and conservation capital, a low output elasticity of the polluting input and a high cost of the polluting input. These conditions mitigate the crowding out effect and the adverse impacts on output of reducing the use of the polluting input while providing resources for investment in capital that stimulates growth while preserving the environment.

The analysis in this paper also shows that the assumptions required for the feasibility of balanced growth paths are fairly restrictive. Further research is needed to examine the impact of environmental policy on growth under more general assumptions about preferences and production functions. Another extension of this paper is to analyze the transitional dynamics of the model and to examine the short run effects of environmental regulations on growth rates and consumption.

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APPENDIX 1

We express the social planner's optimization problem using a current value Hamiltonian using $__1$, $__2$ and $__3$ as the multipliers for the three constraints and obtain the following first order conditions:

$$\mathcal{H} = \boxed{} + \theta_1 / \boxed{} + \theta_2 \boxed{}\boxed{}$$

$$\mathcal{H}_C = 0 \quad \Rightarrow \quad \frac{1}{C} - \theta_1 = 0 \qquad \text{(A.1)}$$

$$\mathcal{H}_M = 0 \quad \Rightarrow \quad -\theta_1 + \theta_2 = 0 \qquad \text{(A.2)}$$

$$\mathcal{H}_X = 0 \Rightarrow \quad \theta_1 [\chi\mu \frac{Y}{X} - w] - \theta_3 \frac{1}{H} = 0 \qquad \text{(A.3)}$$

$$\mathcal{H}_K = 0 \Rightarrow \quad \theta_1 (1-\chi) \frac{Y}{K} = \rho\theta_1 - \dot{\theta}_1 \qquad \text{(A.4)}$$

$$\mathcal{H}_H = 0 \Rightarrow \quad \theta_1 (1-\mu) \frac{Y}{H} \chi - \theta_3 \frac{X}{H^2} = \rho\theta_2 - \dot{\theta}_2 \qquad \text{(A.5)}$$

$$\mathcal{H}_N = 0 \Rightarrow \quad \frac{\phi_N}{N} + \theta_3 R_N = \rho\theta_3 - \dot{\theta}_3 \qquad \text{(A.6)}$$

Optimality conditions under steady state:

In the steady state, using g to denote balanced growth rate for the economic variables, and assuming that N is constant, these conditions can be rearranged to obtain the following:

Using A.1 and A.4 we obtain the Ramsey rule: $g + \rho = r$ (A.7)

Using s_x to denote wX/Y, we can write A.3 as $\mu\chi - s_X = \tau$ (A.8)

This can also be written as $\mu\chi - \tau = wX / Y = wPH / Y$ (A.9)

From A.4 we obtain: $(1-\chi)\frac{Y}{K} = \rho + g = r$ (A.10)

For optimal N we use (A.6) and divide by (A.1) to obtain: $\frac{\phi_N C}{N} + q_3 R_N = \rho q_3 - \frac{\dot{\theta}_3}{\theta_1}$ (A.11)

Note that $\frac{\dot{\theta}_3}{\theta_1} = \dot{q}_3 + \frac{\dot{\theta}_1}{\theta_1}\frac{\theta_3}{\theta_1}$ and $\frac{\dot{\theta}_1}{\theta_1} = -g$ from (A.1) and $\frac{\dot{\theta}_3}{\theta_3} = g$ from Section 2.3. Substituting these into (A.11) and dividing by q_3 we get: $\frac{\phi_N C}{N q_3} + R_N = \rho = r - g$ since $\frac{\dot{q}_3}{q_3} + \frac{\dot{\theta}_1}{\theta_1} = g - g = 0$ (A.12)

Multiplying and dividing the first term by P/Y, (A.12) can be written in terms of $_ = q_3 P/Y$:

$$\frac{\phi_N P(C/Y)}{N\tau} + R_N + g = r \qquad \text{(A.13)}$$

Dividing (A.5) by $__1$ and noting that $__1 = __2$ and $\frac{\dot{\theta}_2}{\theta_2} = g$ from (A.2) we obtain: $r = [\chi(1-\mu) + \tau]\frac{Y}{H}$ (A.14)

The output-capital ratio:

To obtain Y/K in terms of P and H/Y we can rearrange the production function as follows:

$$Y = [H^{(1-\mu)} X^{\mu}]^{\chi} K^{(1-\chi)} \Rightarrow Y = \left[H \left(\frac{X}{H}\right)^{\mu} \right]^{\chi} K^{(1-\chi)} \Rightarrow \left(\frac{Y}{K}\right)^{1-\chi} = \left[\left(\frac{X}{H}\right)^{\mu} \frac{H}{Y} \right]^{\chi} \qquad \text{(A.15)}$$

To solve for Y/K in terms of $_$, we use (A.9) and (A.14) to obtain

$$\frac{X}{H} = \frac{X}{Y}\frac{Y}{H} = \frac{\mu\chi - \tau}{(1-\mu)\chi + \tau}\frac{r}{w} \qquad \text{(A.16)}$$

Substituting for X/H and H/Y in (A.15) we obtain:

$$\frac{Y}{K} = P^{\frac{\mu\chi}{1-\chi}} \left(\frac{s_M}{g}\right)^{\frac{\chi}{1-\chi}} = \left(\frac{\mu\chi-\tau}{(1-\mu)\chi+\tau}\frac{r}{w}\right)^{\frac{\mu\chi}{1-\chi}} \left(\frac{(1-\mu)\chi+\tau}{r}\right)^{\frac{\chi}{1-\chi}} \tag{A.17}$$

Substituting for *r=(1-_)Y/K* and rearranging we obtain the expression for *Y/K* in (2.31)

The output share of conservation capital investment:

We solve for s_M in terms of _ as follows:

$$s_M = \frac{\dot{H}}{Y}\frac{H}{H} = \frac{H}{Y}g = \frac{((1-\mu)\chi+\tau)(r-\rho)}{r} = \frac{((1-\mu)\chi+\tau)}{1} - \frac{((1-\mu)\chi+\tau)\rho}{r} \tag{A.18}$$

We use (2.31) and the equation *r=(1-_)Y/K* to obtain the following expression for *r* in terms of _:

$$r=\left(\frac{\mu\chi-\tau}{w}\right)^{\frac{\mu\chi}{1-\mu\chi}} ((1-\mu)\chi+\tau)^{\frac{\chi(1-\mu)}{1-\mu\chi}} (1-\chi)^{\frac{(1-\chi)}{1-\mu\chi}} \tag{A.19}$$

Substituting for *r* in (A.18) we obtain the expression for s_M in (2.34).

The privately optimal solution:

The consumption share is obtained using the goods market equilibrium:

$$g = \frac{Y}{K}(1 - s_X - s_M - s_C) \Rightarrow s_C = (1 - s_X - s_M) - g\frac{K}{Y} \tag{A.20}$$

Substituting in the expressions in (2.37), (2.41), and (2.42), we have:

$$s_C = \left(1 - \mu\chi - \frac{(r-\rho)(1-\mu)\chi}{r}\right) - g\frac{(1-\chi)}{r} \quad \Rightarrow \quad s_C = \frac{\rho(1-\mu\chi)}{r} \tag{A.21}$$

where *r* is as defined below by using (2.37) to eliminate *Y/K* from the expression on the left in (2.39):

$$r = \left(\frac{\mu\chi}{w}\right)^{\frac{\mu\chi}{1-\mu\chi}} ((1-\mu)\chi)^{\frac{\chi(1-\mu)}{1-\mu\chi}} (1-\chi)^{\frac{(1-\chi)}{1-\mu\chi}} \tag{A.22}$$

To solve for the privately optimal level of pollution in (2.44) we express *P* as follows and substitute for *r* from (A.23) to obtain (2.44) after simplification and rearrangement of terms.

$$P=\frac{X}{H}=\frac{wXYM}{wHYM}=\frac{s_x g}{ws_m}=\frac{\mu\chi gr}{w(1-\mu)\chi g}=\frac{\mu r}{(1-\mu)w} \tag{A.23}$$

APPENDIX B

We obtain an expression for _ by equating the expression for $s_C=\frac{\tau(\rho-R_N)}{\phi_N R_N}$ obtained in (2.32) to the following expression for s_c that can be obtained from (2.29) after substituting for s_M, and s_X and *g* from (2.33), (2.34) and (2.23): $s_C=1-\mu\chi+\tau-\frac{(r-\rho)[(1-\mu)\chi+\tau]}{r}-\frac{(r-\rho)(1-\chi)}{r}=\frac{\rho[1-\mu\chi+\tau]}{r}$ (B.1)

Equating (B.1) with (2.32) and eliminating s_c we obtain $s_C=\frac{\tau(\rho-R_N)}{\phi_N R_N}=\frac{\rho[1-\mu\chi+\tau]}{r}$ (B.2)

Solving for *r*, and equating with the expression for *r* in (A.20) yields:

$$r = \left(\frac{\mu\chi-\tau}{w}\right)^{\frac{\mu\chi}{1-\mu\chi}} ((1-\mu)\chi+\tau)^{\frac{\chi(1-\mu)}{1-\mu\chi}} (1-\chi)^{\frac{(1-\chi)}{1-\mu\chi}} = \frac{\rho\phi_N R_N[1-\mu\chi+\tau]}{\tau(\rho-R_N)} \tag{B.3}$$

This yields the implicit function for _ given in (3.5). Differentiating (3.5) this with respect to τ and other parameters we obtain the following. A sufficient condition for these expressions to have the sign indicated is that __*<1/2*, that is the share of *X* in output in an unregulated economy be less than _. In the following expressions, *A*>0 is equal to the terms on the left hand side of (3.5).

$$\frac{d\tau}{d\phi_N}=\frac{\rho R_N w^{\frac{\mu\chi}{1-\mu\chi}}[1-\mu\chi]}{(\rho-R_N)(1-\chi)^{\frac{(1-\chi)}{1-\mu\chi}}A\chi[\frac{(1-2\mu\chi)+\mu(\mu\chi-\tau)+\tau(1-\tau)}{\tau((1-\mu)\chi+\tau)}]}>0 \tag{B.4}$$

$$\left.\frac{d\tau}{dR_N}\right|_{\tau=0}=\frac{[(\rho-R_N)(1-\chi)^{\frac{(1-\chi)}{1-\mu\chi}}]\rho\phi_N w^{\frac{\mu\chi}{1-\mu\chi}}+\rho\phi_N R_N w^{\frac{\mu\chi}{1-\mu\chi}}(1-\chi)^{\frac{(1-\chi)}{1-\mu\chi}}}{[(\rho-R_N)(1-\chi)^{\frac{(1-\chi)}{1-\mu\chi}}]^2 A\chi[\frac{(1-2\mu\chi)+\mu(\mu\chi-\tau)+\tau(1-\tau)}{\tau((1-\mu)\chi+\tau)}]}>0 \tag{B.5}$$

$$\left.\frac{d\tau}{d\rho}\right|_{\tau=0}=\frac{[(-R_N)(1-\chi)^{\frac{(1-\chi)}{1-\mu\chi}}]\phi_N R_N w^{\frac{\mu\chi}{1-\mu\chi}}}{[(\rho-R_N)(1-\chi)^{\frac{(1-\chi)}{1-\mu\chi}}]^2 A\chi[\frac{(1-2\mu\chi)+\mu(\mu\chi-\tau)+\tau(1-\tau)}{\tau((1-\mu)\chi+\tau)}]}<0 \tag{B.6}$$

$$\left.\frac{d\tau}{dw}\right|_{\tau=0}=\frac{\mu\chi}{1-\mu\chi}\frac{\rho\phi_N R_N w^{\frac{\mu\chi}{1-\mu\chi}}}{w[(\rho-R_N)(1-\chi)^{\frac{(1-\chi)}{1-\mu\chi}}]A\chi[\frac{(1-2\mu\chi)+\mu(\mu\chi-\tau)+\tau(1-\tau)}{\tau((1-\mu)\chi+\tau)}]}>0 \tag{B.7}$$