PROBLEM SET 1

ETHAN LIGON

(1) Find the first derivative for the following functions:

$$f(x, y) = (x^4y^2 + 3)^2x^2 - 4$$
$$f(x) = \frac{(x+1)}{(x-1)}$$
$$h(t) = log(t)$$
$$f(t) = te^{2t}$$
$$f(x) = x + 10^x$$

(2) Find all the partial derivatives for the following functions:

$$f(x, y) = x + x(x - y^2)$$
$$f(x, y) = (y + x)y + y$$
$$f(x, y) = ye^{2x}$$
$$f(x, y) = \frac{1}{x^2 + y^2}$$
$$f(x, y, z) = yz + z^2 + 4xyz$$

(3) Use implicit differentiation to find dy/dx:

$$4y^3 + 7xy^2 - 12x^2 = 71$$
$$\frac{3}{y} + \frac{4}{x} = 2$$
$$y^2 = \frac{x}{xy + 1}$$

(4) Find the total differential dz given:

Date: Due September 13, 2005.

$$z = 5x^3 - 4xy - x - y^2 - 2y$$
$$z = \frac{3xy^2}{x+y}$$
$$z = log(3xy)$$
$$z = x^2ye^x$$

(5) Find the total derivative df/dt:

a)
$$f(x,y) = x - y^2$$
where
$$x(t) = log(2t)$$
and
$$y(t) = e^t$$

b)
$$f(x,y) = log(xy) + e^{x^2+y^2}$$
where
$$x(t) = t + 1$$
and
$$y(t) = e^t$$

c)
$$f(x,y,z) = x^2y^3z^4$$
where
$$x(t) = e^t + t^2$$
$$y(t) = log(t^2)$$
and
$$z(t) = t^3 + t$$

(6) Find the critical points for the following functions. Determine if they correspond to maxima or minima:

$$f(x) = x^2 - x - 5$$
$$f(x) = \frac{x}{x-3}$$
$$f(x) = \frac{x+3}{x^2-16}$$
$$f(x) = \frac{5}{2-2x^2}$$

(7) For each of the following functions, determine the intervals for which they are concave upward and concave downward. Also, determine the points of inflection.

$$f(x) = \frac{x}{1-x}$$
$$f(x) = 3x^2 - 24x^3$$
$$f(x) = \frac{x}{x^2-1}$$

(8) Find the dimensions of a rectangle with perimeter 500 ft whose area is as large as possible.

(9) Find all the points on the curve

$$x^2 - 4y^2 = 16$$

closest to the point (0, 1).

(10) In a certain chemical manufacturing process, the daily weight y of defective chemical output depends on the total weight x of all output according to the empirical formula

$$y = 0.003x + 0.00005x^2$$

If the profit is 1000 dollars per pound of non-defective chemical produced and the loss is 50 dollars per pound of defective chemical produced, how many pounds of chemical should be produced daily in order to maximize profit?

(11) Suppose a firm's total revenues depend on the amount q produced according to the function

$$TR = 24q - q^2$$

Total cost depends on

$$TC = q^2 - 28q + 180$$

a) What levels of q should be produced to maximize total revenue, total profit?
b) Show that the second order conditions for a maximum are satisfied at the output level found in part 11a).

(12) A firm determines that x units of its product can be sold daily at p dollars per unit, where

$$x = 145 - p$$

The cost of producing x units per day is

$$C(x) = 450 + 3x$$

a) Assuming that the production capacity is 300 units per day, determine how many units the company must produce and sell each day in order to maximize its profits.
b) Find the maximum profit.
c) What price per unit must be charged to obtain the maximum profits?