

ARE 251/ECON 270A PROBLEM SET 2

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- (1) Consider a household consisting of only two people, each of whom lives for two periods. Each derives utility from a von Neumann-Morgenstern utility function $U : \mathbb{R} \rightarrow \mathbb{R}$, and discounts future utility by a factor $\beta = 0.9$.

There are two possible states in each period t , denoted by $\omega_t \in \{1, 2\}$. Person i 's endowment in period t and state ω_t is equal to $y_{it}(\omega_t)$. The probability that $\omega_t = 1$ is 0.5. Assume that $U(c) = \log(c)$. There is no storage, and neither person has any interaction with others outside the household.

Each person's endowments in each of the two periods is given by:

| | | Person | |
|----------|-----|--------|---|
| | | 1 | 2 |
| ω | t | | |
| 1 | 1 | 0 | 3 |
| 2 | 1 | 5 | 7 |
| 1 | 2 | 2 | 4 |
| 2 | 2 | 3 | 3 |

- a) Write down an event tree for this economy. Write down the commodity space.
 - b) Calculate the expected discounted utility for agent 2 if he simply consumes his endowment in each period.
 - c) Suppose that in addition to utility from own consumption, person one also enjoys a share θ_2^1 of person two's utility, and conversely person two enjoys a share θ_1^2 of person one's utility. Show that a set of state-contingent altruistic transfers between the two people can yield Pareto optimal outcomes, even in the absence of any other kinds of transfers.
 - d) Show that altruistic transfers are not, however, sufficient for efficient outcomes—in particular, indicate restrictions on altruism necessary to deliver efficiency.
 - e) Show that an income-pooling scheme can yield Pareto optimal outcomes. What is the minimum share that agent two would accept as an inducement to participate in this scheme?
 - f) Using the fact that $\frac{U'(c_{it})}{\beta E U'(c_{it+1})} = 1 + r_t$, compute implicit interest rates given that the couple implements the income pooling scheme.
 - g) Do interest rates depend on shares?
 - h) Would we expect to observe any borrowing or lending between this couple?
- (2) Consider the environment of the previous question, but now suppose that the two agents bargain to determine allocations.
- a) Define the set of bargaining surpluses for the bargaining problem.
 - b) Suppose that the two consult John Nash Jr. What allocation would he recommend?
 - c) How does the Nash solution depend on the discount factor β ?
 - d) How does the Nash solution compare with the income pooling solution you found in the previous problem?