Searching for failures in the Peruvian labor market via mixture models*

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September 22, 2002

Abstract

We use mixture distribution techniques to show that labor market participation is not enough to conclude separability between production and consumption decisions of farm-households. Our findings clearly show the existence of two distinct types of households among net-sellers of labor: those behaving as if unconstrained and hence in a separable way and those that behave as if constrained. In addition, we take advantage the technique to explore and understand the market segmentation and its source. In the case of Peru, ethnic discrimination, differential skills as well as lack of regional opportunities seem to be important deterrents for unconstrained market participation. This work proposes an important refinement to separability studies by exploring unobserved heterogeneity and suggests a methodology that allows us to fully understand market interactions. This allows us to implement more effective policies.

Keywords: Mixture distributions, switching regressions, labor, separability, Peru.

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*We wish to thank Frederick Zimmerman for helpful comments on the econometric methodology. This paper also benefited from participants of the development workshop at the Department of Agricultural and Resource Economics, University of California Berkeley.

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1 Introduction

That institutional failures have important effects in economic behavior has been explored in both theoretical and empirical literature. The existing theoretical literature shows that market failures significantly alter economic behavior; they impede market participation and can affect the response to policies of heterogeneous populations. For example, Roemer [26] analyzes class formation based on differences in assets position. Building on Roemer's work, Eswaran and Kotwal [9] derive a model where differences in asset position and the presence of two market failures (unobserved effort and a non-uniform access to credit) determine the emergence of different market participation regimes.

The implications of these models are multiple. First, they illustrate how social classes or regimes emerge through rational choices. In addition, these choices can help explain efficiency, like the existence of the inverse relation between yields and farm size (Sadoulet et al. [27]). In the context of household economics, the emergence of different regimes has also important policy consequences as they imply differential responses to policy interventions. For example, Sadoulet et al. [27] look at labor markets and show that the presence of a price band for wage, i.e. a positive difference between the wage to pay for hired labor and the effective wage received by family members when working outside, determines endogenous selection of households in labor market participation. Such a difference, induced by fixed transaction costs, determines the appearance of a finite set of self-sufficient farm-households. As a result, market participating households use the market wage as the decision price, while self-sufficient households use an endogenously determined shadow price that also depends on consumption and thus breaking the separability assumption between consumption and production decisions [1].

Empirical literature that looks at the separability hypothesis has focused on developing ways to incorporate and test it. Lopez [21], Jacoby [15], Lambert and Magnac [19] and recently Barrett et al. [1] use a direct approach where they estimate an aggregate production function that is then used to get implicit prices of family labor. These prices are then compared and found different with the market wage, thus rejecting separability. Skoufias [29] uses Jacoby's methodology to find similar results with an ICRISAT data from In-

[1]The separability hyporthesis posits that consumption and production decisions are recursive within the household. First, the households maximizes profits and then allocates its full income for consumption (Singh et al.[28]).

dia. Using demographic variables to explain production decisions, Benjamin [3] finds that they do not affect the demand for pre-harvest labor in rural Java and thus does not rejects separability.

Another approach is to get information directly from the data and to then use it to test separability. Feder et al. [11] use survey questions on access to credit to divide their sample between credit constrained and unconstrained. They find that consumption decisions enter the production decision for constrained households only and use this result to infer non-separability. Finally, Sadoulet's et al. [27] find selective recursiveness for seller and employer regimes but not for self-sufficient households in rural Mexican data.

While these papers make advances in both the theoretical and empirical aspects of separability and market failures, some questions remain unanswered. This paper addresses some of these issues and offers a number of refinements of the previous literature. Using labor markets, we question whether the common conclusion that market failures divide people between market participants and those who choose not to participate is enough. We argue that even among market participants, unobserved heterogeneity (either at the individual or a regional level) can affect the separability assumption. Our attempt is that of explicitly allowing for the fact market participation *per se* does not imply belonging to a particular regime. In particular, the presence of a quantity constraint on the labor market makes it plausible for some farmers to participate in the market and yet be constrained; that is, among those who participate in the market, there are farms that refer to the endogenously determined *shadow wage* as the decision price. These farms are not expected to react to an infinitesimal change in the market price. The practical difficulties of implementing such a model comes from the fact that, usually, the constraints are not observable. Our econometric methodology allows us to identify between constrained and unconstrained households.

In addition, while past work has provided a number of ways to test separability, little has been done to address the source of the market imperfection. The effect of policies does not depend on the knowledge of the existence of market distortions but understanding their source. In this context, our econometric approach allows us to explore and assess market integration and market distortions in more detail. The findings suggest that labor markets in Peru are very segmented and that ethnic and gender discrimination impedes market integration. In addition, on the supply side, the labor market also plays an important role in determining households' equilibria. Regional infrastructure, off-farm opportunities and geographic location affect the probability of finding off-farm work. Policy makers can use such results to target and implement policies that reduce labor market failures and enable households to better anticipate and protect themselves. For example, while our results show that policy schemes that address discrimination and education are important in the Peruvian context, additional programs that enhance off-farm job opportunities are also necessary to complement the labor market development process and ameliorate market participation by marginal groups.

Section 2 develops a household model with a labor market constraint and derives a testable hypothesis for separability among market participants. The econometric approach is presented in section 3. The Peruvian data and findings are presented in section 4. Section 5 concludes.

2 Theory

Building on traditional farm-household models (see Singh et al.[28]), let us consider a farm–household whose objective is to maximize utility. Utility is derived from income, y, and leisure time l^l [2]. The household is endowed with a total amount E of time to be allocated among in–farm work, l^i, off–farm work, l^o, which will be paid a salary w^o, and leisure. Farm labor can be supplied by household members or can be hired on the market, h, at a given wage rate w^h. Finally, there exist an unknown, upper limit to the amount of labor that can be sold on the market, $\bar{L}$.

The household's problem can be represented as follows:

$$\max_{h,l^i,l^o,l^l} U(y, l^l, \mathbf{z^h}) \tag{1}$$

subject to:

$$y = pq(l^i + h, A, \mathbf{z^q}) - w^h h + w^o l^o \tag{2}$$

$$l^l = E - l^o - l^i \tag{3}$$

$$l^l \geq 0 \tag{4}$$

$$h \geq 0 \tag{5}$$

[2] For simplicity, we are assuming no imperfections on the commodity markets, so that it makes sense to include directly income in the utility function, rather than other consumption goods.

$$l^i \geq 0 \tag{6}$$

$$l^o \geq 0 \tag{7}$$

$$\bar{L} \geq l^o \tag{8}$$

where:
U is the household's utility function,
y is total household income,
l^l is leisure,
$\mathbf{z^h}$ is a vector of *household* characteristics relevant in *consumption* decisions,
p is the output price,
q is the quantity produced,
h is the amount of hired labor,
$\mathbf{z^q}$ is a vector of *farm* characteristics relevant in *production* decisions,
w^h is the effective price to be paid for hired labor,
l^i and l^o are the amounts of family labor employed in and off–farm respectively,
w^o is the effective wage received by family labor outside the farm,
A is farm size,
E is the total family labor endowment, and
$\bar{L}$ is the maximum amount of family labor that can find work off-farm.

The utility function $U(\cdot)$ is assumed to be increasing and quasiconcave; the production function $q(\cdot)$ is assumed to be increasing and concave.

Substituting from (2) and (3) for y and l^l the objective function can be written as:

$$\max_{h,l^i,l^o} U(pq(l^i + h, A, \mathbf{z^q}) - w^h h + w^o l^o, E - l^i - l^o, \mathbf{z^h}) \tag{9}$$

subject to the constraints given by equations (5) through (8).

The first order Kuhn Tucker conditions that describe this problem are:

$$\frac{\partial U(\cdot)}{\partial h} : [U_1(\cdot)[pq_L(\cdot) - w^h + \mu^h]]h = 0,\ U_1(\cdot)[pq_L(\cdot) - w^h + \mu^h] \leqq 0,\ h,\ \mu^h \geq 0 \tag{10}$$

$$\frac{\partial U(\cdot)}{\partial l^i} : [U_1(\cdot)pq_L(\cdot) - U_2(\cdot) + \mu^i]l^i = 0,\ U_1(\cdot)pq_L(\cdot) - U_2(\cdot) + \mu^i \leqq 0,\ l^i,\ \mu^i \geq 0 \tag{11}$$

$$\frac{\partial U(\cdot)}{\partial l^o} : [U_1(\cdot)w^o - U_2(\cdot) + \mu^o - \mu^{\bar{L}}]l^o = 0,\ U_1(\cdot)w^o - U_2(\cdot) + \mu^o - \mu^{\bar{L}} \leqq 0 \tag{12}$$

with l^o, $\bar{L}$, μ^o, $\mu^{\bar{L}} \geq 0$, $\bar{L} - l^o \geq 0$.

In addition:

$U_1(\cdot) = U_1(p, w^o, w^h, l^i, h, A, \mathbf{z^q}, E, l^o, \mathbf{z^h})$ and

$U_2(\cdot) = U_2(p, w^o, w^h, l^i, h, A, \mathbf{z^q}, E, l^o, \mathbf{z^h})$

are the marginal utilities of income and leisure respectively,

$q_L(\cdot) = q_L(l^i + h, A, \mathbf{z^q})$ is the marginal productivity of labor and

μ^t, $t = h, i, o, \bar{L}$ are the multipliers associated with the non-negativity constraints.

In this simplified setting and assuming that $w^h > w^o$, that is the price to pay per unit of hired labor is higher than the price received per unit of family labor sold outside, the relative size between farm size A and the total family labor endowment E will determine in which of four possible alternative regimes the farm-household will optimally operate: workers, net sellers, net buyers and self-sufficient in labor. Since the focus of this study is to explain labor allocation decisions and unobserved heterogeneity of small farmer that participate in the market as net sellers, we concentrate the rest of the analysis on them. Appendix B summarizes how these regimes emerge. For a more complete description of this classification process, we refer the reader to earlier work by Roemer [26], Eswaran and Kotwal [9] and Sadoulet et al. [27].

In absence of other constraints (like, for example, consumption cash constraints or food market constraints), for the households who participate in the labor market as net sellers, whether there is *separability* between production and consumption decisions will depend on whether the maximum constraint is binding or not. The simple observation that a household is selling labor will not be sufficient to infer separability.

Proposition 1: For net sellers of labor for whom the off-farm labor constraint is not binding, production and consumption decisions are separable.

To see this, we analyze the Kuhn Tucker first order conditions for net sellers. For these households $h = 0$, $l^i > 0$, $l^o > 0$, $\mu^h = \mu^i = \mu^o = 0$. If, in addition, the off-farm labor constraint (8) is not binding, we also have $l^o < \bar{L}$ and $\mu^{\bar{L}} = 0$ The Kuhn Tucker conditions reduce to:

$$\begin{cases} \frac{\partial U(\cdot)}{\partial l^i} : U_1(\cdot) p q_L(\cdot) - U_2(\cdot) & = 0 \\ \frac{\partial U(\cdot)}{\partial l^o} : U_1(\cdot) w^o - U_2(\cdot) & = 0 \end{cases} \tag{13}$$

where in this case:

$U_1(\cdot) = U_1(p, w^o, l^i, A, \mathbf{z^q}, E, l^o, \mathbf{z^h})$,

$U_2(\cdot) = U_2(p, w^o, l^i, A, \mathbf{z^q}, E, l^o, \mathbf{z^h})$ and
$q_L(\cdot) = q_L(l^i, A, \mathbf{z^q})$.

Combining the two first order conditions we obtain:

$$pq_L(l^i, A, \mathbf{z^q}) = w^o \tag{14}$$

that can be solved for l^i leading to a single, reduced form equation in which the variable l^i is expressed as a function of **only** "production side characteristics" and not of "consumption side characteristics":

$$l^i = f(p, w^o, A, \mathbf{z^q}) \tag{15}$$

In this case, the household will sell in the labor market all the excess labor and the decision price will be the market price w^o.

Proposition 2: For net sellers of labor for whom the off-farm labor constraint is binding, the separability between production and consumption decisions breaks.

If, instead, the quantitative constraint on the labor market is binding (and thus $l^o = \bar{L}$), the separability between production and consumption decisions no longer holds. Using the Kuhn Tucker conditions for this case we have:

$$\begin{cases} \frac{\partial U(\cdot)}{\partial l^i} : U_1(\cdot)pq_L(\cdot) - U_2(\cdot) = 0 \\ \frac{\partial U(\cdot)}{\partial l^o} : U_1(\cdot)w^o - U_2(\cdot) - \mu^{\bar{L}} = 0, \text{ and } \bar{L} = l^o \end{cases} \tag{16}$$

with:
$U_1(\cdot) = U_1(p, w^o, l^i, A, \mathbf{z^q}, E, l^o = \bar{L}, \mathbf{z^h})$,
$U_2(\cdot) = U_2(p, w^o, l^i, A, \mathbf{z^q}, E, l^o = \bar{L}, \mathbf{z^h})$ and
$q_L(\cdot) = q_L(l^i, A, \mathbf{z^q})$.

The household will sell $\bar{L}$ on the labor market and supply labor on-farm up to the point where the marginal product of labor equates the marginal utility from leisure. The decision price becomes a *shadow price*, lower than w^o.

To find the optimal quantity of l^i, the equations above must be solved jointly for l^i and $\mu^{\bar{L}}$, recognizing that for this set of households, constraint (8) holds with equality and thus $l^o = \bar{L}$ (which implies that there are only two unknowns):

$$l^i = l^i(p, w^o, A, \mathbf{z^q}, E, \mathbf{z^h}, l^o = \bar{L}) \tag{17}$$

and

$$\mu^{\bar{L}} = l^i(p, w^o, A, \mathbf{z^q}, E, \mathbf{z^h}, l^o = \bar{L}) \tag{18}$$

As it can be seen, in this case, the constrained optimal allocation for on-farm labor l^i also depends on consumption side parameters, E, $\mathbf{z^h}$, and on the off-farm labor endowment $l^o = \bar{L}$. Therefore, the separability hypothesis breaks.

At this point, a brief discussion about the nature of the off-farm labor constraint is warranted. In particular, from equations (13) and (16) it can be easily verified that the off-farm labor allocation rule is given by:

$$l^o = \begin{cases} l^o(p, w^o, A, \mathbf{z^q}, E, \mathbf{z^h}) & \text{if } l^o < \bar{L} \\ \bar{L} & \text{if } l^o \geq \bar{L} \end{cases} \tag{19}$$

Denoting λ as the probability that a household is constrained we have:

$$\lambda = \lambda(l^o \geq \bar{L}) = \lambda(l^o(p, w^o, A, \mathbf{z^q}, E, \mathbf{z^h}) - \bar{L} \geq 0) \tag{20}$$

which in reduced form becomes:

$$\lambda = \lambda(p, w^o, A, \mathbf{z^q}, E, \mathbf{z^h}, \bar{L}) \tag{21}$$

To conclude, we show that under the assumption of no other market imperfections that might introduce non separability (for example, presence of credit constraints or food market imperfections), a labor selling farm-household will determine the amount of labor employed on farm, l^i, according to one of two alternative regimes, defined by equation (15) and equation (17) respectively. One empirical implication of this is that if the researcher had information on households' classifications, a testable hypothesis on the separability assumption could be directly implemented. However, in most cases, this information is unobserved. The next section addresses this issue of "unknown sample separation".

3 Econometrics

The preceding section establishes that it is conceivable for farm-households to participate in the market as net sellers, and yet be constrained by unobservable quantity limitations or by transaction costs in their ability to respond to price changes. Prior knowledge by the researcher of the sample division could be used to examine each labor regime separately. The problem, however, is that in our case such classification is unknown. This can be translated in econometric terms saying that the farm labor supply response function of a group of market participating households can be represented by a *switching regression model with unobserved sample separation* (Quandt [25], Maddala [22] p.302)[3].

Formally, we can characterize the sample behavior in a three-equation model:

$$l^1 = l_1(\mathbf{x_1}; \boldsymbol{\beta}) + u_1 \tag{22}$$

$$l^2 = l_2(\mathbf{x_2}; \boldsymbol{\gamma}) + u_2 \tag{23}$$

$$\lambda = \lambda(\mathbf{x_\lambda}; \boldsymbol{\xi}) + u_\lambda \tag{24}$$

where using our results from the theoretical part $\mathbf{x_1} = \{p, w^o, A, \mathbf{z^q}\}$ and $\mathbf{x_2} = \{p, w^o, A, \mathbf{z^q}, E, \mathbf{z^h}, \bar{L}\}$, $\mathbf{x_\lambda} = \{p, w^o, A, \mathbf{z^q}, E, \mathbf{z^h}, \bar{L}\}$, β, γ and $\boldsymbol{\xi}$ are coefficients to be estimated and u_j's are normal iid disturbances with zero means and variances σ_j^2.

l^1, l^2 and λ are latent unobserved variables. Instead for observation i, we observe variable l^i, defined by:

$$l^i = \begin{cases} l_1(\cdot) & \text{if } \lambda \leqq 0 \\ l_2(\cdot) & \text{if } \lambda > 0 \end{cases} \tag{25}$$

The problem is that of estimating the parameters $\{\boldsymbol{\beta}; \gamma; \boldsymbol{\xi}; \sigma_1, \sigma_2, \sigma_\lambda\}$ from the sample of N observations on $\{l^i, \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_\lambda\}$, $i = 1, \ldots, N$. Given that a priori we cannot identify the regime composition, a randomly selected observation l^i (household i's on-farm labor supply) will have probability λ of belonging to the first regime, and probability $(1 - \lambda)$ of belonging to the second one. Appendix C provides a complete description of the approach. The main steps are as follows: the conditional density of observation l^i given regime 1 can be written as:

$$f(l^i | regime1) = f(l^i - l_1(\mathbf{x_1}; \boldsymbol{\beta}))/\lambda$$

while the conditional probability of l^i given regime 2 is:

$$f(l^i | regime2) = f(l^i - l_2(\mathbf{x_2}; \boldsymbol{\gamma}))/(1 - \lambda)$$

where $f(\cdot)$ are probability density functions of u_1 and u_2.

[3] Empirical work using a similar approach includes Lee and Porter [20] on cartels, Bash and Paredes-Molina [2] on dual markets in Chile, Murdoch and Stern [23] on sex bias, Pape and van Dijk [24] on growth rates convergence and Conway and Kimmel [7] on moonlighting.

The unconditional density of l^i is then:

$$\begin{aligned} f(l^i) &= \lambda f(l^i|regime1) + (1-\lambda) f(l^i|regime2) = \\ &= f(l^i - l_1(\mathbf{x_1};\boldsymbol{\beta})) + f(l^i - l_2(\mathbf{x_2};\boldsymbol{\gamma})) \end{aligned} \tag{26}$$

that is, the *mixture* of two distributions.

Focusing on the simpler case of mixture of two components, starting from the unconditional density function (26) and assuming that $f(\cdot)$ is normal, $\sigma_1 \propto \sigma_2$ and $\sigma_\lambda = 1$ (required for identification purposes) the likelihood function for a sample of N observations is then:

$$L(\lambda, \boldsymbol{\beta}, \boldsymbol{\gamma}, \sigma_1, \sigma_2) = \prod_N \left[\lambda\phi(l^i - l_1(\mathbf{x_1};\boldsymbol{\beta}), \sigma_1) + (1-\lambda)\phi(l^i - l_2(\mathbf{x_2};\boldsymbol{\gamma}), \sigma_2)\right] \tag{27}$$

where $\phi(\cdot)$ denotes the normal density function. A natural way of estimating the parameters would be that of maximizing (27) with respect to $(\lambda, \boldsymbol{\beta}, \boldsymbol{\gamma}, \xi, \sigma_1, \sigma_2)$.

Properties of mixture distributions and the estimation of their parameters are extensively covered in Titterington [30] and Everitt [10]. Appendix C summarizes some econometric issues and provides insights on how to approach the problem of estimation in practice, as presented in Quandt [25], their extensive commentary (Hartley [13], Bryant [5], Clarke and Heathcote [6], Johnson [16], Hosmer [14], Kiefer [18], Binder [4] and Fowlkes [12]), and summarized in Maddala [22] pp. 302–305.

Agreement seems to have been reached on that estimation of the maximum likelihood, obtained via the so called E-M method (see Hartley [13] and Dempster et al. [8]) is a feasible approach, perhaps after having used other pre-estimation techniques to decide on the initial values of the parameters (Kiefer [17]). Moreover, the E-M approach can be used to estimate the model when λ is considered to be endogenously determined and specified as above. The only required assumption is that the variance of the error term of the classification equation (u_λ) is taken to be one.

The steps of the E-M algorithm are as follows: using starting values for $\beta, \gamma, \xi, \sigma_1, \sigma_2$, we first obtain estimates of the classification vector λ.(the E step). The starting values for the $\beta, \gamma, \sigma_1, \sigma_2$ can be set equal to the estimated values for the pooled sample regression, the rational for which is that, if the observations were truly coming from the same population, those were the values would maximize the likelihood function. Using the estimate for λ to weight the probabilities of each observation to be in each regime and obtain estimates for $\beta, \gamma, \xi, \sigma_1, \sigma_2$ (the M step). This iterative procedure is repeated until the maximum likelihood function (in our case Equation (27)) converges.

4 Data and Results

Data The data come from the 1997 Peruvian LSMS. The survey was contacted on 4500 households. From these, 1131 allocate work between both on and off-farm activities and are used in the analysis. We postpone a discussion on descriptive statistics for now as it will be more relevant to present them after the estimation. We offer instead Figure 1, which plots the distribution of individual off-farm hours worked and seems to suggest the existence of two underlying processes. In particular, there seem to be two distinct subpopulations, one for which individuals work less off-farm and another that work more. What the econometric procedure attempts to do is to explore, among other things, this heterogeneity and characterize the two regimes. We superimpose a normal distribution to hint on the implications of assuming one homogeneous population.

Separability According to a fully separable model, the decision on labor allocation on farm should be purely a production decision, and thus household characteristics (such as E, z^h, $\bar{L}$) should not affect it. Our theoretical model postulates the possibility of the presence of two different regimes among the farm-households in the sample. It also predicts that household characteristics should only affect the constrained regime. We therefore specify one of the two regimes by including variables such as household composition z^h (like children, elder members or ethnicity). We specify the second regime by only including production side characteristics z^q.[4] Apart from this restriction that comes directly from the theoretical model, we do not impose any other restrictions on the parameters of the two models in the belief that, if a dichotomy actually exists, it should be strong enough to let the econometric technique to separate between the two regimes. We apply the maximum likelihood procedure described in the previous section to determine the best way of dividing the observations in two groups. The dependent variable is

[4] Notice that in this specification of the unconstrained regime we exclude any variables that can be argued to affect both consumption and production decisions. On the other hand, our counterfactual estimation (below) includes such variables in both regimes.

the amount of hours allocated by the household for farm activities[5].

Table 1 presents the results. The first column contains the results of an OLS estimation of the model on the whole pooled sample[6]. As expected, production characteristics such as livestock assets, human capital and land assets significantly affect on-farm labor allocation. In addition, these pooled estimates seem to show a significant effect of some household characteristics on the decision of labor use on-farm. This alone provides some indication that the issue of non-separability is important.

The second column in Table 1 presents the results from the maximum likelihood procedure. On average, a household has a probability of 0.48 to be constrained in its labor market participation. As we expected, for those households in the constrained regime, we find that not only production characteristics affect their on-farm labor allocation but consumption ones as well, thus rejecting the separability hypothesis. In particular, we find that small children and the elder significantly increase on-farm labor allocation. In addition, other variables that can be argued as both production and consumption ones also affect on-farm labor allocation. For example, education decreases the on-farm work, implying that human capital is an important asset for off-farm work. In addition regional dummies that capture idiosyncratic effects and shocks of specific geographic areas also affect on-farm labor decisions. Households that live in the mountains work more on-farm compared to Lima, an urban center. This is expected as off-farm opportunities are much more abundant in Lima than in the more isolate mountain regions[7].

For the second group of households estimated by the econometric procedure (and specified as the unconstrained regime), we find that production characteristics such as land and cattle ownership positively affect on-farm labor allocation. In addition, transaction costs in the form of the time to get to the main market, negatively affects work on-farm. One interesting finding is that women have no impact for on-farm labor. One hypothesis is that since these households are not constrained in labor, women may actually have off-farm jobs or do not need to work.

For both regimes, we also expected a significantly different value for the coefficient on the wage rate. For the unconstrained regime it should be sig-

[5]Remember that we do not consider hired labor in this analysis since we are only looking at households that are net sellers of labor.

[6]The coefficients of this OLS estimation were also used as starting values for the likelihood maximization routine.

[7]We also estimated these models using district dummies and got similar results.

nificant and negative, in which the wage rate plays just the role the marginal product of labor. However, for the constrained regime, the relevant price for labor to be used on farm is, instead, a shadow price determined by the subjective equilibrium of the farm-household, and therefore the wage rate will not necessarily reflect the marginal product of labor. Instead the market wage will affect the production decisions only indirectly, through its effect on total family income.

The analysis above would not be correct if any of the consumption side characteristics affect the on-farm labor decision for what we specify as the unconstrained regime. For this, we implement a counterfactual estimation including in the specification of the unconstrained regime all of the consumption characteristics used in the constrained one. The aim is to test whether any of these consumption variables have a significant impact for the unconstrained. Table 2 reports the results of this specification. The results do support the separability hypothesis for the second regime. Specifically, none of the consumption characteristics have a significant impact on the on-farm labor decisions of unconstrained households. Two of the regional dummies affect this decision, but as we discussed above, this could reflect production aspect differences in marketing opportunities among regions.

To recapitulate, applying the maximum likelihood procedure allowed us to separate the sample in two sub-populations of labor market participants (net sellers). We find strong evidence that in one of them, the separability hypothesis between production and consumption decisions is rejected, while for the second it is not. A counterfactual test strengthens our findings by showing that the households in the unconstrained regime behave in a separable fashion.

Understanding the labor constraint The results above provide sensible indication of the presence of a group of farm household which, albeit participating in the labor market, are making their decision on farm activity according to a non-separable model of behavior. This is, of course, not enough to conclude that the *cause* of non separability is the presence of a quantity constraint on the labor market of the kind we described in presenting the theoretical model. The last column of Table 1 contains the estimated coefficients for the equation that determines the group separation. Given the specification of the two regimes, we can interpret this switcher equation to represent the probability of being in the constrained regime. We use equation

21 from the theoretical part to specify the determinants of group separation.

Interesting patterns seem to emerge that relate to market integration and participation. The market wage itself increases the probability of being constrained. A higher wage may make it harder to find an off-farm job due to increased competition (by other workers) and decreased demand by employers. In addition, a larger farm (in terms of land size) lowers the probability to be constrained via the increase in labor demand for on-farm labor. In terms of other household characteristics, a larger number of both male and female members (E) in the household as well as high time costs of accessing markets (p) increase the probability to be constrained, while a larger number of children and elder members (z^h) decrease it.

Interactions between ethnic classification and education are also important to characterize market opportunities. Being indigenous increases the probability to be constrained, perhaps implying the presence of ethnic discrimination. On the other hand, households with higher levels of education are less likely to be constrained. Moreover, an interaction term between the indigenous dummy and education levels captures the importance of education: higher levels of education among indigenous households may mitigate their access to labor markets by reducing the probability to be constrained.

At the regional level, households living closer to mountains and the rainforest (as opposed to Lima) have a higher probability to be constrained as their off-farm opportunities there are more limited. The same is true for the coastal areas but the effect does not seem as strong. We also wanted to explore further the condition of the labor market beyond the regional dummies. While the data set is limited for this, we include two variables that capture market demand and availability for off-farm work at the village level. We differentiate between availability of private and public off-farm job opportunities. Interestingly, while the availability of private jobs decreases the probability to be constrained as expected, the availability of public jobs increase it.

Finally, working more off-farm is negatively related with being constrained. Off-farm work can be thought as representing the off-farm labor constraint ($\overline{L}$) derived in the theoretical model. Relaxing this constraint therefore makes it less likely to be constrained.

The probability to be constrained One of the benefits of our estimation methodology is that we can divide the sample between those that are more likely to be constrained and those that are not (using the predicted probability to be in a given regime $\widehat{\lambda}$). This leads to an ex-post predicted sample of "constrained" and "unconstrained" households. We use different ways to get insights on the structure of the labor market constraint from these sub-samples.

From our theoretical framework, we know that the constrained households' off-farm labor supply l^o is also their binding off-farm allocation labor $\bar{L}$. This distribution is plotted in Figure 2. We would expect that if there was a common market based barrier on the amount of labor to be allocated, it would show up as a very narrow distribution for $l^o = \overline{L}$ for those households in the constrained regime. The fact that this is not true suggests that household idiosyncracies may be more important in terms of accessing the labor market. Figure 2 also plots the distribution of individual off-farm hours for the unconstrained. These individuals overall work more than constrained ones, as seen by the fact that the off-farm hours distribution is shifted to the right. This is consistent with our findings above.

Another way to take of advantage of our methodology is by looking at the predicted probabilities of being constrained. Figures 3 and 4 plot two such distributions by ethnic group and regions respectively. In Figure 3, we see that indigenous people have overall higher probabilities of being constrained, supporting our findings above and the hypothesis of possible ethnic discrimination. While the contrast is not as strong when we look at this in terms of regional differences, there is some evidence to suggest that households living in mountain regions are more likely to be constrained. This may reflect the fact the off-farm labor opportunities in the Peruvian sierra are usually limited and even non-existent.

We also look at the link between being constrained and poverty. We use both income and consumption quintiles to compare the probabilities of being constrained (Table 3). Indeed, there is a strong correlation between being constrained and poverty. For both indicators, being poorer implies having a higher chance of being constrained. This probability decreases at higher levels of consumption and income.

Finally, a more descriptive comparison between "constrained" and "unconstrained" households is presented in Table 4. Overall, constrained households have lower levels of key assets: they own less land, are less educated and are poorer in terms of both income and consumption. Constrained households are also larger and predominantly indigenous. These observations suggest that access to off-farm opportunities may be closely related to access and accumulation of both human and physical assets.

5 Concluding remarks

We use mixture distribution techniques to show that labor market participation is not enough to conclude separability between production and consumption decisions of farm-households. Our findings clearly show the existence of two distinct types of households among net-sellers of labor: those behaving as if unconstrained and hence in a separable way and some that behave as if constrained. These results provide an important refinement of separability studies by extending and expanding the concept of heterogeneity.

In addition, we also take advantage of the econometric technique to explore and understand the market segmentation and its source. In doing this, we try to look at the role of both demand and supply side effects on labor constraints. In the case of Peru, ethnic discrimination, differential education attainments as well as differences in regional opportunities seem to be some important deterrents for market participation.

Some interesting policy implications arise. First, if we are able to characterize well the source of the sample separation, it enables us to assess (at least qualitatively) the impact of different policies. For example, in our case, we find that almost half the sample is likely to be constrained in the labor market, even though they participate in it. Therefore, a wage policy that ignored this observation would not only be incorrect but quite ineffective as well. In particular, policies using pooled wage elasticities would underestimate the effects of a wage change since the constrained share of the pooled sample would not alter its labor market behavior based on a wage policy (also see Figure 5). The methodology proposed here could be then used to correctly estimate more meaningful elasticities based on the unconstrained and thus relevant portion of the population.

The characterization of the source of the market constraint itself also provides us with the means to better understand and implement different policies that can be effective to ameliorate market integration and participation. An exhaustive use of possible individual, regional and market heterogeneity (for example unemployment rates, infrastructure, human capital differences as well as other sources for non separability such as credit and risk) could allow us to make inferences about the structure of the market failure. While the nature of our data and their lack of such a list of supply side variables limits our analysis somewhat, the methodological insights we get are important. Without our data constraints, this methodology can allow future analyses to carefully compare different policy options that may arise which can be used to guide policy makers to devise and target policies that maximize social welfare.

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A Tables and Figures

Table 1. On-farm labor allocation: pooled versus mixture model

		Pooled	Mixture		
			Constrained	Unconstrained	Switcher
Market wage (soles)	w^o	-0.03	-0.14***	-0.05**	0.03**
Land owned (ha.)	A	0.15**	0.92*	0.21***	-1.37***
Adult males (#)	E	25.66***	36.25***	2.54**	19.27***
Adult women (#)	E	25.90***	34.74***	1.24	17.05***
Time to market (min.)	p	0.96	6.20	-3.80***	6.98***
Time to market sq. (min. sq.)	p	0.02	-0.95	0.16***	-0.61***
Cattle owned (#)	z^q	1.30***	0.96***	1.39***	0.06
Coast (dummy)	$z^{q,h}$	13.14	12.02		17.04***
Sierra (dummy)	$z^{q,h}$	28.88***	27.32***		37.95***
Rainforest (dummy)	$z^{q,h}$	25.37***	7.03		32.96**
hh average education (years)	$z^{q,h}$	-10.41**	-12.18**		-7.62***
hh average ed. sq. (years sq.)	$z^{q,h}$	-0.39**	0.67***		-0.01
hh head sex (male=1)	$z^{q,h}$	11.72	14.66		11.08**
hh average ed.* indigenous hh	$z^{q,h}$	-0.71**	-0.42		-0.56***
head age (years)	$z^{q,h}$	0.26	1.35		-0.73
head age sq.(years sq.)	$z^{q,h}$	-0.003	-0.02*		0.002
Boys (#)	z^h	6.51**	18.48***		-9.57***
Girls (#)	z^h	17.18***	38.53***		-2.29
Elder (#)	z^h	12.66**	43.07***		-2.30***
Indigenous hh (yes=1)	z^h	37.97**	19.23		17.38***
Hours worked off-farm	l^o	-0.20***	-0.32***		-0.14***
Availability of private jobs in community	z^q	-49.96	-18.76		-65.10***
Availability of public jobs in community	z^q	8.29	-1.81		18.40***
Constant		130.98**	122.9**	73.98***	33.29
Sample proportion	$\widehat{\lambda}$	1.0	0.48	0.52	1.0
R^2 (pooled)		0.2			
Log likelihood (mixture)		-261			

Dependent variable: household's on-farm work (hours).

Switcher: probability of being constrained.

The coefficients of the switcher equation are all multiplied by 100 ($\widehat{\xi} * 100$).

The missing dummy for regions is Lima.

Significance levels:*: 90%, **: 95%, ***: 99%

Sample size: 1131

Table 2. Counterfactual estimation: On-farm labor allocation using mixture model

		Mixture		
		Constrained	Unconstrained	Switcher
Market wage (soles)	w^o	-0.12***	-0.06**	0.05***
Land owned (ha.)	A	0.89	0.22***	-1.03***
Adult males (#)	E	18.43***	-0.41	32.93***
Adult women (#)	E	26.82***	-0.44	23.14***
Time to market (min.)	p	8.42	-3.61***	2.86
Time to market sq. (min. sq.)	p	-0.59**	0.16***	-0.27***
Cattle owned (#)	z^q	0.66*	0.69***	0.90***
Coast (dummy)	$z^{q,h}$	17.78	1.77	19.05***
Sierra (dummy)	$z^{q,h}$	36.97***	-4.85*	31.91***
Rainforest (dummy)	$z^{q,h}$	13.69	5.49*	32.65***
hh average education (years)	$z^{q,h}$	-16.45***	1.72	-2.41
hh average ed. sq. (years sq.)	$z^{q,h}$	0.72***	-0.06	-0.18
hh head sex (male=1)	$z^{q,h}$	24.13**	1.93	19.07***
hh average ed.* indigenous hh	$z^{q,h}$	-0.31	-0.18	-0.37*
head age (years)	$z^{q,h}$	4.52***	-0.17	-4.35***
head age sq.(years sq.)	$z^{q,h}$	-0.05***	0.001	0.05***
Boys (#)	z^h	9.01**	0.99	-4.92**
Girls (#)	z^h	27.55***	-0.89	4.51**
Elder (#)	z^h	29.12***	-1.23	-13.52***
Indigenous hh (yes=1)	z^h	11.09**	3.70	44.55***
Hours worked off-farm	l^o	-0.17***		-0.18***
Availability of private jobs in community	z^q	-31.25	-36.26	-27.35
Availability of public jobs in community	z^q	5.53	0.70	15.12***
Constant		102.91*	107.18***	16.71
Sample proportion	$\widehat{\lambda}$	0.49	0.51	1.0
Log likelihood (mixture)		-254		

Dependent variable: household's on-farm work (hours).
Switcher: probability of being constrained.
The coefficients of the switcher equation are all multiplied by 100 $(\widehat{\xi} * 100)$.
The missing dummy for regions is Lima.
Significance levels:*: 90%, **: 95%, ***: 99%
Sample size: 1131

Table 3: Probabilities to be constrained by quintiles ($\widehat{\lambda}$)

	First (lowest)	Second	Third	Fourth	Fifth (highest)
Income	53	49	43	43	37
Consumption	48	50	45	42	36

Table 4: Household descriptive statistics by predicted constrained regime

		Constrained	Unconstrained	Pooled
hh on-farm work (hours per week)	l^i	153	105*	124
hh off-farm work (hours per week)	l^o	112	108	110
Per hh member on-farm work (hours per week)	l^i	39	44*	42
Per hh member off-farm work (hours per week)	l^o	27	42*	36
Market wage (soles)	w^o	189	219*	207
Adult men in hh (#)	E	2.1	1.3*	1.6
Adult women in hh (#)	E	2.2	1.4*	1.7
Land owned (hectares)	A	0.7	2.8*	1.9
Cattle owned (#)	z^q	3.2	0.9*	1.8
Time to market (min.)	p	54	66*	60
Average hh education (years)	$z^{q,h}$	9.5	9.9*	9.6
hh head education (years)	$z^{q,h}$	8.6	11.2*	10.2
head age (years)	$z^{q,h}$	55	46*	50
hh head sex (male=1)	$z^{q,h}$	0.8	0.9*	0.9
Coastal (%)	$z^{q,h}$	20	26*	24
Sierra (%)	$z^{q,h}$	30	21*	25
Rainforest (%)	$z^{q,h}$	17	19	18
Lima (%)	$z^{q,h}$	31	34	33
Boys in hh (#)	z^h	1.1	1.2*	1.1
Girls in hh (#)	z^h	1.2	1.0*	1.0
Elder in hh (#)	z^h	0.4	0.4	0.4
hh size (#)	z^h	7.1	5.3	6.0
Dependency ratio ($\frac{\#\ children\ +\ elder}{\#\ adults}$)	z^h	0.7	1.1*	0.9
hh indigenous (%)	z^h	32	7*	17
Per capita income (soles)	y	1111	1557*	1362
Per capita consumption (soles)	C	2467	3532*	3101
	obs	458	673	1131

Note: * means that there is a significant difference between the unconstrained and constrained groups at the 90% level or more

Figure 1: Off-farm hours worked per individual

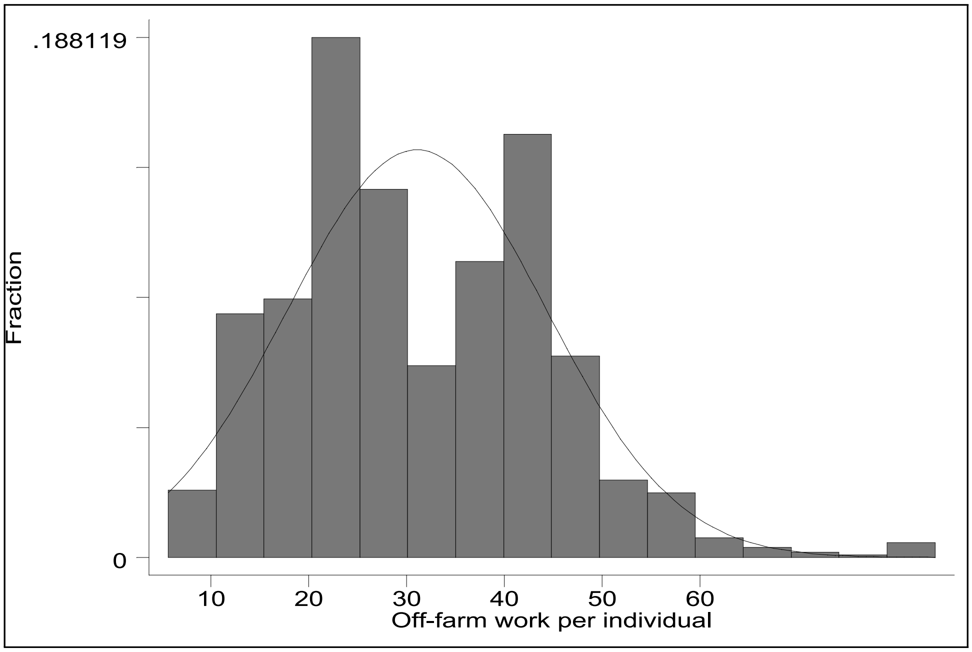


Figure 2: Kernel densities of off-farm hours worked per individual

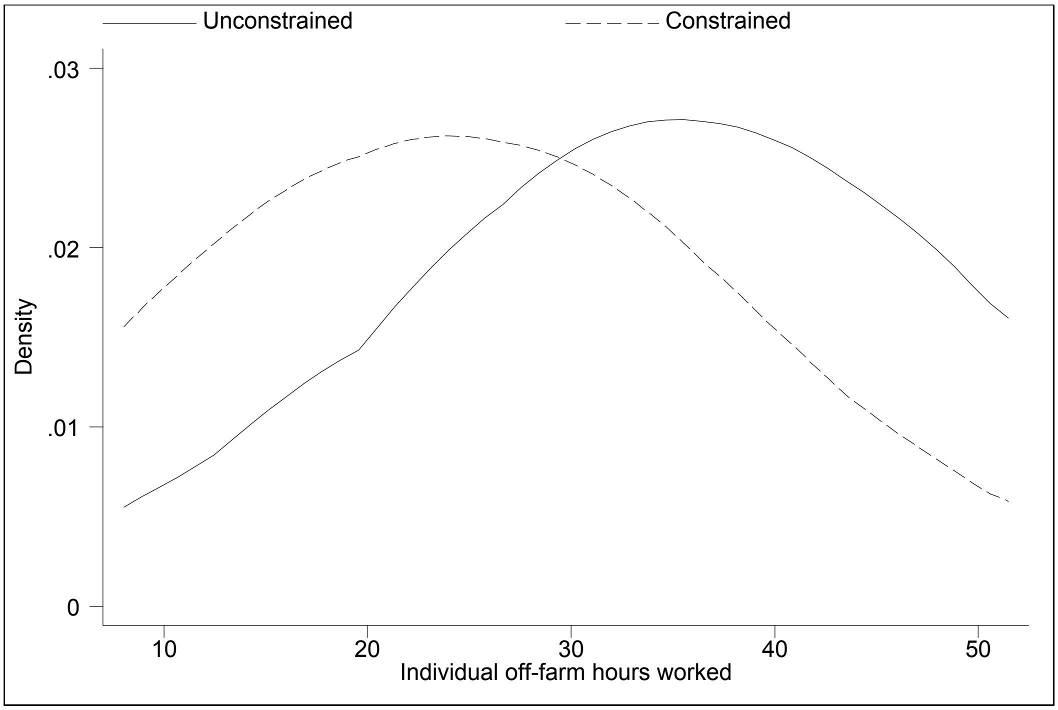


Figure 3: Kernel densities of the probability to be constrained by ethnicity

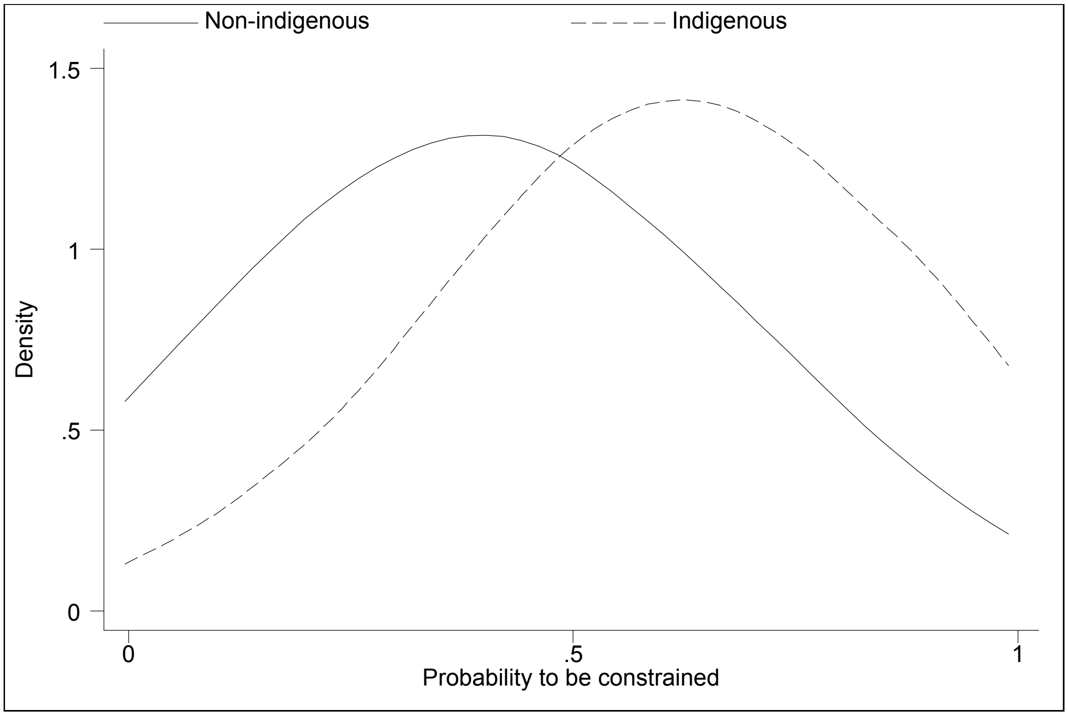


Figure 4: Kernel densities of the probability to be constrained by geographic region

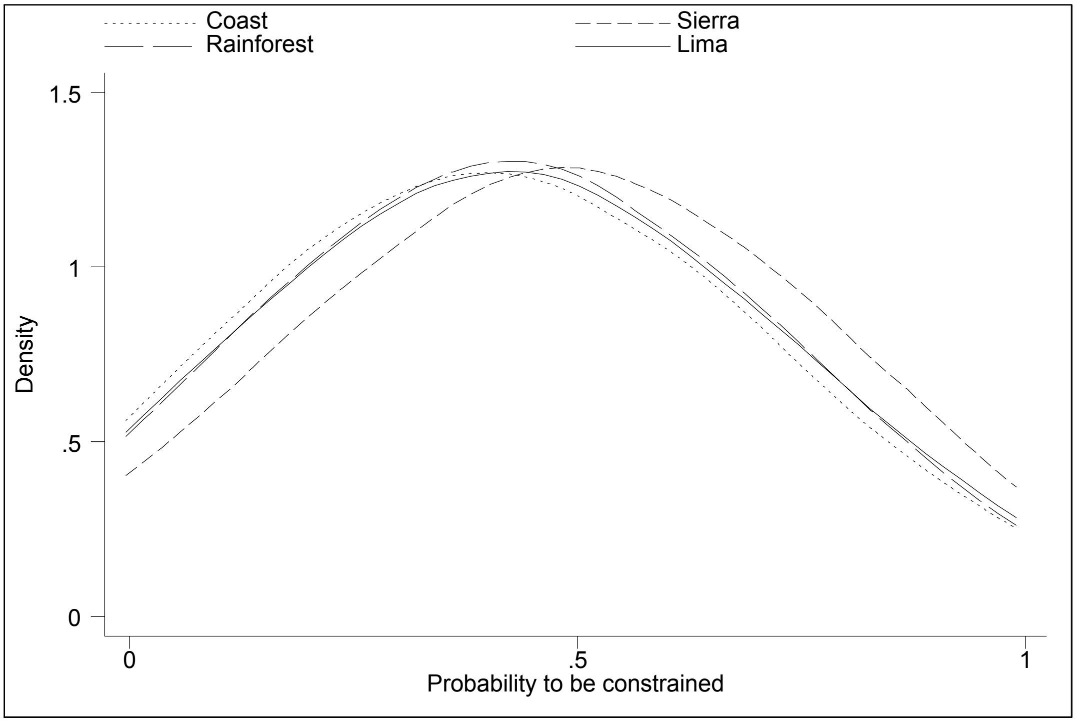
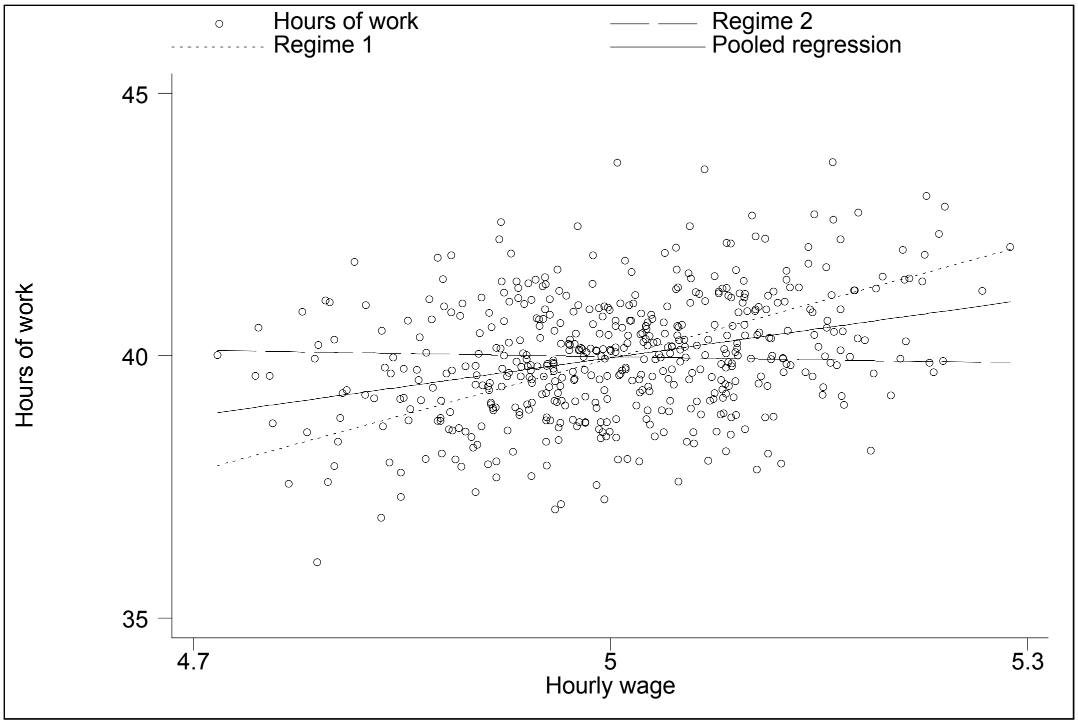


Figure 5: Pooling effects



B Regime Classification

In order to classify farm households in labor participation regimes, we first observe that for a farm-household that does not participate in the labor market at all, that is, for a self-sufficient (in labor) farm-household:

$$h = l^o = 0$$

Then, from the Kuhn-Tucker conditions (equations (10), (11) and (12)), the quantity of labor on-farm $\underline{l^i}$ is given by solving:

$$U_1(pq_L(\underline{l^i}, A), E - \underline{l^i}) \; pq_L(\underline{l^i}, A) = U_2(pq_L(\underline{l^i}, A), E - \underline{l^i}) \tag{28}$$

Equation (28) states that the household will allocate labor for on-farm activities until the value of the marginal product of an additional unit of work equates the marginal utility of income. The shadow wage $\underline{w}$ can be defined as the value of the marginal product of labor at $\underline{l^i}$:

$$\underline{w} = pq_L(\underline{l^i}, A) = \underline{w}(p, E, A) \tag{29}$$

Sadoulet et al. [27], show that $\underline{w}$ is an increasing function of A and a decreasing function of E :

(i) $\frac{\partial \underline{w}}{\partial A} > 0$

(ii) $\frac{\partial \underline{w}}{\partial E} < 0$

Define A_o to be the farm size such that:

$$pq_L(0, A_0) = w^o$$

Then, farm-households are classified in regimes as follows:

1. Worker

   If farm size $A < A_o$, $(i), (ii) \Rightarrow pq_L(l^i, A) < w^o$. Then, $l^o > 0, l^i = h = 0$. In this case, it is not worthwhile to engage in farm activities as the market wage $w^o$is higher. The household will only work off-farm in the labor market. If the labor constraint for this household is binding, the household will only allocate $\bar{L}$ in the market. If the constraint is not binding then it will allocate all the available labor.

2. Net seller

   If $A > A_o$ and $\underline{w} < w^o$ and using (ii): $l^o > 0$, $l^i > 0$, $h = 0$.

   For A small enough, employing all family labor in the farm would make the marginal productivity of labor lower than the effective wage w^o, so that it becomes convenient to divert part of the available family labor from farm to off-farm activities until $\underline{w} = w^o$. However, it is at this point that the maximum constraint, $\bar{L}$, becomes relevant: if the excess supply of family labor is smaller that $\bar{L}$, the household will sell in the labor market all the excess labor and the decision price will be the market price w^o. If, instead, the excess supply is larger than $\bar{L}$, the household will sell on the market all the labor it can, $\bar{L}$, and then supply more labor in the farm up to the point where the marginal product of labor equates the marginal utility from leisure. The decision price becomes a *shadow price*, lower than w^o.

3. Self-sufficient

   If $A > A_o$ and $w^o < \underline{w} < w^h$ and using (ii): $l^o = 0$, $l^i > 0$, $h = 0$.

   For intermediate values of A, the household will find itself to be self sufficient in labor use. Employing all available family labor in the farm will make the marginal product of labor low enough to make unprofitable the hiring of labor at the market wage w^h, but not low enough to make it convenient to divert family labor from farm operation to market. The decision price will thus be a *shadow price*, whose value is bounded above by w^h and below by w^o.

4. Net buyer

   If $A > A_o$ and $\underline{w} > w^h$ and using (ii): $l^o = 0$, $l^i > 0$, $h > 0$.

   For A relatively large, the household will be a net buyer of labor. Even employing all available family labor on farm, marginal productivity of labor is still higher than the wage rate w^h, so that it is profitable to hire labor. The decision price to determine how much labor to utilize in the production activity, and thus how much to produce and how much to work will be the *market price* w^h.

C Statistical Appendix: Mixture Distributions

Consider the three equation regression model:

$$l^1 = l_1(\mathbf{x_1};\boldsymbol{\beta}) + u_1 \tag{30}$$
$$l^2 = l_2(\mathbf{x_2};\boldsymbol{\gamma}) + u_2 \tag{31}$$
$$\lambda = \lambda(\mathbf{x_\lambda};\boldsymbol{\xi}) + u_\lambda \tag{32}$$

where $u_1 \sim \mathrm{N}(0,\sigma_1^2)$, $u_2 \sim \mathrm{N}(0,\sigma_2^2)$ and $u_\lambda \sim \mathrm{N}(0,\sigma_\lambda^2)$ are iid disturbances, and where l^1, l^2 and λ are *latent* variables. Suppose that the *observed* variable l^i is defined by:

$$l^i = \begin{cases} l_1(\cdot) & \text{if } \lambda \leqq 0 \\ l_2(\cdot) & \text{if } \lambda > 0 \end{cases} \tag{33}$$

Then the problem is that of estimating the parameters $\{\boldsymbol{\beta};\boldsymbol{\gamma};\boldsymbol{\xi};\sigma_1,\sigma_2,\sigma_\lambda\}$ from the sample of N observations on $\{l^i,\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_\lambda\}$, for $i = 1,2,\ldots,N$.

Let

$$f_n(l^i) = \frac{1}{(2\pi)^{1/2}}\sigma_n \exp\left\{-\frac{1}{2}(l^i - \mathbf{x}_n{}'\tau)^2\right\}$$

for $n = 1,2,\lambda$ and $\tau = \boldsymbol{\beta},\boldsymbol{\gamma},\boldsymbol{\xi}$ respectively.

The joint pdf of the latent variables l^1, l^2, and λ is given by:

$$g(l^1,l^2,\lambda) = f_1(l^1)\cdot f_2(l^2)\cdot f_\lambda(\lambda) \tag{34}$$

whereas the pdf of the *observed* dependent variable l^i is given by:

$$h(l^i) = \lambda^*\cdot f_1(l^1) + (1-\lambda^*)f_2(l^2) \tag{35}$$

with

$$\lambda^* = \mathrm{Prob}[\lambda \leqq 0] = \Phi(-\mathbf{x}_\lambda{}'\boldsymbol{\xi}) \tag{36}$$

in which we are assuming that $\sigma_\lambda = 1$ for identification, and where $\Phi(\cdot)$ denotes the standard normal cdf.

Using the above, the *conditional* pdf of l^1, l^2 and l^λ, given the observed value of l^i is:

$$g(l^1,l^2,\lambda|l^i) = \begin{cases} g(l^i,l^2,\lambda)/h(l^i) & \text{if } \lambda \leqq 0 \\ \\ g(l^1,l^i,\lambda)/h(l^i) & \text{if } \lambda > 0 \end{cases} \tag{37}$$

Hartley [13] shows that the partials of the log-likelihood function:

$$L(\boldsymbol{\beta}, \gamma, \boldsymbol{\xi}, \sigma_1^2, \sigma_2^2) = \sum_{i=1}^{N} \log(h(l_i)) \tag{38}$$

with respect to $\boldsymbol{\beta}$ and γ can be written as:

$$\frac{\partial L}{\partial \boldsymbol{\beta}} = \frac{1}{\sigma_1^2} \sum_{i=1}^{N} w_1(l^i) \cdot (l^i - \mathbf{x}_1^{'} \boldsymbol{\beta}) \cdot \mathbf{x}_1 = 0 \tag{39}$$

and

$$\frac{\partial L}{\partial \gamma} = \frac{1}{\sigma_2^2} \sum_{i=1}^{N} w_2(l^i) \cdot (l^i - \mathbf{x}_2{}' \gamma) \cdot \mathbf{x}_2 = 0 \tag{40}$$

while the partial of the log-likelihood function with respect to ξ can be written as:

$$\frac{\partial L}{\partial \xi} = \sum_{i=1}^{T} (E[\lambda | l^i] - \mathbf{x}_\lambda{}' \boldsymbol{\xi}) \cdot \mathbf{x}_\lambda = 0 \tag{41}$$

where $E[\lambda | l^i]$ is the conditional expectation of λ given the observed value of l^i, which is equal to:

$$E[\lambda | l^i] = \mathbf{x}_\lambda{}' \boldsymbol{\xi} - w_1(l^i) \cdot \frac{f_\lambda(0)}{\lambda^*} + w_2(l^i) \cdot \frac{f_\lambda(0)}{1 - \lambda^*} \tag{42}$$

and where $w_1(l^i)$ and $w_2(l^i)$ are weights defined as:

$$w_1(l^i) = \lambda^* \cdot [f_1(l^i)/h(l^i)] \tag{43}$$

$$w_2(l^i) = (1 - \lambda^*) \cdot [f_2(l^i)/h(l^i)] \tag{44}$$

Also, the partials of the log-likelihood with respect to σ_1^2 and σ_2^2 can be written as:

$$\frac{\partial L}{\partial \sigma_1^2} = -\frac{1}{\sigma_1^4} \sum_{i=1}^{N} w_1(l^i)[\sigma_1^2 - (l^i - \mathbf{x}_1{}' \boldsymbol{\beta})^2] = 0 \tag{45}$$

$$\frac{\partial L}{\partial \sigma_2^2} = -\frac{1}{\sigma_2^4} \sum_{i=1}^{N} w_2(l^i)[\sigma_2^2 - (l^i - \mathbf{x}_2{}' \gamma)^2] = 0 \tag{46}$$

The first order conditions in $(39 - 41)$ and $(45 - 46)$ can be numerically solved to find the values of $\{\boldsymbol{\beta}, \gamma, \boldsymbol{\xi}, \sigma_1, \sigma_2\}$ that maximize (38).