Estimating a Demand System with Choke Prices

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Abstract

We present a new, information-theoretic approach for estimating a system of many demand equations where the unobserved reservation or choke prices vary across consumers. We illustrate this method by estimating a nonlinear, almost ideal demand system (AIDS) for four types of meat using cross-sectional data from Mexico, where most households did not buy at least one type of meat during the survey week. The system of demand curves vary across demographic groups.

keywords: demand system, choke prices, generalized maximum entropy, meat

Estimating a Demand System with Choke Prices

If a consumer's reservation or choke price for a good—the intercept of that person's demand curve with the price axis—is below the observed price, the consumer does not purchase the product. To estimate a system of demand curves efficiently requires that we also estimate individual consumers' choke prices. We present a practical method of estimating a system of demand equations where individuals' choke prices vary. We apply our method to estimate a system of meat demand curves for Mexican consumers.

Over the years, economists have tried many approaches to deal with the "zero" problem, that some consumers do not buy any units of a good. Many early studies included observations with zero consumption and ignored the distributional implications, which results in biased estimates. Other early studies threw out such observations, deleting important information and biasing their results both because of sample selection and because some of these consumers will start buying as the price falls. A still common approach is to aggregate the data over time or over goods to eliminate zero observations. Doing so causes aggregation bias, generally precludes one from imposing theoretical restrictions, and force one to make implausible assumptions.

Since the 1990s, researchers have developed several methods to estimate demand systems using individual data with zeroes. These approaches fall into three general categories: two-step censored models, Kuhn-Tucker models, and generalized maximum entropy (GME) models.[1]

A number of studies use a two-step censored model to account for zero expenditure, such as Heien and Wessells (1990) and Shonkwiler and Yen (1999). Consumers decide whether to

[1] One other approach is to use count data models (Haab and McConnell, 1996).

purchase the good or not in the first stage, and how much to buy in the second stage using Amemiya- or Heckman-like estimators. The two benefits of this approach are that it can include various random disturbances such as errors in maximization and measurement errors and it can estimate demand systems with many goods. However, the estimation procedure ignores utility theory and the role of choke prices. Usually, the estimation uses imputed prices based on demographic information for zero observations instead of using choke prices and results in inconsistent estimates (Arndt 1999).

In the utility maximization or Kuhn-Tucker framework, two approaches are used. In the primal approach, the research specifies the form of the random utility function and then derives the Kunh-Tucker conditions to obtain the demand equations (Wales and Woodland 1983). In the dual approach, the researcher specifies the functional form of the random indirect utility function and obtains the demand equation from the Kuhn-Tucker conditions (Lee and Pitt 1986).

In these models, corner solutions (zero consumption) vary across the population, whose preferences are randomly distributed, and the choke prices are unobserved. The advantage of this approach is that it is consistent with utility theory. However, deriving and imposing demand theory regularity conditions is difficult for many functional forms and may require conflicting or overly restrictive conditions (Van Soest, et al., 1993).

The biggest problem with this approach has been computational difficulties because one faces multiple integrals and has to evaluate cumulative joint distribution functions. As a result, these methods may only be practical for systems with a small number of goods (Phaneuf 2000, Kim et al. 2002). Recent use of Bayesian, generalized method of moments, Gibbs sampling, jackknife techniques, and other techniques have made this approach more computationally feasible (e.g., Perali and Chavas 2000; Kao et al. 2001; Kim et al. 2002; Dong et al. (2004);

Millimet and Tchernis 2008; Li et al. 2014; Mehta 2014).[2] However, many of these approaches introduce new complications, such as linearity assumptions. These approaches require one to use a specific demand system.

A third approach (Arndt 1999, Golan et al. 2001) uses generalized maximum entropy (GME) to estimate a Kuhn-Tucker model. The GME has five advantages over traditional maximum likelihood methods.

First, unlike most of the maximum-likelihood Kuhn-Tucker models that work with only one objective function, this approach easily works with any demand system merely by writing the demand system. Second, imposing any equality or inequality constraint, including regularity conditions, is much easier than with classical maximum likelihood or Bayesian techniques.

Third, because maximizing the GME objective does not involve numerical integration, one can use a standard nonlinear optimization package to estimate relatively large demand systems. As a result, the GME approach practically handles a larger number of censored equations than most maximum likelihood approaches.

Fourth, GME performs well with both ill-posed problems (such as small data sets, underdetermined problems, and high levels of collinearity) and well-posed problems. Fifth, because the GME estimator does not require assumptions about the error structure and because it uses all the data, it is more robust and efficient than are maximum likelihood estimators. According to Arndt (1999), based on a root mean square error criterion, a Monte Carlo simulation shows that the GME approach performs better than the Lee and Pitt (1986) approach (which in turn performs much better than a two-step procedure). Similarly, Golan et al. (1997) find that the GME estimator has

[2] For example, Li et al. (2014) derive closed-form equations and avoid solving high-dimensional integration by using changes of variables, conditioning steps, and numerical approximations of integrals and normal density functions.

lower empirical mean square error than does the maximum likelihood tobit estimator in small samples regardless of whether errors are normal or not.

A weakness of many of these earlier studies, including the GME studies, is that they estimate models with non-negativity constraints without explicitly estimating choke prices, which reduces their efficiency. In this paper, we extend the Kuhn-Tucker/GME approach in Golan et al. (2001) to estimate choke prices of each household explicitly as well as the demand system coefficients. Not only is it relative easy to use this approach to impose nonnegativity constraints, but this approach can employ any utility function, rather than being limited to a single specification.

We apply our technique to estimate a four-equation AIDS Mexican meat demand system using the same data as Golan et al. (2001), for comparison purposes. In our empirical application, we concentrate on comparing the elasticities of demand for the choke-price and no-choke-price models. We find that the choke-price model estimates of the Hicksian and Marshallian own-price elasticities are much closer to zero elasticity than those from the no-choke-price model.

I. AIDS Random Utility Model with Zeroes

Although we can use any utility function, for specificity, we assume that we have an incomplete, nonlinear almost ideal demand system (AIDS, Deaton and Muellbauer 1980) random utility model (RUM). Generally, the indirect utility function is

$$v(\boldsymbol{p}, m, \boldsymbol{d}, \boldsymbol{\varepsilon}) = \left[\ln E - \phi - \boldsymbol{b}'\boldsymbol{d} - (\boldsymbol{a} + \boldsymbol{A}\boldsymbol{d} + \boldsymbol{\varepsilon})' \ln \boldsymbol{p} - \tfrac{1}{2} \ln \boldsymbol{p}' \boldsymbol{B} \ln \boldsymbol{p}\right] e^{-\boldsymbol{g}' \ln \boldsymbol{p}}, \tag{1}$$

where E is the consumer's total expenditure on meat; $\boldsymbol{p}$ is the price vector; $\boldsymbol{d}$ is a vector of demographic variables; ϕ is a parameter; $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{g}$ are vectors of parameters; $\boldsymbol{A}$ is a matrix of parameters; and $\boldsymbol{B}$ is a symmetric matrix of parameters.

Given this indirect utility function, the interior solutions for the demand equations in share form are

$$\boldsymbol{s} = \boldsymbol{a} + \boldsymbol{Ad} + \boldsymbol{B}\ln\boldsymbol{p} + \boldsymbol{\gamma}\left[\ln E - \phi - \boldsymbol{b}'\boldsymbol{d} - (\boldsymbol{a} + \boldsymbol{Ad})'\ln\boldsymbol{p} - \tfrac{1}{2}\ln\boldsymbol{p}'\boldsymbol{B}\ln\boldsymbol{p}\right] + (\boldsymbol{I} - \boldsymbol{g}\ln\boldsymbol{p}')\tilde{\boldsymbol{\varepsilon}}, \qquad (2)$$

where each element of $\boldsymbol{s}$, $s_i = p_i q_i / E$, $i = 1,\ldots, n$, is the budget share of the i^{th} good. Let $\boldsymbol{\iota}$ be a vector of ones. The RUM errors enter Equation (2) linearly and $\boldsymbol{\iota}'(\boldsymbol{I} - \boldsymbol{g}\ln\boldsymbol{p}')\tilde{\boldsymbol{\varepsilon}} = 0,$ due to the adding up constraints and homogeneity. Because $\boldsymbol{\iota}'\boldsymbol{s} = 1$, it follows that $\boldsymbol{\iota}'\boldsymbol{a} = 1$, $\boldsymbol{A}'\boldsymbol{\iota} = \mathbf{0}$, $\boldsymbol{B}\boldsymbol{\iota} = \mathbf{0}$, and $\boldsymbol{\iota}'\boldsymbol{g} = 0.$ The covariance matrix, $\tilde{\Sigma} = \mathrm{E}(\tilde{\boldsymbol{\varepsilon}}\tilde{\boldsymbol{\varepsilon}}'),$ therefore is singular, $\tilde{\Sigma}\boldsymbol{\iota} = \mathbf{0},$ and $\Sigma = (\boldsymbol{I} - \boldsymbol{g}\ln\boldsymbol{p}')\tilde{\Sigma}(\boldsymbol{I} - \ln\boldsymbol{p}\boldsymbol{g}')$ also satisfies $\Sigma\boldsymbol{\iota} = (\boldsymbol{I} - \boldsymbol{g}\ln\boldsymbol{p}')\tilde{\Sigma}(\boldsymbol{I} - \ln\boldsymbol{p}\boldsymbol{g}')\boldsymbol{\iota} = \mathbf{0}.$

We do not know the underlying primitive structure of $\tilde{\Sigma}$, so we focus on the “conditional” RUM errors by using $\boldsymbol{\varepsilon} = (\boldsymbol{I} - \boldsymbol{g}\ln\boldsymbol{p}')\tilde{\boldsymbol{\varepsilon}}$ in Equation (2) and working directly with the joint distribution of $\boldsymbol{\varepsilon}$ in the simpler AIDS model specification,

$$\boldsymbol{s} = \boldsymbol{a} + \boldsymbol{Ad} + \boldsymbol{B}\ln\boldsymbol{p} + \boldsymbol{g}\left[\ln E - \phi - \boldsymbol{b}'\boldsymbol{d} - (\boldsymbol{a} + \boldsymbol{Ad})'\ln\boldsymbol{p} - \tfrac{1}{2}\ln\boldsymbol{p}'\boldsymbol{B}\ln\boldsymbol{p}\right] + \boldsymbol{\varepsilon}. \qquad (3)$$

To model the *conditional choke prices* for goods not consumed by an individual in a given period, we reorder the goods as necessary and divide the goods into those the consumer buys and those that the consumer does not buy. That is, we partition $\boldsymbol{s}$ into two sub-vectors, $\boldsymbol{s}_{(1)} \gg \mathbf{0}_{(1)}$ and $\boldsymbol{s}_{(2)} = \mathbf{0}_{(2)},$ so that $\boldsymbol{s} = [\boldsymbol{s}'_{(1)}\ \boldsymbol{s}'_{(2)}]'.$ We also partition all other terms consistently, so that,

$$\mathbf{0}_{(1)} \ll \boldsymbol{s}_{(1)} = \boldsymbol{a}_{(1)} + \boldsymbol{A}_{(1)}\boldsymbol{d} + \left[\boldsymbol{B}_{(1,1)}\boldsymbol{B}_{(1,2)}\right]\begin{bmatrix}\ln\boldsymbol{p}_{(1)}\\ \ln\tilde{\boldsymbol{p}}_{(2)}\end{bmatrix}$$

$$+\boldsymbol{g}_{(1)}\left\{\ln E - \phi - \boldsymbol{b}'\boldsymbol{d} - (\boldsymbol{a} + \boldsymbol{Ad})'\begin{bmatrix}\ln\boldsymbol{p}_{(1)}\\ \ln\tilde{\boldsymbol{p}}_{(2)}\end{bmatrix} - \frac{1}{2}\begin{bmatrix}\ln\boldsymbol{p}_{(1)}\\ \ln\tilde{\boldsymbol{p}}_{(2)}\end{bmatrix}'\begin{bmatrix}\boldsymbol{B}_{(1,1)} & \boldsymbol{B}_{(1,2)}\\ \boldsymbol{B}_{(2,1)} & \boldsymbol{B}_{(2,2)}\end{bmatrix}\begin{bmatrix}\ln\boldsymbol{p}_{(1)}\\ \ln\tilde{\boldsymbol{p}}_{(2)}\end{bmatrix}\right\} + \boldsymbol{\varepsilon}_{(1)}, \qquad (4)$$

and

$$\mathbf{0}_{(2)} = \boldsymbol{s}_{(2)} = \boldsymbol{a}_{(2)} + \boldsymbol{A}_{(2)}\boldsymbol{d} + \left[\boldsymbol{B}_{(2,1)}\boldsymbol{B}_{(2,2)}\right]\begin{bmatrix}\ln \boldsymbol{p}_{(1)} \\ \ln \tilde{\boldsymbol{p}}_{(2)}\end{bmatrix}$$
$$+\boldsymbol{g}_{(2)}\left\{\ln E - \phi - \boldsymbol{b}'\boldsymbol{d} - (\boldsymbol{a}+\boldsymbol{A}\boldsymbol{d})'\begin{bmatrix}\ln \boldsymbol{p}_{(1)} \\ \ln \tilde{\boldsymbol{p}}_{(2)}\end{bmatrix} - \tfrac{1}{2}\begin{bmatrix}\ln \boldsymbol{p}_{(1)} \\ \ln \tilde{\boldsymbol{p}}_{(2)}\end{bmatrix}'\begin{bmatrix}\boldsymbol{B}_{(1,1)} & \boldsymbol{B}_{(1,2)} \\ \boldsymbol{B}_{(2,1)} & \boldsymbol{B}_{(2,2)}\end{bmatrix}\begin{bmatrix}\ln \boldsymbol{p}_{(1)} \\ \ln \tilde{\boldsymbol{p}}_{(2)}\end{bmatrix}\right\} + \boldsymbol{\varepsilon}_{(2)}, \qquad (5)$$

where Equation (5) defines the conditional choke prices for $\tilde{\boldsymbol{p}}_{(2)}$ as the implicit solution to the system of quadratic equations,

$$\ln \boldsymbol{p}_{(2)} \geq \ln \tilde{\boldsymbol{p}}_{(2)} = -\left[\boldsymbol{B}_{(2,2)} - \boldsymbol{g}_{(2)}(\boldsymbol{a}_{(2)} + \boldsymbol{A}_{(2)}\boldsymbol{d} - \boldsymbol{B}_{(2,1)}\ln \boldsymbol{p}_{(1)})'\right]^{-1}\left\{\boldsymbol{a}_{(2)} + \boldsymbol{A}_{(2)}\boldsymbol{d} + \boldsymbol{B}_{(2,1)}\ln \boldsymbol{p}_{(1)}\right.$$
$$\left. +\boldsymbol{g}_{(2)}\left[\ln E - \phi - \boldsymbol{b}'\boldsymbol{d} - (\boldsymbol{a}_{(1)} + \boldsymbol{A}_{(1)}\boldsymbol{d})'\ln \boldsymbol{p}_{(1)} - \tfrac{1}{2}\ln \boldsymbol{p}'_{(1)}\boldsymbol{B}_{(1,1)}\ln \boldsymbol{p}_{(1)} - \tfrac{1}{2}\ln \tilde{\boldsymbol{p}}'_{(2)}\boldsymbol{B}_{(2,2)}\ln \tilde{\boldsymbol{p}}_{(2)}\right] + \boldsymbol{\varepsilon}_{(2)}\right\}. \qquad (6)$$

In our application, we have four goods. Because a consumer either buys or does not buy each of the four goods, we have $2^4 = 16$ possible regimes or combinations of positive and zero purchases of the various goods. However, because each individual has a positive total expenditure, we do not observe the regime in which no goods are consumed, so that the number of different estimating systems is 15.

Equations (4) and (6) are the generic estimating equations for each consumption regime. The errors $\boldsymbol{\varepsilon}_{(1)}$ are censored from below. The errors

$-\left[\boldsymbol{B}_{(2,2)} - \boldsymbol{g}_{(2)}(\boldsymbol{a}_{(2)} + \boldsymbol{A}_{(2)}\boldsymbol{d} - \boldsymbol{B}_{(2,1)}\ln \boldsymbol{p}_{(1)})'\right]^{-1}\boldsymbol{\varepsilon}_{(2)}$ are censored from above.

This presentation highlights three issues. First, nonlinearity in parameters and interactions with choke prices complicate the estimation problem substantially. Second, nonlinearity in prices can lead to multiple solutions for the choke prices. Third, a large number of goods, *n*, results in up to 2^n regimes, and could require one to have to evaluate probability integrals of up to *n* dimensions were one to use a maximum likelihood technique.

II. Estimation and Inference

To estimate this system of censored demand equations together with the choke prices, we generalize the GME method for estimating a single, censored equation given in Golan et al. (1997). We start by providing some intuition as to how the traditional maximum entropy approach works. Then, we show how to estimate the AIDS with choke prices using that approach.

A. Maximum Entropy and Generalized Maximum Entropy: A Brief Overview

The GME approach (Golan et al., 1996) has its roots in information theory and builds on the entropy measure of Shannon (1948) and the classical maximum entropy (ME) principle of Jaynes (1957a, 1957b). The maximum entropy procedure is an inversion procedure for inferring an unknown probability distribution function from incomplete information.

In this approach, one maximizes Shannon's entropy subject to constraints representing the available (observed and unobserved) information. These constraints connect the unobserved entities of interest, say the unknown probabilities or economic parameters of interest, with the observed information, say the sample's information.

Heuristically, Shannon's entropy measures our uncertainty about the occurrence of a collection of events. Letting X be a discrete, random variable with possible outcomes x_s, $s = 1, 2, \ldots, N$, with probabilities $\boldsymbol{\pi} = (\pi_1, \pi_2, ..., \pi_N)'$ such that $\boldsymbol{\iota}'\boldsymbol{\pi} = 1$, where $\boldsymbol{\iota}'$ is a vector transposed of 1's, the *entropy* of the distribution $\boldsymbol{\pi}$ is

$$S(\boldsymbol{\pi}) = -\boldsymbol{\pi}' \ln \boldsymbol{\pi}, \tag{7}$$

where $0\ln 0 \to 0$. The function S is zero when $\pi_s = 1$ for some value of s, and it reaches a maximum of $\ln(N)$ for the uniform distribution $\pi_1 = \pi_2 = \ldots = \pi_N = 1/N$. This function is not a function of the events x_s themselves, but rather it's a function of their respective probabilities.

To infer the unknown probabilities $\boldsymbol{\pi}$ that characterize $M < N$ pieces of information, say M expected values, or moments of a distribution, Jaynes (1957a, 1957b) proposed maximizing the entropy, subject to the available moment information (constraints) and the requirement that the probabilities are normalized (add to one). The basic axioms of the maximum entropy (ME) method are well-developed (see Shore and Johnson, 1980 and Skilling, 1989).

The maximum entropy approach uses only the information provided in the constraints. No other hidden information is imposed. In that regard, it can be thought of as the most uninformed solution out of all solutions that satisfy the available information—the constraints. This most uninformed solution can be viewed in the following way. If no constraints are imposed, $S(\boldsymbol{\pi})$ reaches its maximum value and the π's are distributed uniformly. All possible realizations of the random variable are equally likely.

If constraints (information) are imposed, the chosen solution is the one that is "closest" to the state where no constraints are imposed. Thus, the maximum entropy procedure finds the solution that keeps us as close as possible to a state of complete uncertainty (or to our initial prior information). In statistical terms, the solution is the flattest possible likelihood that is consistent with the observed expectation values (constraints). For a detailed discussion of maximum entropy and information theoretic modeling, motivations, foundations and examples see Golan (2018).

The traditional ME approach assumes that the information (or sample information) in the form of moment conditions holds exactly. In contrast, the generalized maximum entropy (GME) method (Golan et al., 1996) uses each observation (or moment) directly and allows these conditions to be stochastic restrictions that hold only approximately.

The GME uses a flexible, dual-loss objective function: a weighted average of the entropy of the systematic part of the model and the entropy from the error terms. The ME is a special case of the GME that places no weight on the entropy of the error terms and where the data are represented in terms of exact moments. By varying the weight in the GME objective, we can improve either our precision or predictions. Here, we use a balanced approach where we give equal weight to both objectives.[3]

B. Estimating an AIDS Model Without Choke Prices

To estimate the AIDS model without choke prices, we follow the approach in Golan et al., 2001. We rewrite the system of AIDS share equations in Equation 3 in non-vector form as the individual share equation for a good for a particular household, *h*, taking account of whether a good is purchased or not:

$$s_{ih} = \alpha_i + \sum_{l=1}^{12} a_{il} d_{lh} + \sum_{j=1}^{4} \beta_{ij} \ln \tilde{p}_{jh} + \gamma_i \ln(E_h / P_h) + \varepsilon_{ih}, \quad \text{for } s_{ih} > 0, \tag{8}$$

$$s_{ih} > \alpha_i + \sum_{l=1}^{12} a_{il} d_{lh} + \sum_{j=1}^{4} \beta_{ij} \ln \tilde{p}_{jh} + \gamma_i \ln(E_h / P_h) + \varepsilon_{ih}, \quad \text{for } s_{ih} = 0, \tag{9}$$

where P_h is the AIDS aggregate, nonlinear price index

$$\ln P_h = \phi + \sum_{j=1}^{4} \alpha_j \ln \tilde{p}_{jh} + \sum_{j=1}^{4} \sum_{l=1}^{12} a_{jl} d_{lh} \ln \tilde{p}_{jh} + \tfrac{1}{2} \sum_{j=1}^{4} \sum_{k=1}^{4} \gamma_{jk} \ln \tilde{p}_{jh} \ln \tilde{p}_{kh}, \tag{10}$$

where $\tilde{p}_{ih}$ equals the actual price if the household purchases meat *i* and equals the conditional choke price, if it is not purchased.

Because entropy is defined over a proper probability distribution, we must transform all of the parameters in Equation (8) – (10) into probability distributions. We can then express the entropy measure for each one of these terms. For example, to transform α_j, for all *j*, we start by

[3] The results in our empirical application are not sensitive to the weight. For example, raising the weight from 0.5 to 0.9 on the systematic measure causes the estimated coefficients and the correlation between actual and estimated values to changes by less than 1%.

choosing a set of discrete points in its support space, $(z_1^{\alpha_j}, z_2^{\alpha_j}, \ldots, z_M^{\alpha_j})'$ of dimension $M \geq 2$ that are at uniform intervals, symmetric around zero, and span an interval [–*c*, *c*]. The dimension *M* can differ for each parameter or error term, but we use *M* = 3 in our application. Thus, $(z_1^{\alpha_j}, z_2^{\alpha_j}, \ldots, z_M^{\alpha_j})' = (-c,\ 0,\ c)'$. Our results change negligibly if we increase *M*.

Corresponding to these discrete points in the support space is a vector of unknown weights $(x_1^{\alpha_j},\ x_2^{\alpha_j},\ x_3^{\alpha_j})'$ such that $x_1^{\alpha_j} + x_2^{\alpha_j} + x_3^{\alpha_j} = 1$. That is, α_j is the expected value $\sum_{m=1}^{3} z_m^{\alpha_j} x_m^{\alpha_j} = \alpha_j = -c x_1^{\alpha_j} + c x_3^{\alpha_j}$. We use the same approach for the other coefficients.

Similarly, we treat the errors ε_{ih} as unknown entities and define a transformation matrix *V* that converts the possible outcomes for ε_{ih} to the interval [0, 1]. This transformation is done by defining a vector of *M* = 3 discrete points $(v_1, v_2, v_3)'$, distributed evenly and uniformly about zero, and a corresponding vector of proper (i.e., positive) unknown weights $(w_{ti1}, w_{ti2}, w_{ti3})'$ such that $\sum_{m=1}^{3} v_m w_{imh} = \varepsilon_{ih}$. We make no assumption about the distribution of the error terms ε_{ih}'s, or the probabilities **w**.

Let $\mathbf{x} = (\mathbf{x}^{\alpha'}, \mathbf{x}^{a'}, \mathbf{x}^{\beta'}, \mathbf{x}^{\gamma'}, \mathbf{x}^{\phi'})'$, where $\mathbf{x}^{\alpha'}$ is a vector of all the weights for all the α_i terms, and so forth. Similarly, ***w*** is a vector of the all error term weights, w_{ijh}. Our GME estimator is

$$\max_{\boldsymbol{q},\boldsymbol{w}} S(\mathbf{x}, \boldsymbol{w}) = -\mathbf{x}' \ln \mathbf{x} - \boldsymbol{w}' \ln \boldsymbol{w} \qquad (11)$$

subject to the budget-share, the nonlinear price index Equation (10), the GME adding-up conditions that the weights for each parameter or error term sum to one and the consumer-theory adding-up, homogeneity, and symmetry conditions (and any other regularity conditions one wishes to impose).

Forming the Lagrangian and solving for the first-order conditions yields the optimal solution $\hat{\mathbf{x}}$ and $\hat{\mathbf{w}}$, from which we derive the point estimates for the AIDS coefficients.

That this GME estimator is consistent follows immediately by extending the proof in Golan et al. (1997) that a censored GME estimator for a single equation is consistent (see the Appendix to Golan et al. 2001).

C. Estimating the AIDS Model with Choke Prices

We use an iterative procedure to estimate the AIDS model with choke prices.

First, we estimate the AIDS model imposing the nonnegativity constraints but ignoring the choke prices, as described in the previous section. This estimate provides the starting values for the model with choke prices.[4]

Second, given these starting value, we use our GME method to estimate the choke prices for goods with zero shares.

Third, we replace the actual prices with the estimated choke prices for those goods with zero shares and re-estimate the original AIDS model.

Fourth, we repeat the second and third steps until the model converged (no changes with a tolerance of 0.0001). Our Mexican four-meat AIDS model converged in six iterations.

Fifth, we estimate the standard errors using a stratified pairs bootstrap with 1,000 replications. Each replication uses a representative subsample of 500 households with replacement. To select a representative sample, we draw from each of the 15 consumption groups (determined by which of the four meats a household purchased in the sample week) in

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[4] As a sensitivity check, we started with different initial values and re-estimated the model. We did not find that the initial starting values play an important role.

proportion to the group's share of the total sample. Then, we re-estimate the model for each subsample using as starting values the parameter estimates from the entire sample.

In the second step of the iterative process in which we estimate the choke price, an analytic solution could produce multiple, or negative, roots for the possible choke prices, because the choke price equations are nonlinear, as we discussed earlier. Thus, we need a method to choose a single choke price for each one a household's good that has a zero share. Following on the same logic that directed Jaynes (1957a, 1957b) to use the entropy as the criterion function for selecting a single solution out of the many solutions that are consistent with the constraints (the information we have), we use a GME approach.

Without the AIDS model's constraints, the GME method would estimate choke prices uniformly distributed within the support. However, given the constraints, the estimated choke prices are conditional on the other prices, income and the other demographic variables, and their own error terms.

To obtain the GME estimates for the choke-price model, we maximize the entropy of the noise and the choke prices subject to normalization and Equations 8–10, which capture the AIDS structure conditional on individual purchases and characteristics. The inferred choke prices are the most uniformed positive prices that satisfy all the required conditions. This approach is necessary to prevent solutions (the roots of the equations) that are negative, a mathematical possibility due to the nonlinearities.

D. Supports

We set our support wide enough to include all the possible outcomes. The natural support vector for the error terms is (–1, 0, 1), because the dependent variables are shares, s_i, which lie between 0 and 1.

A variety of AIDS empirical studies (see the survey in Golan et al., 2001) estimated coefficients on the ln price terms, γ_{ij}, that lie within the interval (–0.2, 0.2). Their estimated coefficients on the ln real expenditure, β_i, and the intercept, α_{ti}, lie within the interval of (–1, 1). We chose support vectors that are 100 times wider than these intervals: (–20, 20) for γ_{ij}. The support is $(-100, 100)$ for β_i and α_{ti} (and hence for ρ_{ik}). Making a moderately large change in these support vectors, while keeping the center of the support unchanged, has negligible effects on the estimated coefficients and elasticities.

The support of the choke prices includes possible values from zero through the observed price, because the choke price cannot exceed the observed price. Let $\boldsymbol{q}^{c\prime}$ be the vector of the GME weights for the choke prices for the four goods and $\boldsymbol{u}$ be the vector of weights for the error terms associated with the choke prices. The supports are [−1, 0, 1] for v_j^u and [ln 1.1, ½ (ln 1.1 + ln p_{ti}), ln p_{ti}] for v_{tim}^p. As a result, the choke prices is between zero and the observed price for a given good and location.

III. Data

So that we can compare our results to the earlier Golan et al. (2001) study, we use the same World Bank data set. It is based on a cross-sectional Mexican household survey conducted in the fourth quarter of 1992 by the National Institute of Statistics, Geography and Informatics (INEGI), an agency of the Ministry of Budgeting and Programming in Mexico. INEGI used a stratified and multi-stage sampling method to produce a representative sample for the entire population and for urban and rural households. The data cover 31 states and a Federal District.

The database has detailed information about consumption during a one-week survey period and demographic characteristics by household. The database has 581,027 observations of purchasing events by about 10,500 households for at least 205 foods.[5]

We look at four meat product aggregates: beef, pork, chicken, and processed meat. We restrict our sample to those households that bought at least one of these four categories of meat. Of the 7,591 households that bought some meat during the sample week, 32% did not buy beef; 70%, pork; 34%, chicken; and 57%, processed meat.[6]

The corresponding prices are also aggregates. For example, the price of beef is an expenditure weighted average of beefsteak, pulp, bone, fillet, special cuts, and ribs and other. The prices of various meats vary geographically.[7]

Because the data set reports prices only if purchases are made, for the "actual" price of a good that a household did not purchase we need a proxy. We assume that the household faces the average price level in its geographic location: a rural or urban area in a particular state or Federal District.

Table 1 shows that the means and standard deviations for the 7,591 households. (Golan et al. 2001 used only a subset of the data: 1,000 observations.) Table 1 also provides summary statistics for the consumption shares of the four meats, the corresponding prices, expenditures on

[5] The quantity measures reported below also include own-produced and consumed goods as well as purchased goods.
[6] Golan et al. (2001) also included fish. We dropped that category because only 13% of consumers purchased fish (and few did in the interior of the country) out of the 7,897 households that bought some meat. Eliminating fish does not greatly affect the non-choke price estimates of the elasticities of the other meats.
[7] We conducted pairwise tests of the hypothesis that the prices are drawn from the same normal distribution across the 129 locations (urban and rural areas within states). Based on t-tests, we rejected the hypothesis at the 5% level that the average prices are homogeneous in 54% of the comparisons.

meats, and the 12 demographic variables (including ln expenditure) that we use in our GME nonlinear AIDS Model.

IV. Empirical Results

We estimated the non-choke-price and the choke-price models using the nonlinear-optimization program GAMS (Generalized Algebraic Modeling System) and Python. Table 2 shows our estimates of the AIDS model without choke prices. Table 3 presents the estimates for the model with choke prices.[8]

A. Tests

We tested for homogeneity in the first stage AIDS model, which is the model without the choke prices. We do that because for homogeneity requires testing each equation separately (see Deaton and Mulbaeur, 1980). We estimated each equation with and without the homogeneity restrictions and then calculated the entropy-ratio statistic, which has a limiting χ^2 distribution. These tests fail to reject the homogeneity hypothesis at the 5% (or even 1%) significance level for all four goods. The differences in the objective values with homogeneity imposed is negligible. Thus, using the entropy-ratio test, we fail to reject the homogeneity hypothesis at the 5% (or 1%) significance level. These test result are similar to those of Golan et al. (2001).

We tested the symmetry requirement for the entire system for both the first stage and the choke price models. The differences in the objective values with symmetry imposed (in both the first stage and choke price models) is negligible. Thus, using the entropy-ratio test, we fail to reject the symmetry hypothesis at the 5% significance level. These test results are very similar to those of Golan et al. (2001).

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[8] The choke-price mode has a larger entropy value, 32,947.3, than does the no-choke-price model, 31,387.5.

We also tested the hypothesis that the errors for each equation are independently and identically distributed. Given the estimated residuals, we used Wooldridge's (1990) robust test for heteroscedasticity (his Equations 3.22, 3.23, and 3.24). We cannot reject the hypothesis of homoscedastic errors for beef, pork, and processed meat at the 5% significance level. The results for chicken were inconclusive.

B. Predictions

The estimated model with choke prices has greater predictive power—the correlation between the observed and predicted shares—than the one without choke prices. Table 4 shows the correlation for both models. In the model without choke prices, the correlations for each share equation range between 0.14 and 0.27, while the correlation for the system is 0.44. In contrast, the correlations of the choke-prices model's shares range between 0.80 and 0.91 and the system correlation is 0.89. Of course, part of the superior fit is due to estimating additional parameters in the choke-price model. This predictive power is surprisingly high given that the estimates use cross-sectional data with measurement errors in the price data.

C. Choke Price

We can illustrate why the non-choke-price and choke-price models differ by showing that the "data" sets for the four goods differ substantially. The non-choke-price model uses the observed prices; whereas, the choke-price model replaces the observed prices with estimated choke prices for goods with zero shares. Figure 1 shows the histograms for the actual prices and the estimated choke prices for the four goods.

In each choke price histogram, the largest mode occurs at about the half the mode of the observed prices. Each of the beef choke-price histograms has three modes (most clearly seen for beef), with the smaller modes to the right of the largest one.[9] Consequently, each of the choke-price histograms is skewed, with slightly more weight to the right of the largest mode.

D. Elasticities

We need to take into account choke prices in determining the Marshallian and Hicksian price elasticities. We calculate the average price using the actual price if the quantity is positive and the conditional choke price otherwise.

The "mean" quantity, $\bar{q}_{ih}$, for good $i = 1,\ldots, 4$ and household $h = 1,\ldots, H$ is

$$\bar{q}_{ih} = \tilde{p}_{ih}^{-1} E_h \Big[\alpha_i + \sum_{\ell=1}^{L} a_{i\ell} d_{\ell h} + \sum_{j=1}^{N} \beta_{ij} \ln \tilde{p}_{jh} + \gamma_i \Big(\ln E_h - \phi - \sum_{j=1}^{N} \alpha_j \ln \tilde{p}_{jh} - \sum_{j=1}^{N} \sum_{\ell=1}^{L} a_{j\ell} d_{\ell h} \ln \tilde{p}_{jh} - ½ \sum_{j=1}^{N} \sum_{k=1}^{N} \beta_{jk} \ln \tilde{p}_{jh} \ln \tilde{p}_{kh} \Big) \Big],$$

where $\tilde{p}_{ih}$ equals the actual price if the household purchases meat i and equals the conditional choke price, if it is not purchased. If $q_{ih} > 0$, or $q_{ih} = 0$ and directions of any price changes are restricted such that $\Delta q_{ih} \geq 0$ (e.g., $\Delta \tilde{p}_{ih} \leq 0$), the Marshallian own- and cross-price derivatives are:

$$\frac{\partial \bar{q}_{ih}}{\partial \tilde{p}_{ih}} = -\tilde{p}_{ih}^{-1} \bar{q}_{ih} + \tilde{p}_{ih}^{-2} E_h \Big[\beta_{ii} - \gamma_i \Big(\alpha_i + \sum_{\ell=1}^{L} a_{i\ell} z_{\ell h} + \sum_{j=1}^{4} \beta_{ij} \ln \tilde{p}_{jh} \Big) \Big],$$

$$\frac{\partial \bar{q}_{ih}}{\partial \tilde{p}_{jh}} = \tilde{p}_{ih}^{-1} \tilde{p}_{jh}^{-1} E_h \Big[\beta_{ij} - \gamma_i \Big(\alpha_j + \sum_{\ell=1}^{L} a_{j\ell} z_{\ell h} + \sum_{j=1}^{4} \beta_{jk} \ln \tilde{p}_{kh} \Big) \Big].$$

Averaging these directional derivatives, quantities (either q_{ih} or $\bar{q}_{ih}$), and modified prices, $\tilde{p}_{ih}$, over all H households in the sample, we obtain the matrix of aggregate ordinary own- and cross-price elasticities defined by,

[9] Changing the dimension of the support space from three to five has virtually no effect on these histograms.

$$E_{\bar{q}_i}^{\tilde{p}_j} = \left(\sum_{h=1}^{H} \frac{1}{H} \frac{\partial \bar{q}_{ih}}{\partial \tilde{p}_{jh}} \right) \left(\sum_{h=1}^{H} \frac{1}{H} \tilde{p}_{jh} \right) \Bigg/ \left(\sum_{h=1}^{H} \frac{1}{H} \bar{q}_{ih} \right), \ i, j = 1, \cdots, 4.$$

To obtain the Hicksian derivatives, we must add the (directionally constrained) expenditure effects

$$\frac{\partial \bar{q}_{ih}}{\partial m_h} \bar{q}_{jh} = \left(\frac{\bar{q}_{ih}}{m_h} + \frac{\gamma_i}{\tilde{p}_{ih}} \right) \bar{q}_{jh}.$$

Averaging the Hicksian own- and cross-price derivatives across all *H* households and multiplying these by the above averages of prices and quantities, we obtain the corresponding elasticities.

Table 5 reports the Hicks-compensated price elasticities for each type of meat and the corresponding standard errors. All the elasticities are statistically significantly different from zero at the 0.05 level for the choke-price model. All the elasticities are statistically significant for the model without choke prices except for the pork price-processed meat, and the processed meat price-pork elasticities.

In both models, all the own-price elasticities are negative, of course. In the non-choke-price model, all the cross-price elasticities are positive, so all the meats are substitutes. In the choke-price model, the cross-price elasticities are positive except for the pork price-chicken, pork price-processed meat, and the processed meat price-pork elasticities. These three negative cross-price elasticities are close to zero.

Table 6 shows our estimated Marshallian own-price elasticities for the choke-price model. Our technique estimates the elasticities for both models very precisely.

All the Hicksian (Table 5) and Marshallian (Table 6) own-price elasticities for the choke-price model are substantially smaller in absolute value than for the non-choke-price model. The choke-price Hicksian elasticities are between 26% and 51% as large as the non-choke-price model elasticities in absolute value. The choke-price Marshallian elasticities are between 34% and 79% as large as the non-choke-price model elasticities. Thus, we conclude that the failure to account for choke prices leads to overestimates of the own-price-responsiveness of demand.

Our meat elasticities estimates are similar to those in the literature based on aggregate data for roughly the same period (see Golan et al. 2001). The simulated Marshallian price elasticities in Dong et al. 2004, which uses a Kuhn-Tucker approach and slightly more recent Mexican data, are closer to our choke-price elasticities than to our no-choke-price elasticities.

V. Conclusions

A major challenge facing empirical economists is to estimate demand systems with non-negativity constraints. Our generalized maximum entropy (GME) approach practically and efficiently estimates a demand system with many goods and the choke prices.

The GME approach has many advantages over traditional maximum likelihood (ML) methods for estimating demand systems with non-negativity constraints. Key among these are the ability to apply this technique easily to any demand system, to impose any equality or inequality constraints (such as regularity conditions) easily, to be able to estimate relatively large demand systems, and to perform well with ill-behaved problems (such as small data sets and high levels of collinearity).

The main innovation in this paper over earlier GME (and most other Kuhn-Tucker) approaches is that we explicitly estimate the choke or reservation prices. We extended the model in Golan et al. (2001) to estimate the choke prices as well as the AIDS model Kuhn-Tucker equations.

The choke-price model estimates of the Hicksian and Marshallian own-price elasticities are much closer to zero than those from the no-choke-price model.

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Table 1. Summary Statistics

	Mean	S.D.
Beef share of meat expenditure	0.41	0.35
Pork share of meat expenditure	0.13	0.24
Chicken share of meat expenditure	0.32	0.33
Processed meat share of meat expenditure	0.15	0.25
Natural log of price of beef	9.55	0.27
Natural log of price of pork	9.43	0.24
Natural log of price of chicken	8.95	0.31
Natural log of price of processed meat	9.48	0.30
Natural log of expenditure on all meat	15.98	0.88
Household lives in urban area	0.63	0.48
Household head is female	0.13	0.33
Household head is in school	0.02	0.14
Household head attended		
Primary	0.52	0.50
Secondary	0.18	0.38
Preparatory	0.08	0.27
College	0.10	0.29
Share of household members		
Between 0 and 5 years old	0.14	0.17
Between 6 and 15	0.21	0.21
Between 16 and 28	0.26	0.25
Between 29 and 45	0.21	0.20
Between 46 and 60	0.10	0.20

Number of observations = 7,591

Table 2. GME Estimates of the AIDS without Choke Prices

	Beef	Pork	Chicken	Processed Meat
Intercept	–0.011	0.073	0.572	0.366
	(0.884)	(0.187)	(0.000)	(0.000)
Beef price	–0.021	0.003	–0.005	0.023
	(0.114)	(0.716)	(0.594)	(0.004)
Pork price	0.003	0.018	–0.020	–0.001
	(0.716)	(0.069)	(0.008	(0.850)
Chicken price	–0.005	–0.020	0.042	–0.017
	(0.594)	(0.008)	(0.000)	(0.024)
Processed meat price	0.023*	–0.001	–0.017	–0.005
	(0.004)	(0.850	(0.024)	(0.559)
Expenditure	0.092	0.023	–0.038	–0.077
	(0.000)	(0.000)	(0.000)	(0.000)
Urban	0.060	–0.045	–0.044	0.029
	(0.000)	(0.000)	(0.000)	(0.000)
Household head is female	–0.009	–0.016	0.004	0.021
	(0.448)	(0.069)	(0.748)	(0.016)
Household head is in school	–0.047	0.014	0.046	–0.013
	(0.090)	(0.480)	(0.085)	(0.529)
Household head attended				
Primary	0.003	–0.037	–0.012	0.046
	(0.783)	(0.000)	(0.292)	(0.000)
Secondary	0.005	–0.065	–0.036	0.096
	(0.755)	(0.000)	(0.014)	(0.000)
Preparatory or vocational	0.009	–0.083	–0.049	0.123
	(0.628)	(0.000)	(0.006)	(0.000)
College	0.019	–0.089	–0.069	0.139
	(0.289)	(0.000)	(0.000	0.000)
Share of household members				

Between 0 and 5 years	–0.117	–0.020	0.067	0.070
	(0.000)	(0.320)	(0.013)	(0.001)
Between 6 and 15	–0.105	0.025	0.030	0.049
	(0.000)	(0.143)	(0.191)	(0.006)
Between 16 and 28	0.001	0.064	–0.080	0.015
	(0.967)	(0.000)	(0.000)	(0.380)
Between 29 and 45	–0.004	0.019	–0.064	0.049
	(0.894)	(0.324)	(0.012)	(0.012)
Between 46 and 60	–0.013	–0.009	0.0004	0.021
	(0.621)	(0.648)	(0.989)	(0.268)

Φ = 2.265 (1.027)

Table 3. GME Estimates of the AIDS with Choke Prices

	Beef	Pork	Chicken	Processed Meat
Intercept	0.202	0.302	0.331	0.165
	(0.000)	(0.000)	(0.000)	(0.000)
Beef price	0.125	–0.034	–0.056	–0.034
	(0.000)	(0.000)	(0.000)	(0.000)
Pork price	–0.034	0.084*	–0.032	–0.018
	(0.000)	(0.000)	(0.000)	(0.000)
Chicken price	–0.056	–0.032	0.115	–0.027
	(0.000)	(0.000)	(0.000)	(0.000)
Processed meat price	–0.034	–0.018	–0.027	0.079
	(0.000)	(0.000)	(0.000)	(0.000)
Expenditure	0.012	–0.016	–0.001	0.005
	(0.000)	(0.000)	(0.000)	(0.000)
Urban	0.015	0.007	–0.020	–0.002
	(0.000)	(0.028)	(0.000)	(0.602)
Household head is female	–0.006	0.001	0.0003	0.004
	(0.285)	(0.815)	(0.963)	(0.429)
Household head is in school	–0.023	–0.004	0.023	0.004
	(0.052)	(0.715)	(0.075)	(0.755)
Household head attended				
Primary	0.007	–0.008	–0.011	0.012
	(0.176)	(0.082)	(0.061)	(0.036)
Secondary	0.009	–0.005	–0.022	0.018
	(0.165)	(0.360)	(0.002)	(0.008)
Preparatory	0.016	–0.0004	–0.033	0.018
	(0.044)	(0.944)	(0.000)	(0.035)
College	0.022	0.007	–0.043	0.014
	(0.004)	(0.305)	(0.000)	(0.075)
Share of household members				

Between 0 and 5 years	–0.008	0.005	–0.003	0.005
	(0.522)	(0.606)	(0.830)	(0.677)
Between 6 and 15	–0.002	0.018	0.004	–0.020
	(0.815)	(0.052)	(0.698)	(0.072)
Between 16 and 28	0.011	0.002	–0.020	0.007
	(0.256)	(0.821)	(0.060)	(0.491)
Between 29 and 45	–0.007	0.011	–0.015	0.011
	(0.530)	(0.269)	(0.233)	(0.346)
Between 46 and 60	–0.002	–0.001	0.001	0.001
	(0.863)	(0.958)	(0.914)	(0.922)

$\Phi = 4.006$ (9.463)

Table 4. Correlations between Observed and Predicted Shares

	Without Choke Prices	With Choke Prices
Beef	0.27	0.91
Pork	0.14	0.86
Chicken	0.21	0.88
Processed Meats	0.25	0.80
Demand System	0.44	0.89

Table 5. Estimated Hicks Price Elasticities (Standard Errors)

(a) Without Choke Prices

	Beef	Pork	Chicken	Processed Meat
Beef Price	–0.551	0.158	0.266	0.127
	(0.044)	(0.024)	(0.028)	(0.023)
Pork Price	0.506	–0.711	0.128	0.076
	(0.105)	(0.088)	(0.079)	(0.057)
Chicken Price	0.341	0.052	–0.531	0.138
	(0.039)	(0.026)	(0.037)	(0.024)
Processed Meat Price	0.346	0.068	0.291	–0.705
	(0.101)	(0.060)	(0.081)	(0.062)

(b) With Choke Prices

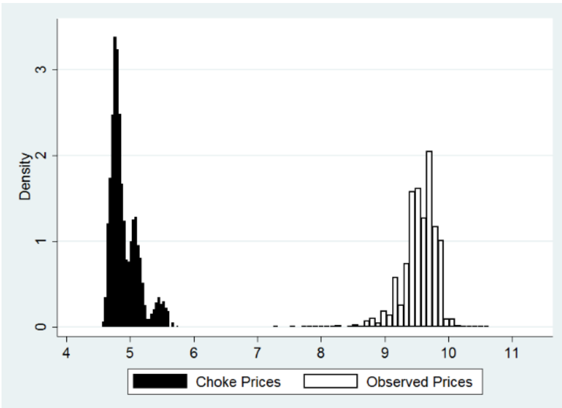
	Beef	Pork	Chicken	Processed Meat
Beef Price	–0.317	0.037	0.214	0.066
	(0.001)	(0.0004)	(0.001)	(0.0005)
Pork Price	0.271	–0.184	–0.086	–0.0004
	(0.002)	(0.001)	(0.004)	(0.002)
Chicken Price	0.232	0.028	–0.324	0.065
	(0.001)	(0.0004)	(0.001)	(0.001)
Processed Meat Price	0.142	–0.002	0.178	–0.318
	(0.002)	(0.001)	(0.004)	(0.002)

Table 6. Estimated Marshallian Own–Price Elasticities (Standard Errors)

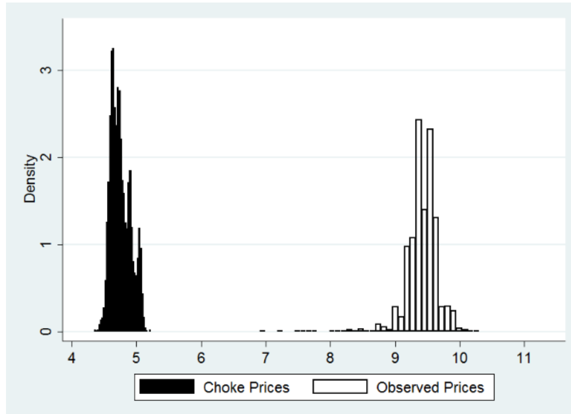
	Beef	Pork	Chicken	Processed Meat
GME without Choke Prices	–1.050	–0.862	–0.809	–0.776
	(0.037)	(0.080)	(0.040)	(0.072)
GME with Choke Prices	–0.736	–0.296	–0.640	–0.472
	(0.001)	(0.001)	(0.001)	(0.001)

Figure 1. Histograms of Choke Prices and Actual Prices

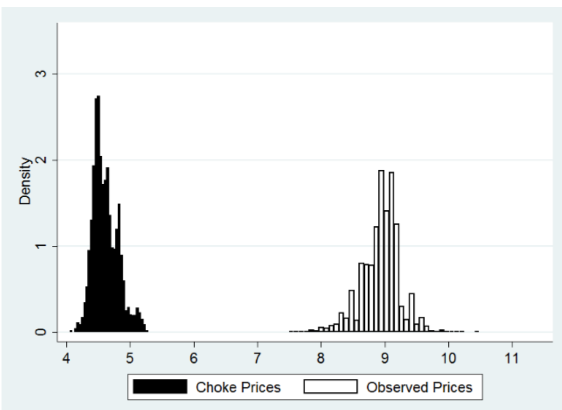
(a) Beef



(b) Pork



(c) Chicken



(d) Processed Meat