1.

$$Max_{\{q_i\}}\ \text{NPV} = \frac{1}{(1+\text{r})^{\text{i}}}\left(B(q_i) - C(q_i)\right) \text{subject to } \sum_{i=0}^{1} q_i = S_0 \ \text{ for i} = 0{,}1$$

$$Max_{\{q_i, l\}}\ \text{L} = \frac{1}{(1+\text{r})^{\text{i}}}\left(B(q_i) - C(q_i)\right) + l\left(\text{S}_0 - \sum_{i=0}^{1} q_i\right)$$

F.O.C

$$\begin{aligned}
\frac{\partial L}{\partial q_0} &= B'(q_0) - C'(q_0) - l = 0 \\
\frac{\partial L}{\partial q_1} &= \left(\frac{1}{1+r}\right)(B'(q_1) - C'(q_1)) - l = 0 \qquad (1) \\
\frac{\partial L}{\partial l} &= S_0 - q_0 - q_1 = 0
\end{aligned}$$

From which we get the following important inter-temporal equilibrium conditions

$$\begin{aligned}
B'(q_0) - C'(q_0) &= \left(\frac{1}{1+r}\right)B'(q_1) - C'(q_1) \qquad (2) \\
q_0 + q_1 &= S_0
\end{aligned}$$

Perfect Competition

In perfect competition $P_i = B'(q_i)$ for $i = 0{,}1$

Since we know that $P_i = 1000 - .5q_i$ for $i = 0{,}1$, $C'(q_i) = 400$ for $i = 0{,}1$, and $r = 10\%$, then (2) becomes

$$\begin{aligned}
600 - 0.5q_0 &= \left(\frac{1}{1.1}\right)(600 - 0.5q_1) \qquad (3) \\
q_0 + q_1 &= 1000
\end{aligned}$$

The solution to the above simultaneous system of equation is

$$\begin{aligned}
q_0{*} &= 533.33 \\
q_1{*} &= 466.66
\end{aligned}$$

To find the market price in each period just substitute q into the demand equations.

$$\begin{aligned}
P_0 &= 1000 - 0.5(q_0{*}) = 1000 - .5(533.33) = 733.33 \\
P_1 &= 1000 - 0.5(q_1{*}) = 1000 - .5(466.66) = 766.66
\end{aligned}$$

To find l use the first equation of the F.O.C system (1)

$$l = B'(q_0{*}) - C'(q_0{*}) = P_0 - C'(q_0{*}) = p_0 - C'q_0) = 733.33 - 400 = 333.33$$

Monopolist

Under the monopolist the marginal benefit is not equal to the price.

Competitive Market:

B(q) = pq (price is given)
B'(q) = p

Monopolist:

B(q) =p(q)q (price is affected by monopolist's output)
B'(q) = p(q) + p'(q)q

Therefore, in this case we have the following:

$$B'(q_i) = 1000 - q_i \text{ for } i = 1,2$$
$$C'(q_i) = 400 \text{ for } i = 1,2$$
$$r = 10\%$$

So (2) becomes

$$600 - q_0 = \left(\frac{1}{1.1}\right)(600 - q_1) \qquad (4)$$
$$q_0 + q_1 = 1000$$

The solution to the above simultaneous system of equation is

$$q_0^m = 504.76$$
$$q_1^m = 495.23$$

To find the market price in each period just substitute q into the <u>demand</u> equations.

$$P_0 = 1000 - 0.5(q_0^m) = 1000 - .5(504.76) = 747.62$$
$$P_1 = 1000 - 0.5(q_1^m) = 1000 - .5(495.23) = 752.38$$

To find l^m use the first equation of the F.O.C system (1)

$$l^m = B'(q_0*) - C'(q_0*) = 1000 - q_0^m - C'(q_0^m) = 1000 - 504.76 - 400 = 95.23$$

Open Access

In an open access situation anyone who could derive a positive profit out of extraction will extract the resource form the ground. There is no way of preventing anyone from doing this. Consequently, nobody really cares about the future. All what each individual cares is to extract as long as there is a positive profit to be made. Hence, l =0, there is no user cost because no one gives much importance to the future, it is like having an infinite discount rate.

The equilibrium condition is real simple, produce while profits are positive and stop when there are no more profits.

$p = B(q_{oa}) - C(q_{o\,a}) = 0$

In Perfect Competition:

$$p_0 = P_0 q_{oa} - C(q_{o\,a}) = 0 \rightarrow \mathrm{P}_0 = \mathrm{C'(q)}$$
$$1000 - .5\mathrm{q}_0 = 400$$
$$\mathrm{q}_0 = 1200$$

But you can’t extract 1,200 units since $S_0 = 1000$. Therefore, The open access extraction is $qoa = 1000$.

The open access equilibrium condition for first period is

$$P_0 - C'(q_0) = 0 \qquad (5)$$

You will recall that the equilibrium condition for the optimal first period extraction under competition is

$$P_0 - C'(q_0) = l \qquad (6)$$

If under open access we impose a tax equal to l , then we would achieve optimality.

<u>Question 1 Answer Summary</u>

a) $q_0{*} = 533.33, q_1{*} = 466.66, P_0 = 733.33, P_1 = 766.66$

b) $q_0{}^m = 504.76, q_1{}^m = 495.23, P_0{}^m = 747.62, P_1{}^m = 752.38$

c) $l = 333.33, l^m = 95.23$. The user cost is higher under competition because competition will extract more in the present and leave less for the future. Therefore, one unit in the future becomes more valuable (because there will be less of it). Under the monopolist, however, current extraction is lower and more is left for the future, so one extra unit in the future is not as valuable than under competition

d) Open access equilibrium is q =1000. Obviously, this is not optimal because the optimal extraction in the present is q =533.33

e) Optimality is achieved when we set a tax equal to $l = 333.33$.

2.

a) $TB_1 = \int_0^9 MB_1(q)dq = \int_0^9 (20 - q_1)dq_1 = 20q_1 - 0.5q_1^2\Big|_0^9 = 139.5$

$TB_2 = \int_0^9 MB_2(q)dq = \int_0^9 (20 - 2q_2)dq_2 = 20q_2 - q_2^2\Big|_0^9 = 99$

$TotalBenefits = TB = TB_1 + TB_2 = 139.5 + 99 = 238.5$

b) There will be a total of 18 permits. Permits will be traded until the marginal benefits of each of the firms are equated. Thus, combining these two facts, we have that

$MB_1(q_1) = MB_2(q_2)$
$20 - q_1 = 20 - 2q_2$
$20 - q_1 = 20 - 2(18 - q_1)$ (since $q_2 = 18 - q_1$)
$q_1^* = 12$
$q_2^* = 18 - q_1^* = 18 - 12 = 6$

Firm 1 will optimally generate 12 units of pollution, and firm 2 will generate 6 units of pollution. Therefore, firm 1 will buy 3 permits from firm 2. The price of the permits is determined by the marginal benefit.

$P = MB_1(q_1^*) = MB_2(q_2^*)$
$P = 20 - 12$
$P = 8$

c) Gains from trade.

<u>Firm 1</u> (Bought 3 permits)

Gain from polluting more = $\int_9^{12} MB_1(q)dq = \int_9^{12} (20 - q_1)dq_1 = 20q_1 - 0.5q_1^2\Big|_9^{12} = 28.5$
Cost of permits = 3x8 = 24
Net gains from trade = 28.5-24 = 4.5

<u>Firm 2</u> (Sold 3 permits)

Gain = 3x8 = 24

Cost from polluting less = $\int_9^6 MB_2(q)dq = \int_9^6 (20 - 2q_2)dq_2 = 20q_2 - q_2^2\Big|_9^6 = -15$
Net gains from trade = 24 – 15 =9

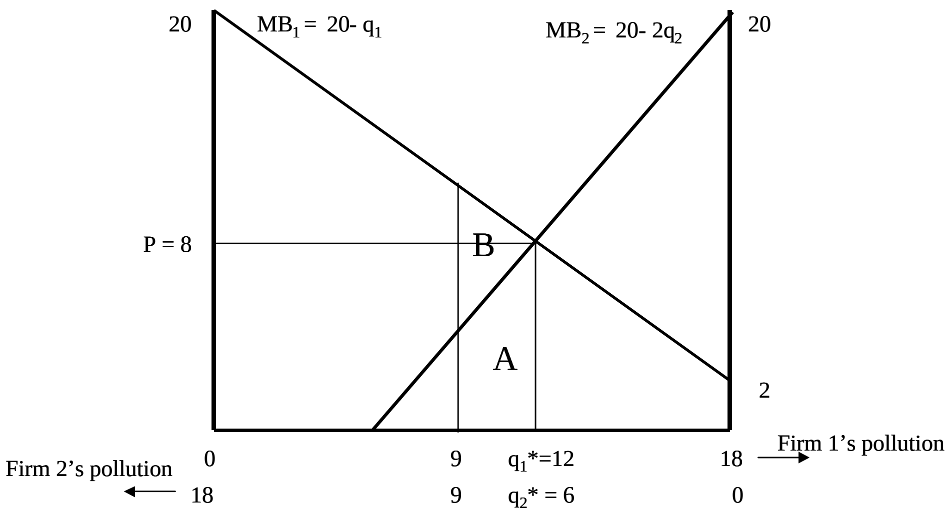
Total net gains from trade = 4.5 + 9 = 13.5

Another way to view this is to look at the total benefits derived by the two firms under the permit system.

$TB = TB_1 + TB_2$

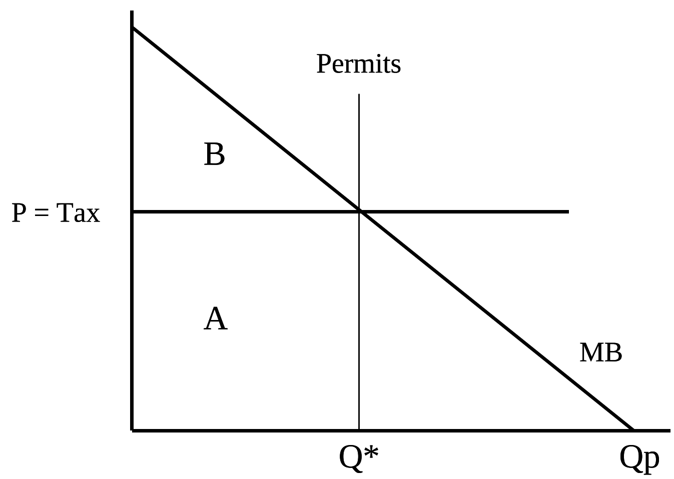
$TB = \int_0^{12} (20 - 0.5q_1)dq_1 + \int_0^6 (20 - 2q_2)dq_2 = 168 + 84 = 252$

The difference between the total benefits under the permit system (optimal) and the benefits under the uniform-standard system (sub-optimal) is precisely equal to 252-238.5 = 13.5. In other words, the deadweight loss of the uniform-standard is DWL = 13.5, which is precisely the efficiency gains that you achieve by moving to the optimal system.



Society's benefit from moving from uniform standard to permits = Area A+B
Society's cost from moving from uniform standard to permits = Area A
Net gains = Area B = 0.5 (11 - 2)(3) = 13.5

d) Yes, a tax of $8 per unit of pollution will achieve the optimal levels because each firm will equate its marginal benefit to the tax. Hence, the marginal benefits of the two firms will be equated. This is the same condition that we had under the permit system. The industry will clearly prefer the permit system because the rents (the value of the permits) will remain within the industry. In a tax system, these rents will be transferred to the government as revenues from the tax.



If the social optimum, Q*, is achieved through a tax, then the government receives area A in revenues and the firm is left with a surplus equal to area B. If on the other hand Q* is achieved through a permit system, then the firm will still enjoy a surplus of area B, but it will also have Q* permits that are worth P dollars each. Hence, the firm will enjoy a rent equal to area A, which would have otherwise gone to the government in a tax system. The overall surplus of the firm is area A+B. The firm (and the industry) will prefer permits to taxes.