Diversification Done Right

Let x be the random return to a security, possibly holding a timber stand. Assume that the return to one security is independent of the return to another. That is, they are stands far apart and so aren't affected by the same fires or bugs.

The mean of x is μ and the variance of x is σ^2.

The variance ~~is~~ of x is calculated as $\sigma^2 = \frac{1}{T} \sum_{i=1,T} (x_i - \mu)^2$.

Here x_i is a realization of x – ~~that~~ for instance ~~is~~ it is the observed return in year i.

The variance of $\frac{x}{n}$ is σ^2/n^2. ($\frac{x}{n}$ has mean μ/n and use formula above.)

Now what is the return to buying $1/n$ of each of n of the securities or stands

$$S_n = \frac{x^1}{n} + \dots \frac{x^n}{n}$$

$$\text{Mean}(S_n) = \frac{\mu}{n} + \dots \frac{\mu}{n} = \mu$$

$$\text{Var}(S_n) = n \frac{\sigma^2}{n^2} = \frac{\sigma^2}{n} \quad \text{(independence)}$$

So as $n \to \infty$ there is no risk to this portfolio

Diversification.

Now lets suppose that $Cov(X_i, X_j) = \sigma^2$ as well. The risks are perfectly correlated!

~~ta~~ mean $S_n = \frac{\mu}{n} + \dots \frac{\mu}{n} = \mu$ as before

~~variance =~~

look at $S_2 = Var\left(\frac{X_i}{2} + \frac{X_j}{2}\right) =$

$\sigma^2 + \sigma^2 + 2Cov(X_i, X_j) = \frac{4\sigma^2}{4} = \sigma^2$

$Var\ S_n = \frac{n^2\sigma^2}{n^2} = \sigma^2$

So even with a lot of stands or stocks

$\underline{\mu}$ & $\underline{\sigma^2}$ stay the same

for all S_n

Chapt Basic Financial Analysis

I PV All Types inc Faustman, SEV
II About interest rates
III Taxes

PV (p. 326 + Seq

V_0 value now, present value
V_n value at time N

e.g. $0 \to 2000$ $n = 5$ so $n \to 2005$

i interest rate

$$V_n = (1+i)^n V_0 \quad \text{future value}$$

value of \$1

$$V_0 = (1+i)^{-n} V_n \quad \text{present value}$$

$$(i+1)^n = V_n / V_0 \quad \text{so} \quad i = \left(\frac{V_n}{V_0}\right)^{1/n} - 1$$

called internal rate of return (irr)

Net present value

R_t revenue at time t
C_t costs
$NR_t \equiv R_t - C_t$

$$NPV_0 = \sum_{t=0,T} \frac{R_t - C_t}{(1+i)^t}$$

e.g. C_0 could be planting costs

Example
P Pine

$C_0 = \$-100/ac$ plant
$C_{10} = \$-50/ac$ thin
$R_{90} = 40 \frac{MBF}{ac} \cdot \frac{\$200}{MFC} = 8000$

$i = 4\%$

$$PV = -100 \frac{-50}{(1+i)^{10}} + \frac{8000}{(1+i)^{90}}$$

$$= -100 - 33.78 + 234 = +100$$

comment:

1) price isn't going up over 90 yrs
2) i reflects this by not including inflation, either
3) ~~always~~ same answer if $i = 7\%$ and $p + c$ go up at 3% / yr

$$\frac{p \cdot (1.03)^{90}}{(1.07)^{90}} \approx \frac{p}{(1.04)^{90}}$$

4) Suppose there was a 20% chance of blowdown, fire, or bark beetle?

Formulas

for a recurring payment of R starting in 1 yr

$$V_0 = \sum_{n=1,\infty} R(1+i)^{-n}$$

$$V_0 = \frac{R}{1+i} + \sum_{n=2,\infty} R(1+i)^{-n}$$

$$V_0 = \frac{R}{1+i} + (1+i)^{-1} \sum_{n=1,\infty} R\,(1+i)^{-n}$$

$$V_0 = \frac{R}{(1+i)} + \frac{V_0}{1+i}$$

$$V_0(1+i) - V_0 = V_0 i = R$$

$$V_0 = R/i$$

Sights

a) $\frac{R}{1+i}$ immediate return

b) V_0 value function

Regular periodic harvest
- The Faustman Problem

harvest every t ('rotation age') years forever

$$V_0 = \sum_{n=1,\infty} \frac{R}{(1+i)^{nt}}$$

let $1+\delta = (1+i)^t$

$$V_0 = \sum_{n=1,\infty} \frac{R}{(1+\delta)^n}$$

$$V_0 = \frac{R}{\delta} = \frac{R}{(1+i)^t - 1}$$

Sights

a) here V_0 is called the Soil Expectation Value SEV

b) suppose R is $R(t)$ and is given by a stand table + price projection

Continuos time discounting

~~$e^{3t} = (1+i)^t$~~

$$e^{mt} = (1+i)^t \qquad mt = t\ln(1+i)$$

$$m = \ln(1+i)$$

So e^{mt} gives same values as $(1+i)^t$

Faustman, (1849)

$$V_0 = \frac{R(t)}{e^{mt}-1}$$

$R(t)$ is net revenue ~~of~~ at time t.

e.g. $P_t \cdot H_t - C_0(1+i)^t$ (volume)

notice ability to future value costs

$$V_0 = R(t)(e^{mt}-1)^{-1}$$

What t maximizes $R(t)$?

$$\frac{dV_0}{dt} = \frac{dR^*}{dt}(e^{mt}-1)^{-1} - (e^{mt}-1)^{-2} R m e^{mt}$$

$$\frac{dR/dt}{R} = \frac{\cancel{R}\, m e^{mt}}{e^{mt}-1}$$

Faustman Formula

$$\frac{dR/dt}{\cancel{d}R} = \frac{m}{1-e^{-mt}}$$

The One Rotation Problem

$$V = \frac{R}{(1+i)^t} = Re^{-mt}$$

$$0 = dV/dt = R'(t)e^{-mt} - mRe^{-mt}$$

$$\frac{\frac{dR}{dt}}{R} = m$$

Q: If $t = 90$ and $i = 4$ is the one period and Faustman problem so different

How about $t = 30$ and $i = 4$

e^{-mt} 30 yr $\approx \frac{1}{3}$

DAVIS p. 329 has table   90 yrs $\approx \frac{1}{30}$

N Period formulas

$$V_0(N) = \sum_{m=1,N} \frac{R}{(1+i)^m}$$

$$V_0(\infty) = \sum_{m=1,\infty} \frac{R}{(1+i)^m}$$

$$V_0(\infty) = \sum_{m=1,N} \frac{R}{(1+i)^m} + \sum_{m=N+1,\infty} \frac{R}{(1+i)^m}$$

$$V_0(\infty) = V_0(N) + \frac{1}{(1+i)^N} \sum_{m=1,\infty} \frac{R}{(1+i)^m}$$

$$V_0(\infty) = V_0(N) + \frac{1}{(1+i)^N} V_0(\infty)$$

$$V_0(N) = \frac{R}{i} - \frac{R}{i(1+i)^N} \cancel{= \frac{R}{i^2}}$$

$$= \frac{(1+i)^N - 1}{i\,(1+i)^N} R$$

<u>Sights</u>

Same basic trick as finding V_0.

Sinking fund:

I need $V_n(n)$ in year n
how much to pay per year? Let
that be R

<u>Recall</u>:

$$V_0(n) = \frac{R[(1+i)^N - 1]}{i(1+i)^N}$$

$$V_n(n) = (1+i)^N \cdot V_0(n) \quad \underline{\text{future value}}$$

$$V_n(n) = \frac{R[(1+i)^N - 1]}{i}$$

(Comment: you can
check this with a
spread sheet.)

$$R = \frac{V_n(n)\, i}{(1+i)^N - 1}$$

(Comment: you ~~hav~~ are
just putting this R in
the bank + waiting till
you have \$100)

Infinite series starting at time 0
with first payment at zero

$$= \frac{R}{i} + R = \frac{(1+i)R}{i}$$

Key ~~Points on~~ formulas:

$$V_0(\infty) \quad \sum_{m=1,\infty} R/(1+i)^m = P/i$$

$$V_0(N) \quad \sum_{m=1,N} R/(1+i)^m = \frac{((1+i)^N - 1)\,R}{i}$$

$$SEV = \sum_{m=1,\infty} \frac{R}{(1+i)^{tm}} = \frac{R}{(1+i)^t - 1}$$

Origins of i

1. Your boss tells you.
   Corporate hurdle rate.
   Carters (Arrow + Linds) 7% real
   for water projects
2. Opportunity cost of capital
   $\approx$ return on S&P 500
   $\approx$ 10% nominal
3. Idiosyncratic to individual
   Teenager: Now, please. $i = 50\%$

Comment: Number 3 brings up a harder problem.

I can borrow or lend at 7% (S&P real)
I prefer ~~\$~~ happiness today to
happiness tomorrow at the
rate of $\rho = 1\%$
Happiness is $u(c)$ where c
is consumption
When you get a client like
this I can help you for
a fee....

Interest rates

1. Nominal rates are the rates quoted. I will pay you \$1.05 next year for each dollar you give me today. \$1.05 may not buy anything next year, but that is a different problem

2. CPI. How much more will the same bundle of consumer goods cost next year? ?

   e.g. CPI = 100 in 1982
   = ___ now

   So ___ now ~~costs~~ buys same as 100 in 1982.
   inflation.

3. Real rate = Nominal Rate − Rate of inflation

Comment (again) M% inflation

$$\frac{P(1+M)}{1+(i+M)} \overset{?}{=} \frac{P}{1+i}$$

real, ~~real rate~~, inf

i+M nominal

4. RoR on S&P500 > RoR on 1 yr Treasury
Equity premium
Payment for Risk

T called risk free rate

5. TIPS
Real Risk free rate.

6. Term structure
short
long (30 yr)
Expect of lots of shorts
but rate expected to rise
long > short

7. Liquidity
Big debt issues like T-bills
have lots of traders always
Small debt issues do not
+ have a liquidity premium

Q: How does the market value the risk of being in the timber business?

A: Two major types of risk

1. Idiosyncratic - Plum Creek Timber has risk of blowdown. <u>Not</u> correlated with other business risks like USU's cost overruns on its uranium enrichment plant

Math: Two firms both with expected earnings of \$10. Both with variance σ^2. ~~On~~ Risks are independent. Buy one share of each.

$$\text{Expectation} = \$20$$
$$\text{Variance} = 2\sigma^2$$
$$\text{SD} = \sqrt{2}\,\sigma^2$$
$$95\%\text{ confidence } ? \quad \$20 \pm 2\sqrt{2}\cdot\sigma$$

<u>N Firms</u>

$$n \cdot 10 \pm \cancel{2\sqrt{n}}\, 2\sigma\sqrt{n}$$

Return per firm w/ n firms

$$10 \pm \frac{2\sigma}{\sqrt{n}}$$

$n \Rightarrow \infty$ Return is 10!

<u>Conclusion</u>: The return ~~to~~ per firm invested in <u>does not depend</u> on idiosyncratic risk.

The market does not 'price' the risk of blowdown. (Just its expected value)

2. Systematic risk. The economy has a recession and timber prices fall.

Math: Two firms. Perfectly correlated so Variance $= \sigma_1^2 + \sigma_2^2 + 2Cov(1,2)$
$= 4\sigma^2$!

N Firms .

$n \cdot 10 \pm 2N\sigma$ ~ 95% confidence

So no change with diversification

What about the privately owned firm, like SPI?

Other sights

$$(1+RRR)^N = \frac{FV \text{ of benefit}}{FV \text{ of costs}}$$

is one way to compare investments that end at different times.

For forestry it makes more sense to think about how the land will be used for a long time - e.g. infinity

P 371,

7.16a $R(s)-C(s)$

7.16b $R'-C'=0$

P 380-379

Doug fir
yield, rotation age,
Note that SEV has a
lower rotation age
than age of NPV

The 'Hartman Age'

Suppose old trees have value just standing there. e.g. bio diversity. Then rotation age is also longer.

TAXES!

Income $\approx 30\%$

Long term capital gains at $\approx 15\%$

Corporate tax + Individual Tax

REIT passes value to individual without corp tax

Comment on REITS
- sorting timber
- transfer price
- taxes
- diversification

Capital gain:

Selling price less cost basis

reforest is part of cost base

The odd bit: Suppose pre commercial thin is 'maintenance'.

Then it reduces income + income tax immediately + increase capital gain with a lag.

(17)

Best case:

Cost \$100 now

State & Fed tax rate $\approx$ 35

Cost now after tax \$65

Benefit PV to present \$100

\$100 & cap gains tax $\approx$ 17%

$= \$100 + \$17 = 83$

So a 'breakeven' before tax

becomes 83/65 return!

p. 386 for some formulas

also see website if it's still there

Property tax. value of forest grows + so does tax
Ad valorem (like on a house)
expressed in Mills = $\frac{1}{10}\%$

<u>Fact</u> The present value of property subject to a m percent tax rate is the same as the property discounted at $\delta + m\%$

<u>Proof</u>:

$$V_t = \int_t^\infty (R - mV(z)) e^{-\delta(z-t)} dz$$

take time derivative

$$\dot{V} - \delta V = R - mV$$

$$\dot{V} - (m+\delta V) = v \quad \text{integrate w/}$$

$$V_t = \int_t^\infty R\, e^{-(m+\delta)(t-z)} dz$$

can be <u>very</u> steep. Shortens rotation

<u>Yield tax</u>

Just a percent of the sales price

Fixed tax on land

Percent of SEV calculated independent of management.

Conclusion

1. It is PV that matters. Make it big
2. Compare two management regimes with PV.
3. Keep it all real or nominal
4. Watch your taxes - income + property do matter a lot
5. Learn to derive your basic value formulas

8 Valuation

Marketed goods - timber, hunting rights, etc
Non Market Goods - arm chair environmentalist

Appraisal

"amount paid by willing buyer to willing seller"

Why

1) going to buy or sell and don't want to get taken
2) tax reasons (basis on day of death) Property tax
3) planning - whats there?
4) damages - fire
5) taking
6) transfer price among related companies (capital gain vs. ordinary income
7) collateral for loans
8) marking investment to market
e.g. pension fund owns timber

Methods of Appraisal

Market evidence - comparable sales - is king.
few sales same size, place, time, species, volume
lots of room to argue
300/MBF or 600/MBF ~~ROW~~
less than fee simple

Capitalized Income (future prices)
Estimate costs + revenues
discount same
even more room to argue
what will be price of redwood in 50 yrs

Conversion Return (current prices)
Log value - costs

Market Quantification
Use supply + demand eq to estimate price + do Capitalized income

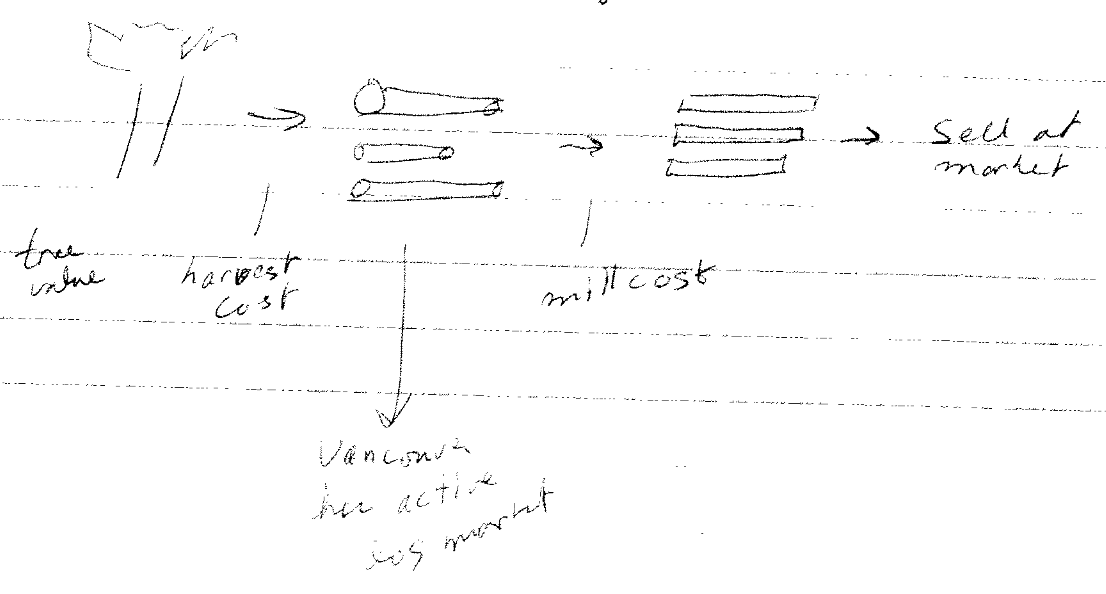
Replacement Cost
no good for timber where it isn't standardized

Expert judgement
forget it.

"Now You Never for the Second Tree"

Valuing a tree – better be a big tree + cost $700/MBF

1) Could work from tree measurement to estimate log size and grade for butt or all potential logs
2) If you could estimate the log size + grade, you could estimate the ~~timber~~ lumber yiel



p 409

Step 1 — get lumber $/MBF

2 grade recovery from logs

e.g. log grade 1, 18" log in yields 33% #1 common

volume - log to lumber scaler

depends on mill

3 Costs of hauling, skidding — roads etc price

Costs of logging

** Costs include fixed and variable costs

A. If Stump price < 0 when using variable cost, don't cut

B If Stump price < 0 when using full cost won't make a profit.

USDA Stump Appraisal

1) include "profit"

2) for average operator

If correct but a little random then lots of sales wouldn't sell.

Half of all operators would go under each period for lack of timber

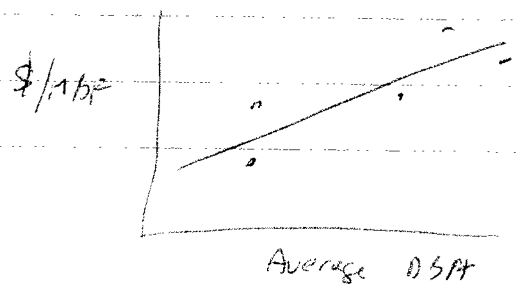
None of this happens. They are and need to be lower than average break even.

4/6

Ca values from
~~Bureau~~ State Board of Equalization Yield Tx
Consulting Co's (HJW) keep records of
what their client timber sold for

p. 417

Using sales data in a regression



$\$/MBF = 1.74 \cdot ADSH + \ldots$

Want to see if you can estimate
$/MBF out of sample
Want Bias and RMSE

on average my regression
is $12 high

$\sqrt{\Sigma (E\$ - Actual\ \$)^2}$

Caution: USFS sales have a
dominant species that is bid
and many non-dominant that are
priced. e.g. alder is $10/MBF what
will you bid for Doug fir?

Road Costs

FS bids timber as follows
We will give you $X and require
you to build a road if you win
How much will you bid for timber?
Ans. If $X really is road
cost bid really is timber value
USFS nets Bid x Volume - $X

Time - A 5 year sale
I think inflation will be 8%/year
You think it will be 4%/yr.
Otherwise we are the same.
Who wins?
1981, I think

More $ for big stands?
Hotelling valuation principle
Truth - costs level out in
medium size

Nantimba outputs.

Marketed – hunting, skiing, communications towers, vacation homes

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CU + KU lectures