Privatization, Market Liberalization and Learning in Transition Economies

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ABSTRACT: Privatization and market liberalization are widely considered to be complementary reforms in transition economies. This article challenges this view and the closely related "big bang" approach: when pursued too vigorously, privatization may impede the transition process following liberalization. Our result is based on an explicit model of market learning. Compared to a mature market, a market in transition is characterized by greater uncertainty regarding market conditions, including equilibrium prices and quantities. Economic actors must learn about these conditions through their participation in the market process. Less than full privatization is optimal if the costs of learning are sufficiently important.

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Privatization and market liberalization are widely considered to be complementary reforms in transition economies. This article challenges this view and the closely related “big bang” approach to economic reform. Our analysis suggests that when pursued too vigorously, privatization may actually impede the transition process following market liberalization and reduce social welfare. Our result is based on an explicit model of the market learning process, which is an intrinsic component of any transition from a socialist economy—in which markets and market institutions are either nonexistent or highly distorted by government interventions—to a fully-functioning market economy. The theoretical literature to date on the transition in Central and Eastern Europe has ignored the need for individuals to simultaneously learn, through their participation in the market process, about the features of a market in transition and the effects of government-instituted reforms, e.g. Murrell. In what follows, we will argue that, insofar as it fails to take account of the learning process, the policy advice provided by Western experts to transition economies may be seriously flawed.

An urgent task facing policymakers in a small transition economy is to identify those subsectors of the economy in which their country will have a comparative advantage.[1] Typically, very little information about the identity of these subsectors is provided by relative prices from the pre-transition era, since these were hugely distorted by production quotas, taxes and subsidies, and other nonmarket influences. So what economic policies will best facilitate the process of acquiring the necessary information? The standard economic advice proffered by Western economists has been to follow a “big bang” approach of simultaneous, and rapid, liberalization and privatization. Proponents of this approach rely on the efficacy of Adam Smith’s “hidden hand” as a vehicle for achieving the optimal reallocation of resources: their belief is that newly privatized producers, who will be highly responsive to the newly liberalized market signals, have the best chance of identifying the optimal path of adjustment to the new market realities.

We investigate the relationship among learning and adjustment and the degree of privatization in an extremely stylized model of the transition process. We divide the production sector into privatized and nonprivatized firms (*parastatals*). The fraction of privatized firms is viewed as a one-time policy choice, which remains fixed for the duration of the transition period. Our privatized firms are modeled as responsive to market signals. Specifically, they base their production decisions on their private signals about market conditions and previously realized market prices. Parastatals simply select a fixed level of production. The firms produce for a world market, with deterministic world price w^*. Two inputs are required for the production process: the first is available on world markets and is perfectly elastically supplied at a price of unity; the second is nontradable, with a stochastic, upward-sloping residual supply curve. The source of the stochasticity is transition-related uncertainty about the demand for the input by other sectors, which are also adjusting to the transition process and are simultaneously undergoing a similar learning process. An unusual aspect of our model is that producers are required to make input decisions *before* the price uncertainty has been resolved. We impose this assumption because of its convenience: together with our assumption of risk neutrality, it insulates *expected* welfare from the randomness in supply. One interpretation of the assumption is that input decisions are sequential and there is relatively little substitutability among inputs. For example, the nontradable input might be labor: labor requirements are typically determined at the beginning of the production cycle, while actual services are paid for at the end of the cycle, by which time the price uncertainty is resolved.

More generally, by modelling firms as bidding against other sectors for a non-tradable input, we are able to address the issue of comparative advantage in a reduced-form way within a one sector model. This partial equilibrium orientation is, of course, a serious limitation of our model as a tool for welfare analysis, especially because we are implicitly assuming that market failures are simultaneously occuring in other sectors of the economy. Nonetheless, we believe that our partial equilibrium orientation is warranted by its simplicity relative to the general equilibrium alternative.

The only information that our producers have about the input, in addition to their own individual signals, is its past realized prices. In particular, our producers know neither the expected intercept of the input supply curve, the number of nonresponsive producers nor the amount produced by each. Further, they do not know the structure of the market. Rather than attempt to learn the parameters of an unknown structural model, our responsive producers simply predict market prices using an adaptive expectations-style learning rule.

Since Lucas, models of expectation formation such as the one we present here have been widely criticized on the grounds that they postulate non-“rational” behavior by economic agents. If agents behaved in the manner we postulate, the argument runs, then arbitrage possibilities would arise and remain unexploited, due to the ‘ad hoc’ nature of agents’ price expectation formation rule. This critique is certainly compelling when applied to models of long-run or steady-state behavior. Because in such contexts an abundance of econometric data would be available, agents should be able to “reverse engineer” the economic environment within which they are operating, and then base their price predictions and production decisions on an empirically validated structural model of this environment. This critique has much less force when applied to models of short-run—and, in particular, *transition*—behavior. Because they are operating in a transition environment, the agents in our model have had neither the time, the data nor the experience to “master the model” to the extent required by the rational expectations hypothesis. Given the inevitable uncertainty about market structure that characterizes all transition economies, and the inevitable transition-related noise that contaminates whatever data is available, it seems reasonable to suppose that producers might use past price observations as a forecasting tool, rather than relying upon some structural model in which they have no basis for confidence. A related point is frequently made by econometricians in defense of their use of reduced form time-series models for short-term forecasting (see, for example, Judge et al, p. 675). Indeed, as an empirical matter, it is well known that those very arbitrage opportunities on which the rational expectations critique is based are in fact *extremely* widespread in the early stages of transition economies. While these opportunities will no doubt be exploited eventually, if they have not already disappeared, our focus in this paper is on the period during which agents have insufficient information to exploit them.

Our approach to the gradualism versus big-bang controversy differs from the approaches that have dominated the economic transition literature e.g., Gates, Milgrom and Roberts; and Murphy, Shleifer and Vishny.

Rather than modeling a centrally-manipulated process, in which market participants respond perfectly to incentives set by government, we focus specifically on the functioning of transition markets when information and incentives are imperfect. We ignore political-economic considerations such as those raised by Laban and Wolf. In contrast to studies such as Dewatripont and Roland, we treat uncertainty as an integral component of the market transition process, and consider how individuals' responses to market signals affect production, profits, prices and social welfare.

The policymaker in this paper is modeled as choosing a constant level of privatization for the entire transition. In the interests of tractability, we do not attempt to identify the optimal *rate* at which nonprivatized firms should be converted into privatized firms. While this is a fascinating and important policy issue, it is also a *much* more difficult one in the context of an explicit model of learning. In order to address it, we would have to formalize the dynamic optimization problem facing the policymaker and to address the issue of how beliefs regarding the country's comparative advantage in production should be updated within this context.

Whereas we focus on the importance of uncertainty and the functioning of transition markets when information and incentives are imperfect, we nonetheless presume that the policymaker has the capacity to manipulate the transition process. Formally, we model the policymaker as choosing, once and for all, the fraction of firms that will be privatized. This modeling approach suggests that the policymaker must possess information regarding the transition process that firms do not, and uses this information to optimize its one-time privatization decision. One might wonder, then, why the policymaker does not simply share its information with the industry and thereby mitigate some of the uncertainty of the transition process. [2] We prefer to interpret our model as formalizing the privatization policy that would be selected by an omniscient (but constrained) observer, i.e., one much better informed than the actual policymaker. An alternative interpretation is that the choice of a specific *number* by the policymaker is a convenient way of representing the much more qualitative type of policy decision that policymakers are actually required to make. Specifically it is much easier to formalize the problem we pose than the more realistic, but less concrete one of how supportive of privatization the government should be. The policy implications we derive regarding the effects of learning are no less relevant because of this.

First, we construct a “modified cobweb model” with time varying parameters. Second, we distinguish three phases of the dynamic adjustment path: (i) a phase of explosive oscillations in prices and production; (ii) a phase of damped oscillations; and (iii) a phase of monotone convergence to perfect information prices. We refer to the first two phases as the *short-run* and the last phase as the *long-run*. The results in this section focus on the relationship between price and production volatility and the fraction of privatized producers. In the short-run, volatility increases with the degree of privatization while in the long-run, additional privatization reduces volatility. Moreover, the length of the short-run increases with the number of responsive producers. Third, we specify the policymaker’s performance function and examine how the optimal level of privatization depends on the various parameters of the model.

Increasing the degree of privatization has short-run costs and long-run benefits. Price volatility results in welfare losses relative to the perfect information equilibrium: our responsive producers base their production decisions on estimated prices and hence misallocate resources when these prices differ from realized prices. A more vigorous privatization program increases volatility both in the short-run and the early long-run, and hence exacerbates this first kind of resource misallocation. On the other hand, our parastatal producers are misallocating resources by ignoring market signals, and as the number of parastatals declines, this second kind of misallocation becomes less important. Because the costs of privatization decline in the long-run, while the benefits remain constant over time, the optimal level of privatization depends on the policymaker’s rate of time preference. We prove that if the short run is sufficiently important to the policymaker, there is a unique optimal level of privatization, which falls short of full privatization; on the other hand, if policymakers are sufficiently patient then full privatization is optimal.

1. A MODEL OF LEARNING IN A TRANSITION ENVIRONMENT

We consider a partial-equilibrium model in which producers learn about the market price of one of their inputs. We adopt the linear-quadratic model which is the standard for learning-theoretic papers (see Townsend, Rausser and Hochman, Bray and Savin, etc.). We assume that market demand for *output* is perfectly elastic at the world price. The production of q units of output requires $0.5q^2$ units of a tradable input and q units of a nontradable input. While the tradable input is elastically supplied at a world price of unity, the supply of the other input is upward sloping with a random intercept. An interpretation of the randomness is that the *residual* supply of the input is stochastic due to stochastic demand for the input by other sectors, which are also adjusting in the course of the transition. At the start of the transition, each price-responsive producer has a point estimate of the market-clearing price for the input. As the transition progresses, producers revise their estimates of this price, based on the unfolding path of realized prices. Thus, our producers are learning about the cost of doing business in this particular sector: because of competing pressures for resources, a key component of their cost structure is unknown. With this formulation we can address the policy question of how a country in transition identifies those sectors in which it has a comparative advantage.

The total number of producers, denoted by N, will be held fixed for now. All producers are risk-neutral and have identical cost functions, but a fraction $\alpha = \frac{n}{N}$ are privatized and responsive to market signals, while the remaining fraction $(1-\alpha)$ are nonresponsive parastatals. Each parastatal produces the quantity $\bar{q}$, so that aggregate parastatal output is $(1-\alpha)N\bar{q}$. Producers' common cost function is denoted by $C(q) = pq + \frac{1}{2}q^2$, where p is the (unknown) price of the nontradable input. Thus in period t, each privatized producer's estimated profit maximizing level of output is identically equal to the difference between the commonly known world price of output, w^*, and her (subjective) estimate of the market clearing price of the input, $\hat{p}_{ti}$. It follows that at anticipated prices $\{\hat{p}_{ti}\}_{i=1}^{n}$, aggregate demand for the input is $N\big((1-\alpha)\bar{q} + \alpha w^*\big) - \sum_{i=1}^{n} \hat{p}_{ti}$. Supply of the input in period t at price p is equal to $(a - \delta_t + bp)$, where $a < 0$, $b > 0$ and δ_t is a *quantity* shock.

We consider two kinds of restrictions on supply shocks. The first are maintained throughout.

Assumption 1. *The δ_t's are independently distributed. For each t, the distribution of δ_t is symmetric about zero and has bounded support. For every t, $a - \delta_t \leq 0$ for all possible realizations of δ_t.*

The latter assumption ensures that the price of the input will be positive (since the vertical intercept of the inverse input supply curve is $(\delta - a)/b > 0$.) In addition, we will assume either Assumption 2 or Assumption 2′ below. Assumption 2 states that the supply shocks are essentially transitional in nature, and so eventually shrink to zero. That is, letting $\bar{\delta}_t$ denote the upper boundary of the support of δ_t, we assume:

Assumption 2. $\lim_{t\to\infty} \sum_{\tau=1}^{t} \bar{\delta}_\tau$ *is finite.*

An implication of this assumption is that the sum of the variances of the δ_t's is finite also.[3] Our alternative assumption is:

Assumption 2′. *The δ_t's are identically distributed.*

The main difference between the alternative assumptions is that under Assumption 2, anticipated prices asymptotically *coincide* with the perfect information price, whereas under Assumption 2′, anticipated prices are asymptotically unbiased predictors of the perfect information price.

The market clearing price of the input in period t is $p_t = b^{-1}\Big(\delta_t + N\big((1-\alpha)\bar{q} + \alpha w^*\big) - a - \sum_{i=1}^{n} \hat{p}_{ti}\Big)$. Observe that p_t depends only on the *sum* of price-responsive agents' anticipated prices. To highlight this, we define the *average anticipated price* in period t, $\hat{p}_t = n^{-1} \sum_{i=1}^{n} \hat{p}_{ti}$, and rewrite the expression as:

$$p_t \; = b^{-1}\Big(M(\alpha) \; + \; \delta_t - \alpha N \hat{p}_t\Big). \tag{1}$$

where $M(\alpha) = N\big((1-\alpha)\bar{q} + \alpha w^*\big) - a > 0$.

For each $\alpha \in [0, 1]$, we define a benchmark input price $p^*(\alpha)$ with the following property: if each private producer anticipates this price and produces accordingly, and *if there were no supply shocks*, then the market clearing price of the input would indeed be $p^*(\alpha)$. It is defined as follows:

$$p^*(\alpha) \quad = \frac{M(\alpha)}{b + \alpha N} \tag{2}$$

Henceforth, we will refer to $p^*(\alpha)$ as the *perfect information input price* and suppress references to α except when necessary. A special case is $p^*(1) = (Nw^* - a)/(N + b)$, which we refer to as the *Walrasian input price*, p^W, since this is the input price that would prevail in the Walrasian equilibrium of the perfect information version of our model with no parastatal firms. We assume that $(w^* - p^W) \neq \bar{q}$, i.e., that parastatals' production level differs from the level that would be Pareto optimal if all firms were responsive.

Before any production takes place, each producer has a point estimate, $\hat{p}_{1i}$, of the market clearing price of the input. One possible interpretation is that $\hat{p}_{1i}$ is the view of market conditions that i acquires during her pre-transition experience. These estimates are private information. We make no assumptions at this point about the statistical distribution of producers' estimates. In particular, they may or may not be unbiased estimates of the perfect information price $p^*(\alpha)$. We will, however, maintain throughout that producers have no idea whether or not their estimates are unbiased. Indeed, producers have no other prior information about market conditions. In particular, the magnitudes α, N, a and b are unknown, as are the parameters governing the distribution of the δ_t's. Furthermore, producers do not know the structure of the market. That is, they do not know that input supply is linear, or that other firms have linear supply curves. These assumptions reflect the lack of market knowledge that characterizes economies at the outset of a transition.

In period $t > 1$, i's estimate of the t'th period input price, denoted by $\hat{p}_{ti}$, is a convex combination of realized market prices in previous periods and her original private signal, with higher weights placed on more recent price realizations: $\hat{p}_{ti} = \left(\sum_{\tau=0}^{t-1} \gamma^\tau\right)^{-1} \left[\sum_{\tau=0}^{t-2} \gamma^\tau p_{t-\tau-1} + \gamma^{t-1} \hat{p}_{1i}\right]$. Here, γ is not a rate of *time* preference but rather reflects the rate at which producers discount past price information. We assume that γ is identical for all individuals. To avoid dealing with certain special cases (see p. 12 below) we impose additional bounds on the size of γ.

Assumption 3. *(a)* $0 < \gamma < b^{-1}$*; (b)* $\lim_{t\to\infty} \sum_{\tau=1}^{t} \gamma^{\tau} > b^{-1} N$.

Note that

$$\hat{p}_{t+1,i} = \left(\sum_{\tau=0}^{t} \gamma^{\tau}\right)^{-1} \left[p_t + \sum_{\tau=1}^{t} \gamma^{\tau} \hat{p}_{ti} \right] \tag{3}$$

Again aggregating anticipated prices, setting $\Gamma_t = \sum_{\tau=0}^{t} \gamma^{\tau}$ and observing that $\Gamma_t - 1 = \sum_{\tau=1}^{t} \gamma^{\tau}$, we obtain the following relationship between average anticipated prices in successive periods:

$$\hat{p}_{t+1} = (\Gamma_t)^{-1} \Big(p_t + (\Gamma_t - 1)\hat{p}_t \Big) \tag{4}$$

Observe from equations (1) and (2), we have for all $t \geq 1$:

$$(p_t - p^*) = \frac{n}{b} \Big(\frac{\delta_t}{n} - (\hat{p}_t - p^*) \Big) \tag{5}$$

The learning rule we specify derives from the adaptive expectations literature. Muth (1960) shows that such rules are optimal prediction rules when the effect of uncertainty on a system has both a temporary and a permanent component. In the classical literature on adaptive expectations, the individuals who are predicting the system's behavior do not interact with the system. In our model, by contrast, agents' expectations influence their production decisions, which *in the aggregate* affect the behavior of the system. Nonetheless, given the pattern of behavior we assume for our agents, an econometrician who does not know the structure of the model but only knows that agents utilize adaptive expectations, so that there is both a permanent and a transitory component to shocks, cannot do a better job of predicting prices than by estimating coefficients on lagged prices. Indeed, it is difficult to imagine a more sophisticated rule that producers might adopt, given their total ignorance about the parameters that determine market conditions and the structural model. Note in particular that at least in the early stages of the transition, it would be a challenging statistical problem to disentangle the effects of the per-period supply shocks from those of agents' private initial signals. For example, suppose that the first few realized prices exceed $\hat{p}_{1i}$. Even if she knew the underlying structure of the sector, producer i would have no way of knowing whether to attribute these unexpectedly high prices to: (a) a large negative value of $(\hat{p}_{1i} - p^*)$; (b) a large negative value on average of $(\hat{p}_{1j} - p^*)$, $j \neq i$, resulting in underproduction; or (c) a sequence of positive δ_t's.

2. The dynamics of prices and production

2.1. **Production and Price Paths for fixed α.** In this subsection we first derive an expression for average anticipated price in period t. We then fix an arbitrary vector of private market signals and a sequence of $s-1$ supply shocks, and consider the dynamic path of realized input prices from period s into the future.

When $t = 1$, private producer i's anticipated input price is just her initial private signal of the market price, $\hat{p}_{1i}$. As noted above (equation (5)), whether the difference, $(p_1 - p^*)$, between the market clearing price and the perfect information price is positive or negative depends jointly on whether private producers have *on average* under- or over-estimated the perfect information price—i.e., on the relationship between $\hat{p}_1$ and p^*— and on the sign of δ_1. In period $t = 2$, i's updated estimate of the market price, $\hat{p}_{2i}$, is a weighted average of her initial signal and the previous period's realized price, p_1. From (4) and (5), the expression $(\hat{p}_2 - p^*)$, which is the divergence from the perfect information price of the average anticipated price in period two is $(\hat{p}_2 - p^*) = \frac{1}{1+\gamma}\left(\frac{\delta_1}{b} + \left(\gamma - b^{-1}n\right)(\hat{p}_1 - p^*)\right)$

As the transition progresses, private producers sequentially revise their estimates of the market price. While earlier price observations are increasingly discounted, each new price observation has an increasingly small role in determining producers' estimates. Combining (3) and (5), we obtain the following relationship between aggregate anticipated prices in periods $t-1$ and t:

$$(\hat{p}_t - p^*) = \frac{\delta_{t-1}}{b\sum_{\tau=0}^{t-1}\gamma^\tau} + \frac{\sum_{\tau=1}^{t-1}\gamma^\tau - b^{-1}n}{\sum_{\tau=0}^{t-1}\gamma^\tau}(\hat{p}_{t-1} - p^*) \tag{6}$$

By recursively substituting, we can express $(\hat{p}_t - p^*)$ in terms of the realized supply shocks up to period $t-1$ and the gap between the average initial signal and the perfect information price:

$$(\hat{p}_t - p^*) \quad = \sum_{\tau=1}^{t-1} \frac{\Phi(\tau+1, t-1)}{b\Gamma_\tau}\delta_\tau \; + \; \Phi(1, t-1)(\hat{p}_1 - p^*) \tag{7}$$

where $\Phi(\tau, \tau') = \prod_{m=\tau}^{\tau'} \frac{\left(\Gamma_m - 1 - b^{-1}n\right)}{\Gamma_m}$ if $\tau \leq \tau'$ and 1 otherwise. For future reference, note that $\frac{\partial\Phi(\tau,\tau')}{\partial n} = -b^{-1}\Phi(\tau, \tau') \sum_{m=\tau}^{\tau'} \left(\Gamma_m - 1 - b^{-1}n\right)^{-1}$.

Let $\bar{t}(b, \gamma, n)$ denote the smallest t such that $\sum_{\tau=1}^{t-1} \gamma^\tau \geq b^{-1}n$. Note that $\bar{t}(b, \gamma, n)$ increases with n. Assumption 3 guarantees that $\bar{t}(b, \gamma, n) > 1$ for all n. To ensure that certain critical derivatives exist—specifically expression (11) below—we impose the following technical assumption:

Assumption 4. *For all natural numbers n, $\sum_{\tau=1}^{\bar{t}(b,\gamma,n)} \gamma^\tau \neq b^{-1}n$.*

Observe in equation (7) that for $m \in [\tau+1, t-1]$, the m'th element of the product $\Phi(\tau+1, t-1)$ will be positive iff $m \geq \bar{t}(b, \gamma, n)$. An important property of our model is that the coefficients on each of the random terms in expression (7) shrink to zero as t increases:

Lemma 1. *For all τ, $\lim_{t\to\infty} \frac{\Phi(\tau+1,t-1)}{b\Gamma_\tau} = 0$.*

The proofs of this lemma and the following propositions are gathered together in the appendix.

We can now construct the sequence of gaps between realized prices and the perfect information price, starting from an arbitrary vector of private market signals. First observe from (5) and (7) that for all $t \geq 1$, the gap between the *realized* price at t and the perfect information price is

$$(p_t - p^*) \quad = \frac{n}{b}\left(\frac{\delta_t}{n} - \sum_{\tau=1}^{t-1} \frac{\Phi(\tau+1, t-1)}{b\Gamma_\tau}\delta_\tau \; - \; \Phi(1, t-1)(\hat{p}_1 - p^*)\right) \tag{8}$$

If Assumption 2 holds, expression (8) and Assumption 3 imply that *every* sequence of *realized* prices converges to the perfect information price, p^*. This result requires no restrictions on the statistical distribution of agents' initial signals. An immediate corollary is that with certainty, average *anticipated* price will asymptotically coincide with p^*. If Assumption 2′ holds rather than Assumption 2, then the best we can say is that, conditional on any vector of initial market signals and supply shocks up to time s, the *expected* paths of actual and anticipated prices, starting from period $s+1$, converge to the perfect information price p^*.

Proposition 1. *(a) If Assumption 2 holds, then for any vector of initial market signals and sequence of supply shocks,* $\lim_{t\to\infty}(p_t - p^*) = 0$ *and* $\lim_{t\to\infty}(\hat{p}_t - p^*) = 0$. *(b) If Assumption 2′ holds, then for any vector of initial market signals and sequence of supply shocks up to period s,* $\lim_{t\to\infty} E_s(p_t - p^*) = 0$ *and* $\lim_{t\to\infty} E_s(\hat{p}_t - p^*) = 0$.

2.2. **Qualitative properties of production and price paths.** Our goal in this and the following subsection is to study "the shape" of the production and price paths generated by an arbitrary vector of private market signals and supply shocks over time, and to investigate how this shape changes with n. Unless restrictions are imposed on supply shocks, however, very little can be said about any *given* path. Accordingly, we assume initially that all supply shocks are zero, which allows us to illustrate the factors influencing the effects of the initial uncertainty.

We begin by analyzing the sequence of average anticipated prices. In period one, private producer i's anticipated input price is just her initial private signal of the market price, $\hat{p}_{1i}$. In period two, i's estimate of the market price, $\hat{p}_{2i}$, is a weighted average of her initial signal and the previous period's realized price. Consider the expression $(\hat{p}_2 - p^*)$, which is the divergence from the perfect information price of the average anticipated price in period two assuming no supply shocks: $(\hat{p}_2 - p^*) = \frac{1}{1+\gamma}\left(\gamma - b^{-1}n\right)(\hat{p}_1 - p^*)$. Note that because $b\gamma < 1$ (p. 8 above), the sign on $(\hat{p}_2 - p^*)$ is different from the sign on $(\hat{p}_1 - p^*)$.

Now consider the behavior of the average anticipated price in period t as a function of the preceding period's average anticipated price: $(\hat{p}_t - p^*) = \frac{\sum_{\tau=1}^{t-1}\gamma^\tau - b^{-1}n}{\sum_{\tau=0}^{t-1}\gamma^\tau}(\hat{p}_{t-1} - p^*)$. Under assumption 3, we can distinguish three cases, depending on whether: (i) $(\sum_{\tau=1}^{t-1}\gamma^\tau - b^{-1}n) < -\sum_{\tau=0}^{t-1}\gamma^\tau$ or equivalently $1 + 2\sum_{\tau=1}^{t-1}\gamma^\tau < b^{-1}n$; (ii) $(\sum_{\tau=1}^{t-1}\gamma^\tau - b^{-1}n) \in [-\sum_{\tau=0}^{t-1}\gamma^\tau, 0]$ or equivalently $\sum_{\tau=1}^{t-1}\gamma^\tau < b^{-1}n < 1 + 2\sum_{\tau=1}^{t-1}\gamma^\tau$; (iii)

$\sum_{\tau=1}^{t-1}\gamma^{\tau} > b^{-1}n$. In case (i), the coefficient on $(\hat{p}_{t-1} - p^*)$ is less than -1; in case (ii) it belongs to $[-1, 0]$, while in case (iii) it belongs to $[0, 1]$. Let $\underline{t}(b, \gamma, n)$ denote the largest t such that case (i) holds, and $\bar{t}(b, \gamma, n)$ denote the smallest t such that case (iii) holds. It follows from the preceding observation that in the absence of supply shocks, the path of anticipated prices generated by any vector of initial market signals can be divided into at most three phases: phase (i) runs from period 1 to $\underline{t}(b, \gamma, n)$, phase (ii) from $\underline{t}(b, \gamma, n) + 1$ to $\bar{t}(b, \gamma, n) - 1$ and phase (iii) from $\bar{t}(b, \gamma, n)$ on. Phase (i) is characterized by explosive oscillations, phase (ii) by damped oscillations and phase (iii) by monotone convergence. We shall refer to phases (i) and (ii) as the *short-run*, and to phase (iii) as the *long-run*. Thus, in the short-run the paths of production and anticipated prices exhibit the familiar cobweb pattern, except that the underlying parameters vary with time.

Once supply shocks are introduced, a "representative price path" is, of course, no longer meaningful. Certainly, we can no longer proceed as above and partition *any given* price sequence into three phases with qualitatively different dynamic properties.[4] For example, there are sequences of supply shocks whose associated price paths alternate forever between oscillatory and monotone phases. In a probabilistic sense, however, the properties of the model with supply shocks mirror the characteristics described above. For example, if $\underline{t}(b, \gamma, n) > 1$, then the gap between $\hat{p}_{\underline{t}(b,\gamma,n)}$ and p^* will more likely than not be wider than the gap between $\hat{p}_1$ and p^*. Similarly, the gap between $\hat{p}_{\bar{t}(b,\gamma,n)}$ and p^* will more likely than not be narrower than the gap between $\hat{p}_{\underline{t}(b,\gamma,n)}$ and p^*.[5]

2.3. **The effect of increasing the number of price-responsive producers.** In the absence of supply uncertainty, an increase in n, the number of private producers, has three consequences. First, there is an increase in the magnitude of oscillations during the short-run. Second, the duration of the short-run increases. More precisely, both $\underline{t}(b, \gamma, \cdot)$ and $\bar{t}(b, \gamma, \cdot)$ increase with n (p. 11), but $\underline{t}(b, \gamma, \cdot)$ increases by more than $\bar{t}(b, \gamma, \cdot)$ so that phase (i) is extended and phase (ii) is squeezed. Third, once the long-run is reached, prices and production converge to perfect information levels at a faster rate. To see this, consider the ratio $\frac{\left(\Gamma_t - 1 - b^{-1}n\right)}{\Gamma_t}$. In the short-run, when this ratio is negative, an increase in n makes it more negative, increasing the magnitude of oscillations. Also, an increase in n postpones the date at which the ratio turns positive. In the long-run, when it is positive, an increase in n makes it less positive, increasing the rate of convergence.

Now suppose that supply shocks are non-zero. Again, the effects of n are comparable to those above, but only in a probabilistic sense. For example, if n increases to n', then the probability that the the gap between $\hat{p}_{\underline{t}(b,\gamma,n')}$ and p^* is wider than the gap between $\hat{p}_1$ and p^* will exceed the probability that the gap between $\hat{p}_{\underline{t}(b,\gamma,n)}$ and p^* is wider than the gap between $\hat{p}_1$ and p^*. Now consider the long-run, and suppose that the gap between $\hat{p}_{\underline{t}(b,\gamma,n')}$ and p^* is equal to the gap between $\hat{p}_{\underline{t}(b,\gamma,n)}$ and p^*. In this case, an increase in n increases the likelihood of smooth convergence to the perfect information price, since for $t > \bar{t}(b,\gamma,\cdot)$, the coefficients on the δ_t's decline as n increases.

2.4. **The variance of market prices.** In the preceding subsections, we considered the the shape of individual dynamic price and production paths and the effect of n on these shapes in the absence of supply uncertainty. We now examine the statistical properties of these paths and the effect of n on these properties. To simplify the analysis in this section, we assume that the average initial private signal is an unbiased estimator of the perfect information price.[6]

Assumption 5. $E(\hat{p}_1 - p^*) = 0$.

Under Assumption 5, (7) implies that for every t, $\hat{p}_t$ is an unbiased estimator of the perfect information price. The variance of $\hat{p}_t$ is obtained directly from the same expression. Letting ς^2 denote the variance of the average initial signal and σ_t^2 denote the variance of δ_t, we obtain:

$$\mathrm{Var}(\hat{p}_t) \;=\; \left\{ \sum_{\tau=1}^{t-1} \left(\frac{\Phi(\tau+1, t-1)}{b\Gamma_\tau} \right)^2 \sigma_\tau^2 \;+\; (\Phi(1, t-1))^2 \varsigma^2 \right\} \tag{9}$$

To economize on notation, we set $\Gamma_0 = b^{-1}$ and $\sigma_0^2 = \varsigma^2$. We can now rewrite (9) as

$$\mathrm{Var}(\hat{p}_t) \;=\; \sum_{\tau=0}^{t-1} \left(\frac{\Phi(\tau+1, t-1)}{b\Gamma_\tau} \right)^2 \sigma_\tau^2 \tag{9'}$$

We calculate the variance of the t-period'th *realized* price from expressions (8) and (9)

$$\mathrm{Var}(p_t) \;=\; b^{-2}\left\{\sigma_t^2 \;+\; \sum_{\tau=0}^{t-1}\left(\frac{n\,\Phi(\tau+1,\,t-1)}{b\Gamma_\tau}\right)^2\sigma_\tau^2\right\} \tag{10}$$

Holding n fixed, the effect of time on the variances of both $\hat{p}_t$ and p_t will be immediately apparent from expressions (9) and (10). The turning point between phases (i) and (ii) in the zero supply shock case here determines the behavior of the variances of $\hat{p}_t$ and p_t. In the very short-run (phase (i)), each period an additional term with magnitude greater than 1 is multiplied by the $t-1$ pre-existing terms and another term is added to the sum, so that both variances increase with t. In phase (ii) and the beginning of phase (iii), an additional term with magnitude less than one is multiplied by the pre-exisiting terms, but an additional term is added, so the effect of t is indeterminate. In the very long term, however, both variances shrink to zero.

Proposition 2. *For $t < \underline{t}(b,\gamma,n)$, Var$(p_t)$ > Var(p_{t-1}) and Var$(\hat{p}_t)$ > Var$(\hat{p}_{t-1})$. For any given $t \geq \underline{t}(b,\gamma,n)$, the relationship between variances in successive t's cannot be determined. However, if Assumption 2 holds, then* $\lim_{t\to\infty}$ *Var*$(\hat{p}_t) = \lim_{t\to\infty}$ *Var*$(p_t) = 0$.

We now consider the relationship between n and the variance of the average anticipated price in period t. We find that in the short-run, an increase in n increases volatility, while in the long-run, the effect of n is indeterminate. Recalling from p. 11 the expression for $\frac{\partial\Phi(\tau,\tau')}{\partial n}$, we obtain the following expressions for the first and second derivatives of $\mathrm{Var}(\hat{p}_t)$ with respect to n:

$$\frac{d\mathrm{Var}(\hat{p}_t)}{dn} \;=\; -\frac{2}{b}\left\{\sum_{\tau=0}^{t-1}\left(\frac{\Phi(\tau+1,\,t-1)}{b\Gamma_\tau}\right)^2\left(\sum_{m=\tau+1}^{t-1}\left(\Gamma_m-1-b^{-1}n\right)^{-1}\right)\sigma_\tau^2\right\} \tag{11}$$

$$\begin{aligned}\frac{d^2\mathrm{Var}(\hat{p}_t)}{dn^2} \;=\; \frac{2}{b^2}\Bigg\{\sum_{\tau=0}^{t-1}\left(\frac{\Phi(\tau+1,\,t-1)}{b\Gamma_\tau}\right)^2\Bigg[2\left(\sum_{m=\tau+1}^{t-1}\left(\Gamma_m-1-b^{-1}n\right)^{-1}\right)^2 \\ -\left(\sum_{m=\tau+1}^{t-1}\left(\Gamma_m-1-b^{-1}n\right)^{-2}\right)\Bigg]\sigma_\tau^2\Bigg\}\end{aligned} \tag{12}$$

Under Assumption 3, $\Phi(\tau+1, t-1)$ is the product of terms which for sufficiently large t are eventually all less than unity. Note the sequence $\left(\left(\Gamma_m - 1 - b^{-1}n\right)^{-1}\right)_{m=1}^{\infty}$ is bounded. These facts[7] together with Assumption 2, imply that the sequences $\left(\frac{d\text{Var}(\hat{p}_t)}{dn}\right)_{t=1}^{\infty}$ and $\left(\frac{d^2\text{Var}(\hat{p}_t)}{dn^2}\right)_{t=1}^{\infty}$ are bounded.

Under Assumption 2, the comparative statics of volatility with respect to n are determinate only in the short-run, when anticipated (and hence realized) prices become more volatile as n increases. Under Assumption 2′, they are also determinate in the extremely long-run, when price volatility declines as n increases.

Proposition 3. *(a) In the short-run (i.e., for $t < \bar{t}(b, \gamma, n)$), Var$(\hat{p}_t)$ is increasing and convex in n. (b) Under Assumption 2, for any given $t \geq \bar{t}(b, \gamma, n)$, the derivative of Var$(\hat{p}_t)$ with respect to n cannot be signed. (c) Under Assumption 2, $\lim_{t\to\infty} \frac{dVar(\hat{p}_t)}{dn} = 0$, for all n. (d) Under Assumption 2′ there exists T such that for all n and all $t > T$, $\frac{dVar(\hat{p}_t)}{dn}$ is negative.*

3. Expected social surplus

3.1. **Expected social surplus in period t.** *Expected social surplus* in period t, V_t, is defined as as the sum of the expected producer surpluses accruing to private producers and parastatals and the surplus that accrues to suppliers of the nontradable input. We compare V_t to *Walrasian social surplus*, SS^W, which is the surplus that would arise if there were no uncertainty and if all production occured at Walrasian levels. It is useful to introduce an intermediate level of social surplus, *perfect information social surplus*, SS_α, which is the surplus that would arise if private producers responded optimally to parastatal production levels and there were no uncertainty (thus $\text{SS}^W = \text{SS}_{(1)}$). We can now decompose V_t into three components: SS^W plus a (negative) *misallocation effect*, $\Delta^M\text{SS} = \text{SS}_\alpha - \text{SS}^W$, which measures the deadweight loss due to parastatals' non-Walrasian production levels, plus a (negative) *uncertainty effect*, $\Delta^U\text{SS} = \text{V}_t - \text{SS}_\alpha$, which measures the loss due to private producers' imperfect information about market conditions.

This formulation allows us to highlight a tradeoff that arises each period. While the tradeoff is starkest in the short-run, it also applies to the early stages of the long-run. (Under either Assumption 2 or Assumption 2′, the tradeoff becomes one-sided in the extremely long-run.) For standard reasons, the misallocation effect declines as n increases: since parastatals misallocate resources, an increase of n (or, equivalently, in $\alpha = n/N$) moves the perfect information equilibrium price $p^*(\alpha)$ closer to the Walrasian price $p^*(1)$. On the other hand, an increase in n exacerbates the uncertainty effect. Private producers' profits are negatively related to price variance, which increases with n. Since parastatal members are assumed to be risk neutral, their expected surplus is independent of the degree of price variance. We will show that both effects are convex in n, so that expected social surplus attains a unique maximum. These effects depend not only on the number of private producers, n, but also on the *total* number of producers in the sector, N. Accordingly, we treat the total number of producers, N, as a variable rather than a parameter.

Walrasian social surplus is the sum of aggregate profits and input producer surplus in the Walrasian equilibrium with no uncertainty. Since the Walrasian output level is $(w^* - p^W)$, aggregate producer profits are $N(w^* - p^W)^2/2$, and total input producer surplus is $N(w^* - p^W) \times (p^W + a/b)/2$. Thus $\text{SS}^W = \frac{N(a+bw^*)^2}{2b(N+b)}$.

The misallocation effect, $\Delta^M\text{SS}$, is a function of $\Delta\bar{q} = \bar{q} - (w^* - p^W)$, the difference between the parastatal output level and the Walrasian level, and of ς^2. $\Delta^M\text{SS}$ is obtained by summing the areas of three deadweight loss triangles due to parastatal misallocation under perfect information with deadweight loss due to total private production not being produced in the cost-minimizing fashion across firms. Aggregating the areas of the parastatal triangles yields $-0.5\Delta\bar{q}^2\frac{(N-n)(N+b)}{n+b}$. The loss due to differences in price forecasts across private firms is equal to $-0.5n\mathbb{E}[(\hat{p}_{ti} - \hat{p}_t)^2]$, so that, since producers' initial price signals are independent and identically distributed, $\Delta^M\text{SS} = -0.5\left(\Delta\bar{q}^2\frac{(N-n)(N+b)}{n+b} + (\sum_{\tau=0}^{t-1}\gamma^\tau)^{-2}\gamma^{2(t-1)}n(n-1)\varsigma^2\right)$. Note that $\Delta^M\text{SS}$ is concave with respect to n. Since the first term increases with n, while the second term decreases with n, the sign of $\Delta^M\text{SS}$ is indeterminate.

The uncertainty effect, $\Delta^U\text{SS}$, is the sum of two terms with opposite signs. Private producers are negatively affected by uncertainty. Whenever they over-produce, the input price exceeds $p^*(\alpha)$ and whenever they under-produce, the input price falls short of $p^*(\alpha)$. In either case, profits fall short of perfect information levels. Input producers, in contrast, are positively affected by uncertainty. Their sales exceed the perfect information level whenever the input price exceeds $p^*(\alpha)$ and fall short of this level whenever the input price is below $p^*(\alpha)$. Parastatal producers are unaffected by uncertainty, since the quantity they produce is independent of price.

$$\mathrm{E}\left[\mathrm{PS}_t - \mathrm{PS}^*(\alpha)\right] \;=\; \frac{2n+b}{2b}\mathrm{E}(p^*)^2 \;-\; \frac{2n+b}{2b}\mathrm{E}\hat{p}_t^2$$

Similarly, denoting the input producer's actual surplus in period t by IS_t and its perfect information surplus given α (and $\bar{q}$) by $\mathrm{IS}^*(\alpha)$, we have:

$$\mathrm{E}\left[\mathrm{IS}_t - \mathrm{IS}^*(\alpha)\right] \;=\; (2b)^{-1}n^2\mathrm{Var}(\hat{p}_t)$$

Summing the two expected differences yields $\Delta^U\mathrm{SS} = -\frac{n}{2}\left(1+b^{-1}n\right)\mathrm{Var}(\hat{p}_t)$. The input quantity shock has no further effect on expected social surplus, since firms are risk neutral and make their input use decisions before the uncertainty is resolved.

Summarizing, expected social surplus in period t as a function of n is:

$$\begin{aligned}\mathrm{V}_t(n) \;=0.5\Bigg\{\frac{N(a+bw^*)^2}{b(N+b)} \quad &- \quad \Delta\bar{q}^2\frac{(N-n)(N+b)}{n+b} \quad - \quad (\sum_{\tau=0}^{t-1}\gamma^\tau)^{-2}\gamma^{2(t-1)}n(n-1)\varsigma^2 \\ &- \quad n\left(1+b^{-1}n\right)\mathrm{Var}(\hat{p}_t)\Bigg\}\end{aligned} \tag{13}$$

3.2. **Present discounted value of expected social surplus.** So far, we have considered the relationship between the size of the private sector and expected social surplus at each given point in time. However, the key policy issue our analysis addresses is: what fraction of firms should be privatized, assuming that this fraction will be fixed for the entire transition period? To answer this question, we consider the decision problem facing a policymaker with discount rate ρ, whose objective is to maximize the present discounted value of expected social surplus, defined as $\mathrm{V}(n) = (1-\rho)\sum_{t=1}^{\infty}\rho^{t-1}\mathrm{V}_t(n)$, and whose only policy instrument is the level of n. Note that $\mathrm{V}(n)$ is a convex combination of the per-period values of expected social surplus (i.e., the weights on the per period values sum to one). Substituting from expression (13), we obtain:

$$\mathrm{V}(n) \;=0.5\left\{\frac{N(a+bw^{*})^2}{b(N+b)} \;-\; \Delta\bar{q}^2\frac{(N-n)(N+b)}{n+b} \;-\; n(n-1)\varsigma^2(1-\rho)\sum_{t=1}^{\infty}\rho^{t-1}\left[(\sum_{\tau=0}^{t-1}\gamma^{\tau})^{-2}\gamma^{2(t-1)}\right]\right.$$
$$\left. -\; n\left(1+\frac{n}{b}\right)(1-\rho)\sum_{t=1}^{\infty}\rho^{t-1}\mathrm{Var}(\hat{p}_t)\right\} \tag{14}$$

We identify conditions under which a unique solution exists for the policymaker's task of maximizing $\mathrm{V}_t(\cdot)$ with respect to n. In general, we cannot do this because discounted expected social surplus is not in general globally concave. In the short-run,however, the per-period ESS's *are* concave, so that a sufficient condition for global concavity is that short-run considerations are sufficiently important to the policymaker. The following proposition makes this precise: there will be a unique optimal level of privatization provided that the policymaker is sufficiently impatient. The result holds under either Assumption 2 or Assumption $2'$.

Proposition 4. *Given any values of b, N and γ, there exists $\bar{\rho} > 0$ such that if the policymaker's discount rate ρ is less than $\bar{\rho}$, then there is a unique level of privatization that maximizes discounted expected social surplus.*

The proposition established conditions under which a unique level of privatization will exist, provided that the policymaker is sufficiently impatient. This unique level may be an interior solution or a corner solution; the proposition does not distinquish between them. An alternative way to guarantee uniqueness is to identify conditions under which a corner solution must obtain. Under Assumption $2'$, full privatization will be optimal provided that the policy-maker is sufficiently *patient*. The key to this result is that in the long-run, the variance of anticipated prices actually *decreases* with n and hence one aspect of the tradeoff between misallocation and uncertainty evaporates, while the other aspect becomes more and more one-sided. Hence if the policy-maker is sufficiently patient, long-run considerations will eventually dominate short-run concerns, and full privatization will be optimal. If the policymaker was infinitely patient, with a social discount rate of 1, then full privatization would always be optimal, since finite short-term losses would be more than offset by infinite long-term gains.[8] By continuity, we can argue that there are social discount rates close to 1 for which full privatization is optimal, which allows us to state the following proposition:

Proposition 5. *Given any values of b, N and γ, if Assumption 2 holds, there exists $\tilde{\rho} < 1$ such that if the policymaker's discount rate ρ exceeds $\tilde{\rho}$, then full privatization will be the unique maximizer of discounted expected social surplus.*

4. CONCLUSION

This paper is premised on the idea that learning is an integral part of the transition from central planning to a market economy. Our focus on learning reveals a welfare tradeoff associated with privatization policy in transition economies, when market liberalization is accompanied by uncertainty over market conditions. A more vigorous privatization program increases both short-run price and production volatility as well as the time it takes for this volatility to works its way out of the system. These effects diminish welfare, so if policymakers are sufficiently concerned with the short-run a policy of less than complete privatization will be optimal. On the other hand, an increase in the number of responsive private producers reduces the misallocation effect due to parastatals' distorted production levels, which is welfare-enhancing. Thus, if policymakers are sufficiently patient a policy of full privatization will be optimal.

The magnitude and the distribution of transition-related uncertainty affect the optimal level of privatization. A uniform increase in the variance of producers' initial signals and all supply shocks reduces the optimal degree of privatization. Reducing the share of total uncertainty borne in the early stages of the transition process increases the optimal level of privatization. In the case with independent identically distributed supply shocks, an increase in the total size of the sector increases the optimal level of privatization if and only if the variance of the initial signals is less than the variance of the supply shocks. The interaction between the input price uncertainty faced by the sector and the optimal level of privatization indicates that uncertainty regarding the effects of government policies in the transition period will affect the optimal level of privatization, possibly making it more costly to privatize in a given sector.

While transition governments are more concerned with dynamic issues, such as the optimal *rate* at which parastatals should be privatized, than with static ones, such as the optimal *level* of privatization, our static analysis has some clear dynamic implications. Specifically, it suggests that the greater the degree of initial uncertainty about market conditions, the more gradually should the privatization process begin. Also, government policies that support information provision and institution-building will be particularly important in the earliest stages of transitions, when their benefits are largest. In addition, information provision will be more important in industries with more privatized producers.

Rather than supporting either side of the big-bang vs. gradualism debate, our analysis adds a new dimension to the debate by emphasizing the learning process. The tradeoff we derive favors gradualism under some circumstances and big-bangs under others. Even when the learning considerations addressed in this paper would suggest a gradualist approach, gradualism may not be optimal when broader considerations, particularly political-economic ones, are taken into account. Regardless of these considerations, however, our analysis indicates that because the big-bang approach fails to acknowledge the costs of rapid privatization in a uncertain environment, its predictions will be likely to be overly optimistic except when uncertainty is minimal or policymakers are very patient and learning is correspondingly unimportant.

APPENDIX

Proof of Lemma 1. Since $\Gamma_\tau \geq 1$, Lemma 1 can be verified by showing that $\lim_{t\to\infty} \Phi(\tau+1, t-1) = \lim_{t\to\infty} \prod_{m=\tau+1}^{t-1} \frac{\left(\Gamma_m - 1 - b^{-1}n\right)}{\Gamma_m} = 0$. Observe that for each $m < \bar{t}(b, \gamma, n)$, $\frac{\left(\Gamma_m - 1 - b^{-1}n\right)}{\Gamma_m} \in [-b^{-1}n, 0)$. Moreover, from Assumption 3, there exists $\zeta(n) \in (0, 1)$ such that for all $m \geq \bar{t}(b, \gamma, n)$, $\frac{\left(\Gamma_m - 1 - b^{-1}n\right)}{\Gamma_m} \in (0, \zeta(n))$. Hence for all τ and $t > \tau$, $|\Phi(\tau+1, t-1)|$ is bounded above by $\left(b^{-1}n\right)^{\bar{t}(b,\gamma,n)}$. Moreover, for $t > \tau > \bar{t}(b, \gamma, n)$, $|\Phi(\tau+1, t)| < \zeta(n)\, |\Phi(\tau+1, t-1)|$, from which Lemma 1 follows. ■

Proof of Proposition 1. Fix $\epsilon > 0$. To prove part (a) of the proposition, we will show that

$$\text{there exists } T > 0 \text{ such that for all } t > T,\ |(\hat{p}_t - p^*)| < \epsilon \tag{15}$$

From Assumption 3, there exists $\alpha \in (0, 1/3)$ satisfying $(1 - 3\alpha) = \lim_{t\to\infty} \frac{\sum_{\tau=1}^{t-1} \gamma^\tau - b^{-1}n}{\sum_{\tau=0}^{t-1} \gamma^\tau}$. Thus, we can pick T' sufficiently large that for all $t > T'$, $(1 - 2\alpha) > \frac{\sum_{\tau=1}^{t-1} \gamma^\tau - b^{-1}n}{\sum_{\tau=0}^{t-1} \gamma^\tau}$. By assumption 2, we can assume additionally that T' is sufficiently large that for all $t > T'$, $\left|\frac{\delta_{t-1}}{b\sum_{\tau=0}^{t-1} \gamma^\tau}\right| < \alpha\epsilon$. It follows from (6) that for $t > T'$, if $|(\hat{p}_{t-1} - p^*)| > \epsilon$, then

$$\begin{aligned} |(\hat{p}_t - p^*)| &= \left|\frac{\delta_{t-1}}{b\sum_{\tau=0}^{t-1} \gamma^\tau}\right| + \frac{\sum_{\tau=1}^{t-1} \gamma^\tau - b^{-1}n}{\sum_{\tau=0}^{t-1} \gamma^\tau}|(\hat{p}_{t-1} - p^*)| \\ &< \alpha\epsilon + (1 - 2\alpha)|(\hat{p}_{t-1} - p^*)| < (1 - \alpha)|(\hat{p}_{t-1} - p^*)| \end{aligned}$$

It follows that there exists T such that $|(\hat{p}_T - p^*)| = (1-\alpha)^{T-t+1}|(\hat{p}_{t-1} - p^*)| \leq \epsilon$, since $\lim_{T\to\infty}(1-\alpha)^{T-t} = 0$. Finally, observe that if $|(\hat{p}_{t-1} - p^*)| \leq \epsilon$, then

$$\begin{aligned} |(\hat{p}_{t+1} - p^*)| &= \left|\frac{\delta_t}{b\sum_{\tau=0}^{t} \gamma^\tau}\right| + \frac{\sum_{\tau=1}^{t} \gamma^\tau - b^{-1}n}{\sum_{\tau=0}^{t} \gamma^\tau}|(\hat{p}_t - p^*)| \\ &< \alpha\epsilon + (1 - 2\alpha)\epsilon = (1 - \alpha)\epsilon < \epsilon \end{aligned}$$

Thus we have established that statement (15) is true, which completes the proof.

To prove part (b), observe that $E_s(\hat{p}_t - p^*) = \sum_{\tau=1}^{s-1} \frac{\Phi(\tau+1,t-1)}{b\Gamma_\tau}\delta_\tau + \sum_{\tau=s}^{t-1} \frac{\Phi(\tau+1,t-1)}{b\Gamma_\tau}E\delta_\tau + \Phi(1, t-1)(\hat{p}_1 - p^*)$. Since $E\delta_\tau = 0$, for all τ, part (b) follows immediately from Lemma 1. ■

Proof of Proposition 2. Suppose $t < \underline{t}(b, \gamma, n)$. Each increment in t adds another positive term to expressions (9) and (10). Moreover, the coefficients on the common terms are larger at t than at $t-1$. Hence $\text{Var}(p_t) > \text{Var}(p_{t-1})$ and $\text{Var}(\hat{p}_t) > \text{Var}(\hat{p}_{t-1})$. For $t \geq \underline{t}(b, \gamma, n)$, each increment in t again adds another positive term to the expressions but the coefficients on common terms are *smaller* at t than at $t-1$. Hence the indeterminacy. Now assume that Assumption 2 holds, let t increase without bound and let $S =$

$\lim_{t\to\infty}\sum_{\tau=1}^{t}\sigma_\tau^2$. (The existence of S is guaranteed by Assumption 2.) Fix $\epsilon > 0$ and $T' > \bar{t}(b,\gamma,n)$ such that $\sum_{\tau=1}^{T'-1}\sigma_\tau^2 > S - 0.5\epsilon b^2$. Now pick $T > T'$ such that $\sum_{\tau=1}^{T'-1}\left(\frac{\Phi(\tau+1,T-1)}{b\Gamma_\tau}\right)^2 + (\Phi(1,t-1))^2 < 0.5\epsilon/S$. T exists because we are summing a fixed number (T') of terms, each of which goes to zero as T increases. Since $T' > \bar{t}(b,\gamma,n)$, $\left(\frac{\Phi(\tau+1,t-1)}{b\Gamma_\tau}\right)^2 < b^{-2}$, for all $t > \tau+1 > T'$ (since $\Phi(\tau+1,t-1)$ is the product of terms, all of which are less than unity, and Γ_τ exceeds unity.) Hence for $t > T$,

$$\begin{aligned}\mathrm{Var}(\hat{p}_t) &= \sum_{\tau=T'}^{t-1}\left(\frac{\Phi(\tau+1,t-1)}{b\Gamma_\tau}\right)^2\sigma_\tau^2 + \sum_{\tau=0}^{T'-1}\left(\frac{\Phi(\tau+1,t-1)}{b\Gamma_\tau}\right)^2\sigma_\tau^2 \\ &< b^{-2}\sum_{\tau=T'}^{t-1}\sigma_\tau^2 + S\left(\sum_{\tau=0}^{T'-1}\left(\frac{\Phi(\tau+1,T-1)}{b\Gamma_\tau}\right)^2\right) < 0.5\epsilon b^2 + 0.5\epsilon b^2 = \epsilon\end{aligned}$$

This proves that $\lim_{t\to\infty}\mathrm{Var}(\hat{p}_t) = 0$. It now follows from (10) that in addition $\lim_{t\to\infty}\mathrm{Var}(p_t) = 0$. ■

Proof of Proposition 3. (a) By definition of $\bar{t}(b,\gamma,n)$, $\left(\Gamma_m - 1 - b^{-1}n\right)$ is negative for every $m < \bar{t}(b,\gamma,n)$. Hence for every $\tau < t < \bar{t}(b,\gamma,n)$, $\sum_{m=\tau+1}^{t-1}\left(\Gamma_m - 1 - b^{-1}n\right)^{-1}$ is negative, so $\frac{\partial\mathrm{Var}(\hat{p}_t)}{\partial n}$ is positive. To see that $\frac{d^2\mathrm{Var}(\hat{p}_t)}{dn^2} > 0$, observe that in the short-run all terms in the summation $\left(\sum_{m=\tau+1}^{t-1}\left(\Gamma_m - 1 - b^{-1}n\right)^{-1}\right)^2$ are negative, so that the square of the sum exceeds the sum of the squares. (b) For $t \geq \bar{t}(b,\gamma,n) > \tau$, $\sum_{m=\tau+1}^{t-1}\left(\Gamma_m - 1 - b^{-1}n\right)^{-1}$ includes both positive and negative terms, so that the coefficients on the σ_τ's cannot be signed in general. (c) Consider the expression for $\frac{d\mathrm{Var}(\hat{p}_t)}{dn}$ in (11). Observe that for each τ, $\lim_{t\to\infty}\sum_{\tau=0}^{t-1}\left(\frac{\Phi(\tau+1,t-1)}{b\Gamma_\tau}\right) = 0$. Also, by assumption, $\lim_{\tau\to\infty}\sigma_\tau^2 = 0$. Finally, as noted in the text below (11), the sequence $\left(\left(\Gamma_m - 1 - b^{-1}n\right)^{-1}\right)_{m=1}^{\infty}$ is bounded. It follows that for every $\epsilon > 0$, there exists T such that for $t > T$, $\left|\frac{d\mathrm{Var}(\hat{p}_t)}{dn}\right| < t\epsilon$. Moreover, there exists $\delta \in (0,1)$ and T' such that for $t > T'$ and every $\tau < t$, $\left|\left(\frac{\Phi(\tau+1,t)}{b\Gamma_\tau}\right)\right| < (1-2\delta)\left|\left(\frac{\Phi(\tau+1,t-1)}{b\Gamma_\tau}\right)\right|$. Hence for t sufficiently large $\left|\frac{d\mathrm{Var}(\hat{p}_{t+1})}{dn}\right| < (1-2\delta)(t+1)\epsilon < (1-\delta)t\epsilon$. That is, $\left|\frac{d\mathrm{Var}(\hat{p}_{t+1})}{dn}\right|$ is eventually dominated by a geometrically decreasing series, and hence converges to zero. (d) Let σ^2 denote the common variance of the δ_τ's and consider the expression inside the curly brackets in display (11). Note that all of the terms in the summation over τ are positive except for when $\tau < \bar{t}(b,\gamma,n)$. Now consider the last term in the summation over τ, $\tau = t-1$. We can pick $\epsilon > 0$ such that for each $t > \bar{t}(b,\gamma,N) \geq \bar{t}(b,\gamma,n)$, the coefficient on

σ^2_{t-1}, $\left(\frac{\Phi(t,t-1)}{b\Gamma_\tau}\right)^2 \left(\Gamma_m - 1 - b^{-1}n\right)^{-1}$, exceeds ϵ. Moreover, from Lemma 1, there exists T such that for $t > T$, $\left|(\Phi(1,t-1))^2 \sum_{m=1}^{t-1} \left(\Gamma_m - 1 - \frac{n}{b}\right)^{-1} \varsigma^2 - \sum_{\tau=1}^{\bar{t}(b,\gamma,n)-1} \left(\frac{\Phi(\tau+1,t-1)}{b\Gamma_\tau}\right)^2 \left(\sum_{m=\tau+1}^{t-1} \left(\Gamma_m - 1 - \frac{n}{b}\right)^{-1}\right) \sigma^2\right|$ is smaller than $\epsilon\sigma^2$. Thus, for $t > T$ all of the negative terms within $\{\cdot\}$ are exceeded in absolute value by the single positive term $\left(\frac{\Phi(t,t-1)}{b\Gamma_\tau}\right)^2 \left(\Gamma_m - 1 - b^{-1}n\right)^{-1}\sigma^2$. Because $\{\cdot\}$ is preceded by a minus sign, it follows that $\mathrm{Var}(\hat{p}_t)$ decreases with n for $t > T$. ■

Proof of Proposition 4. To prove the proposition, it is sufficient to prove that for sufficiently small ρ, $\mathrm{V}(\cdot) = (1-\rho)\sum_{t=1}^{\infty} \rho^{t-1}\mathrm{V}_t(\cdot)$ is strictly concave on the interval $[1, N]$.[9]

$$\frac{d\mathrm{V}(n)}{dn} = 0.5\left\{\Delta\bar{q}^2\left(\frac{N+b}{n+b}\right)^2 - (1-\rho)(2n-1)\varsigma^2\sum_{t=1}^{\infty}\rho^{t-1}\left[(\sum_{\tau=0}^{t-1}\gamma^\tau)^{-2}\gamma^{2(t-1)}\right] - (1-\rho)\sum_{t=1}^{\infty}\rho^{t-1}\left[\left(1+2\frac{n}{b}\right)\mathrm{Var}(\hat{p}_t) + n\left(1+\frac{n}{b}\right)\frac{d\mathrm{Var}(\hat{p}_t)}{dn}\right]\right\} \tag{16}$$

$$\frac{d^2\mathrm{V}(n)}{dn^2} = -\Delta\bar{q}^2\frac{(N+b)^2}{(n+b)^3} - (1-\rho)2\varsigma^2\sum_{t=1}^{\infty}\rho^{t-1}\left[(\sum_{\tau=0}^{t-1}\gamma^\tau)^{-2}\gamma^{2(t-1)}\right] - (1-\rho)\times \sum_{t=1}^{\infty}\rho^{t-1}\left[\frac{\mathrm{Var}(\hat{p}_t)}{b} + \left(1+2\frac{n}{b}\right)\frac{d\mathrm{Var}(\hat{p}_t)}{dn} + \frac{n}{2}\left(1+\frac{n}{b}\right)\frac{d^2\mathrm{Var}(\hat{p}_t)}{dn^2}\right] \tag{17}$$

Consider the expression for $\mathrm{V}_t(n)$ given by (13). Clearly for all n, $\frac{(N-n)}{n+b}$ and $n(n-1)$ are convex in n. Moreover, $\frac{n}{2}\left(1+b^{-1}n\right)$ is both convex and increasing in n. The product $\frac{n}{2}\left(1+b^{-1}n\right)\mathrm{Var}(\hat{p}_t)$ will be convex in n, and hence $\mathrm{V}_t(\cdot)$ will be concave, provided that $\mathrm{Var}(\hat{p}_t)$ is convex and increasing at t. Now Proposition 3 establishes that $\mathrm{Var}(\hat{p}_t)$ is convex and increasing in n for $t < \bar{t}(b,\gamma,n)$. Part (a) of Assumption 3 guarantees that $\bar{t}(b,\gamma,n) > 1$ for all n. Moreover, $\bar{t}(b,\gamma,\cdot)$ increases with n (p. 11). Therefore $\mathrm{V}_t(\cdot)$ is concave on $[1, N]$, for all $t < \bar{t}(b,\gamma,1)$. Indeed, there exists $\bar{\epsilon} > 0$ such that for all $t < \bar{t}(b,\gamma,1)$, $\left|\frac{d^2\mathrm{V}_t(\cdot)}{dn^2}\right| > \bar{\epsilon}$ on $[1, N]$. On the other hand, since both $\left(\frac{d\mathrm{Var}(\hat{p}_t)}{dn}\right)$ and $\left(\frac{d^2\mathrm{Var}(\hat{p}_t)}{dn^2}\right)$ are bounded above (p. 16), it follows that if $\bar{\rho}$ is sufficiently small

$$\frac{d^2\mathrm{V}(\cdot)}{dn^2} = (1-\bar{\rho})\left\{\sum_{t<\bar{t}(b,\gamma,1)} \bar{\rho}^{t-1}\frac{d^2\mathrm{V}_t}{dn^2}(\cdot) + \sum_{\tau>\bar{t}(b,\gamma,1)} \bar{\rho}^{t-1}\frac{d^2\mathrm{V}_t}{dn^2}(\cdot)\right\}$$

$$< -(1-\bar{\rho})\left\{\sum_{t<\bar{t}(b,\gamma,1)} \bar{\rho}^{t-1}\bar{\epsilon} + \bar{\epsilon}\right\} \leq 0.$$

■

Proof of Proposition 5. To prove the proposition it is sufficient to show that for all $n \in [1, N]$, $\frac{d\mathrm{V}(n;\rho)}{dn}$ is positive (see expression (16)). This first term on the right hand side is positive and bounded away from zero. The second term approaches zero as t approaches infinity. From Proposition 2, $\lim_{t\to\infty} \mathrm{Var}(\hat{p}_t) = 0$, while from Proposition 3, $\lim_{t\to\infty} \frac{d\mathrm{Var}(\hat{p}_t)}{dn} = 0$, so that the weighted sum of the $\mathrm{Var}(\hat{p}_t)$'s and the second term can be made arbitrarily small by choosing ρ sufficiently close to zero. ■

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