Regulating Multiple Polluters: Deterrence and Liability Allocation

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ABSTRACT

We consider the problem of regulating many polluting firms when their individual emissions are unobservable. The tension between the dual regulatory goals of pollution deterrence and funding of remediation is examined under two different constraints: that penalty revenues be sufficient to fund remediation costs; and that transfers from firms to the regulator must be nonnegative. To isolate the pure effect of increasing the number of polluting firms, we compare an industry consisting of a single large firm with another in which many small firms in aggregate mimic the characteristics of the large firm. Contrary to previous findings, we show that both the number of firms and the ability to monitor individual firms significantly affect the welfare of a wide class of types of regulator.

1. Introduction

It often happens that multiple firms pollute a common site but their individual contributions cannot be distinguished. For example, when farmers sharing a common watershed all use similar pesticides it is expensive, if not impossible, to trace the resulting pollution of the water supply to individual farmers. It is widely recognized by practitioners that in such instances, increasing the number of polluters exacerbates the complexity of the regulation problem. In these situations, it is difficult to allocate responsibility between parties because of the difficulty of accurately determining the relative culpability of each. This paper examines the consequences of this difficulty for the regulator. Specifically, we will say that our regulator faces a *multiplicity problem* if it is more difficult to regulate an industry consisting of a large number of small firms than a comparable industry with a small number of large firms. We will show that depending on which restrictions are imposed on the regulator, a multiplicity problem may or may not arise.

While regulation of nonpoint pollution is an important issue in environmental economics, this topic is also closely related to the issue of "team production" in agency theory, in that the individual contributions and efficiency of individual workers are often unobservable (Holmstrom, 1982; Rasmusen, 1987; McAfee and McMillan, 1991). In these papers, it has been observed that there is in fact no multiplicity problem—under an optimally designed incentive scheme, the regulator's expected utility is unaffected by the number of team members. Indeed, even if the regulator in these models could costlessly observe individual members' contributions, this additional information would be of no value. The literature does of course recognize that "[i]n the world, ... inefficiencies are undoubtedly inherent in team production." It concludes, however, that

> "the source of team problems is not the unobservability of team members' efforts or abilities per se. The source of team inefficiencies must be sought elsewhere, in features assumed away [such as] team members' risk aversion, or collusion among team members." (McAfee and McMillan (1991), p. 571).

When these analyses are transplanted to our context, they yield the result that the regulator can do equally well, whether there is a large number of small contributors or a small number of large ones. Our primary objective is to demonstrate that in the context of environmental regulation, the multiplicity problem *does* arise if certain realistic restrictions are imposed on the kinds of regulatory instruments that are available to the regulator, and so can be explained without resort to factors such as collusion or risk aversion. This is consistent with the observations of Rasmusen and Zenger (1990), who examine organizational diseconomies of scale, and other work cited within. As a corollary, the capacity to monitor the performance of individual firms reemerges as a valuable regulatory tool. On the other hand, we also identify conditions under which, counterintuitively, there is a "reverse" multiplicity problem: multiplicity actually benefits the regulator.[1] The two constraints on penalties we consider are: that penalties must be fair in the sense of not applying arbitrary punishments; and that penalties must be politically feasible in the sense of not rewarding firms for polluting. These constraints are absent in the related analyses cited above.

As a byproduct of our analysis, we show that multiplicity problems arise for reasons that differ from the ones alluded to in the opening paragraph. Given a particular penalty scheme, the incentive for an individual firm to free-ride does *not* increase as the number of firms increase. Rather, the effect in our model of increasing multiplicity is to increase the cost to the regulator of *implementing* any given penalty scheme. Thus, in equilibrium, firm emissions increase with the number of firms because the regulator *chooses* to offer lower-powered incentives for abatement and, by implication, to set higher targets for emissions.

A distinguishing feature of our model is that penalties serve the dual function of deterring firms from polluting and also allocating financial responsibility for cleanup (remediation) ex post. The regulatory task of remediation appears not to have been modeled previously. Although the moral hazard problem has been examined in a nonpoint pollution context (Segerson, 1988), we are unaware of any treatment of both moral hazard and adverse selection issues in this context.

The informational structure of our model is that only *aggregate* emissions are observable to the regulator. Consequently, the regulator is confronted with both a moral hazard problem

[1]A similar kind of result has been observed by Paroush (1993) in a model in which teams of decision-makers choose levels of investment in human capital.

(individual firms' emissions are not observable due to co-mingling) and an adverse selection problem (firms have private information about their abatement costs). Thus, our model reflects two important aspects of reality: regulators are required to regulate nonpoint pollution, and they are often not well informed about the characteristics of firm-specific production technologies. Efficiency considerations dictate that firms with less costly abatement technologies undertake relatively more abatement. The regulator's problem is to design penalties that elicit an optimal abatement response from each firm subject to the informational constraints outlined above and given that the objective of the regulator is to maximize the weighted sum of penalty revenues and firm profits less remediation costs and environmental damage.

Our model is based on McAfee and McMillan (1991) (hereafter MM). Recasting their model in our framework, optimal penalties are continuous and linear in aggregate environmental damage. In our context, however, MM's model has limited applicability, for three reasons. First, while their principal is a self-interested firm, ours is a regulator. It is therefore appropriate to assume that our principal's self-interested motivations are at least partially tempered by a a concern for society's welfare. Second, MM place no budgetary restriction on the regulator, so the optimal penalties could require the regulator to partially fund remediation. In reality, however, regulators (for example, the EPA) typically have limited internal budgets, and depend heavily on contributions by firms to fund remediation. Third, MM's penalty structure involves a two-part tariff, consisting of a *variable penalty* (linear in aggregate damage) and a *fixed transfer* (dependent only on firm abatement costs). The optimal penalty may require the regulator to make a positive fixed transfer to some firms. It seems unlikely, however, that such penalties would be politically feasible, since the regulator would be required to make payments to polluting firms.[2] We show that for very general specifications of the regulator's objective function that the multiplicity problem does indeed arise when additional constraints are imposed to reflect the latter two considerations.

In section 2, we develop the *unconstrained model* in which the regulator is constrained only by the standard incentive compatibility and participation constraints. This model is closely

[2]This issue is discussed further in footnote 16 below.

related to that in MM, though differs in that our regulator's payoff is a weighted average of self-interested and social concerns. A regulator who heavily weights social concerns—in our context, firm profits—will be referred to as *benevolent*; one who heavily weights selfish concerns—i.e., penalty revenues net of environmental and remediation costs—will be referred to as *revenue-maximizing*. In section 3 we analyze the *self-funding* model, which imposes upon the unconstrained model the additional constraint that aggregate penalty revenues levied by the regulator equal the total cost of remediation. In section 4 we analyze the *no-subsidy* model, which imposes upon the unconstrained model the restriction that the regulator must offer penalties that involve nonnegative fixed transfers from firms to the regulator. For each model we examine the multiplicity problem by *fissioning* a small number of large firms to create a large number of small firms, thus ensuring that the aggregate scale of the problem is unchanged. In section 5, we investigate the value to the regulator of firm-specific emissions data under each of the additional constraints.

We identify conditions under which imposing a binding self-funding constraint results in net emissions (i.e., emissions remaining after remediation), and hence environmental damage, being greater than in the unconstrained model. The conditions are that either the regulator places sufficiently low weight on firm profits, or that informational rents are sufficiently independent of the level of prescribed emissions. To see this, note that a revenue-maximizing regulator will seek to minimize both the budget deficit (remediation costs less penalty revenues) and environmental damage. Hence a constraint that eliminates the former must exacerbate the latter; otherwise the regulator would be better off than in the unconstrained model. On the other hand, if firms' profits are essentially unaffected by a change in prescribed emissions, then the regulator's choice of emissions levels will be essentially unaffected by the weight she assigns to firms' profits. Our model thus suggests that if either of these conditions is satisfied, environmentalists should *oppose* initiatives that require industry to fully fund remediation of pollution. In the no-subsidy model, we identify conditions under which net emissions will either increase or decline following a prohibition on positive transfers to firms. These conditions will thus determine whether environmentalists can be expected to either oppose or favor such a prohibition. The conditions we identify involve the particular specification of the remediation cost function, but not the specification of the regulator's objective function. In particular, our results for the no-subsidy model do not depend on whether the regulator is revenue-maximizing or benevolent.

Our main results concern the effect of fissioning firms on the regulator's payoff. Once again, these results are robust with respect to variations within the class of objective functions that we consider. In the self-funding model, the multiplicity problem does not arise if emissions are deterministic, while if emissions are stochastic it has an ambiguous effect on the regulator's expected utility. Thus, the regulator's payoff may actually *increase* as the number of firms increase, although we show that it will eventually decrease once the number of firms is sufficiently large, provided that firms have sufficient control over emissions. In the no-subsidy model, however, the multiplicity problem *always* arises.

Because the intuition for our multiplicity result in the self-funding model is rather more technical, it is deferred to section 3. In the no-subsidy model, assume that there is only one large firm whose transfer to the regulator is positive. Now "fission" this firm, i.e., replace it with m smaller firms whose characteristics in the aggregate mimic those of the the original firm. By construction, this change will leave aggregate industry emissions and profits unchanged if each of the m firms is confronted with the penalty originally designed for the large firm. However, aggregate variable penalties will increase by a factor of m since the m small firms pay the same marginal penalty as the large firm (since they are of the same abatement cost type), and the penalty is levied on *aggregate* damage. Because it cannot violate the m small firms' participation constraints, the regulator must then decrease the size of the fixed transfer that these firms must pay. Hence, fissioning decreases the size of the fixed transfers that firms must make, and for sufficiently large m the regulator will be required to make *positive* transfers to some firms in the unconstrained equilibrium. Since such transfers are infeasible in the no-subsidy model, the regulator's expected payoff in this model must be lower.

In section 5 we examine the benefit to the regulator of a technology that permits monitoring of individual firm emissions. The results mirror those obtained in sections 3 and

4, as individual monitoring resolves precisely those difficulties that underly the multiplicity problem. In particular, the regulator can now penalize firms on the basis of individual contributions to environmental damage, allowing the same incentives to be achieved with smaller transfers, and thus effectively relaxing the no-subsidy constraint. Hence, technologies that facilitate individual firm monitoring are valuable under precisely the same conditions as those that gave rise to the multiplicity problem in the first place.

2. The Unconstrained Model

2.1. **Basic Structure.** A regulator has jurisdiction over a population of a set $N = \{1, ..., n\}$ of polluting firms. Each firm $i \in N$ chooses a target level of emissions y_i, with actual emissions being random and conditional on y_i. The regulator observes only the aggregate emissions of the n firms, denoted by x. Specifically, x is a random variable with density function $f(x|\boldsymbol{y}) \equiv \frac{dF(x|\boldsymbol{y})}{dx}$, with expectation conditional on $\boldsymbol{y} \equiv (y_1, \ldots, y_n)$ equal to $\sum_i y_i$.

Each firm's cost of abatement, indexed by θ_i, is private information—for example, θ_i could be an index of the age of the abatement technology being used. All other firms and the regulator perceive θ_i to be drawn independently from a distribution with density $g(\theta) \equiv \frac{dG(\theta_i)}{d\theta_i}$ and support $[\underline{\theta}, \bar{\theta}]$. Denote by $\underline{\theta}$ the most efficient abatement technology possible. The amount of abatement effort undertaken by each firm is private information, and is implicitly defined through the relationship between abatement cost and emissions since each pair (y_i, θ_i) defines a unique level of abatement effort. The vector of types of the n firms is denoted by $\boldsymbol{\theta} \equiv (\theta_1, \ldots, \theta_n)$.

The abatement cost function, $c(y_i, \theta_i)$, satisfies $c_y < 0$, $c_\theta > 0$, $c_{yy} > 0$, $c_{y\theta} < 0$ and $c_{yy\theta} > 0$—that is, the marginal cost of abatement increases more rapidly with abatement for high cost firms. Remediation is determined and undertaken by the regulator after actual emissions are observed. Given remediation r, *net emissions* equals $x - r$, resulting in environmental damage $D(x - r)$, where $D' > 0$ and $D'' > 0$. Remediating an amount r, given emissions of x, costs $R(x, r)$, where $R_r > 0$, $R_{xr} < 0$, $R_{xx} > 0$, $R_{rr} > 0$, and $R_{rrx} < 0$—that is, the marginal cost of remediation increases less with remediation as emissions increase. Attaining zero net emissions is prohibitively expensive in that $R(x, x) = \infty$ for all $x > 0$, so $r \leq x$.[3]

Regulation involves two tasks: designing and imposing penalties on firms for observed (aggregate) emissions, and determining the level of remediation. The regulator offers a menu of penalties, $\boldsymbol{s} \equiv \{s(x, \hat{\boldsymbol{\theta}}) : \hat{\boldsymbol{\theta}} \in \prod_{i=1}^{n} \theta_i\}$, consisting of a penalty for each conceivable combination of reports, $\hat{\boldsymbol{\theta}}$, one from each firm. The function $s_i(x, \hat{\boldsymbol{\theta}})$ defines the penalty for a firm that reports its type to be $\hat{\theta}_i$ while the other $n-1$ firms report $\hat{\boldsymbol{\theta}}_{-i}$ and aggregate emissions are observed to be x. The regulator can credibly commit to any menu $\boldsymbol{s}$.

Each firm produces a fixed, identical level of output $\bar{q}$, implying that emissions are unrelated to output. While this assumption will not hold in many cases, it is not needed to obtain our results and it simplifies the analysis.[4] One interpretation is that all firms are capacity constrained and remain so even when the optimal emissions tax is imposed, because production constraints are independent of abatement expenditures. Demand is perfectly elastic with output price equal to one. Both firms and regulator are risk neutral. Each firm's payoff is random and given by

$$\pi_i(x, \boldsymbol{\theta}, y_i) \quad = \quad \bar{q} \; - \; c(y_i, \theta_i) \; - \; s_i(x, \boldsymbol{\theta}). \tag{1}$$

The regulator's problem is to design a menu of penalties that induces each firm to choose the level of emissions that maximizes the regulator's expected utility, subject to ensuring that all firms participate and truthfully reveal their private information.[5] As noted above, we consider regulators whose objective functions range from *benevolent* (social-surplus maximizing) to *revenue-maximizing* (where revenue is defined as penalty receipts less net environmental damage). Deadweight losses are incurred whenever transfers are made, whether these are negative or positive.

[3]An example of a remediation function satisfying all these restrictions is $R(r, x) = \frac{r}{x-r}$, for $r < x$.

[4]Our results also hold under a more general specification involving variable production and a positive relationship between output and pollution.

[5]Environmental regulation is not typically voluntary—regulators can enforce participation. Thus, nonparticipation could be interpreted as bankruptcy, with the participation constraint implying that the regulator is politically constrained from forcing firms into bankruptcy. It may even be optimal for the regulator not to induce bankruptcy, as bankruptcy will result in a lower total contribution by firms toward remediation costs, leaving the regulator a larger "orphan share" of the costs to fund itself.

Using the revelation principle, we identify a Bayes Nash equilibrium. An equilibrium consists of a menu of penalties, $\boldsymbol{s}$, and a vector of reports $\hat{\boldsymbol{\theta}} = (\hat{\theta}_i)_{i=1}^n$, which implicitly determine aggregate emissions and remediation. The time line is as follows. The regulator commits to a menu of penalties. Each firm then reports its type, the marginal penalty for each firm being collectively determined by all reports. Firms then choose their emissions (or, equivalently, abatement effort). The regulator then observes aggregate emissions and determines the level of remediation. Lastly, the transfers implied by the menu of penalties together with reported types and observed aggregate emissions are implemented.

Define $U(x)$ to be the regulator's *command and control utility*—the regulator would maximize this function if there were no incentive or participation constraints (and hence no need for penalties).

$$U(x) \quad \equiv \quad \phi \sum_i \pi_i(x, \boldsymbol{\theta}) \; - \; R(x, r(x)) \; - \; D(x - r(x)),$$

where $\phi \geq 0$ is the welfare weight that the regulator places on firm profits. Let $E_{\boldsymbol{\theta}}(\cdot) \equiv \int \cdots \int_{\underline{\theta}}^{\bar{\theta}} (\cdot) \, dG(\boldsymbol{\theta})$ and $E_{-i}(\cdot) \equiv \int \cdots \int_{\underline{\theta}}^{\bar{\theta}} (\cdot) \, dG(\boldsymbol{\theta}_{-i})$. Given a function $\xi(x|\boldsymbol{y})$, denote by $E_x \xi(x)$ the conditional expectation $\int \xi(x) f(x|\boldsymbol{y}) \, dx$. The regulator's problem is to choose a penalty scheme $\boldsymbol{s}$ to solve:

$$\max_{\boldsymbol{s}} \; E_{\boldsymbol{\theta}} \, E_x \left[U(x) \; + \; \sum_{i \in N} \eta_i s_i(x, \boldsymbol{\theta}) \right] \tag{2}$$

$$\text{s.t.} \qquad \forall\, x \qquad \tilde{r}(x) \quad = \quad \operatorname*{argmax}_r \; [-D(x, r) - R(x, r)] \tag{3}$$

$$\forall\, i \in N, \, \boldsymbol{\theta}, \, \theta_i', y_i' \qquad E_x \, E_{-i} \, \pi_i(x, \boldsymbol{\theta}, y_i(\boldsymbol{\theta})) \quad \geq \quad E_x \, E_{-i} \, \pi_i(x, (\theta_i', \boldsymbol{\theta}_{-i}), y_i') \tag{4}$$

$$\forall\, i \in N \qquad E_x \, E_{-i} \; \pi_i(x, \boldsymbol{\theta}, y_i(\boldsymbol{\theta})) \quad \geq \quad 0 \tag{5}$$

Expression (2) states that the regulator's objective is to maximize the expected value, over types and emissions levels, of the command and control utility plus penalty revenues. This expression implicitly assumes truthful reporting ($\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}$). For a benevolent regulator, $\phi = 1$;

for a revenue-maximizer, $\phi = 0$. The deadweight loss associated with transfers is given by

$$\eta_i \quad \equiv \quad \begin{cases} 1-\lambda & \text{if } s_i \geq 0 \\ (1-\lambda)^{-1} & \text{if } s_i < 0, \end{cases} \qquad \text{where } \lambda \in [0,1).$$

Under this specification, deadweight losses are incurred regardless of the direction in which transfers flow: whether firms are taxed or subsidized, a loss of λ times the transferred amount is incurred.[6] Note that, *ceteris paribus*, a benevolent regulator prefers not to penalize firms, in order to avoid the deadweight losses.

Equation (4) is the incentive compatibility constraint, ensuring that each firm does indeed truthfully report its type and choose its emissions level optimally. Equation (5) is the participation constraint which ensures that each firm's equilibrium expected payoff is at least as high as its reservation payoff, assumed equal to zero. Equation (3) says that the regulator chooses the level of remediation optimally given emissions, x.

The optimal remediation level given x, $\tilde{r}(x)$, is defined by Equation (3), so $\tilde{r}(x)$ is stochastic. The regulator specifies a level of remediation such that, given x, the marginal cost of remediation is just equal to the marginal benefit from lower environmental damage. Totally differentiating the first order condition associated with Equation (3), we see that remediation increases with emissions in equilibrium:

$$\frac{d\tilde{r}(x)}{dx} \quad = \quad \frac{-(R_{ry} - D'')}{R_{rr} + D''} \quad > \quad 0. \tag{6}$$

Since $\tilde{r}(x)$ is monotonic there always exists a unique optimal choice of $\tilde{r}$.

Denote the regulator's optimal vector of emissions, given $\boldsymbol{\theta}$, by $\tilde{\boldsymbol{y}}(\boldsymbol{\theta}) \equiv (\tilde{y}_1(\boldsymbol{\theta}), \ldots, \tilde{y}_n(\boldsymbol{\theta}))$.[7] Following MM, we assume that U, c, and g are such that $\tilde{\boldsymbol{y}}(\boldsymbol{\theta})$ is well-defined and continuously differentiable. The regulator's problem is to design a menu of penalties which implements $\tilde{\boldsymbol{y}}(\boldsymbol{\theta})$. MM have shown that under certain conditions $\tilde{\boldsymbol{y}}(\boldsymbol{\theta})$ for a revenue-maximizing regulator

[6]At first sight it might seem more natural to set $\eta_i = 1 + \lambda$, regardless of the direction of the transfer flow. Under this specification, it would cost the regulator \$$(1+\lambda)$ to subsidize firms by \$1, but the regulator would earn \$$(1+\lambda)$ for every dollar of taxes paid by the firm. Clearly this makes no sense.
[7]If there were no deadweight losses associated with transfers (i.e., $\lambda = 0$), the complete information outcome would be achievable in equilibrium (see p. 164 of Laffont (1990) for a discussion of this well known result).

can be implemented by penalties that are linear in the extent of environmental damage (but not linear in type). Such penalties are attractive to the extent that they lower computational requirements. A similar, but more general, result is obtained here for a regulator whose objective function is given by Equation (2).

Proposition 0. (McAfee & McMillan, 1991) *If for all* i*, (a)* $\frac{\partial \beta_i}{\partial \theta_i} \leq 0$ *and (b) for all* $j \neq i$*,* $\frac{\partial \tilde{y}_j}{\partial \theta_i} \leq 0$*, then the penalty scheme is an optimal contract:*[8]

$$s_i(x, \boldsymbol{\theta}) \quad = \quad \beta_i(\boldsymbol{\theta})[D(x) - \bar{D}(\tilde{\boldsymbol{y}}(\boldsymbol{\theta}))] \; + \; T \; - \; c(\tilde{y}_i(\boldsymbol{\theta}), \theta_i) \; - \; \int_{\theta_i}^{\bar{\theta}} c_\theta(\tilde{y}_i(t, \boldsymbol{\theta}_{-i}), t)\, dt, \quad (7)$$

where,

$$\beta_i(\boldsymbol{\theta}) \quad = \quad \frac{-c_y(\tilde{y}_i(\boldsymbol{\theta}), \theta_i)}{\bar{D}'(\tilde{\boldsymbol{y}}(\boldsymbol{\theta}))} > 0, \qquad \bar{D}(\boldsymbol{y}) \quad \equiv \quad E_x[D(x)|\boldsymbol{y}], \qquad \text{and} \qquad T \quad \leq \quad \bar{q}.$$

The proof of this proposition, along with the proofs of most others, is relegated to the Appendix.

The regulator bases firm i's marginal penalty, β_i, not only on i's reported type, but also on the reports of all other firms. This is necessary because the environmental damage function is convex, implying complementarities between firms in the effect of their emissions on environmental damage. The penalty has several components. A firm of type θ_i pays a positive *variable penalty*, $\beta_i D(x)$ to the regulator. This firm also makes a *fixed transfer* (independent of x) to the regulator, which has four components and is equal to $T - c(\tilde{y}_i, \theta_i) - \beta_i \bar{D} - \int_{\theta_i}^{\bar{\theta}} c_\theta(\tilde{y}_i(t, \boldsymbol{\theta}_{-i}), t)\, dt$. The first term, T, is identical for all firm types, while the last three are type-specific. The magnitude of T depends on the relative weight that the regulator assigns to appropriating profits versus minimizing deadweight losses. The third term simply ensures that if firms truthfully reveal their type, then the variable penalty has no net effect on their expected profit. The fourth term is the *information rent*, the amount required to

[8]Thus, the constrained efficient solution, $\tilde{\boldsymbol{y}}(\boldsymbol{\theta})$, can be implemented by penalties that are linear in aggregate emissions provided that: (a) marginal penalties are decreasing in abatement cost; and (b) an increase in one firm's abatement cost decreases the regulator's prescribed emissions level (i.e., increases the marginal penalty) for all other firms. Increasing each other firm's penalty as a firm's abatement cost increases is optimal because it ensures that relatively more efficient firms undertake a larger share of the total abatement burden. It is not necessary that the penalty be a function of damage, $D(x)$—it could also be conditioned on emissions, x, for example.

induce i to truthfully report its type. In sum, these payments ensure that in equilibrium each firm participates and truthfully reports its abatement cost. A firm of type $\bar{\theta}$ (the least efficient type) receives an expected profit of $\bar{q} - T$ (cf. zero in the MM model). Hence, the participation constraint implies that $T \leq \bar{q}$. All firms except the least efficient (i.e., types $\theta_i < \bar{\theta}$) earn an expected profit of $\bar{q} - T$ plus the information rent corresponding to their type.

Although we assume a more general objective function, the basic structure of the optimal linear penalties in our model is identical that in MM.[9] This can be explained as follows. First, Theorem 2 in MM shows that, provided the two sufficient conditions identified in Proposition 0 are satisfied, the functional form (though not the particular parameterization) of the optimal linear penalty is not dependent upon assumptions about the regulator's objective function. Thus, the general structure of their penalty is also optimal in our context. Second, the only difference between Equation (7) and MM's solution is that we replace $\bar{q}$ with the choice variable T. This difference reflects the fact that it is not optimal for all types of regulator to appropriate all of firm profits (subject to incentive and participation constraints). In MM's model, the regulator is a revenue-maximizer who places zero value on firms' retained profits. Thus, subject to satisfying incentive and participation constraints, it is optimal for the regulator to fully appropriate all firm profits, implying that $T = \bar{q}$. However, in the presence of deadweight losses from transfers, a benevolent regulator prefers, *ceteris paribus*, not to penalize firms so as to avoid incurring deadweight losses, and thus may set $T < \bar{q}$.[10]

Note that because T is a lump-sum transfer and identical for all firms, it has no effect on firms' choices of emissions or the incentive compatibility of the penalty—it can only

[9]It should be noted, however, that the two sufficient conditions in Proposition 0 may be satisfied only for some subset of the feasible space of objective functions.

[10]For example, suppose $\phi > 0$ and $\lambda > 0$, and there is a single firm of type θ_i for which the optimal emissions level, $\tilde{y}_i(\theta_i)$ is such that $c(\tilde{y}_i, \theta_i) + \int_{\theta}^{\bar{\theta}_i} c_\theta(\tilde{y}_i(t), t)\, dt < \bar{q}$. The cost to the regulator of appropriating (in expectation) $\Delta\pi_i$ from the firm, by choosing T such that $E_x s_i(x, \theta_i) > 0$, is $\phi \Delta E_x \pi_i + \lambda \Delta E_x \pi_i$—the first term is the value the regulator attributes to the firm's lost profits, while the second is the deadweight loss. The expected benefit to the regulator is simply $\Delta E_x \pi$. Hence, if $1 - \phi - \lambda < 0$, then the regulator optimally chooses T to ensure that $E_x s_i(x, \theta_i) \leq 0$. It follows from above that in this case $T < \bar{q}$. If, however, $1 - \phi - \lambda > 0$ and the above condition on $\tilde{y}_i$ is satisfied, then in equilibrium $T > c(\tilde{y}_i, \theta_i) + \int_{\theta}^{\bar{\theta}_i} c_\theta(\tilde{y}_i(t), t)\, dt$.

affect firms' participation decision. Since MM show that penalties of the form given by Equation (7) are incentive compatible in their context when $T = \bar{q}$, this remains true in our setting for any choice of T. Finally, note that while the *form* of the penalty function remains unchanged, the *equilibrium values* of its component parts, such as β_i and T, and hence the equilibrium value of emissions, $\tilde{\boldsymbol{y}}(\boldsymbol{\theta})$, vary with ϕ and λ and other primitives of the problem. This follows straightforwardly from the fact that the regulator's optimal level of emissions depends on the weight it attaches to firms' profits, the nature of firms' cost functions, the magnitude of deadweight losses, and other factors.

2.2. **Fissioning and the Multiplicity Problem in the Unconstrained Model.** To analyze the multiplicity problem, we introduce the notion of *fissioning*. We say that a firm is *fissioned into m firms* if each of these m firms produces $\bar{q}/m$ units of output, and has a cost function $c^m(\cdot,\cdot)$ satisfying

$$c^m(\frac{1}{m}y_i, \theta_i) \quad = \quad \frac{1}{m}c(y_i, \theta_i). \tag{8}$$

That is, if each of the m small firms produce exactly $1/m$ times the original firm's emissions, then the cost of abatement for each fissioned firm is exactly $1/m$ times that incurred by the original firm. It follows that after fissioning

$$c_y^m(\frac{y_i}{m}, \theta_i) \quad = \quad c_y(y_i, \theta_i) \qquad \text{and} \qquad c_\theta^m(\frac{y_i}{m}, \theta_i) \quad = \quad \frac{1}{m}c_\theta(y_i, \theta_i). \tag{9}$$

If n' of the original n firms are fissioned, the set of firms under the regulator's jurisdiction will be augmented from N to $N' = (m-1)n' + n$. The regulator's program will be identical to the one specified above [(2)-(5)] except that N in these expressions will be replaced by N'.

This fissioning procedure provides a pure test of the multiplicity problem in the sense that it captures the effect on the regulator of increasing the number of firms without changing the aggregate scale of the industry. We now show that the multiplicity problem does not arise in the unconstrained model—the aggregate properties of the model, including total emissions, are unaffected by fissioning.

Proposition 1. *In the unconstrained model, when a single firm is fissioned into m firms, all aggregate properties of the equilibrium are unaffected. In particular: (a) the equilibrium marginal penalty,* $\beta_i(\boldsymbol{\theta})$*, for the original large firm is the same as for each of the m fissioned firms; (b) equilibrium aggregate emissions and remediation are unchanged; and (c) the regulator's equilibrium expected payoff is unchanged.*

If the marginal penalty that was applied to the large firm is also applied to the m fissioned firms, it follows from Equation (9) that aggregate emissions remain unchanged. By assumption, aggregate production also remains unchanged, implying that all aggregate properties of the model, and thus the regulator's expected payoff, remain unchanged. This in turn implies that the original penalty for the large firm remains optimal after fissioning.

Note that since the same marginal penalty is applied to fissioned firms, aggregate variable penalty revenue increases by $(m-1)\beta_i(\theta_i)D(x)$, which is offset by an increase in fixed transfers of $(m-1)\beta_i\bar{D}$. However, the cost reimbursement and the information rent for each of the fissioned firms are only $1/m$'th of the size of those for the original firm. Lastly, the second component of the fixed transfer, T, is also $1/m$'th of the size of that for the original firm.

The fissioning process (Equation (8)) clearly imposes a very specific relationship between the abatement cost functions of small and large firms. In particular, we assume a constant returns to scale abatement technology when varying firm size. It might be argued that in reality there are economies of scale in abatement—a large firm can more efficiently abate the same amount as two small firms. Clearly, the presence of such economies will only exacerbate the negative impact of fissioning on the regulator's problem. However, it would be quite inappropriate to introduce this additional factor in the present context, since the exercise of fissioning is strictly a conceptual one: its goal is to isolate the pure effect of increasing firm numbers on the regulator's payoff, holding all other factors constant.

3. The Self-Funding Model

In this section, we impose the additional constraint on the regulator that aggregate penalty revenues must equal remediation costs. We refer to this as the *self-funding* constraint. The restriction is a natural one given that governments are often faced with limited budgets. Indeed, for simplicity we assume here that penalty revenues are the regulator's sole source of funds so that these revenues must be no less than remediation costs.[11] The addition of this constraint is interesting only if it is violated in the unconstrained equilibrium, i.e., if $R(\tilde{\boldsymbol{y}}, \tilde{r}) > \sum_i \eta_i s_i(\tilde{\boldsymbol{y}}, \boldsymbol{\theta}))$. From Equation (10) below, it clearly will be violated if R_r is sufficiently large for all $r < x$, and if $\bar{q}$ is sufficiently small (since $T \leq \bar{q}$).

Constrained equilibria for the self-funding model are denoted by $(\hat{\boldsymbol{y}}, \hat{r})$, in order to distinguish them from the equilibria of the unconstrained model, denoted by $(\tilde{\boldsymbol{y}}, \tilde{r})$.[12] The only difference between the self-funding model and the unconstrained model in the preceding section is that Equation (3) in the latter—which required that remediation levels be chosen optimally given observed aggregate emissions—is replaced by constraint (10) below, which states that for every realization of x, the regulator must choose the remediation level, r, at which remediation costs $R(x, r)$ exactly exhaust net penalty revenues, $\sum_i \eta_i s_i(x, \boldsymbol{\theta})$:[13]

$$\sum_{i=1}^{n} \eta_i \left[T + \beta_i [D(x) - \bar{D}(\hat{\boldsymbol{y}})] - c(\hat{y}_i, \theta_i) - \int_{\theta_i}^{\bar{\theta}} c_\theta(\hat{y}_i(t, \boldsymbol{\theta}_{-i}), t)\, dt \right] = R(x, r). \quad (10)$$

To introduce the effect of self-funding in our model, we assume for the moment that x is degenerately distributed, with point mass on $\sum_i y_i$ (i.e., emissions are deterministically related to abatement efforts). Figures 1(a) and (b) illustrate the possible effects of imposing the self-funding requirement. The functions labeled $r_b(x; b')$ represent iso-budget loci for the regulator, i.e., points (x, r) for which $b(x, r) \equiv \sum_{i=1}^{n} \eta_i s_i(x, \boldsymbol{\theta}) - R(x, r)$ is equal to the constant b'. That is, the function $r_b(x; b')$ assigns to each emissions level the remediation level that the regulator must implement in order to maintain a constant net budget level of b'. The slope of $r_b(x; \cdot)$ is necessarily positive, since both abatement costs and information rents decrease with $x = \sum_i y_i$, causing $s_i(\cdot, \boldsymbol{\theta})$ to *increase* with x. Moreover, iso-budget lines further to the southeast represent higher (less negative) levels of b. The 45° lines denote a iso-damage loci, i.e., sets of (x, r) pairs such that $x - r$, and hence environmental damage, is constant. Iso-damage lines further southeast represent higher damage levels. The functions labeled $r_h(x; h')$ represent iso-*payoff* loci for the regulator, that is, points (x, r)

[11]The nature of the results are unchanged if it is assumed that the regulator has some positive amount of exogenous funds to contribute toward cleanup costs.
[12]Note that "hats" are also used to denote reported type.
[13]Clearly it is always the case that $\eta_i = 1 - \lambda$ in this case since $R(x, r) \geq 0 \implies s_i \geq 0$.

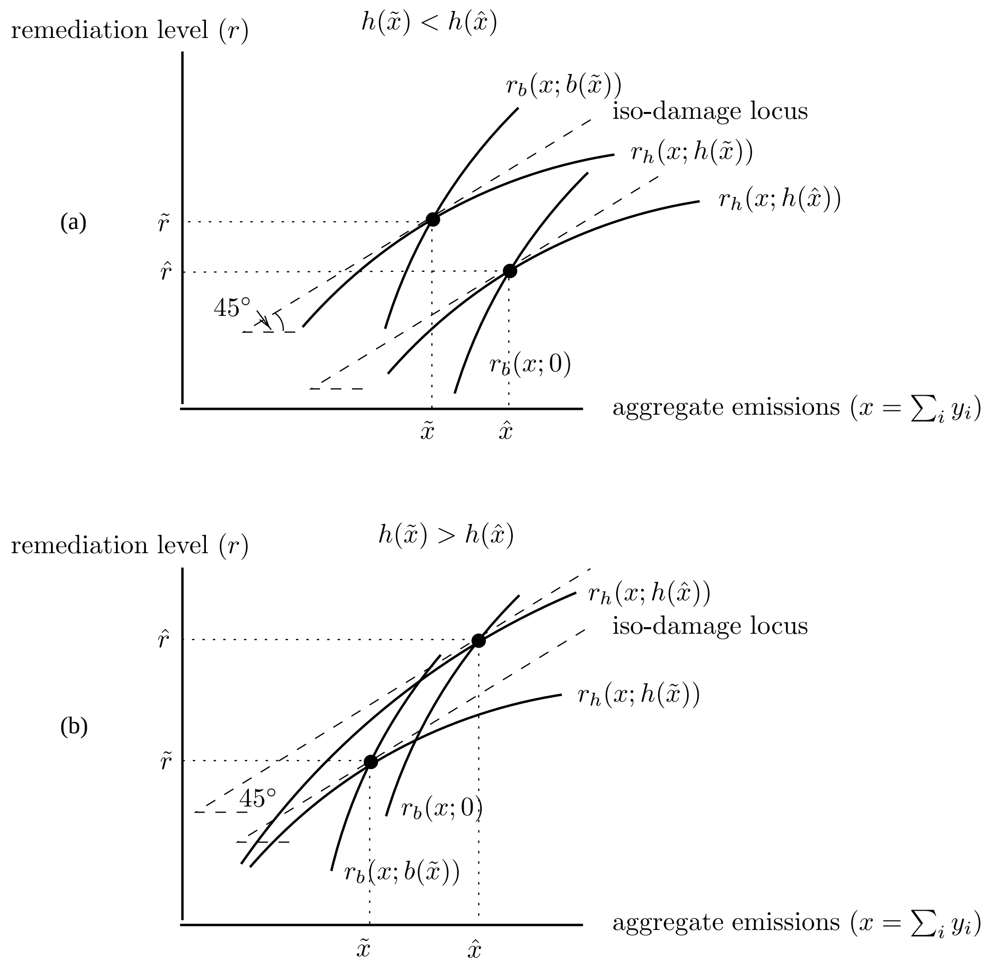

FIGURE 1. Effect of Imposing the Self-Funding Constraint.

such that $h(x, r) \equiv \sum_{i=1}^{n} [\phi \pi_i(x, \boldsymbol{\theta}) + \eta_i s_i(x, \boldsymbol{\theta})] - R(x, r)$ remains constant at h'. Clearly $\lim_{\phi \to 0} r_h(x; k) = r_b(x; k)$ for any k. It is also straightforward to show that $r'_h(x; \cdot) < r'_b(x; \cdot)$. When self-funding is imposed, the solution to the regulator's problem is given by $(\hat{x}, \hat{r})$, the point of tangency between the zero iso-budget locus and the regulator's highest indifference curve. Proposition 2 discusses the effect of imposing the self-funding requirement on the regulator.

Proposition 2. *Assume that x is degenerately distributed with point mass on $\sum_i y_i$, and that remediation costs exceed penalty revenues in the equilibrium of the unconstrained model. For a revenue-maximizing regulator, equilibrium net emissions are higher in the self-funding model than the unconstrained model. More generally, this result holds if ϕ or $c_{\theta y}$ is sufficiently close to zero.*

Proof. Observe that $r_h'(\tilde{x}; h(\tilde{x}, \tilde{r})) = 1$, where the second argument of $r_h(\cdot;\cdot)$ refers to the value of h defining the level set. This is proved formally in the Appendix. Informally, suppose that $r_h'(\tilde{x}; h(\tilde{x}, \tilde{r})) \neq 1$. In this case, $(\tilde{x}, \tilde{r})$ cannot be optimal, since there would exist some (x, r) which lies on $r_h(x; h(\tilde{x}, \tilde{r}))$ but which gives a lower level of net emissions. More generally, using the same reasoning, $r_h(\cdot; h(\tilde{x}, \tilde{r}))$ lies everywhere below the iso-damage locus passing through $(\tilde{x}, \tilde{r})$. Also, by definition, $r_h(\tilde{x}; h(\tilde{x}, \tilde{r})) = r_b(\tilde{x}; b(\tilde{x}, \tilde{r}))$. Now denote the solution to the no-subsidy model by $\hat{x}$. Using the same reasoning as above, $r_h'(\hat{x}; h(\hat{x}, \hat{r})) = 1$, and $r_h(\tilde{x}; h(\hat{x}, \hat{r})) = r_b(\tilde{x}; 0)$. These properties are illustrated in Figures 1(a) and (b).

Suppose $\phi = 0$, in which case $r_h(x; \cdot) \equiv r_b(x; \cdot)$. All the properties ascribed above to $r_h(x; \cdot)$ now apply also to $r_b(x; \cdot)$. In particular, $r_b(x; b(\tilde{x}, \tilde{r}))$ lies everywhere below the iso-damage line through $(\tilde{x}, \tilde{r})$. Since $b_x > 0$ and $b_r < 0$ and the self-funding constraint is assumed to be binding (i.e., $b(\tilde{x}, \tilde{r}) < 0$), it follows that $r_b(x; 0)$ always lies to the south-east of $r_b(x; b(\tilde{x}, \tilde{r}))$. It follows immediately that $\hat{x} - \hat{r} > \tilde{x} - \tilde{r}$. Continuity of the regulator's objective function in ϕ ensures that this result also holds for values of ϕ sufficiently close to zero. It is straightforward to show that for any ϕ, if $c_{\theta y} = 0$, then $r_h'(x; \cdot) = r_b'(x; \cdot)$ for all x—see Equations (A.1) and (A.2) in the Appendix. In particular, $r_h'(x; h(\tilde{x}, \tilde{r})) = r_b'(x; b(\tilde{x}, \tilde{r}))$, implying that $r_h(x; h(\tilde{x}, \tilde{r})) = r_b(x; b(\tilde{x}, \tilde{r}))$. Thus, the arguments above for $\phi = 0$ apply here also. Once again, continuity implies that for $c_{\theta y}$ sufficiently close to zero, $\hat{x} - \hat{r} > \tilde{x} - \tilde{r}$. □

To see that net emissions may increase if ϕ is large, consider the case where $[\eta_i - \phi] > 0$ for all i, which implies that $h_x > 0$, and $h_r < 0$. It follows then that if $h(\tilde{x}, \tilde{r}) < h(\hat{x}, \hat{r})$, then $r_h(x; h(\hat{x}, \hat{r}))$ lies to the south-east of $r_h(x; h(\tilde{x}, \tilde{r}))$. Since the constrained equilibrium lies on the locus $r_h(x; h(\hat{x}, \hat{r}))$, it follows immediately that $\hat{x} - \hat{r} > \tilde{x} - \tilde{r}$. This case is illustrated in Figure 1(a). If, however, $h(\tilde{x}, \tilde{r}) > h(\hat{x}, \hat{r})$, then $r_h(x; h(\hat{x}, \hat{r}))$ lies to the north-west of $r_h(x; h(\tilde{x}, \tilde{r}))$. Note though, that $\hat{x} - \hat{r} < \tilde{x} - \tilde{r}$ is consistent with, but not necessarily implied by, this condition being satisfied. This is shown in Figure 1(b).

To obtain some intuition for the preceding result, consider the case of a revenue-maximizing regulator, who cares only about minimizing the sum of its budget deficit and environmental damage. Clearly the regulator's expected payoff in the solution to the constrained model is bounded above by the expected payoff in the solution to the unconstrained model. Since the budget is larger (less negative) in the former, it follows that environmental damage, and hence net emissions, must also be higher under the self-funding restriction. Similarly, if the information rents earned by firms are little affected by a change in prescribed emissions, imposing a self-funding constraint results in higher net emissions. The rationale is that when information rents are invariant to changes in prescribed emissions, the only effect of increasing y_i is to decrease penalty revenues—profits for firm i are unchanged. Thus, for the purposes of choosing the optimal $\boldsymbol{y}$ to satisfy the self-funding constraint, the regulator will act *as if* she is unconcerned about firm profits. Thus, the same results hold as for when the regulator really does not care about firm profits (i.e., $\phi = 0$). It does not appear possible to rule out the possibility that imposing self-funding restrictions on a benevolent regulator may result in *lower* environmental damage. That is, the least cost means of satisfying the self-funding constraint may involve increasing prescribed emissions, but increasing remediation by even more, causing environmental damage to decrease.

Proposition 2 demonstrates that if a budget deficit prevails and the regulator places sufficiently low weight on the welfare of firms, then requiring self-funding can be harmful to the environment. However, it seems plausible that in reality environmentalists would favor self-funding restrictions. Perhaps paradoxically, our model suggests the opposite conclusion: to the extent that environmentalists assign a *higher* value to a clean environment than do regulators, the model predicts that they would prefer to *increase* the budget deficit. The most plausible explanation of this apparent paradox is that the incomplete information concerns which drive our formal model are considered by environmentalists to be of secondary importance relative to the pressing need that environmentalists perceive for obtaining funding sources that will finance environmental cleanup.

We noted in the preceding section that an increase in the number of polluting firms does not *per se* exacerbate the regulatory problem. That is, if in the standard agency framework we fission a single firm, creating two identical firms that each produce only half the level of emissions at half the cost of the original firm, then the regulator's welfare remains unchanged. We now demonstrate that, regardless of whether the regulator is revenue-maximizing or benevolent, the model remains invariant to fissioning even in the presence of the self-funding constraint provided that aggregate emissions are deterministically produced (i.e., $x \equiv \sum_i y_i$). This invariance does not hold, however, if the production of emissions is random. In this case we observe that, for all objective functions in the class considered, provided the support of f is not too large (i.e., firms have "enough" control over the level of emissions produced), then if the number of firms is sufficiently large, any further increase in the number of firms by fissioning lowers the regulator's equilibrium expected payoff.[14] On the other hand, we also identify sufficient conditions under which fissioning actually *benefits* the regulator. The following proposition formalizes these results.

Proposition 3. *If emissions are deterministic (i.e., $x \equiv \sum_i y_i$), then all aggregate properties of the self-funding model are unchanged by fissioning. If, however, emissions are stochastic, then:*

(a) *there always exists some $\epsilon > 0$ such that if supp $f \subset [\sum_i \hat{y}_i - \epsilon, \sum_i \hat{y}_i + \epsilon]$, then, for sufficiently large initial number of firms, fissioning one firm into any $m \in \mathbb{N}$ firms* decreases *the regulator's equilibrium expected payoff.*

(b) *if*

i) $R_r(x, r_b) + R_x(x, r_b) > \sum_i^n \beta_i D'(x)$, *for all $x \in$ supp f,*

ii) $D''(x - r_b)$ *is sufficiently close to zero, for all $x \in$ supp f, and*

iii) $R_{rr}(x, r_b) < |R_{rx}(x, r_b)|$ *for all $(x, r_b(x))$ such that $x \in$ supp $f(x|\hat{\boldsymbol{y}})$,*

then increasing the number of firms by fissioning one firm into any $m \in \mathbb{N}$ increases *the regulator's equilibrium expected payoff provided that supp $f(x|\hat{\boldsymbol{y}})$ is sufficiently small.*

First consider the case where emissions are deterministic. The only difference between the regulator's problem in this and the unconstrained model is that the equilibrium remediation level is now determined by the constraint $\sum_i \eta_i s_i - R(x, r) = 0$. But it has been shown in the proof of Proposition 1 that when $x \equiv \sum_i y_i$, that $\sum_i \eta_i s_i(x)$, and hence $\sum_i \eta_i s_i - R(x, r)$, is invariant with respect to the fission operation. Since the other terms in the regulator's objective function are also invariant to fissioning, it follows that fissioning has no effect on the regulator's expected payoff.

Now suppose that emissions are stochastic. Since penalty revenues are, by assumption, identically equal to remediation expenditures, and expected firm profits invariant to fissioning, fission benefits the regulator if and only if expected environmental damage decreases.

[14]The restriction on the support of $f(x|\cdot)$ ensures that self-funding is feasible for each possible realization of x, given n. This restriction is necessary because for each x there exists some n above which self-funding cannot be implemented because $\sum_i \eta_i s_i(x, n, \cdot) < 0$.

Part (a) of Proposition 3 says that if the initial population of firms is sufficiently large, and a technical condition is satisfied, further fissioning of a firm increases expected environmental damage. Part (b) states that if initially there are only a few firms, then under certain conditions fissioning will *decrease* expected environmental damage.

The explanation for these results is as follows. First note that the *only* effect of fissioning is to increase (decrease) the level of remediation at emissions levels satisfying $D(x) > \bar{D}(\hat{\boldsymbol{y}})$ ($D(x) < \bar{D}(\hat{\boldsymbol{y}})$). This is because for a given level of emissions, x, the only effect of an increase of Δn in the number of firms of type i is to change aggregate penalty revenues by the amount $\Delta n \beta_i [D(x) - \bar{D}(\hat{\boldsymbol{y}})]$—by construction, the remaining terms on the left hand side of Equation (10) are unaffected by the fission. To maintain self-funding, remediation must change in such a way that the change in remediation costs exactly offset the above change in penalty revenues. The critical question is: does the decrease in remediation when $x = x' - \epsilon$ exceed the increase in remediation when $x = x' + \epsilon$, where x' satisfies $D(x') = \bar{D}(\hat{\boldsymbol{y}})$. If this is the case, then *expected* remediation must decrease with fission.

The answer to the above question is indeterminate if the initial number of firms is small, but if there are sufficiently many firms, then the decrease in remediation at $x' - \epsilon$ dominates the increase in remediation at $x + \epsilon$. The latter is true because the required increase in remediation following fission at high emissions levels is decreasing in the number of firms, whereas the required decrease in remediation at low emissions level is increasing—this follows from the convexity of $R(r, \cdot)$. Specifically, as r_b increases with n for $x > x'$, so also does R_r. This implies that smaller increases in r_b are required to achieve self-funding as the number of firms increases. Similarly, larger decreases in r_b are required as n increases when $x < x'$. Thus for sufficiently large n, the net effect of increasing firm numbers through fission is to decrease the expected level of remediation.

To complete the argument, we need to relate changes in expected remediation level to changes in expected environmental damage. For large values of n, fission also increases the variance of expected remediation. Since the damage function is convex, the increase in variance reinforces the effect of the decrease in expected remediation—expected environmental damage must increase. If n is small, then expected environmental damage may increase or decrease due to fissioning. We show in part (b) of Proposition 3 that it will *decrease* if: (i) remediation costs are suffiently more sensitive than the level of emissions to an increase in the level of remediation and if the marginal cost of abatement is sufficiently small;[15] (ii) the relative magnitudes of R_{rr} and R_{rx} are such that expected remediation *increases* with fission; and (iii) the damage function D is sufficiently close to linear that the negative effect of increased variance is small relative to the positive effect of increased expected remediation. Although near-linearity of the damage function is a rather restrictive condition, there is no reason to expect that the conditions on the remediation and abatement technologies will not hold in practice.

4. The No-Subsidy Model

Recall that the optimal incentive scheme in our model consists of a lump-sum transfer and a variable penalty. In the solution to the unconstrained model, it may happen that the transfer is negative, i.e., the regulator is required to make a lump-sum payment to some firm. For many reasons, it seems highly implausible that such contracts will be implementable in practice.[16] Accordingly, in this section, we investigate the implications of prohibiting the regulator from making positive lump-sum transfers to any firm.

In the solution $(\tilde{\boldsymbol{y}}, \tilde{r})$ to the unconstrained model, the regulator receives a fixed transfer from a firm of type θ_i equal to the following amount:

$$T \quad - \quad \left\{ \beta_i \bar{D}(\tilde{\boldsymbol{y}}) + c(\tilde{y}_i, \theta_i) + \int_{\theta_i}^{\bar{\theta}} c_\theta(\tilde{y}_i(t, \boldsymbol{\theta}_{-i}), t)\, dt \right\} \tag{11}$$

If expression (11) is negative, the regulator must make a positive payment to firm i. This condition is more likely to occur when the cost of abatement is higher and the greater is the dispersion of firm types. To exclude this possibility, we add to the regulator's unconstrained

[15]This interpretation of condition (i) of Proposition 3(b) is due to the observation that for small supp f, D approaches $\bar{D}$, and from Proposition 0 we know that $\bar{D}'\beta_i = -c_y$.

[16] Although firms do receive various environmental incentives and credits from regulators, they are typically conditioned on performance rather than firm characteristics as in our model—deposit refund schemes are an example Swierzbinski (1994). Contracts involving fixed payments to firms are likely to encourage inefficiency due to rent-seeking activity. Such contracts might also lack credibility unless effected prior to the observation of actual emissions—otherwise the regulator has an ex post incentive to reneg on promised payments. Budgetary constraints on the regulator could thus limit the size of the fixed payments possible.

maximization problem, as defined by Equations (2)–(5), the following *no-subsidy* constraint:

$$T \quad \geq \quad \beta_i \bar{D}(\hat{\boldsymbol{y}}) + c(\hat{y}_i, \theta_i) + \int_{\theta_i}^{\bar{\theta}} c_\theta(\hat{y}_i(t, \boldsymbol{\theta}_{-i}), t)\, dt, \ \ \forall\, i. \tag{12}$$

This constraint states that the gross fixed amount owed to firm i by the regulator—the right hand side of (12)—cannot exceed the gross amount owed by the firm to the regulator (the left hand side). With this restriction, our penalties look more familiar: a two part tariff with a nonnegative fixed cost of participation and a variable penalty that is linear in observed aggregate emissions.

For the remainder of this section we will assume that in the solution to the unconstrained model, this condition is violated for at least some type θ_i. Since $T \leq \bar{q}$, a sufficient condition for this property to hold is that $\bar{q}$ be sufficiently small. In Proposition 4 we show that for both benevolent and revenue-maximizing (and intermediate) types of regulator, the effect of imposing the no-subsidy constraint is to increase the levels of both gross emissions and remediation. The effect on *net* emissions, however, is indeterminate, but dependent on an easily interpretable condition.

Proposition 4. *Gross emissions and remediation are higher in the equilibrium of the no-subsidy model than the unconstrained model. Net emissions are higher iff* $R_{rr} > |R_{ry}|$.

The generality of Proposition 4 is at first sight surprising. Since $\bar{D}(\cdot)$ is an *increasing* function of $\sum_i y_i$, it is not obvious that an increase in gross emissions will reduce the right hand side of inequality (12). It turns out, however, that the first order conditions for the regulator's problem impose a restriction on the term $\beta_i \bar{D} + c(\cdot, \theta_i)$—the increase in damage as emissions increase is more than offset by the decrease in the marginal penalty and the abatement cost. The magnitude of the information rent also declines as emissions increase, since at lower abatement levels, abatement costs are less sensitive to firm type. Thus, imposing the no-subsidy constraint must increase gross emissions.

In order to simplify the exposition, once again we assume that x is degenerately distributed, with point mass on $\sum_i y_i$. Figure 2 illustrates that net emissions can either increase or decrease. Again we use "hats" to denote constrained equilibrium levels. As in section 2, the

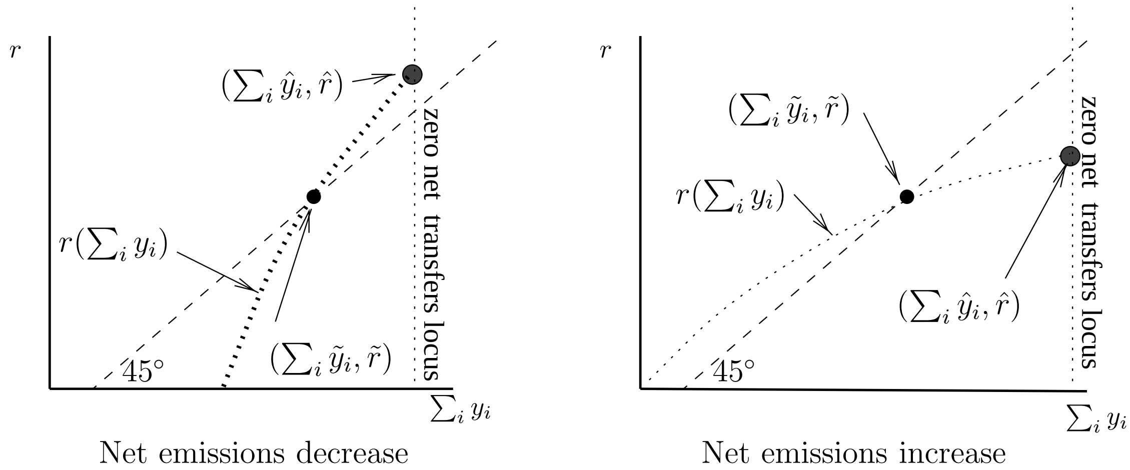

FIGURE 2. Effect of Imposing the No-Subsidy Constraint

function $r(\cdot)$ is defined by Equation (3). Since the no-subsidy constraint is independent of the remediation level, the zero-net-transfer locus is a vertical line in $(\sum_i y_i, r)$ space which, if the constraint is binding, lies to the right of the unconstrained solution $(\sum_i \tilde{y}_i, \tilde{r})$. Thus, if the constraint is binding, the solution to the no-subsidy model $(\sum_i \hat{y}_i, \hat{r})$, is a point on this vertical line. Graphically, it will be clear from figure 2 that whether the net emissions are higher or lower in the no-subsidy model depends entirely on whether the slope of $r(\cdot)$ at $(\sum_i \tilde{y}_i, \tilde{r})$ is less than or greater than unity. That is, net emissions decrease if and only if the marginal cost of additional remediation is more responsive to changes in the level of emissions than to changes in the level of remediation.

The intuition is that if the cost of additional remediation falls sufficiently rapidly with emissions, then the increase in prescribed aggregate emissions when the no-subsidy constraint is imposed will be associated with an even larger increase in remediation, causing net emissions to decline. Thus, our analysis shows that if moral hazard and adverse selection are important factors in pollution regulation, then imposing the no-subsidy constraint on regulators may or may not benefit the environment, depending on the particular structure of the remediation cost function. The structure of the damage and abatement cost functions and the particular choice of weight ϕ by the regulator turn out to have no bearing on this issue.

Finally, we show that in the no-subsidy model, fissioning always decreases the regulator's payoff. This result holds for the entire class of regulator objective functions considered, and the rate of this decrease is independent of the stochastic nature of emissions. Thus, the multiplicity problem is inescapable once the no-subsidy constraint is imposed.

Proposition 5. *Fissioning a firm for which the no-subsidy constraint is binding results in a reduction in the regulator's equilibrium payoff, regardless of the stochastic process by which emissions are determined. The effect on the level of net emissions depends on the properties of the remediation technology: if $\tilde{r}'(\cdot) < 1$ on the interval $(\sum_i \tilde{y}_i, \infty]$, then net emissions increase; if $\tilde{r}'(\cdot) > 1$ on this interval, net emissions decrease.*

To see that an increase in the number of firms increases gross emissions in equilibrium, suppose that there is a single firm, of type θ_i, and the constraint (12) is binding for this firm, i.e.,

$$T \quad = \quad \beta_i \bar{D}(\hat{\boldsymbol{y}}) + c(\hat{y}_i, \theta_i) + \int_{\theta_i}^{\bar{\theta}} c_\theta(\hat{y}_i(t, \boldsymbol{\theta}_{-i}), t)\, dt, \tag{13}$$

If the same menu of penalties is offered after fissioning the original firm, then aggregate output is unchanged in equilibrium. As explained in section 2.2, for each of the m fissioned firms, all terms above except $\beta_i \bar{D}(\hat{\boldsymbol{y}})$ are reduced to $1/m$'th of their size prior to fissioning. Fissioning thus causes the constraint to be violated, and an increase in gross emissions, $\boldsymbol{y}$, is required to restore equality. An interesting possibility in this case is that environmental damage may actually *decrease* as a result of the fission, because remediation may increase by more than the original increase in emissions.[17] Hence, although the regulator is adversely affected by fissioning, environmental lobbies may actually benefit.

The regulator's expected payoff decreases following fissioning because, as shown above, fissioning makes the no-subsidy constraint bind more tightly. That is, the set of feasible choices available to the regulator is strictly smaller following fissioning. If prior to fissioning the no-subsidy constraint is binding in equilibrium, then after fissioning the original solution is no longer implementable. As discussed above, the regulator responds to fissioning by

[17] This would be less likely if environmental damage was modeled to depend, more realistically, on both gross and net emissions, since in general environmental damage will accumulate between the time that pollution is generated and remediated. In this case, a decrease in net emissions would not necessarily imply a decrease in environmental damage.

increasing prescribed gross emissions, which requires that the marginal penalty, β_i, decrease. The change in β_i has no *direct* effect on the regulator's expected payoff. The effect is indirect, through the impact that the increase in y_i, and associated change in the remediation level, has on the regulator's payoff.

5. The Value of Firm-Specific Emissions Data

While it seems intuitive that the regulator should benefit from knowledge of firm-specific emissions, the received literature concludes differently. MM show that in their model the regulator's equilibrium utility is the same regardless of whether individual contributions or only aggregate output is observed. Once additional constraints are imposed on the regulator, however, the value of disaggregated information becomes apparent. In this section we provide an heuristic discussion of this issue. Formal derivations are omitted since they involve relatively straightforward applications of results obtained in preceding sections. We show that firm-specific emissions data are valuable to the regulator precisely when fissioning would reduce its payoff.

Our results can be summarized as follows. In the self-funding model, firm-specific emissions data has zero value if aggregate emissions are deterministically related to individual emissions (i.e., if $x(\boldsymbol{y}) \equiv \sum_i y_i$). Now suppose that x is stochastically determined. In some instances, the value of firm-specific data may actually be *negative*, i.e., if it were available, the regulator would prefer to suppress it! Finally, firm-specific data will *always* be valuable when the no-subsidy constraint is binding.

Formally, assume that the regulator observes each firm i's individual emissions, $x_i = y_i + \epsilon_i$, where ϵ_i represents the stochastic component of i's emissions, and let $x = \sum_i x_i$. To maintain conformity with the preceding sections, assume that the n distributions of ϵ_i give rise to the same conditional distribution of aggregate emissions, $x = \sum_i (y_i + \epsilon_i)$ as the distribution assumed in previous sections, i.e., $f(x, \boldsymbol{y})$. Define $S_i(x_i, x|\boldsymbol{y}) \in \mathbb{R}^n$ by $S_i(x_i, x|\boldsymbol{y}) = \frac{x_i D(x)}{x}$ and note that $\sum_i S_i = D(x)$. Thus, the vector $S = (S_1, \ldots, S_n)$ can be interpreted as allocating to each firm a responsibility share for environmental damage, based on its relative contribution x_i/x to aggregate emissions. We now compare the original penalty, $\boldsymbol{s}$, defined in expression (7) above, to an alternative, $\boldsymbol{\sigma} = (\sigma_i)_{i=1}^n$, based on S_i, which is defined as follows:

$$\begin{aligned}\sigma_i(x_i, x, \boldsymbol{\theta}) \quad &= \quad \alpha_i(\boldsymbol{\theta})[S_i(x_i, x|\boldsymbol{y}(\boldsymbol{\theta})) - \bar{S}_i(\tilde{\boldsymbol{y}}(\boldsymbol{\theta}))] \quad + \quad T \quad - \quad c(\tilde{y}_i(\boldsymbol{\theta}), \theta_i) \\ & \qquad\qquad - \quad \int_{\theta_i}^{\bar{\theta}} c_\theta(\tilde{y}_i(t, \boldsymbol{\theta}_{-i}), t)\, dt, \end{aligned} \tag{14}$$

where,

$$\alpha_i(\boldsymbol{\theta}) \quad = \quad \frac{-c_y(\tilde{y}_i(\boldsymbol{\theta}), \theta_i)}{\frac{\partial}{\partial y_i}\bar{S}_i(\boldsymbol{y}(\tilde{\boldsymbol{\theta}}))} > 0, \qquad \text{and} \qquad \bar{S}_i(\boldsymbol{y}(\boldsymbol{\theta})) \equiv E_x S_i(x_i, x|\tilde{\boldsymbol{y}}(\boldsymbol{\theta})).$$

It is straightforward to verify that $\boldsymbol{\sigma}$ is an optimal contract whenever $\boldsymbol{s}$ is optimal and hence, that the two schemes prescribe the same vector of emissions $\tilde{\boldsymbol{y}}$ under these conditions. Moreover, in the unconstrained model, the schemes $\boldsymbol{s}$ and $\boldsymbol{\sigma}$ yield the regulator the same *expected* payoff.[18] We now compare these two penalty schemes in the self-funding and no-subsidy models, imposing the simplifying assumption that $D(x)$ is approximately linear, i.e., $D(x) \approx \gamma x$, for some $\gamma \in \mathbb{R}_+$. In this case, $S_i(x_i, x|\tilde{\boldsymbol{y}}(\boldsymbol{\theta})) \approx \gamma(y_i + \epsilon_i)$. Also, $\frac{\partial \bar{S}_i(\tilde{\boldsymbol{y}}(\boldsymbol{\theta}))}{\partial y_i} \approx \frac{\partial \bar{D}(\tilde{\boldsymbol{y}}(\boldsymbol{\theta}))}{\partial y_i} \approx \gamma$, so that $\alpha_i(\boldsymbol{\theta}) \approx \beta_i(\boldsymbol{\theta}) \approx \frac{-c_y(\tilde{y}_i(\boldsymbol{\theta}), \theta_i)}{\gamma}$. Moreover, we assume that θ_i (and hence α_i) is the same for each i.

First, compare the penalty schemes $\boldsymbol{s}$ and $\boldsymbol{\sigma}$ in the self-funding model. If emissions are deterministic ($\epsilon_i \equiv 0$, $\forall\, i$), then clearly the regulator gains nothing by shifting to penalties based on $\boldsymbol{\sigma}$, since both $[S_i(x_i, x|\boldsymbol{y}(\boldsymbol{\theta})) - \bar{S}_i(\tilde{\boldsymbol{y}}(\boldsymbol{\theta}))]$ in (14) and $[D(x|\boldsymbol{y}(\boldsymbol{\theta})) - \bar{D}(\boldsymbol{y}(\boldsymbol{\theta}))]$ in (7) are identically equal to zero in equilibrium, and since the penalties are otherwise identical. If emissions are stochastic, however, then either the $\boldsymbol{\sigma}$-penalty or the $\boldsymbol{s}$-penalty may dominate. To see why, note that the comparison between penalties based on individual and aggregate emissions is similar to the comparison between an industry before and after fission. In both cases, the *only* thing that changes is the sensitivity of aggregate penalty revenues to x. Specifically, under the assumptions above, if penalties are linear in individual responsibility shares then an increase in aggregate emissions of dx generates an increase in penalty revenues of approximately $\alpha_i D'(x)dx$. When penalties are (close to) linear in aggregate damage,

[18]The variance of the regulator's payoff is greater under $\boldsymbol{s}$ than $\boldsymbol{\sigma}$, but since the regulator is risk neutral, this is immaterial.

the increase in revenues generated by dx is n times this amount, while expenditure on remediation must likewise increase in order to satisfy self-funding. This is, in essence, the same as comparing stochastic emissions before and after fission in section 3. Just as we could not conclude that fissioning always reduces the regulator's payoff under the self-funding constraint (the initial number of firms was required to be sufficiently large), so too here we cannot conclude that the regulator is always better off being able to observe individual emissions data. It is possible for the regulator to be *worse* off from using firm-specific data due to it imposing, via self-funding, a regime of remediation that gives the regulator a lower expected payoff (due to higher expected damage) than if aggregate emissions were used.

Finally, we compare the $\boldsymbol{\sigma}$ and $\boldsymbol{s}$ penalties in the no-subsidy model, where the comparison is more transparent. Recall that under the $\boldsymbol{s}$-penalty the no-subsidy constraint is given by expression (12). Under the $\boldsymbol{\sigma}$-penalty, the corresponding constraint is

$$T \quad \geq \quad \alpha_i \bar{S}_i(\boldsymbol{y}) + c(y_i, \theta_i) + \int_{\theta_i}^{\bar{\theta}} c_\theta(y_i(t, \boldsymbol{\theta}_{-i}), t)\, dt, \;\; \forall\, i. \tag{15}$$

Thus, under our simplifying assumptions, $\alpha_i \approx \beta_i$. Moreover, $\sum_i \bar{S}_i(\boldsymbol{y}) = \bar{D}(\boldsymbol{y})$. It follows immediately that constraint (12) is more stringent than (15). In conclusion, whenever there are multiple firms in the no-subsidy model and the constraint is binding on at least some θ_i, the regulator's utility under the $\boldsymbol{\sigma}$-penalty exceeds the utility under the $\boldsymbol{s}$-penalty.

Appendix A.

Proposition 0: We need to show that Equation (7) maximizes the regulator's expected payoff, subject to the incentive and participation constraints. McAfee and McMillan (1991) prove in Theorem 2 the incentive compatibility of Equation (7) in their setting, provided that the two sufficient conditions in Proposition 0 of our paper are satisfied. Since we assume $\bar{q} > 0$ while MM implicitly assume $\bar{q} = 0$, our firms' profit functions differ from MM's only by a constant. Also, since in general $T \neq 0$ in our model while $T = 0$ in MM, the optimal penalty function in the two models differs only by the constant T, which is identical for each firm. Incentive compatibility requires simply that firms prefer not to misrepresent their type. It follows that penalty functions that differ from an incentive compatible penalty function only by a constant, where this constant is identical for all firms, are also incentive compatible. Moreover, this remains true if the firms' profit functions are translated by some additive constant—lump-sum taxes have no effect on profit maximizing behavior. It follows then that Equation (7) is incentive compatible in the context of our model.

The participation constraint implies that any optimal contract satisfies $T \leq \bar{q}$. If not, then $E_x\pi(x, \bar{\theta}) < 0$, implying that firms of type $\bar{\theta}$ would prefer not to participate in the regulatory scheme (thereby obtaining $\pi(\bar{\theta}) = 0$).

Finally, by an appropriate choice of T, the regulator ensures that her (incentive and participation constrained) expected payoff is maximized by implementing Equation (7). For example, if $\phi = 0$, as in MM, the regulator optimally chooses $T = \bar{q}$, appropriating all firm profits subject to satisfying Equations (4) and (5). This completes the proof. □

Proposition 1: It is first shown that (b) and (c) are true assuming that (a) is also true. It is then shown that it follows then that (a) must also be true. We simplify here by assuming that the original single firm is the only firm, so that $y_i \equiv \boldsymbol{y}$. The emissions of each of the m smaller firms obtained from subdividing a single firm of type θ_i is denoted as y_j, $j = \{1, \ldots, m\}$. Aggregate prescribed emissions of the m firms in equilibrium is denoted $\sum_{j=1}^{m} \tilde{\tilde{y}}_j$, while that of the single large firm is denoted $\tilde{y}_i$.

Suppose the regulator offers the equilibrium menu of penalties derived in the original setting, $\boldsymbol{\beta}$, to the population of m small firms. Since by construction we have $c_y(y_i, \theta_i) = c_y^m(\frac{1}{m}y_i, \theta_i)$, provided that aggregate prescribed emissions of the m small firms is the same in equilibrium to that of the equivalent large firm (i.e., $\tilde{y}_i = \sum_j \tilde{\tilde{y}}_j$), then β elicits a level of intended emissions from each of the m firms of $\tilde{\tilde{y}}_j = \frac{1}{m}\tilde{y}_i$ in equilibrium. The requirement that $\tilde{y}_i = \sum_j \tilde{\tilde{y}}_j$) ensures that the value of D in the denominator of the expression for β_i does not change. But it follows immediately that this requirement is satisfied since $\sum_{j=1}^{m} \tilde{\tilde{y}}_j = m\frac{1}{m}\tilde{y}_i = \tilde{y}_i$. Thus, aggregate emissions are invariant to the fissioning if the same menu of penalties is offered.

Now consider the effect of the fissioning on the equilibrium level of remediation. Since the optimal remediation level is a function only of x, and the invariance of $\sum_j \tilde{\tilde{y}}_j$ to m implies that x is also invariant to m, it follows that the remediation level is also invariant to choice of m. Formally, since $\beta_i = \beta_j$ implies $\sum_j \tilde{\tilde{y}}_j = \tilde{y}_i$, it follows that

$$R_r(x(\sum_j \tilde{\tilde{y}}_j), \tilde{\tilde{r}}) - D''(x(\sum_j \tilde{\tilde{y}}_j) - \tilde{\tilde{r}}) = R_r(\tilde{y}_i, \tilde{r}) - D''(\tilde{y}_i - \tilde{r}) = 0$$

$$\Rightarrow \quad \tilde{\tilde{r}} = \tilde{r}.$$

This completes the proof of (b).

Using the results above, it remains only to show that $\sum_j[\phi\pi_j + \eta_j s_j]$ is unaffected by fissioning. Since $Ex = \sum_{j=1}^m \tilde{\tilde{y}}_j = \tilde{y}_i$ in equilibrium, the variable and fixed payments associated with $\boldsymbol{\beta}$ cancel out in equilibrium and aggregate penalties are equal to

$$\begin{aligned}\sum_{j=1}^m s_j(\tilde{\tilde{y}}_j, \theta_i) &= m\frac{T}{m} - \sum_{j=1}^m c^m(\tilde{\tilde{y}}_j, \theta_i) - \sum_{j=1}^m \int_{\theta}^{\bar{\theta}_i} c_\theta^m(\tilde{\tilde{y}}_j(t), t)\, dt \\ &= T - mc^m(\tilde{\tilde{y}}_j, \theta_i) - m\int_{\theta_i}^{\bar{\theta}} c_\theta^m(\tilde{\tilde{y}}_j(t), t)\, dt \\ &= T - mc(\frac{1}{m}\tilde{y}_i, \theta_i) - \int_{\theta_i}^{\bar{\theta}} mc_\theta^m(\frac{1}{m}\tilde{y}_i(t), t)\, dt \\ &= T - c(\tilde{y}_i, \theta_i) - \int_{\theta_i}^{\bar{\theta}} c_\theta(\tilde{y}_i(t), t)\, dt.\end{aligned}$$

Hence, expected penalty revenues are also invariant to the fissioning. By definition, total operating costs, $\sum_j c_j(y_j, \theta_i)$, and total revenues, $\sum_j \bar{q}_j$ are invariant to fissioning, so it follows that net firm profits are also invariant. Thus, $\sum_j[\phi\pi_j + \eta_j s_j]$ is unaffected by fissioning, and it follows immediately that (c) is true.

Finally, we have shown that implementing the menu $\boldsymbol{\beta}$ for the m smaller firms leaves the regulator's expected payoff unchanged. Suppose there exists some other menu $\boldsymbol{\beta}'$ such that the regulator does even better offering this to the m small firms. Then, by previous arguments, this same menu could be offered to the original large firm and it would implement the same $\sum_i y_i$ and r as the m small firms in aggregate. Moreover, the regulator's expected utility must also increase by offering $\boldsymbol{\beta}'$ to the single large firm. But this contradicts the optimality of $\boldsymbol{\beta}$. Hence (a) is true also. □

Proposition 2: The slope of the locus $r_h(x; \cdot)$ is given by

$$r_h'(x; \cdot) \;=\; \sum_{i=1}^n \left[\eta_i c_y(y_i, \boldsymbol{\theta}) + [\eta_i - \phi]\left(-\int_{\theta_i}^{\bar{\theta}} c_{\theta y}(y_i(t), t)\, dt\right) - R_y\right] R_r^{-1}. \quad \text{(A.1)}$$

The slope of the locus $r_b(x; \cdot)$ is given by

$$r_b'(x; \cdot) \;=\; \sum_{i=1}^n \left[-(\eta_i \left(c_y(y_i, \boldsymbol{\theta}) + \int_{\theta_i}^{\bar{\theta}} c_{\theta y}(y_i(t), t)\, dt\right) - R_y\right] R_r^{-1}. \quad \text{(A.2)}$$

Note that, by assumption, $x \equiv \sum_i y_i$ here. Also, a change in x is assumed to be optimally allocated across firm types. Since $c_{\theta y} < 0$, it follows immediately that $r_h'(x; \cdot) < r_b'(x; \cdot)$.

The equation characterizing the solution to the self-funding model is

$$\sum_{i=1}^n \left[\eta_i c_y + [\eta_i - \phi]\left(T_y + \int_{\theta_i}^{\bar{\theta}} c_\theta(y_i(t), t)\, dt\right)\right] + R_y - R_r r'(x) - D'[1 - r'(x)] = 0,$$

which by definition is satisfied at $\tilde{x}$. Combining Equation (A.1) and the expression above gives $r_h'(\tilde{x}) = 1$. □

Proposition 3: To prove the case for non-stochastic emissions, it is sufficient to show that, holding β fixed, that the self-funding constraint is unaffected by fissioning. Since the self-funding model differs from the unconstrained model only by the addition of the self-funding constraint, it follows then from Proposition 1 that all aggregate properties of the self-funding model are unaffected by fissioning.

Without loss of generality, assume initially only one firm of type θ_i. Equation (10) becomes

$$T - c(\hat{y}_i, \theta_i) - \int_{\theta_i}^{\bar{\theta}} c_\theta(\hat{y}_i(t, \boldsymbol{\theta}_i), t)\, dt \quad = \quad R(\hat{y}_i, r).$$

Fissioning into m smaller firms and imposing the same penalty as prior to fissioning gives, by Equation (8),

$$\begin{aligned} m\left[\frac{T}{m} - c^m(\frac{\hat{y}_i}{m}, \theta_i) - \int_{\theta_i}^{\bar{\theta}} c_\theta^m(\frac{\hat{y}_i}{m}(t), t)\, dt\right] &= R(\sum_{i=1}^{m} \frac{\hat{y}_i}{m}, r) \\ \Leftrightarrow \quad m\left[\frac{T}{m} - \frac{1}{m}c(\hat{y}_i, \theta_i) - \frac{1}{m}\int_{\theta_i}^{\bar{\theta}} c_\theta(\hat{y}_i(t), t)\, dt\right] &= R(\hat{y}_i, r) \\ \Leftrightarrow \quad T - c(\hat{y}_i, \theta_i) - \int_{\theta_i}^{\bar{\theta}} c_\theta(\hat{y}_i(t, \boldsymbol{\theta}_i), t)\, dt &= R(\hat{y}_i, r). \end{aligned}$$

Thus, the self-funding constraint is independent of fissioning.
(a) Denote by n the initial number of firms. Under the self-funding restriction, the regulator chooses r_b such that, given $\hat{\boldsymbol{y}}$ and n, for each x, $\sum_i \eta_i s_i(x) = R(x, r_b)$. It follows that, for given $\boldsymbol{y}$, the effect on the regulator's payoff of a change in n is fully captured by the change in expected firm profits and environmental damage, $\sum_i \phi\pi_i(x, \boldsymbol{\theta}) - E_x\, D(x - r_b)$. However, as argued in the proof of Proposition 1, expected firm profits are unaffected by fissioning. By the envelope theorem, it is necessary and sufficient to show that, evaluated at $\hat{\boldsymbol{y}}$, expected environmental damage decreases with n for the regulator's expected equilibrium payoff to increase with n.

Expected damage can be written as

$$\begin{aligned} E_{x \in \text{supp } f} D(x - r_b(x, n, \hat{\boldsymbol{y}})) &= \int_{\text{supp } f(x|\hat{\boldsymbol{y}})} D(x - r_b(x, n, \hat{\boldsymbol{y}}))\, dF(x, \hat{\boldsymbol{y}}) \\ \therefore \quad \frac{d}{dn} E_{x \in \text{supp } f} D(x - r_b(x, n, \hat{\boldsymbol{y}})) &= -\int_{\text{supp } f(x|\hat{\boldsymbol{y}})} D'(\cdot)\frac{d}{dn} r_b(x, n, \hat{\boldsymbol{y}})\, dF(x, \hat{\boldsymbol{y}}). \end{aligned}$$

Let x' be defined by $D(x') = \bar{D}(\hat{\boldsymbol{y}})$. Clearly $x' > \sum_i \hat{y}_i$. Without loss of generality assume that there is only a single type of agent, θ_i, so that Equation (10) can be rewritten as $R = nT + n\{\beta_i[D(x) - \bar{D}] - c(y_i, \cdot) - \int_{\theta_i}^{\bar{\theta}} c_\theta(t, \cdot)\, dt\}$. Denoting the support of $f(x|\boldsymbol{y})$ as $[a, b]$, the previous expression can be rewritten as

$$\frac{d}{dn} E_x D(x - r_b(x, n, \hat{\boldsymbol{y}})) = -\int_a^{x'} D'(x - r_b)\frac{dr_b}{dn}\, dF - \int_{x'}^{b} D'(x - r_b)\frac{dr_b}{dn}\, dF. \text{ (A.3)}$$

The function $r_b(x, n, \hat{\boldsymbol{y}})$ has the following properties:

$$\frac{d}{dn} r_b(x, n, \hat{\boldsymbol{y}}) = \frac{1}{R_r} \beta_i [D(x) - \bar{D}(\hat{\boldsymbol{y}})] \lesseqqgtr 0, \ \ \forall\, x \lesseqqgtr x', \tag{A.4}$$

$$\frac{d^2}{dn^2} r_b(x, n, \hat{\boldsymbol{y}}) = \frac{-1}{(R_r)^2} \beta_i [D(x) - \bar{D}(\hat{\boldsymbol{y}})] R_{rr} \frac{dr_b}{dn} \leq 0, \tag{A.5}$$

and

$$\lim_{n \to \infty} \frac{dr_b}{dn} = 0, \ \ \forall\, x \in (x', b]. \tag{A.6}$$

Equation (A.6) follows from the fact that for given x, as $n \to \infty$, $r_b \to x$, implying that $R_r \to \infty$. The result follows then from Equation (A.4). The fact that r_b asymptotes to x and not some $\bar{r} < x$ can easily be verified by showing that for any r arbitrarily close to given x, there exists some value of n such that the penalty revenues associated with n and x is exactly equal to the associated remediation costs—which is the definition of r_b.

Note that for each $x \in [a, x')$ there exists some $N_x < \infty$ such that for any $n > N_x$, $\sum_i \eta_i s_i(x, n, \boldsymbol{\theta}) < 0$, in which case self-funding cannot be satisfied since both r_b and $R(x, r)$ are defined only on $\mathbb{R}_+$. Thus, Equations (A.4)–(A.5) show that while for each $x \in (x', b]$, $\frac{dr_b}{dn}$ converges continuously to zero as n increases, for each $x \in [a, x')$, $|\frac{dr_b}{dn}|$ increases continuously with n until it reaches some finite maximum value—for higher values of n, self-funding cannot be satisfied.

From Equations (A.4)–(A.6), for any $\delta > 0$, there exists some smallest integer $n_x \in \mathbb{N}_+$ for each $x \in (x', b]$ such that $\frac{d}{dn} r_b(x, n_x, \cdot) < \delta$, $\forall\, n > n_x$. Let $\bar{n} \equiv \sup\{n_x : x \in (x', b]\}$ and $k \equiv \sup\{D'(x - r_b(x, \bar{n}, \cdot)) : x \in (x', b]\}$. Clearly $0 < k < \infty$ and $\frac{d}{dn} r_b(x, \bar{n}, \cdot) < \delta$. Likewise, for each $x \in [a, x')$, there exists some smallest integer $m_x \in \mathbb{N}_+$ such that $|\frac{d}{dn} r_b(x, m_x, \cdot)| > \delta \frac{k}{\ell}$, $\forall\, m > m_x$, where ℓ is a constant yet to be defined (we ignore for the moment the possibility that m_x may be inconsistent with satisfying self-funding for some $x \in [a, x')$). The existence of some such m_x for each $x \in [a, x')$ follows from Equations (A.4) and (A.5). Choose $\bar{m} \equiv \sup\{m_x : x \in [a, x')\}$ and $n^* \equiv \max\{\bar{m}, \bar{n}\}$. Replace both $\bar{n}$ and $\bar{m}$ with n^* in the preceding analysis—the established conditions continue to hold. Now define $\ell \equiv \sup\{D'(x - r_b(x, n^*, \cdot)) : x \in [a, x')\}$—note that ℓ is bounded away from zero, so that $\delta \frac{k}{\ell}$ is finite.

Now define $a' \equiv \sup\{x : r_b(x, n^*, \cdot) = 0, x > 0\}$. If $a' \leq a$, then the analysis above stands unchanged, as self-funding can be implemented for each $x \in [a, b]$. However, if $a' > a$, then self-funding is not implementable for those $x \in [a, a')$. In this case, define a new support $[a', b']$ for $f(x|\cdot)$ which is symmetric around $\sum_i y_i$, and recalculate all previous steps, defining $\bar{n}'$ and $\bar{m}'$ similarly. Since $[a', b'] \subset [a, b]$, it follows that $\bar{n}' \leq \bar{n}$ and $\bar{m}' \leq \bar{m}$, implying that $n^{*\prime} < n^*$. In turn this implies that $r_b(x, n^{*\prime}, \cdot) > r_b(x, n^*, \cdot) > 0$ for all $x \in [a', x')$, so that self-funding is implementable for all $x \in [a', b']$. Thus, it is without loss of generality that we assume in the remainder of the proof that self-funding is implementable for all $x \in [a, b]$.

It follows that, given n^*,

$$
\begin{aligned}
-\int_a^{x'} D'(x - r_b(x, n^*, \cdot))\frac{d}{dn} r_b(x, n^*, \cdot)\, dF &> \int_a^{x'} \ell \frac{\delta k}{\ell}\, dF \\
&= \int_a^{x'} \delta k\, dF \\
&> \int_{x'}^{b'} \delta k\, dF \\
&> \int_{x'}^{b} D'(x - r_b(x, n^*, \cdot))\frac{d}{dn} r_b(x, n^*, \cdot)\, dF, \\
\Rightarrow \quad \frac{d}{dn} E_x\, D(x - r_b(x, n^*, \cdot)) < 0.
\end{aligned}
$$

Note that although we implicitly treat n as a continuous variable here, which strictly speaking is inconsistent with the model structure, it remains true that fissioning a single firm into $1 < m < \infty$ firms, where $m \in \mathbb{N}$, results in a decrease in the regulators expected payoff under the identified conditions. In particular, by choosing ϵ sufficiently small, the above results hold for the fission of one firm into any $m \in \mathbb{R}_+$ firms, where $1 < m < m^u$ and m^u is decreasing in ϵ—m is simply interpreted as a number here. Thus, it is simply a matter of choosing ϵ sufficiently small such that $m^u > 2$ i.e., so that there exist integers in the interval $[1, m^u]$.

(b) Suppose that $D''(\cdot) = 0$. It follows that $x' = \sum_i \hat{y}_i$ and $x' - a = b - x'$. Also,

$$
n\beta_i D'(x) < R_x(x, r) + R_r(x, r) \quad \Rightarrow \quad \frac{d}{dx} r_b(x, \cdot, \cdot) \leq 1, \quad \forall\, x \in [a, b].
$$

Condition (iii) and $\frac{d}{dx} r_b(x, \cdot, \cdot) \leq 1$ implies that $R_r(x'+\Delta, r_b(x'+\Delta)) < R_r(x'-\Delta, r_b(x'-\Delta))$, for each $\Delta \in (0, \frac{b-a}{2}]$. This and Equation (A.4) imply that

$$
\frac{d}{dn} r_b(x' + \Delta, n, \cdot) > \left| \frac{d}{dn} r_b(x' - \Delta, n, \cdot) \right|, \quad \forall\, \Delta \in \left(0, \frac{b-a}{2}\right].
$$

It follows immediately then that under conditions (i), (iii) and $D''(\cdot) = 0$, that

$$
\begin{aligned}
\frac{d}{dn}\, E_{x \in [a,b]} D(x - r_b(x, n, \hat{\boldsymbol{y}})) &= -\int_a^{x'} D'(x - r_b)\frac{dr_b}{dn}\, dF - \int_{x'}^{b} D'(x - r_b)\frac{dr_b}{dn}\, dF \\
&< 0.
\end{aligned}
$$

Standard arguments can be used to show that the above result must also hold for functions $D(\cdot)$ such that $D''(\cdot) > 0$ but sufficiently close to zero. Hence, expected damage is decreasing in n, implying that the regulator's expected equilibrium payoff increases due to the fission of one firm into m. As in part a, supp f must be chosen sufficiently small to ensure that for given n, m and $\hat{\boldsymbol{y}}$, that self-funding is implementable for all $x \in$ supp f. □

Proposition 4: We first show that gross emissions increase when the constraint (12) is added to the unconstrained problem. Let $\ell_i(\boldsymbol{y})$ denote the amount owed by the regulator to firm i given the emissions profile $\boldsymbol{y}$:

$$\ell_i(\boldsymbol{y}) = \beta_i \bar{D}(\boldsymbol{y}) + c(y_i, \theta_i) - T + \int_{\theta_i}^{\bar{\theta}} c_\theta(y_i(t, \theta_{-i}), t)\, dt, \quad \forall\, i, \tag{A.7}$$

Differentiating this expression and inverting, we obtain

$$\frac{d\ell_i}{d\boldsymbol{y}} = \frac{\bar{D}(c_y \bar{D}'' - c_{yy}\bar{D}')}{(\bar{D}')^2} + \int_{\theta_i}^{\bar{\theta}} c_{\theta y}(y_i(t), t)\, dt < 0. \tag{A.8}$$

That is, the amount owed by the regulator to firm i (the right hand side of Equation (A.7)) is everywhere decreasing in $\boldsymbol{y}$. It follows that if the no-subsidy constraint is imposed, the regulator must assign to each firm a higher level of emissions, y_i. Hence, imposing a binding no-subsidy constraint implies that the prescribed level of emissions for each firm must increase.

As in the unconstrained problem, the regulator chooses the equilibrium level of r according to Equation (3), where x is conditioned on the $\boldsymbol{y}$ implied by the no-subsidy constraint. If the no-subsidy constraint is binding, then it is sufficient that $r'(x) < 1$ for net emissions to increase. That is, if

$$\begin{aligned} r(y) \equiv \frac{-(R_{yr} - D'')}{R_{rr} + D''} &< 1 \\ \Leftrightarrow \quad -R_{yr} &< R_{rr} \\ \Leftrightarrow \quad |R_{yr}| &< R_{rr}, \end{aligned}$$

then net emissions always increase in response to the imposition of the no-subsidy constraint. □

Proposition 5: By assumption, the no-subsidy constraint is binding prior to fission, implying

$$\begin{aligned} T - \beta_i \bar{D}(\hat{y}_i) - c(\hat{y}_i, \theta_i) - \int_{\theta_i}^{\bar{\theta}} c_\theta(\hat{y}_i(t), t)\, dt &= 0 \\ \Rightarrow \quad \frac{1}{m}\left[T - \beta_i \bar{D}(\hat{y}_i) - c(\hat{y}_i, \theta_i) - \int_{\theta_i}^{\bar{\theta}} c_\theta(\hat{y}_i(t), t)\, dt\right] &= 0 \\ \Rightarrow \quad \frac{1}{m}T - \beta_i \bar{D}(\hat{y}_i) - \frac{1}{m}c(\hat{y}_i, \theta_i) - \frac{1}{m}\int_{\theta_i}^{\bar{\theta}} c_\theta(\hat{y}_i(t), t)\, dt &< 0. \end{aligned}$$

By Equation (8), this implies

$$\frac{1}{m}T - \beta_j \bar{D}(\hat{\boldsymbol{y}}) - c^m(\hat{y}_j, \theta_i) - \int_{\theta_i}^{\bar{\theta}} c_\theta^m(\hat{y}_j(t), t)\, dt \quad < \quad 0.$$

Since the no-subsidy constraint is violated following fissioning, $\boldsymbol{\beta}$ must not be optimal. Moreover, the optimal penalty, $\hat{\boldsymbol{\beta}}$, following fissioning must give the regulator a strictly lower expected payoff. If it gave a higher expected payoff, this would contrdict the assertion that $\boldsymbol{\beta}$ was optimal prior to fissioning. $\square$

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