## Key to PROBLEM SET 2

1. There are three polluting firms in the economy with marginal benefit curves given by: $\mathrm{MB}_{1}=75$ $(5 / 4) q_{1}$ for firm $1, \mathrm{MB}_{2}=75-\mathrm{q}_{2}$ for firm 2, and $\mathrm{MB}_{3}=75-(3 / 4) \mathrm{q}_{3}$ where $\mathrm{q}(\mathrm{i}=1,2,3)$ is the amount of emissions produced by each firm
a) The MB curve tracks the aggregate level of emissions. At any level of valuation $\left(\mathrm{MB}_{1}=\mathrm{MB}_{2}\right.$ $=\mathrm{MB}_{3}=\mathrm{MB}$ ), aggregate emissions can be obtained by summing horizontally the three firms' curves. Analytically, you need to find the inverse of the MB equations

$$
\begin{gathered}
\mathrm{q}_{1}=60-4 / 5 \mathrm{MB} ; \mathrm{q}_{2}=75-\mathrm{MB} ; \text { and } \mathrm{q}_{3}=100-4 / 3 \mathrm{MB} \text {, and sum quantities: } \\
\mathrm{q}_{1}+\mathrm{q}_{2}+\mathrm{q}_{3}=\mathrm{Q}=235-(47 / 15) \mathrm{MB} ; \text { or inversely, } \mathrm{MB}=75-(15 / 47) \mathrm{Q}=75-0.32 \mathrm{Q}
\end{gathered}
$$

(decimals are rounded off) Your graph should look like the following:

b) Remember that " $q_{i}$ " does not represent the output of each firm, but instead the quantity of pollution it emits. Firm 1 is more efficient in reducing that pollution. Intuitively, it is less dependent on emitting pollution than others to create profits (think of emissions as inputs). Alternatively, reducing emissions would cost it the least.
c) The optimal level of emissions ( $\mathrm{Q}^{*}$ ) can be obtained by equating the marginal social cost curve with the aggregate MB curve: $\mathrm{MSC}=30+\mathrm{Q}=\mathrm{MB}=75-(15 / 47) \mathrm{Q}$, which implies $Q^{*}=34.1$ (and MB* $=64.1$ ).
d) Since marginal benefits of emissions are already net of marginal costs, the optimal tax on emissions is simply equal to the aggregate MB at the optimum quantity, which is $\mathrm{MB}^{*}=64.1$. To find out the amount of emissions produced by each firm ( $\mathrm{q}_{1}{ }^{*}, \mathrm{q}_{2}{ }^{*}$, and $\mathrm{q}_{3} *$ ) given the tax, simply substitute $\mathrm{MB}^{*}=\operatorname{tax}=64.1$ into the equations you calculated in part a. On the graph, those $\mathrm{q}_{\mathrm{i}}{ }^{*}$ quantities (8.7, 10.9, and 14.5, respectively) are at the intersections of the horizontal line running from the equilibrium point $B$ to the left and each of the three $M B_{i}$ curves.
e) Instead of a tax, government may decide to use a uniform (non-transferable) standard equal to $Q^{*} / 3=34.1 / 3=11.4$. Notice that at that level, each firm values marginal emission differently: those $\mathrm{MB}_{\mathrm{i}}(11.4)$ values $(60.8,63.6$, and 66.5 , respectively) are at the intersection of the vertical line above $\mathrm{Q}^{*} / 3=11.4$ and each of the three $\mathrm{MB}_{\mathrm{i}}$ curves.
f) With the non-transferable permits, firms 1 and 2 are now emitting more than they would choose to when charged unit tax as in part d. Firm 3 is emitting less.
g) First of all, "gains from trade" does not refer to revenues and expenses from the sale or purchase of permits, but rather to the net increase in benefits that result from such trades.

It can be shown mathematically that individual emissions ( $\mathrm{q}_{\mathrm{i}}{ }^{*}$ ) would be identical whether government introduces a system of transferable permits (where each firm is given permits for Q*/3 units of emissions) or a tax on emissions. In the latter case, each firm would be maximizing its own benefits minus tax payments $\left[B\left(q_{i}\right)-\left(q_{i} t\right)\right]$. First-order conditions give us three equations $\left(\mathrm{MB}_{\mathrm{i}}-\mathrm{t}=0\right.$, one per firm), and we also know that the tax " t " has been set such that emissions sum to the optimal level: $q_{1}+q_{2}+q_{3}=Q^{*}=34.1$. That represents four equations in four unknowns ( $\mathrm{q}_{1}, \mathrm{q}_{2}, \mathrm{q}_{3}$, and t$)$.

If there is instead a transferable permit system, each firm is maximizing its own benefits plus permit revenues $\left[B\left(q_{i}\right)+v\left(Q^{*} / 3-q_{i}\right)\right]$. Think of " $v$ " as the market value of permits traded among firms. For firms buying permits $\left(Q^{*} / 3<q_{i}\right)$, revenues will be negative---i.e. expenses. First order conditions are three equations of the form $\mathrm{MB}_{\mathrm{i}}-\mathrm{v}=0$, and the total amount of permits must still be equal to $Q^{*}$. Now substitute " $t$ " for " $v$ ", and you have the exact same equations in both alternatives! Solutions for $\mathrm{q}_{1}, \mathrm{q}_{2}$, and $\mathrm{q}_{3}$ must be identical, and the optimal market value $\mathrm{v}^{*}$ must be equal to optimal tax $\mathrm{t}^{*}$.

The numbers you found in part $d$ are still valid: $q_{1}=8.7, q_{2}=10.9$, and $q_{3}=14.5$. If we assume competitive markets for permits, they would be traded at $v^{*}=64.1=t^{*}$, the optimal level of tax of part d.

The gains of trade can be found by summing benefits when firms are restricted to $q_{i}=Q^{*} / 3=$ 11.4 (part e), and comparing to the optimal situation when permits are traded. Benefits for each firm are given by the areas under the $\mathrm{MB}_{\mathrm{i}}$ curves between 0 and the quantity actually emitted, $\mathrm{q}_{\mathrm{i}}$. In the non-transferable permits scheme, they are all producing $\mathrm{Q}^{*} / 3=11.4$, so we need to sum the three trapezoids $(1 / 2)\left[75+\mathrm{MB}_{\mathrm{i}}(11.4)\right]$ (11.4). That amounts to $2,364.6$.

If the permits are transferable, each firm produces up to the point where $q_{i}=v^{*}$, so we sum the trapezoids $(1 / 2)(75+64.1)$ [ $\left.\mathrm{q}_{\mathrm{i}}(64.1)\right]$. That amounts to $2,373.3$. The "gains to trade" are given by the increase in benefits (welfare) allowed by trading: 2,373.3-2,364.6 = 8.7. (of course, 8.7 is relatively small, but that is simply a function of the numbers we have cooked up, not a reflection of actual potential gains to trade)

You could also have calculated directly the areas of the "little triangles", which represent differences between permit expenses/revenues and actual benefit of emitting more/less. Using the formula area $\mathrm{a}_{\mathrm{i}}=\left|64.1-\mathrm{MB}_{\mathrm{i}}(11.4)\right|(1 / 2) 11.4-\mathrm{q}_{\mathrm{i}}(64.1) \mid$ for each firm, you get $4.5,0.1$, and 3.7 , respectively, which sums up to 8.3 (close enough considering all the rounding off)
2. "Motor vehicles, power plants, factories, chemical solvents, combustion products from various fuels, and some consumer products release nitrogen oxide ("NOX") and volatile organic compound gases ("VOCs") that react with sunlight to produce ground-level ozone. Ground-level ozone is the primary ingredient in smog and can have harmful effects on human health and the environment.

In choosing between incentive-based instruments (taxes) and direct control measures (standards) to reduce ozone-causing pollution, what considerations would influence the EPA's decision in favor of one type of policy or the other? Be sure to describe the welfare implications under the two types of policies."

As usual, we concentrate on efficiency rather than on equity. Not because it is more important, but because we have more to say about it (politicians should feel free to redistribute income once the most efficient policy has been put in place. They usually don't, but that is another story.) If the world were a perfect, simple place, taxes and standards would enable the EPA to achieve the optimal amount of smogcausing pollutants with equal efficiency. A standard would increase producers' surplus, leaving the government treasury unaffected. A tax would reduce producers' surplus, but improve the Treasury's position, which may allow a reduction of distortionary taxes elsewhere. Consumers' surplus is reduced in either case.

The world, however, is not such a simple place. Assuming there is no Coasian solution to this problem - because of the large number of people affected, the heterogeneity of the polluters and the physical difficulty in determining the perpetrators - there still remain important considerations. They include:

- Uncertainty (Weitzman)

When there is uncertainty, the tax/standard the government sets is likely to be suboptimal, resulting in too much or too little pollution and hence some deadweight loss. Whether standards are preferred to taxes depends on the relative slope of the uncertain marginal benefit and cost curves. If the marginal benefit curve is relatively more elastic, then standards lead to a lower deadweight loss. Intuitively, imposing a price-distorting measure like a tax if demand is price-sensitive may not be a great idea. If the marginal benefit curve is relatively inelastic, taxes result in a lower social loss and are the preferred tool.

- Heterogeneity of the abatement costs

Given all the different possible sources of ozone-causing pollution, this is likely to apply. If firms have different pollution-control costs, it makes sense to have the firms that can most efficiently reduce pollution do so before asking less efficient firms. This will minimize the social costs of controlling pollution. A uniform standard in which all firms are allowed to pollute equally (which amounts to a non-transferable permits scheme) does not recognize this principle. Taxes would be preferred to standards if firms have different marginal costs of controlling pollution. Notice that combining the standard with tradable permits achieves the same result if the market for these permits is efficient and no transaction costs are involved in buying and selling permits.

- Dynamic efficiency (clean technologies and conservation technologies)

At a given point in time, there are a number of production technologies available. These technologies give firms different marginal benefits from being able to pollute. Cleaner technologies allow firms to reduce their pollution, but in the absence of environmental policies, firms have no reason to adopt such technologies. Under a standard, firms have some reason to adopt since their costs of achieving the set standard will be reduced. Under the tax, not only would they reduce costs of attaining the desired level of emissions, they also reduce their tax bill.

- Market structure

Returning to the perfect world we had in the introduction, we notice that the argument presented there assumed a perfectly competitive market. If the market is not perfectly competitive, it is no longer obvious that a tax is the appropriate instrument. The optimal tax may be a negative tax, i.e., a subsidy. It depends on whether the market (monopoly, monopsony, middle-man) is producing too much or too little. If the optimal policy is a subsidy, then firms in the industry realize extra profits. This may entice new entrants (entry may or may not be possible) and result in too many firms producing too much output and too much pollution. The optimal standard will be just as effective here as it was above and there is some justification for a standard over an incentive-based system.

Less formal considerations relate to the enforcement and monitoring costs of the two policies and whether efficiency is the goal rather than political-economic factors related to employment and prices. Suppose firms are heterogeneous not only in terms of their marginal abatement costs, but also in terms of their laborintensity and their pollution-intensity per unit of output. Then, the Hochman-Zilberman model shows that a standard that achieves the same level of aggregate pollution as a tax will have a higher level of output and more labor-intensive firms will remain in the industry. The higher level of output implies lower prices of the produced good (Supply is higher, so we move down the demand curve. Price must fall to encourage people to buy the extra output)

