

Introduction to auctions

What is an Auction?

1. A public sale in which property or merchandise are sold to the highest bidder.
2. A market institution with explicit rules determining resource allocation and prices on the basis of bids from participants.
3. Games: The bidding in bridge, for example.

Examples of Auctions

■FCC Spectrum

McMillan, 1994, Selling Spectrum Rights, JEP.

<http://www.paulklempere.org>

■Procurement Auctions

■Treasury Bills

■Internet

■Wine Options

■Quota Rights, Auctioning countermeasures in WTO

Working paper, Bagwell K., Staiger R., et al:

<http://www.ssc.wisc.edu/~rstaiger/auctionation071803.pdf>

Lots of good theory and empirical work.

- Game is simple with well defined rules
- Actions are observed directly
- Payoffs can sometimes be inferred

Also, a lot of data

- Government sales:
 - Timber rights, mineral rights, oil and gas, treasury bills, spectrum auctions, emission permits, electricity
- Government sales:
 - Defense, construction, school milk
- Private sector:
 - Auctions houses, agriculture, real estate, used cars, machinery
- Online auctions:

Many possible mechanisms

- Open versus sealed
- First price versus second price
- Secret versus fixed reserve price

Several Formats:

4 auction types:

- First-price sealed-bid auction: you don't see your opponents' bids. Highest bid wins. Winner pays her bid, b . The winner's profit is: $v - b$. Losers get nothing.
- Second-price sealed-bid auction: you don't see your opponents' bids. Highest bid wins. Winner pays the second highest bid in the auction. Therefore the winner's profit is: v minus the second highest bid. Losers get nothing.
- English, or Ascending-price drop-out auction: Sort of like the game of chicken. Price starts ticking upwards from 0 until... wherever. You're all paying the price until you decide to drop out! As soon as the second-to-last person drops out, the auction ends. Winner's profit is v minus the price at which the second-to-last guy dropped out.
- Dutch, or Descending price auction: Game of chicken – in reverse! Price starts at \$11, ticks down until somebody says "I'll take it." The winner's profit is v minus the price at which she called "I take it."

		Bidding Process	
		Open	Closed
Price Determination	First price	Dutch	FPSB (*)
	Second Price	English (*)	SPSB Vickrey auction

Legend:

- FPSB: first price sealed bid
- SPSB: second price sealed bid
- (*) empirical work

Most often studies are about FPSB or English Auctions. Recent E-bay studies are an exception. These auctions are like an English auction with a time constraint. These are often modeled as Second Price Sealed Bid.

- ⇔ strategically equivalent (under certain circumstances)

Note: The researcher can also look at variations with multi-unit auctions and or sequential auctions.

English Auction Ascending Bid

- Bidders call out prices (outcry)
- Auctioneer calls out prices (silent)
- Highest bidder gets the object
- Pays a bit over the next highest bid

Sealed Bid

- Bidders submit secret bids to the auctioneer
- The auctioneer determines the highest and second highest bids
- Highest bidder gets the object
- Pays the next highest bid
(Second-Price Vickrey)
- Pays own bid (First-Price)

Dutch (Tulip) Auction Descending Bid

- “Price Clock” ticks down the price
- First bidder to “buzz in” and stop the clock is the winner
- Pays price on clock

- Dutch Auction ~ First Price
- English Auction ~ Second Price

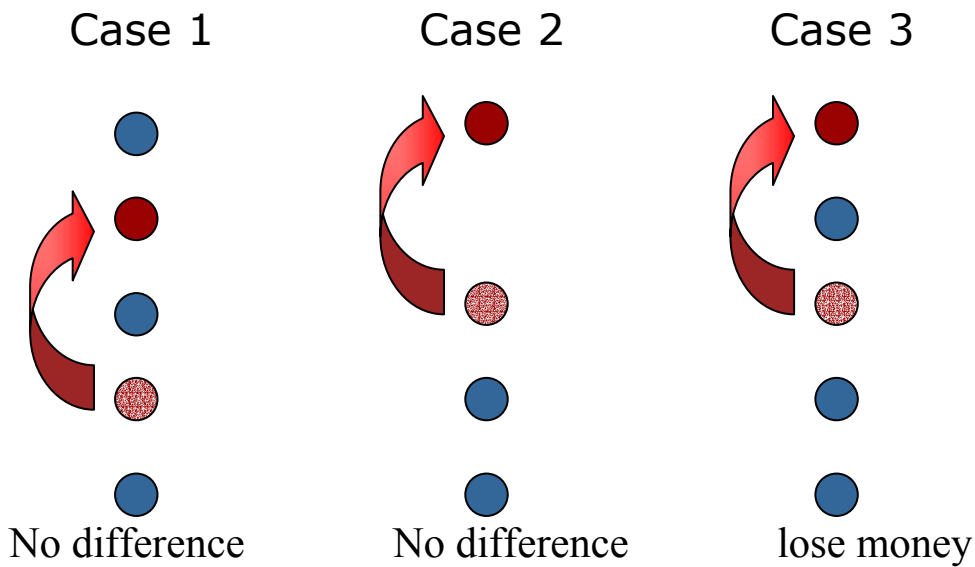
Other Auction Formats

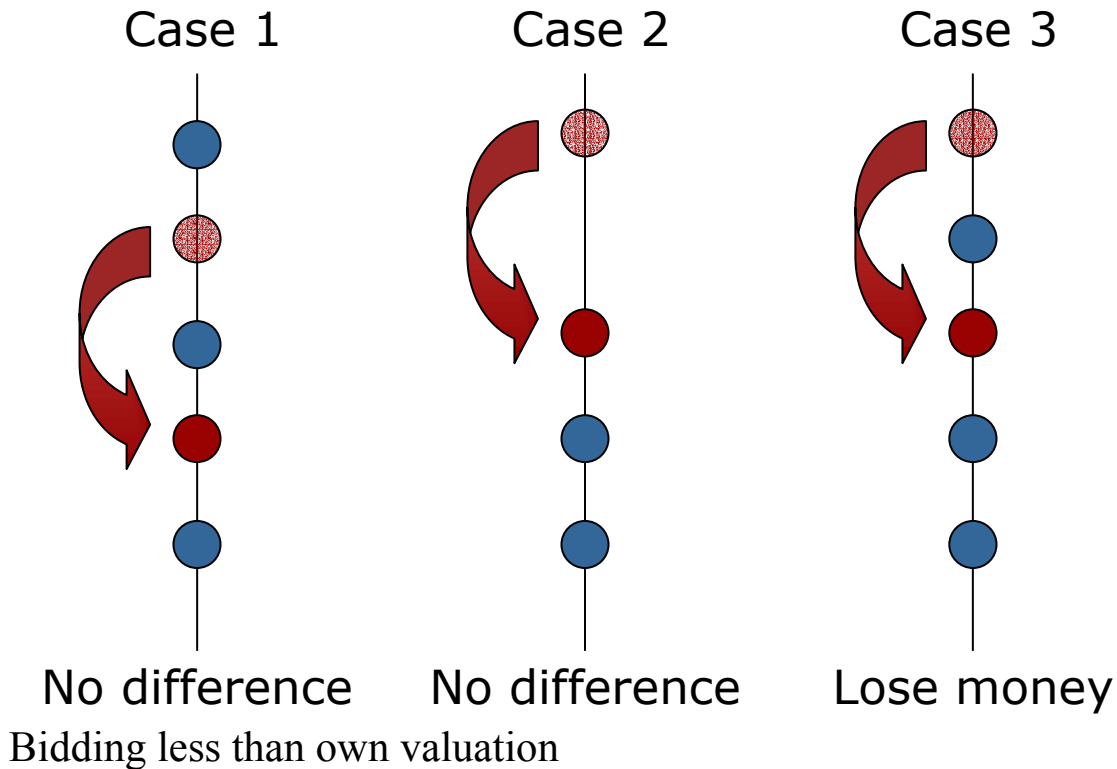
- Double auction
 - Buyers and sellers bid
 - Stock exchanges
- Reverse auction
 - Single buyer and multiple sellers
 - Priceline.com
- Multiunit auction
 - Seller has multiple items for sale
 - FCC spectrum auctions

The Vickrey Second Price Auction

- Why are they so popular?
- Bidding strategy is easy!
- Bidding one's true valuation is a (weakly) dominant strategy
 - Proxy bidding on eBay
- Intuition: the amount a bidder pays is not dependent on her bid

Bidding Higher Than My Valuation





So in a Second Price Auction

- always bid your true valuation
- Winning bidder's surplus = Difference between the winner's valuation and the second highest valuation
- Surplus decreases with more bidders

First Price Auction

- First price auction presents tradeoffs
- If bidding your valuation – no surplus
- Lower your bid below your valuation
 - Smaller chance of winning, lower price
- Bid shading
 - Depends on the number of bidders
 - Depends on your information
 - Optimal bidding strategy complicated!

Revenue Equivalence

- Assume an auction with the following rules:
 - The prize always goes to the person with the highest valuation
 - A bidder with the lowest possible valuation expects zero surplus
- All such auctions yield the *same expected revenue!*

Comparative Statics – More Bidders:

- More bidders leads to higher prices
- More bidders leads to less surplus

■ Example (second price auction):

Each bidder has a valuation of either \$20 or \$40, each with equal probability

Number of Bidders = Two bidders

$$\Pr\{20,20\} = \Pr\{20,40\} = \Pr\{40,20\} = \Pr\{40,40\} = 1/4$$

$$\text{Expected price} = 3/4 (20) + 1/4 (40) = 25$$

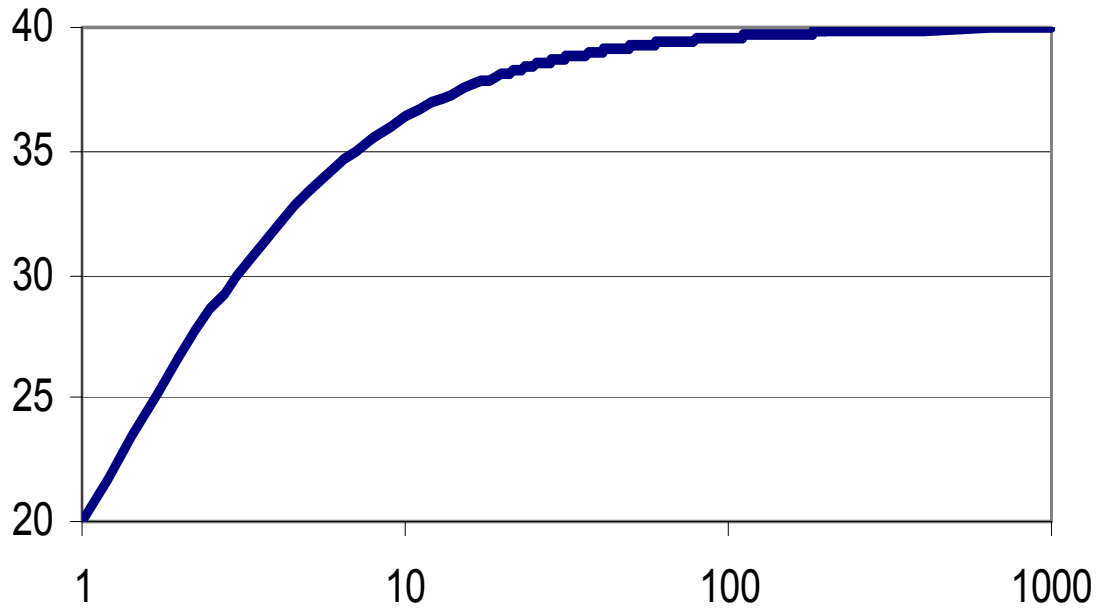
Number Bidders = Three bidders

$$\begin{aligned} & \Pr\{20,20,20\} = \Pr\{20,20,40\} = \Pr\{20,40,20\} \\ = & \Pr\{20,40,40\} = \Pr\{40,20,20\} = \Pr\{40,20,40\} \\ = & \Pr\{40,40,20\} = \Pr\{40,40,40\} = 1/8 \end{aligned}$$

$$\text{Expected price} = (4/8) (20) + (4/8) (40) = 30$$

Number of Bidders (general assumption)
Valuations are drawn uniformly from $[20,40]$

■ Expected price:



■



Goals of empirical work:

- Positive: How do agents bid?
Are valuations correlated? If so how?
Are observed bids consistent with Bayesian Nash Equilibrium (BNE)?
Are bidders colluding?(we'll see an empirical paper:Porter and Zona (JPE, 1993)).
- Normative: Identify optimal reserve price
Identify revenue maximizing efficient auction

Two kinds of approach for empirical work:

- Reduced form: Test theory predictions
Make inferences about behavior and bidding environment
- Structural form: Assume the theory holds and estimate Data generating process

Designing Auctions

- How many objects are to be auctioned?
- Is there a reserve price?
- Is the reserve price known to bidders?
- How are bids collected?
- Who is the “winner”?
- How much does the winner have to pay?

Auction Rules

- Bidding
 - Who is allowed to bid?
 - How are bids presented?
 - How much must bids be beaten by?
 - Is bidding anonymous or favored?
- Information
 - Are current bids revealed?
 - Are winners identified?
- Clearing
 - Who gets what?

Designing an Efficient Auction

- Highest valuation receives the object
 - If highest valuation is greater than seller's value, sale is consummated , that is, if there are gains from trade
- Neither first-price nor second-price auction guarantees both

Inefficiencies

- First-price Auction
 - Highest valuation may be higher than seller's value
 - But: bid-shading results in lower bid
- Second-Price Auction
 - Highest valuation may be higher than seller's value
 - But: second-highest value, which determines the price, might not be => set a reserve price...

Reserve Price

- A reserve price is a “phantom bid” by the seller
- Does not resolve inefficiencies of first-price auction
 - Still have bid-shading
- Does resolve inefficiency in second-price auction
 - Guarantee that sale will occur *at the reserve price*

Sellers driving up the price in 2nd Price auctions:

Second Price Auctions: “Horsing Around”

- A “phantom bid” may be placed *after* the seller knows the other bids
- Place a bid *just below* the highest bid
- Essentially: makes a second-price auction a first-price auction
- English Auction: “plants” fake bidders to drive up the price

Bidders Can “Horse Around Too”

- Cast doubt on object’s value
 - Online auctions
- More bidders drives up prices so...
- Bidders may try to collude
- How likely is such collusion?

Collusion: Bid Rigging

- Difficulties:
 - Collusion must be organized
 - Need to identify bidders
 - Need to award the item to a ring member
 - Need to split up profits among members
- Prisoner’s Dilemma
 - Collusion lowers price
 - but if someone values it more, incentive to bid higher
- Resolution:
 - Hold private secondary auction

Collusion

- Other ways to collude:
 - Bid rotation (construction)
 - Subcontracting (Treasury Bills)
- Discouraging Collusion
 - Sealed-bid Auctions
 - Anonymity – do not release results

Sources of Uncertainty

■ Private value auction

- Inherent differences among bidders
- Item is for personal use – not resale
- Values of other bidders unknown

■ Common value auction

- Item has single, true value
- Value unknown at time of bidding

Common Value Auctions

■ Example: Offshore oil leases

- Value of oil is same for every participant
- No bidder knows value for sure
- Each bidder has *some* information
- Exploratory drilling

■ Different auction formats are not equivalent

- Oral auctions provide information
- Sealed-bid auctions do not

Levels of Thinking

Would I be willing to pay \$100K, given

what I know before submitting my bid

versus

what I know before submitting my bid, and
that I will only win if no one else is willing to bid \$100K

versus

what I know before submitting my bid, and
that I will only win if no one else is willing to bid \$100K
and the history of bids that I have observed?

In the Common Value Auction

- Learn about value from other bids
- As others drop out – revise own estimate of object’s value
- If cannot revise (sealed bid):
 - Leads to WINNER’S CURSE
- Estimates on average correct
- Winner not picked randomly – highest estimate -> too high

To Avoid Winner’s Curse => COMMANDMENT

The expected value of the object is irrelevant.

To bid, consider only the value of the object if you win!

Avoiding Winner’s Curse

■ Always bid *as if you have the highest signal*

- If you don’t – doesn’t matter, you won’t win
- If you have highest signal – what is the object worth
- Use that as the basis of your bid

■ Numerical example:

- There is a closed box full of money worth \$X
- Five bidders get signals:
- \$X-2K, \$X-1K, \$X, \$X+1K, \$X+2K

Winner’s Curse Example

- Your signal = \$10,000
- So worth 8-12K. On average, 10 (remember the expected value should be irrelevant to your bid!)

■

- Bid 9, make \$1 expected
- If all do this (bid shade \$1) – Then the winner will have gotten the signal \$X+2, bid \$X+1, lost 1.

■ If Instead: I will assume my signal is highest. Then, value is my signal –2, so I will bid $10-2-1=7$. The winner will always have a profit of 1 (the bid shading).

Multiple Unit Auctions (Homogeneous)

■ Extension of Vickrey Auction

All winners pay highest non-winning bids excluding their own

■ Example

- I value goods at 100, 90
- You value at 80, 60
- Why exclude own bids? ...

Highest Non-Winning Bid

■ *If all bid truthfully:*

- 100,90,80,60
- I win both objects at a price of 80
- My Surplus: $100-80 + 90-80 = 30$
- Revenue: $80+80 = 160$

■ *Strategic bidding: I bid 100 and 60*

- 100,80,60,60
- Each of us wins an object at 60
- My Surplus: $100-60 = 40$
- Revenue: $60+60 = 120$

•

Highest Non-Winning Bid

■ Unlike single-unit auction, I might be the highest and second-highest bid

■ Hence, I might set my own price

- Incentive to shade my second bid
- If my own bid cannot set my own price:
 - No incentive to shade my bid
 - All bid truthfully
 - Revenues increase!

Multiple Unit Auctions (Heterogeneous)

- If products are distinct and valuations are independent, can hold distinct auctions
- If values are not independent...
 - Objects interrelated
 - Substitutes
 - Complements
- ... then things get very messy!

(Show spectrum... slide... now)

Summary

- Participating strategies
 - Bid true valuation in 2nd price auctions
 - Shade bids in 1st price auctions
 - In common value auctions, assume that you have the most optimistic estimate
- Designing Auctions
 - Preclude cooperation among bidders
 - Announced reserve price to gain efficiency and relieve doubts
 - In common value auctions, provide information to help with winner's curse

The usual models are

Independent private values and common values

The bottom line, before we formally define these auction models is that, if you see the signals of the other bidders (or their bids) and it causes you to revise your own valuation of the object then it is common value auction, otherwise is private value.

Notation:

r:	reserve price
n:	number of potential bidders
m:	number of bidders
$U_i=U(\sigma_i,v)$:	Utility function of bidder i
v:	Value of good, may or may not be observed by bidder
σ_i :	Private signal of bidder i
F:	Cumulative distribution function of $(\sigma_1, \dots, \sigma_N, v)$
Y_i :	$= \max \{ \sigma_j : j \neq i \}$
W:	winning bid

Assumptions:

1. Each bidder wants only one object
2. U is non-negative, continuous and increasing
3. the signals are one dimensional $\sigma_i \in \mathbb{R}$
4. F is exchangeable across bidder indices, because no bidder has more precise information
5. $(\sigma_1, \dots, \sigma_N, v)$ are positively correlated
6. F, n, U are common knowledge

Independent Private Values (IPV)

$$U_i = \bar{b}_i \text{ for all } i=1, \dots, N$$

\bar{b}_i are all iid

$$F(\bar{b}_1, \dots, \bar{b}_N) = \prod_{i=1}^N F_{\bar{b}_i}(\bar{b}) \quad (\text{product})$$

“Your signal is your payoff.

Other bidder’s signals, \bar{b} s are uninformative to you.”

Pure Common Value (CV)

$$U_i = v$$

cdf is now $F(\bar{b}_1, \dots, \bar{b}_N, v)$

We can have $v = \text{sum of all } \bar{b}'\text{s}$

There is uncertainty about v

“Signals of other bidders causes you to revise your valuation \bar{b}_i .”

Bidding Equilibria (in a nutshell)

Private values:

- Dutch auction: (first price+open bidding or FPSB)
Your bid should shade down from your signal and should be increasing in N .

Example:

Signal $\bar{v}_i = \bar{v}$.

Suppose valuations have support $[0, \bar{v}]$ and are uniformly distributed.

Abstracting from risk aversion, then $E[\text{next highest bid}] = (N-1/N)\bar{v}$

Equilibrium bid: $w = (N-1/N)\bar{v} < \bar{v}$ (so there is bid shading).

- English auction: (second price+open bidding)
Your bid should be higher than the next highest bid, up to your valuation. No strategic behavior here.
- Vickrey auction: (SPSB)
Your bid should be your valuation and the winner pays the second highest price. No strategic behavior here.

Common values:

Your bid should shade down from your signal to account for the winner's curse. Bidder should bid less than $E[v | \bar{v}_i = \bar{v}]$

(remember the numerical example)

Empirical testable implications to distinguish IPV and CV:

In reduced form settings, comparative-statics in N :

FPSB: bid is monotone in N for IPV and non monotone for CV

SPSB: bid independent of N for IPV and decreasing in N for CV.

Off-shore oil: regress bids on N_t conditional on other Z_t - found bid monotone in N_t conditional on Z_t .

2 last concepts

Winner's curse:

A bidder is more likely to bid high and acquire the item when he/she overestimates the item's value.

From Jensen's inequality (convexity of max function)

$$E[\max_i \{ \bar{v}_i \} | v] \geq \max_i \{ E [\bar{v}_i | v] \} = v$$

Intuition:

The person who had the highest expectation was over-optimistic

Revenue Equivalence:

About what type of auction seller prefers, given expected revenues from selling the item. Under some assumptions

(If item goes to the person with highest valuation and bidder with lowest possible valuation expects zero profit)

then the seller may be indifferent between the 4 types of auctions.

Winning bid is the bid of the buyer with highest valuation.

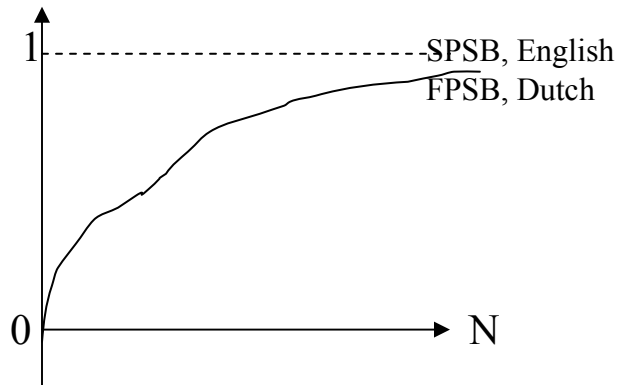
We denoted this by $w = \text{bid}(\bar{v}_{(1)})$, where $\bar{v}_{(1)} > \bar{v}_{(2)} > \dots > \bar{v}_{(N)}$.

For example in SPSB: $w = E[\bar{v}_{(2)} | \bar{v}_{(1)}]$

Final note (and we will see this formally later in this set of notes):

As N increases the bids go to valuation.

Bid/valuation



So far we have covered

- 1- Introduction to Auctions
- 2- IPV and CV – survey

I'll go now a little bit further on formality to introduce the issue of strategic behavior of buyers / collusion in bidding. We'll end with a paper identifying collusion in procurement auctions (Porter and Zona, JPE 1993). So next is:

- 3- Format setup (to get you familiar with notation etc), optimal bids in SPSB and FPSB.
- 4- Define money left on the table, prove Revenue Equivalence Theorem.
- 5- Primer on Structural Auction Estimation – see references in the end (Laffont, Hendricks, Paarsch surveys)
- 6- Introduction to Collusion – see more references in the end of notes.

Some of the many topics we are not covering formally here:

- multi-unit issues (revenue equivalence (see page 34 on – empirical tests... US timber forest service, Hansen 1985)
- introduction of entry
- risk aversion (for SPSB-Vickrey and English still dominant strategy to bid their valuation; but for Dutch and FPSB the bids are more aggressive to increase probability of winning , Holt, JPE 1979- see extra notes page 34)
- budget constraints (common values, Klemperer (EER 98) Wallet Game)
- Asymmetries (Hendricks, Porter AER 88, H,P,Wilson EMA 94, Porter EMA 95 – see references in the end)
- Affiliation, common values (Milgrom Weber, Ecomtca 82, Laffont,1996)
- auction design- seller's management of information

3- Formal setup

Independent Private Value Auctions, single Object (IPV)

Setup:

N potential bidders

Private values: bidder i knows σ_i

Symmetry and independent assumption $\sigma_i \sim \text{iid } F(\sigma)$ on $[\underline{\sigma}, \bar{\sigma}]$, F smooth, with pdf $f(\sigma)$.

Risk neutral: if win item net utility is $\sigma_i - p$,
if does not win and does not pay the net utility is 0.

(no reservation price, no minimum bids, no entry fees)

■ Lets look for symmetric monotone Bayesian–Nash–Equilibria (BNE):

Strategy (a bid) : $\beta(\sigma)$ strictly monotone, increasing, with an inverse $\eta(b)$ defined (valuation associated with the bid).

The probability that bidder i wins given that he submits bid b is given by

$$\Pr\{b > b_j, j \neq i\} = \Pr\{b > \beta(\sigma_j), \forall j \neq i\} = \text{monotone}$$

$$\Pr\{\sigma_j < \eta(b) \forall j \neq i\} = \underset{iid}{F^{N-1}(\eta(b))}$$

If $b = \beta(\sigma_i)$ then $\Pr = F^{N-1}(\sigma_i)$.

The object is assigned to whom values it the most here.

Price and Expected Payoff:

Price pay in event wins $p(b_1, b_2, \dots, b_N)$

Then expected payoffs

$$\Pi(\sigma_i, b) = \{\sigma_i - E[p(b, \beta(\sigma_j))_{j \neq i} | \sigma_j < \eta(b) \forall j \neq i]\} \cdot F^{N-1}(\eta(b))$$

Auction Format 1:

English (cry-out), Vickrey (SPSB)

Claim: dominant strategy equilibrium $\beta(\bar{b}) = \bar{b}$

(i.e., to bid the valuation)

Proof: (see Vickrey, by contradiction)

Strategy: raise price until one is willing to raise the standing highest bid.

(variation: button auction – keep finger on button until $p < \bar{b}_i$)

no regret if $\beta(\bar{b}) = \bar{b}$.

Winning bidder is person with highest valuation.

Lets rank the people by order statistic permutation:

$$\sigma_{(1)} > \sigma_{(2)} > \sigma_{(3)} > \dots > \sigma_{(N)}$$

A person expects to pay in the event that she wins:

$$E[\sigma_{(2)} | \sigma_i = \sigma_{(1)}] \quad (1)$$

And the seller's expected revenues are

$$E[\sigma_{(2)}] = E[E[\sigma_{(2)} | \sigma_i = \sigma_{(1)}]]$$

So in the SPSB the expected payment given by (1) is

$$E[\sigma_{(2)} | \sigma_i = \sigma_{(1)}] = \int_{\underline{\sigma}}^{\sigma_i} s \frac{f(\sigma_{(2)}(s))}{F(\sigma_i)^{N-1}} ds = \int_{\underline{\sigma}}^{\sigma_i} s \frac{(N-1)F(s)^{N-2} f(s)}{F(\sigma_i)^{N-1}} ds$$

Auction Format 2:**Dutch (price goes down and yell stop), First Price SB****Claim: dominant strategy equilibrium no longer $b = \beta(\bar{b}) = \bar{b}$** (indirect proof: McAfee, McMillan, JEL 87: $b < \bar{b}$ and) that $b \rightarrow \bar{b}$ as $N \rightarrow \infty$.FPSB means that $p(b_1, b_2, \dots, b_N) = b_i$

The expected payoff:

$$\Pi(\sigma_i, b) = (\sigma_i - b) \cdot F(\eta(b))^{N-1}$$

Under what circumstance is optimal bid $b = \beta(\bar{b}_i) = \bar{b}_i$?

Solving the First Order Condition (FOC)

$$\frac{\partial \pi(\sigma_i, b)}{\partial b} = 0$$

you get $\beta(\bar{b})$, the optimal bid given the signal \bar{b} .

Let's define

$$\pi^*(\sigma) = \pi(\sigma, \beta(\sigma)).$$

Then taking the derivative with respect to the signal

$$\frac{\partial \pi^*(\sigma)}{\partial \sigma} = \frac{\partial \pi(\cdot)}{\partial \sigma} + \left| \frac{\partial \pi(\cdot)}{\partial \beta} \right|_{\beta=\beta(\sigma)} \frac{\partial \beta(\cdot)}{\partial \sigma}, \text{ by envelope theorem.}$$

So given that

$$\pi^*(\sigma) = \pi(\sigma, \beta(\sigma)) = (\sigma - \beta(\sigma)) F(\eta(\beta(\sigma)))^{N-1}$$

which is equivalent to

$$\pi^*(\sigma) = (\sigma - \beta(\sigma)) F(\sigma)^{N-1}$$

then

$$\frac{\partial \pi^*(\sigma)}{\partial \sigma} = \frac{\partial \pi(\cdot)}{\partial \sigma} = F(\sigma)^{N-1} \quad (\text{eq. 2})$$

Integrating the above equation gives

$$\pi^*(\sigma) = \int_{\underline{\sigma}}^{\sigma} F(s)^{N-1} ds \tag{eq. 3}$$

(eq.2)=(eq.3), and solving for optimal bid

$$\beta(\sigma) = \sigma - \frac{\int_{\underline{\sigma}}^{\sigma} F(s)^{N-1} ds}{F(\sigma)^{N-1}} < \sigma$$

We already knew to expect a mark-down strategy (bid shading) in FPSB, we have now shown this to be optimal strategy, that maximizes the expected payoff.

The bidder, by bid-shading (mark-down strategy), is trading off the probability of getting the good with having a surplus when getting the good. The bidder does not bid its valuation but below.

(We can further show by integrating the optimal bid function by parts that

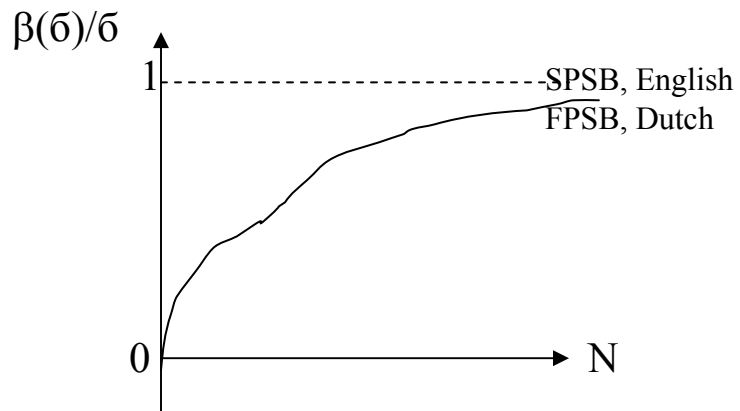
$$\beta(\sigma) = E[\sigma_{(2)} | \sigma_{(1)} = \sigma]$$

that is,

$$\beta(\sigma) = \frac{\int_{\underline{\sigma}}^{\sigma} s(N-1)F(s)^{N-2} f(s) ds}{F(\sigma)^{N-1}} = E[\sigma_{(2)} | \sigma_{(1)} = \sigma] \text{). (see also proof Revenue}$$

Equivalence SPSB and FPSB, see next page)

The bid function is increasing in N and concave in N. As N increases the bids go to valuation.



Two last things before Structural Estimation:

Revenue Equivalence

In terms of seller's revenues. Denote the winning bid by w .

$$\text{Then } w = \beta(\bar{\sigma}_{(1)}) = E[\bar{\sigma}_{(2)} | \bar{\sigma}_{(1)}]$$

$$E[w | \text{FPSB}] = E[E[\bar{\sigma}_{(2)} | \bar{\sigma}_{(1)}]] = E[\bar{\sigma}_{(2)}]$$

$$\text{See from page 24, } E[w | \text{SPSB}] = E[\bar{\sigma}_{(2)}]$$

The Revenue Equivalence Result says that (under certain assumptions made) there is the same expected revenue for seller in the four auction formats.

Money left on the table

$\beta(\bar{\sigma}_{(1)}) - \beta(\bar{\sigma}_{(2)})$ = measure of ex-post regret on part of winner for FPSB.

Sometimes also presented as

$$[\beta(\bar{\sigma}_{(1)}) - \beta(\bar{\sigma}_{(2)})] / \beta(\bar{\sigma}_{(1)})$$

In oil auctions, the money left on the table is almost 50%!

As N increases the amount of regret $\rightarrow 0$.

4- Structural Estimation 101

To estimate the data generation process.

The objective of interest is to recover F .

(Why?

- One possible question, is given F , r optimal?
- Another: Are the bids observed determined by the same data generating process, for example, can we find evidence of a group of bidders strategically submitting bids (bid rigging) and another group not – Porter and Zona (1993), next class
- Estimate the costs of collusion, how much higher are the bids than if there was no collusion
- Given F estimated under a certain auction format what would happen to sellers expected revenues under an alternative format?
- Note that there are lots of reduced form papers also)

Lets consider here IPV.

Maintain risk neutrality assumption, symmetry, privacy, BNE/dominant strategy

N bidders

Valuations \bar{v}_i iid F on $[\underline{v}, \bar{v}^{up}]$, pdf f , F smooth.

Minimum bid r ($>\underline{v}$)

I – SPSB (we'll start with the easiest)

II.A.- English auction where we observe only the winning bid

II.B – English auction where we observe all bids

III – FPSB

Extra- Risk Aversion, Rev. Equiv. k -object; Sequential Auctions.

IV- three approaches to estimation

(i) Simulated NLLS (ii) MLE (iii) non-parametrically

V- Asymmetric Auctions (information (how to use covariates to identify), valuations) – Bayesian Methods.

I- SPSB (easiest)

The dominant strategy: $b_i = \sigma_i$ if $\sigma_i \geq r$
 Else don't bid

The winning bid $w_t = \max[\sigma_{(2:N_t)}, r_t]$

Data: $\{b_{it}\}_{i=1}^{n_t}, n_t, N_t, r_t, t = 1, \dots, T$
 where $\rightarrow n_t = \#\{i \mid b_{it} \geq r_t\}$
 People who submitted bids
 N_t are the potential bidders

The likelihood function:

$$L = \prod_t \{F(r_t)^{N_t - n_t} \cdot \prod_i^{n_t} f(b_{it})\}$$

did not did submit bid

Notes:

- With risk aversion still dominant strategy to bid valuation
- With asymmetry too
- If observable heterogeneity: in auction t , $\exists Z_t : \sigma_{it} \sim iid, F(\sigma \mid Z_t)$
- What may cause problems is unobserved heterogeneity
- If we do not observe N_t but only n_t , then truncation.
- If N_t also depends on unobserved heterogeneity then it is not identified if low bidders (and low number of bids) because N_t low or because lousy realization of unobserved Z 's.

(note for this first part of Structural estimation: We are estimating parameters for the model we have assumed, we are not testing theory here.)

II.A. – English Auction where observe only winning bid

Dominant strategy, the same as SPSB, $b_i = \sigma_i$ if $\sigma_i \geq r$

Data: for the sample where $n_t \geq 1$: $\{w_t, N_t \text{ (debatable), } r_t \}$

One can then infer n_t on the basis of the winning bid as follows:

$n_t = 0$ if no winner

$n_t = 1$ if $w_t = r_t$

$n_t > 1$ if $w_t > r_t$

F smooth, probability of a tie = 0

The winning bid $w_t = \max[\sigma_{(2:N_t)}, r_t], n_t \geq 1$

Likelihood function:

$\text{Prob}\{n_t = 0\} = F(r_t)^{N_t}$

$\text{Prob}\{n_t = 1\} = N_t F(r_t)^{N_t-1} [1 - F(r_t)]$

$\text{Prob}\{n_t > 1\} = h(w_t)$, where the winning bid $w_t = \sigma_{(2:n_t)}$

and $h(w_t)$ is the distribution of the 2nd order statistic

$$h(w_t) = N_t(N_t - 1)F(w_t)^{N_t-2} f(w_t)[1 - F(w_t)]$$

all but 2 highest valuation person than w_t
had lower valuation

Define $D_t = 1$ if $n_t = 1$

and $= 0$ if $n_t > 1$

Then the likelihood is given by

$L = \prod_t \left\{ \frac{\text{Pr}\{n_t = 1\}^{D_t} \cdot h(w_t)^{1-D_t}}{1 - \text{Pr}\{n_t = 0\}} \right\}$, accounting in the denominator for truncation.

$$\text{Rewriting: } L = \prod_t \left\{ \frac{\{N_t F(r_t)^{N_t-1} [1 - F(r_t)]\}^{D_t} \cdot h(w_t)^{1-D_t}}{1 - F(r_t)^{N_t}} \right\}.$$

The seller is not indifferent over the choice of r_t .
 Suppose $\bar{\sigma}_0$ is the seller's valuation in event the item is not sold.

Then the expected revenue of the seller is given by:

$$R = \sigma_0 F^N(r) + r \Pr\{n=1\} + \underbrace{\int_r^{\bar{\sigma}} w \cdot h(w) dw}_{E[w]}$$

The seller wants to maximize this with respect to r .

$$\frac{\partial R}{\partial r} = 0 = \dots \Leftrightarrow (\sigma_0 - r)f + (1 - F) = 0$$

$$\Leftrightarrow r = \sigma_0 + \frac{(1 - F(r))}{f(r)}$$

Once we have estimated F and f , we can choose r .

Note that optimal r is independent of N . The formula is similar to a take it or leave it offer of a firm facing just one buyer.
 IPV is critical here.

II.B- English Auction where all bids are observed

We still do not know N_t .

Bidder i may submit many bids or none.

Let b_{it} be the highest bid submitted by bidder i at t
 $=0$ if no bid was submitted at t

$N_t = \#$ of potential bidders is observed

(if not assume constant and estimate it;

assume $N_t = N = \max\{n_t\}$;

assume $N_t \sim$ poisson truncated below by n_t)

$n_t = \#\{i \mid b_{it} > 0\}$, observed

Order i for t : $b_{1t} > b_{2t} > \dots > b_{nt}$

Redoing the likelihood function as before is a partial likelihood approach because one is ignoring the non-winning bids.

By submitting a bid one gets a lower bound on its valuation.

Inferences on these ordered bidders:

Bidder 1: $\bar{b}_{1t} \geq b_{1t}$, (A)

Bidders 2, ..., n_t : $\bar{b}_{it} \in [b_{it}, b_{1t}]$ (B) (**)

Bidders n_t, \dots, N_t (those that did not bid): $\bar{b}_{it} \leq b_{1t}$ (C)

So the likelihood function is

$$L = \prod_t \left\{ \underbrace{[1 - F(b_{1t})]}_{(A)} \left[\underbrace{\prod_{i=2}^{n_t} (F(b_{1t}) - F(b_{it}))}_{(B)} \right] \underbrace{F^{N_t - n_t}(b_{1t})}_{(C)} \right\}$$

Objective of seller, like FCC in spectrum auctions: efficient allocation vs maximum revenue (reading: Cramton, McMillan)

() Note: If firms are “porking”, we cannot infer this in (B).**

Sometimes bidders bid higher such that the winning bid for a certain spectrum license is higher for the other firm, such that cost of other firm goes up. Firms did this by keeping standing high bids on more than one license in one same area.

III – FPSB

Strategy $b = \beta(\sigma)$, β monotone increasing with inverse η .

$b : \max (\sigma - b) F(\eta(b))^{N-1}$. From the FOC w.r.t. b :

$$b : \frac{1}{\eta'(b)} = [\eta(b) - b](N - 1) \frac{f(\eta(b))}{F(\eta(b))} \tag{D}$$

then we obtain (see derivation pages 25-26):

$$\beta(\sigma) \begin{cases} = \sigma - \frac{\int_r^\sigma F(s)^{N-1} ds}{F(\sigma)^{N-1}}, \sigma \geq r \\ = 0, \text{ow.} \end{cases}$$

In the empirical approaches:

1. Approach :

We know $\sigma \sim F$ unknown

$b = \beta(\sigma)$

change of variable $b \sim G(\sigma) = F(\eta(b)) = \Pr\{\sigma \leq \eta(b)\} = \Pr\{\beta(\sigma) < b\}$

Support of $b = \{0\} \cup [r, \beta(\sigma^{up})]$ (depends on parameters-Donald Paarsch, 1993, pseudo-ML)

G is observable, uncover what F is.

$g(b) = G'(b) = f(\eta(b)) \eta'(b)$.

So equation (D) above becomes

$$1 = [\eta(b) - b](N - 1) \frac{g(b)}{G(b)}$$

$$\Rightarrow \underbrace{\eta(b)}_{\text{valuation_to_be_inferred}} = \underbrace{b}_{\text{bid}} + \frac{G(b)}{(N - 1)g(b)}$$

depends on intensity of competition, # bidders.

Underlying identification issue:

-observe G . Under what conditions G implies a unique F ?

(can show: if $\eta(b)$ strictly increasing, $\lim \eta(b) = r$ if $b \rightarrow r$ from above

(Extra notes: Risk Aversion)

- For SPSB and English, still dominant strategy $\beta(\bar{b})=\bar{b}$.
- For FPSB, Dutch:

$$\pi(\sigma, b = \beta(\sigma)) = U(\sigma - b)F(\eta(b))^{N-1} + \underbrace{U(0)}_{=0, \text{without_loss_gen}} [1 - F(\eta(b))^{N-1}]$$

The bidders will bid more aggressively to increase the probability of winning (Holt, JPE 1979).

Suppose $U(x)=x^\alpha$, $\alpha \in [0,1]$, $\alpha=1$ risk neutral

$$\pi(\sigma, b) = (\sigma - b)^\alpha F(\eta(b))^{N-1}$$

The BNE of the above game is equivalent to the BNE of the game with

$\pi(\sigma, b) = (\sigma - b)F(\eta(b))^{\frac{N-1}{\alpha}}$, again risk neutral and the number of opponents increased by $1/\alpha$, so if N increases, the bid increases.

So $\beta_\alpha() > \beta_1()$.

(Extra notes: Revenue Equivalence in K-object Auction)

$K < N$

Assume: each buyer only wants one item.

The bidders' valuation for one item are IPV and the marginal valuations for additional items are zero.

Theorem: Suppose the auction rules and the BNE are such that the k highest types win and the lowest possible type (\underline{b}) has zero expected pay-off, then expected payoff of bidder i with type \bar{b}_i is

$$T(\sigma) = \int_{\underline{\sigma}}^{\sigma} s \binom{N-1}{K} K \cdot F(s)^{K-N-1} [1 - F(s)]^{K-1} f(s) ds$$

$$\Leftrightarrow T(\sigma) = E[\sigma_{(K+1)} | \sigma_i \in \{\sigma_{(1)}, \dots, \sigma_{(K)}\}]$$

Which is the multi-unit analogy as before.

Auction types:

- discriminatory auction: k highest win, pays own bid
- Vickrey auction: k highest win, pays k+1 highest bid
- Uniform price: k highest win, pay kth highest bid

Empirical paper:

- Reduced Form Test of revenue equivalence, Hansen 1985):

U.S. Forest Service Timber Auctions (English and FPSB at same time): regress $\ln(\text{revenue})_t$ on forest characteristics and Auction format dummy, D_t . OLS-Coeffic on D_t (=1 if FPSB) is 0.1 and significant.

- But D_t not exogenously determined. Two step Heckman:

(i) probit D_t on Z_t and w_t

(ii) step 2

then coeffic on D_t not significantly different from zero.

- Haile (2000, AER), Athey and Levin (JPE, 2001) coefficient on D_t being positive can be rationalized by risk aversion.

(Extra notes: Sequential Auctions)

Suppose N bidders, 2 items

Auction format: sequential FPSB

Key assumptions: the only information revealed after first item sold is the winning bid (not the other bids), bidder only wants one item!

IPV for one item

Suppose BNE

Strategy:

$$\text{item}_1: \beta_{(1)}(\sigma) = E[\sigma_{(3)} | \sigma_{(1)} = \sigma]$$

$$\text{item}_2: \beta_{(2)}(\sigma, b) = E[\sigma_{(3)} | \sigma_{(2)} = \sigma, \underbrace{\sigma_{(1)} = \beta_{(1)}^{-1}(b)}_{\text{winner_1st_item}}]$$

Winning bids:

$$item_1: w_1 = \beta_{(1)}(\sigma_{(1)}) = E[\sigma_{(3)} | \sigma_{(1)}]$$

$$E[w_1] = E[\sigma_{(3)}]$$

$$item_2: w_2 = \beta_{(2)}(\sigma_{(2)}, \underbrace{\beta_{(1)}(\sigma_{(1)})}_{w_1}) = E[\sigma_{(3)} | \sigma_{(2)}, \sigma_{(1)}]$$

Result : $E[w_1] = E[\sigma_{(3)}] = E[w_2] \Rightarrow$ *Martingale*

$$E[w_2 | w_1] = w_1.$$

Same arbitraging condition.

Also true for SPSB.

Empirical Evidence:

■JEP, Ashenfelter, wine

Sequential identical auctions: finds Declining price anomaly.

■Affiliation/CV Milgrom, Weber Ecma, 1982

Prices are super martingale (prices increase since more information about the object)

■Key assumption: M.Perry : if bidders want multiple items, no longer revenue equivalence.

IV – Three approaches to estimation/ Identification issues

1. Simulated NLLS (with winning bids, N_t and r_t , $t=1, \dots, T$)
2. “brute force” MLE (same as above)
3. Non parametrically (needs all bids)

Good survey “Empirical Work concerning auctions” Hendricks and Paarsch, 1995:

<http://www.jstor.org/cgi-bin/jstor/printpage/00084085/ap010113/01a00060/0-150.pdf?userID=8020fb12@berkeley.edu/018dd5533b005012909fc&backcontext=page&config=jstor&dowhat=Acrobat&0-150.pdf>

1. Simulated NLLS (Laffont et al, Econometrica 1995)

- Fitting a moment condition, exploiting revenue equivalence.
- In English auction $w_t = \max \{r_t, \bar{\sigma}_{(2:N_t)}\}$
- A bid $\beta(\bar{\sigma}) = E[\max \{r_t, \bar{\sigma}_{(2:N_t)}\} | \bar{\sigma} = \bar{\sigma}_{(1:N_t)}]$
- Revenue Equivalence:

Winning bid:

$$\beta(\bar{\sigma}_{(1:N_t)}) = E[\max \{r_t, \bar{\sigma}_{(2:N_t)}\} | \bar{\sigma} = \bar{\sigma}_{(1:N_t)}] = E[\max \{r_t, \bar{\sigma}_{(2:N_t)}\}]$$

$$E[w_t] = E[\max \{r_t, \bar{\sigma}_{(2:N_t)}\}] \text{ moment condition they use.}$$

- Observe: $\{N_t, w_t, r_t\}$ $t=1, \dots, T$

■ Simulation:

- 1) posit F (log-normal)
- 2) random draws $\rightarrow E[\max \{r_t, \bar{\sigma}_{(2:N_t)}\}]$
- 3) min weighted average difference of $w_t - E[.]$ to get parameters of F .

■ Problems:

- presumption that r_t is exogenous (r_t is chosen to max something...)
- we don't observe N_t
- alternative hypothesis for F ?
 - there are over-identifying assumptions $\text{var}[w_t]$.
 - one could test if F fits data well.

This method allows for larger families of distribution functions but still a specific distribution assumption is made, not in 3.

2. MLE (Paarsch, Journal of Econometrics, 1992 and Piecewise Pseudo-ML Donald and Paarsch 1993)

(& Donald & Paarsch (93) - support of b depends on parameters:

<http://www.jstor.org/cgi-bin/jstor/printpage/00206598/di008375/00p0026t/0-150.pdf?userID=8020fb12@berkeley.edu/018dd5533b005012909fc&backcontext=page&config=jstor&dowhat=Acrobat&0-150.pdf>

Indiv. Bids: $b_t \sim F(\eta_t(b))$, conditional on N_t, r_t .

Winning bid

$$w_t \sim F(\eta_t(w))^{N_t} = H_t(w_t)$$

$$h_t(w_t) = N_t F^{N_t-1}(\eta_t(w_t)) \cdot \underbrace{\eta_t'(w_t)}_{\text{tricky_control_for_this}} f(\eta_t(w_t))$$

where

$$\eta_t'(w_t) = \frac{N_t}{N_t - 1} \frac{F^{N_t}(\eta_t(w_t))}{(\eta_t(w_t) - w_t)}$$

Requires that joint distribution belongs to particular families of distributions that admit closed form solutions to the bid functions that can be solved using numerical methods. More restrictive than method 1.

3. Non-parametrically (Elyakime et al, 1994):

Need to use all submitted bids.

Sample : $\{b_{it}\}_{t=1, \dots, T, i=1, \dots, N_t}$; r_t ; N_t ; Z_t (co-variates)

We get the inverse bid function from the FOC:

$$\underbrace{\eta_t(b)}_{\text{valuation_to_be_inferred}} = \underbrace{b}_{\text{bid}} + \frac{G_t(b)}{(N-1)g_t(b)}$$

G and g depend on t , because depend on N_t and r_t .

- Consider the sub-sample where N_t is constant and all bidders bid, that is, $r_t \leq \bar{r}_t$ which in turn will keep g and G constant.
- Two stage non-parametric estimation:

1. non-parametrically estimate \hat{g} and \hat{G}

11/2. Form “data” where

$$\hat{\sigma}_{it} = \underbrace{\hat{\eta}_i(b)}_{\text{estimated}} = \underbrace{b}_{\text{bid}} + \frac{\hat{G}(b)}{(N-1)\hat{g}(b)}$$

2. non-parametrically estimate the bid function β by

$$\text{bit} = \beta(\hat{\sigma}_{it})$$

Can then check the identification condition:

- (i) RHS of 1 $\frac{1}{2}$ monotone increasing
- (ii) $\lim \eta(\beta) \rightarrow r$ as $b \downarrow r$.

Problems: If r_i is optimally set, not exogenous!

If there is strategic bidding, collusion, some bids are “not real”.

=> Porter and Zona...

V – Asymmetric auctions (extra)

Theory: Wilson (1967), Milgrom, Weber (1983)

Empirical:

Hendricks/Porter AER 1988, Hendricks/Porter/Wilson, Emta (1994), Porter, Emta (95)

We are in the context of Common Value auctions.

There exist asymmetries of information of bidders and of costs (valuations), think for example auctions for off-shore drill rights, there are neighbors (with adjacent tracts) and non-neighbors of the tract being sold in auction that are possible bidders.

How to use covariates to identify Asymmetry of information from asymmetry of valuation?

Set-up:

FPSB, Common value v

One item sale

Two types of bidders:

- (i) neighbor observes public signal Z and private signal X
- (ii) non-neighbor (NN): observes only public signal Z

risk neutrality

minimum bid R , $E[v] > R$

joint distribution of (v, X, Z, R) is common knowledge.

Strategies:

(ii) Claim: Non-neighbor: will randomize, follow a mixed strategy.

- Why bid? If they did not bid, then the neighbors would bid
- Why randomize? ... explain intuition
- $E[\text{Payoff for all bids}] = 0$ or < 0 if

- (i) neighbor: payoff: $(E[V/X] - b) \text{Prob}\{\text{their bid} > \text{NN bid}\}$
 $= h$

Neighbor has the realization of h , NN form the $E[.]$.

5- Bid rigging

Collusion in Auctions:

Early empirical work:

1. identical bidding – unlikely outcome of un-cooperative bidding
2. Bid rotation, according to phases of the moon
GE/Westinghouse conspiracy.

Empirical paper: Porter and Zona (JPE, 1993)

Detection of bid rigging in procurement auctions

Long-Island highway construction repair.

Suspicion of “complementary bidding”/bid rigging:

Create appearance of competition

Manipulate expectation of seller

	Firms meeting	Firms not meeting
Low bid	√	√
High bid	?	√

Are “?” fake bids?

Procurement auction: lowest bid wins.

Add Model to the board. Setup Likelihood function.

Show table results.

Results: For participants in meetings the higher bids look “phony”
not determined by the same process.

We can go back and check who was in conspiracy.

Other empirical papers: about milk bid rigging Porter, Zona (Rand 99), *Ohio school milk markets: An analysis of bidding*. Pessendorfer 1997, *A study of collusion in first price auctions*.

Does the data exhibit evidence inconsistent with competitive bidding.

It is easier to do bid rigging if FPSB, competition only on price, quality pre-specified, number of firms is stable and trade association.

(far from complete but a good way to start) - **Reading List**

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