Strategic overinvestment and/or strategic underinvestment

Bulow, Geanokoplos, and Klemperer (1985), Fudenberg and Tirole (1984), Tirole (1988)

Basic framework:

Simple two period model

Period 1: incumbent, firm 1, chooses "investment" k1, for example, capacity.
Firm 2, observes k1, and then decides whether to enter.

If 2 does not enter,

$$\pi_2 = 0,$$
$$\pi_1 = \pi_1^m(k_1, X_1^m(k_1))$$

that is, firm 1 enjoys monopoly in the second period, choosing $X_1^m(k_1)$, that is monopoly X given k.

If 2 enters, firms choose simultaneously X_1 and X_2 and the profits are

$$\pi_1(k_1, X_1, X_2)$$
$$\pi_2(k_1, X_1, X_2)$$

where the entry cost of 2 is in there.

Given k1, X_1 and X_2 are determined by Nash equilibrium: { $X_1^*(k_1)$, $X_2^*(k_1)$ }.

We'll look at the effects of changing k1 on the Nash equilibrium, assuming that this NE is unique and stable.

If k1 is chosen such that
$$\pi^2(k_1, X_1^*(k_1), X_2^*(k_1)) \leq 0, \text{ then entry is blockaded}$$
$$\pi^2(k_1, X_1^*(k_1), X_2^*(k_1)) > 0, \text{ then entry is accomodated.}$$

Whether incumbent wants to accommodate or deter depends on the profits' comparison on the incumbent doing one or the other option.

Assumptions:

$\pi^1(k_1, X_1^*(k_1), X_2^*(k_1))$ and $\pi_1^m(k_1, X_1^m(k_1))$

are strictly concave in k_1 and $X_i^*(k_1)$ are differentiable.

Note: we'll ignore the case when entry is blockaded since no strategic issues there.

DETERENCE OF ENTRY

Incumbent chooses k1 such that $\pi^2(k_1, X_1^*(k_1), X_2^*(k_1)) = 0$.

Which strategy relative to choosing for non strategic reasons (assuming firm 2 does not see k1 before she enters , open loop, call that level of investment $k1^{OL}$), should firm 1 use to make firm 2's entry unprofitable?

Lets take the total derivative of firm 2's profits with respect to k_1:

$$\frac{d\pi^2}{dk_1} = \frac{\partial \pi^2}{\partial k_1} + \underbrace{\frac{\partial \pi^2}{\partial X_2}}_{=0}\frac{\partial X_2^*}{\partial k_1} + \frac{\partial \pi^2}{\partial X_1}\frac{\partial X_1^*}{\partial k_1}$$

If $\frac{d\pi^2}{dk_1}$<0 then investment in k1 makes firm 1 tough, if >0 then investment in k1 makes firm 1 soft.

If investing makes the firm look tough then to deter entry the firm should over-invest, $k1>k1^{OL}$, to look "top dog", very big and ready to fight.
If investing makes the firm look soft, then the firm should under-invest to deter entry, $k1<k1^{OL}$ looking "lean and hungry".

ACCOMMODATION

Suppose deterring entry is too costly.
In deterring entry, k1 was determined by post entry 2's profits.
In accommodating firm 1's behavior in first period k1 is dictated by firm 1's profit.

$\pi^1(k_1, X_1^*(k_1), X_2^*(k_1))$.

The incentive to invest is given by the total derivative of the above with respect to k1.

$$\frac{d\pi^1}{dk_1} = \underbrace{\frac{\partial \pi^1}{\partial k_1}}_{direct-effect=DE} + \underbrace{\frac{\partial \pi^1}{\partial X_1}}_{=0} \frac{\partial X_1^*}{\partial k_1} + \underbrace{\frac{\partial \pi^1}{\partial X_2} \frac{\partial X_2^*}{\partial k_1}}_{strategic-effect=SE}$$

The DE exists regardless of firm 2 seeing k1 or not. So, firm should over-invest if SE>0 and under-invest if SE<0.

The key thing is figuring out the sign of SE. The sign(SE)

$$sign\left(\frac{\partial \pi^1}{\partial X_2} \frac{\partial X_2^*}{\partial k_1} \right) \quad ?$$

Given that $\frac{\partial X_2^*}{\partial k_1} = \frac{\partial X_2^*}{\partial X_1} \frac{\partial X_1^*}{\partial k_1} = \underbrace{R'_2(X_1^*)}_{\substack{reaction\_function \\ slope}} \frac{\partial X_1^*}{\partial k_1}$ and $sign\left(\frac{\partial \pi_1}{\partial X_2}\right) = sign\left(\frac{\partial \pi_2}{\partial X_1}\right)$, then

$$sign\left(\frac{\partial \pi^1}{\partial X_2} \frac{\partial X_2^*}{\partial k_1} \right) = sign\left(\underbrace{\frac{\partial \pi^2}{\partial X_1} \frac{\partial X_1^*}{\partial k_1}}_{\substack{entry-\det ernece \\ strategic-effect}} \right) sign\left(\underbrace{R'_2(X_1^*)}_{\substack{slope-reaction-function \\ strategic:complements/substitutes}} \right).$$

In the neighborhood of equilibrium, quantities are strategic substitutes, R’<0 and prices are strategic complements, R’>0.