Problem set 1:

Provision of Quality

Question 1.

Consider a monopolist who sells batteries. Each battery works for h hours and then needs to be replaced. Therefore, if a consumer buys x batteries, he gets $H = xh$ hours of operation. Assume that the demand for batteries can be derived from the preferences of a representative consumer whose utility function is $U = V(H)-px$, where p is the price of batteries and $V' > 0 > V''$. The cost of producing batteries is $C(x) = xc(h)$, where $c', c'' > 0$.

(a) Derive the inverse demand function for batteries and denote it by $p(x)$.

(b) Suppose that the monopolist chooses x and h to maximize his profit. Write down the first order conditions for profit maximization assuming that the problem has an interior solution, and explain the meaning of these conditions.

(c) Write down the total surplus in the market for batteries (i.e., the sum of consumer surplus and profits) as a function of H and h. Derive the first order conditions for the socially optimal x and h assuming that there is an interior solution, and carefully explain in words the economic meaning of these conditions.

(d) Compare the solution that the monopolist arrives at with the social optimum. Prove that the monopolist provides an optimal level of h. Explain the intuition for this result.

Asymmetric information and product quality

Question 2.

Consider the following version of the lemons problem. There is a continuum of buyers and sellers in the market; the total mass of each group is 1. Each seller has one car to sell and each buyer wishes to buy at most one car, but only sellers know the quality of their cars before trading. It is common knowledge however that the quality of cars, denoted *q*, is drawn from a uniform distribution on the interval [0, 1] (hence, the probability that a car's quality is below some number *x* is equal to *x* if $0 \le x \le 1$ and is equal to *1* if *x >= 1*). It is also common knowledge that a fraction α of the sellers are of *type 1* and have a payoff *U1 = p-q/8* if they sell their cars and *0* otherwise, and a fraction *1-*α of the sellers are of *type 2* and their payoff is *U2 = p-q/4* if they sell their cars and *0* otherwise, where *p* is the price of the car (note that the two types of sellers differ only with respect to their payoffs but not with respect to the quality of cars they have to sell). There is a continuum of buyer types: the payoff of a type θ buyer if he buys a car whose quality is *q* is $U(\theta) = q\theta - p$, where θ is distributed uniformly on the unit interval. If a buyer does not buy a car his payoff is *0*. The buyers cannot observe the quality of cars before they buy nor can they observe the type of seller they face.

(a) Compute the supply of cars by type 1 sellers, type 2 sellers, and the aggregate supply of cars (i.e., compute the fraction of cars that will be supplied at a given price by each type of sellers and then add the two to obtain the aggregate supply).

(b) Let $\hat{q}(p)$ denote the average quality of cars supplied on the market as a function of p. Derive that $\hat{q}(p)=2(1+3\alpha)p/(1+\alpha)$. Explain the intuition on how $\hat{q}(p)$ varies with p and with α.

(c) Assume that buyers correctly anticipate $\hat{q}(p)$ and compute the demand for cars (i.e., the fraction of buyers that will wish to buy a car at a given price). Explain the shape of the demand function.

(d) Assume that the market is perfectly competitive and solve for the equilibrium price, p*, given that $S(p)=4p(1+\alpha)$ *(from (a))*.