When Promoters Like Scalpers*

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Abstract

If a monopoly supplies a perishable good, such as tickets to a performance, and is unable to price discriminate within a period, the monopoly may benefit from the potential entry of resellers. If the monopoly attempts to intertemporally price discriminate, the equilibrium in the game among buyers is indeterminate when the resellers are not allowed to enter, and the monopoly's problem is not well defined. An arbitrarily small amount of heterogeneity of information among the buyers leads to a unique equilibrium. We show how the potential entry of resellers alters this equilibrium.

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"The moment a performance begins, that seat is dead. ... It's like fruit. Its perishable."
(Jeffrey Seller, producer of *Rent*. *New York Times,* July 20, 2003.)

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1 Introduction

A perishable-good monopoly that can charge different prices in different periods but cannot price discriminate within a period may benefit from the existence of a resale market. We illustrate the effects of resellers on the intertemporal pricing strategy of a monopoly that sells perishable tickets that have no value after the event. For specificity, we discuss tickets for a concert or other performance, which are resold by ticket agencies, brokers, "scalpers" (United States), or "touts" (Great Britain).

We study two versions of a two-period model. Consumers have common knowledge about the value of a ticket in one version, and they lack common knowledge in the other. We model the situation without common knowledge by using a "global game" as in Carlsson and Van Damme (1993) and Morris and Shin (1998). Buyers play a pure coordination game when resale agencies (hereafter "scalpers") cannot enter; in the global games setting (i.e., when common knowledge is removed) the equilibrium is unique. When scalpers can enter, buyers' actions are no longer global strategic complements, but in the absence of common knowledge the equilibrium is unique within a particular class of strategies. We examine the equilibrium effect of the potential entry of scalpers.

The historical and the contemporary relation between scalpers and monopoly ticket providers is mixed; in some circumstances the monopoly explicitly cooperates with the scalpers, and in other cases it attempts to eliminate or undermine them. For example, monopolies have successfully lobbied for anti-scalping laws in many jurisdictions (Williams 1994). Thus, there must be circumstances where scalpers either benefit or harm the monopoly. The assumptions of our model imply that scalpers do not harm and possibly benefit the monopoly. Relevant examples include ticket agencies and brokers who explicitly receive cooperation from monopoly ticket providers. For many Broadway (and other) shows, ticket agencies sell a substantial portion of the tickets (Leonhardt 2003).

In our model, some consumers have a relatively high willingness to pay ("high types"), while others have a lower willingness ("low types"). The monopoly cannot distinguish between the two types of consumer except by observing their behavior over time. Consequently, the monopoly cannot price discriminate if it sells all its tickets in a single period, but it can charge different prices in different periods. By incurring a transaction cost, scalpers can distinguish between consumers at a moment in time and hence can price discriminate within a period. The potential entry of scalpers changes the monopoly's pricing problem and the equilibrium of the

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game.

In a one-period model, the monopoly uses (efficient) scalpers as its agents. It sells all of the tickets to them at a price between the high and low willingness to pay. Scalpers resell as many tickets as possible to high types at a relatively high price, and unload the remaining tickets to low types at a lower price, as in Rosen and Rosenfield (1997) and DeSerpa (1994). That is, the monopoly requires that the scalpers buy a "bundle" of tickets; some they can sell at a high price and others they have to dump.

Most previous analyses of scalpers or ticket sales use one-period models; many of these take the price set by the monopoly as given and they ignore how scalpers affect the market.[1] Theil (1993) assumes that the monopoly under-prices tickets (for exogenous reasons) and then examines whether scalpers raise or lower surplus in a one-period model. Swofford (1999) considers price discrimination and explains why different views toward risk, different cost functions, or different abilities to price discriminate may make scalping profitable. Happel and Jennings (1995) discuss anti-scalping laws without using a formal model. In an empirical study, Williams (1994) finds that scalpers increase the average National Football League ticket price by nearly $2, a result that is consistent with scalping benefiting the monopoly.

Courty (2003a) and Courty (2003b) use two-period models in which the monopoly price discriminates over time. Courty's model and ours differ in many respects; a major difference, largely responsible for our differing conclusions, concerns the assumption about when agents know their willingness to pay. In our model, high and low types have the same amount of information about their willingness to pay, and the value of the ticket does not depend on when it is purchased. Courty assumes that consumers with high (potential) willingness to pay do not know whether they actually want to buy a ticket until the final period; consumers with low willingness to pay would buy a ticket only in the first period (because they are not able to make last-minute arrangements, e.g. for the baby-sitter). In Courty's setting – but not in ours – scalpers might harm the monopoly. The differing assumptions and results in Courty's and in our models are consistent with the observation that in some circumstances monopoly ticket sellers cooperate with scalpers and elsewhere they resist them.

Peck (1996) observes in passing that the presence of scalpers may determine whether an

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[1]Some papers on ticket sales discuss price discrimination by the monopoly (but generally do not mention scalpers), e.g., Barro and Romer (1987), Baumol and Baumol (1997), Bertonazzi, Maloney, and McCormick (1993), DeSerpa (1994), Gale and Holmes (1993), DeGraba and Hohammed (1999), Huntington (1993) and Lott and Roberts (1991).

efficient market exists. Van Cayseele (1991) shows that a monopoly with the ability to commit may engage in intertemporal price discrimination in a market with rationing. As in our model, prices are higher in the first period and then fall. Our model differs from his in that we assume that the monopoly cannot price discriminate but that the scalpers can.

The next section analyzes the role of scalpers in a model of common knowledge about payoffs, taking as given the monopoly's first period price. The following section shows how the equilibrium changes when agents have incomplete knowledge of payoffs. We then show how scalpers affect the monopoly's decision in the first period.

2   The model with common knowledge

The first subsection describes the model and introduces the notation. The next subsection considers the second-period equilibrium with and without scalpers, taking as given first-period sales. The following subsection studies the first period equilibrium. When scalpers are excluded from the market, high-types play a coordination game in the first period for a range of first period prices. The ability of scalpers to enter changes the first-period equilibrium by introducing a form of congestion. We then compare the monopoly's first-period pricing decision with and without scalpers.

2.1   Model description

Each potential customer wants to buy one ticket. There are a continuum of buyers, with no "atoms"; that is, all buyers are infinitesimally small relative to the total market. We denote H as the measure of high-type buyers who are willing to pay up to p^h for a ticket and L as the measure of low-type buyers whose willingness to pay is p^l. The difference in willingness to pay between the two types of customers is $D \equiv p^h - p^l > 0$. The monopoly has measure T tickets available and faces excess demand of $E \equiv H + L - T > 0$. All agents are risk neutral.

The second (and last) period occurs shortly before the performance is about to begin; after that time the tickets are worthless. We consider two extreme cases: there are no scalpers or an unlimited number of scalpers with free entry. We study the set of subgame perfect equilibria in which the monopoly chooses each period's price at the beginning of that period; consumers decide whether to buy in a given period; and scalpers (when they are present) decide whether to participate.

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The monopoly cannot distinguish between high and low types or between regular customers and scalpers. If the monopoly sets a price $p \leq p^l$ there is excess demand for tickets; rationing means that each agent has an equal probability of getting a ticket. The monopoly can charge different prices in different periods, as in Dudey (1996) and Rosen and Rosenfield (1997). In each period, consumers take the current price and the actions of all other agents as given. In the first period, consumers have rational point expectations about the second-period price.

Scalpers price discriminate and extract all potential surplus from buyers, but they incur a transaction cost. A scalper that sells a ticket for price p receives revenue net of transactions costs of ϕp, where $\phi < 1$. That is, the scalper's transaction cost per ticket is $(1 - \phi)p$. Due to free entry, risk-neutral scalpers earn zero expected profit.

We show that in equilibrium, there are no first-period sales to scalpers or low types. In the first period, an endogenously determined fraction α of high types choose not to buy (i.e., to "wait"). The high types' payoff from buying in the first period equals the difference between their valuation and the price, $p^h - p_1$. The expected payoff from waiting equals the probability of getting a ticket in the second period, times the difference between p^h and the price they will have to pay. High types who wait to buy must compete with low types and possibly scalpers in the second period. The equilibrium value of α depends on the first-period price and on whether scalpers are allowed to enter in the second period. If $0 < \alpha < 1$ in equilibrium, then all high types must be indifferent between buying or waiting in the first period.

It is worth emphasizing that we consider a restricted, but reasonable set of mechanisms. The monopoly's only decision variables are the prices in the two periods. A richer policy menu might enable the monopoly to achieve higher profits.

2.2 The second-period equilibrium

The second-period equilibrium depends on whether scalpers are allowed to enter. When scalpers are allowed to enter, S scalpers – an endogenous measure – try to buy tickets, and s scalpers – an endogenous measure – actually obtain a ticket. All buyers pay the price p_2 in the second period. The equilibrium number of low types that try to buy a ticket from the monopoly is L^*. Individuals who are indifferent between buying and not buying break the tie by trying to buy a ticket. Thus, $L^* = L$ if $p_2 \leq p^l$, and $L^* = 0$ if $p_2 > p^l$.

Given L^* and s, the probability that a high type is able to buy a ticket from the monopoly in

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the second period is

$$\theta = \frac{T - (1-\alpha)H - s}{\alpha H + L^*}. \tag{1}$$

The numerator of θ is the number of remaining tickets after $(1-\alpha)H$ were sold in the first period and scalpers obtain s tickets. The denominator is the total number of low types and high types trying to obtain a ticket from the monopoly. We adopt

Assumption 1 $p^l T > p^h H$ *(i.e., the monopoly would earn more by selling all tickets at the low price rather than selling only to high-types)*

We have (see the Appendix for all proofs)

Lemma 1 *If scalpers are allowed to enter and Assumption 1 holds, then the optimal second-period price is $p_2^* = \max\left\{p^s\left(\alpha\right), \ p^l\right\}$ where*

$$p^s(\alpha) \equiv \phi \frac{(T-H)\,p^l + \alpha p^h H}{T - (1-\alpha)\,H}. \tag{2}$$

If $p^s\left(\alpha\right) > p^l$, scalpers obtain all the tickets remaining after $(1-\alpha)\,H$ were bought in the first period: $s = T - (1-\alpha)\,H$.

By setting $\alpha = 1$ (i.e., all sales occur in the second period) we obtain the special case of a one-period model in which the monopoly uses scalpers as its agents because of their ability to price discriminate.

The function $p^s(\alpha)$ in Equation (2) is increasing in α. We define $\hat{\alpha}$ as that value of α that satisfies $p^s(\alpha) = p^l$:

$$\hat{\alpha} \equiv \frac{(1-\phi)\left(\frac{T}{H} - 1\right)p^l}{(\phi p^h - p^l)} > 0. \tag{3}$$

A necessary and sufficient condition for $\hat{\alpha} < 1$ is that $\phi > T/(T + DH)$. This inequality states that scalpers capture a large amount of the surplus – their transactions costs are low. If $\alpha = \hat{\alpha}$, the monopoly is indifferent between selling all the tickets to scalpers or to ordinary buyers. We assume that in the case of this tie, ordinary customers obtain all the tickets; the monopoly does not sell to scalpers if $\alpha \leq \hat{\alpha}$.

2.3 The first-period equilibrium

To determine the effect that scalpers have on the equilibrium, we first consider the equilibrium where scalpers are not permitted in the market and then the one in which they may enter. When scalpers are not able to enter, we show the high-type buyers play a coordination game in the first period for a range of prices. The game has two equilibria: Either no high types or all high types buy in the first period. We define the "limit price," $\bar{p}$, as the supremum of prices at which there exists an equilibrium where all high types buy in the first period. We define the "choke price," $\tilde{p}$, as the infimum of prices at which there exists an equilibrium where no high types buy in the first period. We show that *the limit price in this setting is higher than the choke price*. The buyers' behavior is indeterminate for any first-period price that satisfies $\tilde{p} < p_1 < \bar{p}$. Consequently, the monopoly is not able to solve its profit maximization problem because it cannot calculate expected quantity demanded over some range of prices.

We then consider the first-period equilibrium in the game where scalpers are allowed to enter the market. With no loss in generality, we assume that scalpers do not enter in the first period. If the monopoly were to set the first-period price low enough to induce the scalpers to enter, they would buy all of the tickets. The monopoly can achieve the same level of profit by selling all tickets to scalpers in the second period rather than in the first period. That is, it can set the first-period price so high that no one buys in the first period, and then sell all the tickets to scalpers in the second period. Selling all of the tickets to scalpers in the first period and selling all of the tickets to them in the second period are equivalent. However, the monopoly may do better by setting a first-period price that is high enough to discourage scalpers from entering, but low enough to induce some high types to buy in the first period. The same kind of argument shows that selling to low-types in the first period is a dominated strategy; therefore, $p_1 > p^l$.

The possibility that scalpers enter creates a credible threat that high types will receive 0 surplus if they do not buy in the first period. For prices below the limit price, this threat eliminates the equilibrium where no high types buy in the first period. Thus, the potential for scalpers to enter – whether or not they actually appear in equilibrium – removes the first-period indeterminacy for prices $\tilde{p} < p_1 < \bar{p}$. However, for some range of prices there exists no equilibrium with scalpers. The introduction of scalpers removes one source of indeterminacy but creates another.
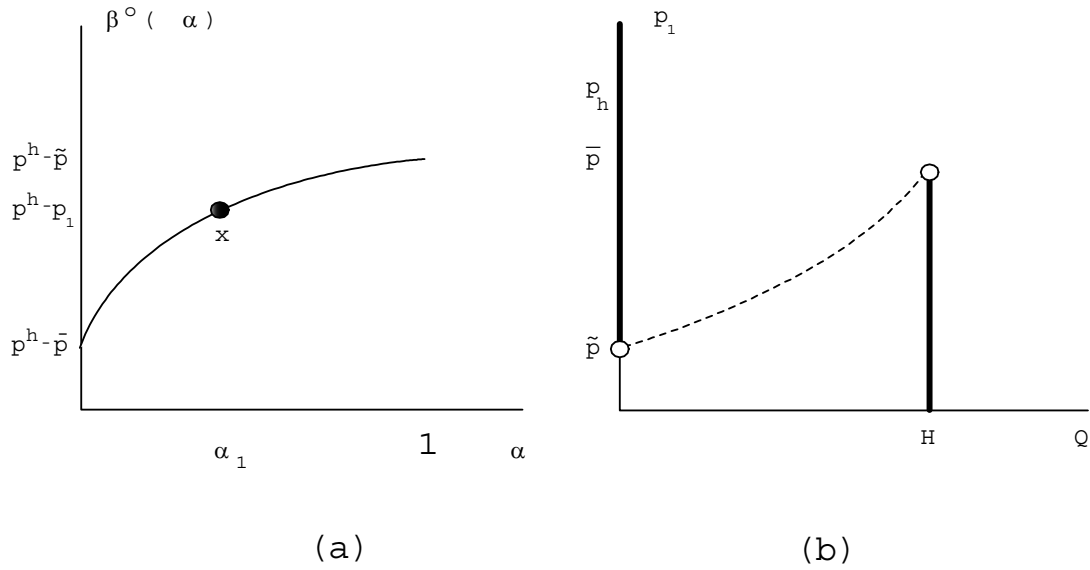
Figure 1: The market without scalpers. (a) Benefit of waiting. (b) Demand

2.3.1 The market without scalpers

We start by examining the market without scalpers. High-type buyers decide whether to buy in the first period or wait until the second period with the hope of obtaining a cheaper ticket. We define the high-type buyer's expected benefit of waiting (not buying in period 1) when scalpers are excluded as $B^0(\alpha)$. Hereafter the superscript "0" means that we assume that scalpers are prohibited from entering the market. We also simplify our notation by adopting the normalizations $T = p^l = 1$, so $p^h = D + 1$.

The expected benefit of waiting equals the consumer surplus, D, obtained from buying at the lower price times the probability, θ, that the high-type buyer will be able to obtain a ticket at this price in the second period. Because the equilibrium values in the second period are $p_2^* = p^l = 1$ and $L^* = L$, the probability is $\theta = [1 - (1 - \alpha)H]/[\alpha H + L]$. Thus, the expected benefit to a high type from waiting is the increasing, concave function

$$B^0(\alpha) = D\frac{1 - (1 - \alpha)H}{\alpha H + L}. \tag{4}$$

Figure 1a graphs $B^0(\alpha)$. If the monopoly sets the low choke price $\tilde{p}$, and if all high types wait to buy ($\alpha = 1$), then the surplus from buying in this period equals the expected benefit of

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waiting: $p^h - \tilde{p} = B^0(1)$. If the monopoly charges the limit price, $\bar{p}$, and if all high types buy in this period ($\alpha = 0$), then $p^h - \bar{p} = B^0(0)$. Using these definitions and our normalizations, we have

$$\bar{p} \equiv 1 + \frac{DE}{L} > \tilde{p} \equiv 1 + \frac{DE}{E+1} \tag{5}$$

Any point on the $B^0(\alpha)$ curve for $0 \leq \alpha \leq 1$ is an *unstable* Nash equilibrium.[2]

Using Figure 1a, we can derive the first-period demand correspondence in the absence of scalpers, Figure 1b. At prices $\tilde{p} < p_1 < \bar{p}$, there are two stable equilibrium demands in the first period, $Q = H$ (where $\alpha = 0$) and $Q = 0$ (where $\alpha = 1$) represented by the heavy vertical lines, and an unstable equilibrium set of demands represented by a dashed curve. (Because the set of unstable equilibrium demands is rising, this "demand curve" is upward sloping.) As more agents buy in the first period, the chance of getting a ticket at the low price in the second period diminishes, making it more attractive to buy early. An additional purchase in the first period lowers both demand and supply by one unit in the second period. The net effect is to lower the probability of being able to buy a ticket in the second period.

Thus, an increase in the measure of agents taking an action (buying or not buying), increases the benefit to other agents of taking the same action. Due to this externality, buyers play a coordination game. At any first-period price that satisfies $\tilde{p} < p_1 < \bar{p}$, the high types' expected payoff is greater in the equilibrium where no one buys ($\alpha = 1$). In the equilibrium where all high types buy, their payoff is $p^h - p_1$, whereas their expected payoff is $B^0(1) > p^h - p_1$ in the equilibrium where no one buys.

Efficiency is greater in the equilibrium where all high types buy. In the absence of scalpers, the only source of inefficiency is that some high types might end up without tickets. Only a

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[2] For example, consider point x in Figure 1a. This point corresponds to first-period price p_1 (where $\tilde{p} < p_1 < \bar{p}$) and a fraction of high types who wait, α_1, with $B^0(\alpha_1) = p^h - p_1$. The proposed equilibrium pair (p_1, α_1) is unstable: agents who believe that a positive measure of high types will deviate from the proposed equilibrium would want to follow that deviation. (A "deviation" means that high types do the opposite of what they are "instructed" to do in equilibrium; they wait when they are supposed to buy, or buy when they are supposed to wait.) If the measure $\mu > 0$ of high types deviate from the proposed equilibrium by buying rather than waiting, then $\alpha = \alpha_1 - \mu < \alpha_1$. With this deviation, all high types prefer to buy in the first period. Similarly, if the measure $\mu > 0$ of high types deviate from the proposed equilibrium by waiting rather than buying, all high types strictly prefers to wait. Thus, a small deviation from the proposed equilibrium generates an increasingly large deviation in the same direction, until a boundary is reached (all wait or all buy). By a similar argument, the two points $(\alpha, p_1) = (0, \bar{p})$ and $(\alpha, p_1) = (1, \tilde{p})$ are unstable; in these cases, we need only consider "one-sided deviations".

fraction $\theta < 1$ of high types obtain tickets when $\alpha = 1$, but they all obtain tickets when $\alpha = 0$.

Figure 1b illustrates why we call $\tilde{p}$ the choke price and $\bar{p}$ the limit price. In Figure 1b, $\tilde{p}$ is the greatest lower bound of the set of prices at which there is a stable equilibrium demand of 0. Similarly, $\bar{p}$ is the least upper bound of prices at which there is a stable equilibrium in which all high types buy.

We summarize the discussion above as:

Proposition 1 *When scalpers are not permitted to enter the market, the equilibrium is indeterminate for first-period prices $\tilde{p} < p_1 < \bar{p}$. At those prices there are two possible equilibria, $\alpha = 1$ and $\alpha = 0$. Buyers prefer the first equilibrium but social welfare (efficiency) is greater in the second. All high types buy in the first period if $p_1 \leq \tilde{p}$ and none buy if $p_1 \geq \bar{p}$. The second-period price is $p^l = 1$, regardless of the first-period price.*

2.3.2   The market with scalpers

Now we describe the equilibrium set when the monopoly is allowed to sell to scalpers in the second period. By Lemma 1, the high type's expected value of waiting is

$$B(\alpha) = \left\{ \begin{array}{c} B^0(\alpha) \text{ for } \alpha \leq \hat{\alpha} \\ 0 \text{ for } \alpha > \hat{\alpha} \end{array} \right\}, \tag{6}$$

where $\hat{\alpha}$ is the critical value of α, above which the monopoly sells only to scalpers.

If $\hat{\alpha} \geq 1$ it never pays the monopoly to sell to scalpers (because α can never be greater than 1), so their ability to enter the market is irrelevant. The only interesting case is when $\hat{\alpha} < 1$, as we hereafter assume. We restate this inequality and Assumption 1 as:

Assumption 2 $\frac{1}{D+1} > H > \frac{1-\phi}{\phi D}$.

If $\alpha > \hat{\alpha}$, high types who did not buy in the first period have to deal with scalpers in the second period, and they capture no surplus. Figure 2a graphs $B(\alpha)$ under Assumption 2, and Figure 2b shows the related demand correspondence. The price $\hat{p}$ is the solution to $p^h - \hat{p} = B(\hat{\alpha})$. Comparison of Figures 1a and 2a shows that when scalpers can enter the market, the benefit of waiting is no longer an increasing function of the number of other high types who wait. As α increases from below $\hat{\alpha}$ to above $\hat{\alpha}$ the action "wait to buy" changes from a strategic complement to a substitute.
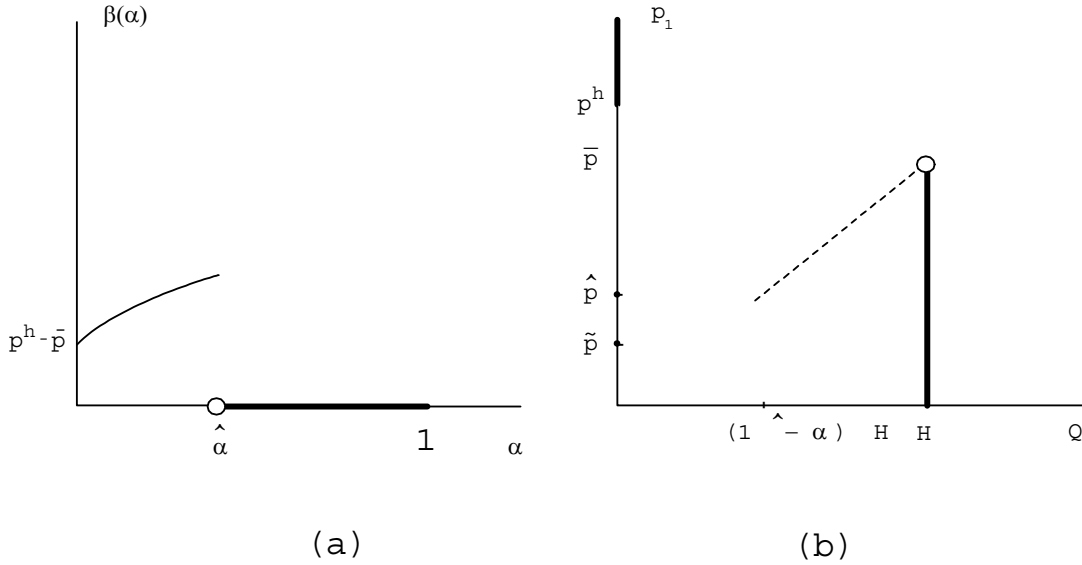
Figure 2: The market with scalpers. (a) Benefit of waiting. (b) Demand.

If scalpers can enter the market, then $\alpha = 1$ is not an equilibrium for prices $p_1 \leq \bar{p}$. At any price $p_1 < \bar{p}$, high types strictly prefer to buy in the first period rather than wait, regardless of whether $\alpha = 0$ or $\alpha = 1$. Interior values of α ($0 < \alpha < 1$) cannot be *stable* equilibria, for the same reason as in the model without scalpers. At $p_1 = \bar{p}$, high types prefer to buy in the first period if $\alpha = 1$, and they are indifferent as to when they buy if $\alpha = 0$. However, $\alpha = 0$ is not a stable equilibrium at $p_1 = \bar{p}$, because other high types would want to mimic the deviation if a positive measure of buyers were to deviate by waiting.

We now show that there is no equilibrium where the first-period price satisfies $\bar{p} \leq p_1 < p^h$. If $\bar{p} < p_1 < p^h$, then in equilibrium it cannot be the case that $\alpha \leq \hat{\alpha}$. When $\bar{p} < p_1 < p^h$ and $\alpha \leq \hat{\alpha}$, the payoff of waiting is strictly greater than the payoff of buying in the first period. High types who were "supposed to buy" would want to deviate by waiting. Similarly, it cannot be the case that $\alpha > \hat{\alpha}$ because then the payoff of buying is strictly greater than the payoff of waiting. Again, if $p_1 = \bar{p}$ there is no stable equilibrium. If $p_1 = p^h$, on the other hand, the only stable equilibrium is $\alpha = 1$.

If the monopoly sets the first-period price at $\bar{p} - \delta$ (where δ is an arbitrarily small positive number), it sells to all high types in the first period, and its total revenue from sales in both periods is $(\bar{p} - \delta) H + (T - H)$. If the monopoly sets the first-period price above p^h, it sells all tickets to scalpers in the second period, and obtains the revenue $p^s(1)$. Using Equation (2),

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we find that the monopoly prefers to sell to scalpers rather than setting $p_1 = \bar{p} - \delta$ if and only if

$$\phi > \frac{DH(L+H-1)+L}{L(1+HD)}. \tag{7}$$

We summarize these results in the following:

Proposition 2 *Under Assumption 2, all high types buy in the first period if $p_1 < \bar{p}$ (the limit price) and none buy if $p_1 \geq p^h$. The monopoly prefers to sell all tickets through scalpers rather than selling to high types at the limit price if and only if scalpers are sufficiently efficient; i.e., if and only if ϕ satisfies Equation (7).*

2.4 A comparison

The monopoly's expected demand function is indeterminate over a range of prices with or without scalpers. However, the source of the problem is different in the two settings. In the absence of scalpers, there are two equilibrium levels of demand for a range of prices $\tilde{p} < p_1 < \bar{p}$. At these prices, the relation between the monopoly's profit and price is a correspondence rather than a function. Consequently, we cannot determine the monopoly's optimal first-period price. With scalpers, the equilibrium, conditional on the price, is unique if it exists. However, for a range of prices, $\bar{p} \leq p_1 < p^h$, the equilibrium does not exist. For these prices, monopoly profits are not defined. Nonetheless, we can show that the ability of scalpers to enter the market increases monopoly profits, reduces high type consumer welfare, and has ambiguous effects on efficiency.

Scalpers do not reduce and may increase monopoly profits. If the monopoly charges $\bar{p} - \delta$ and scalpers cannot enter, the supremum monopoly profit is $\bar{p}H + (1 - H)$ when all high types buy. When scalpers can enter, the monopoly is guaranteed approximately the supremum level of profit, because the threat of scalpers eliminates the equilibrium where all high types wait. If ϕ satisfies the inequality (7), the monopoly does even better by selling to scalpers.

Scalpers reduce consumer welfare. We noted in Proposition 1 that consumer welfare without scalpers is greatest in the equilibrium where all high types wait ($\alpha = 1$). Their surplus is smallest when the monopoly charges slightly less than $\bar{p}$ and all high types buy in the first period. If the inequality (7) is not satisfied, the monopoly can charge (approximately) $\bar{p}$ and eliminate the equilibrium $\alpha = 1$. Consumers get the minimum payoff they would have received in the absence of scalpers. If the inequality (7) is satisfied, consumers receive zero surplus.

Numerical experiments show that scalpers have an ambiguous effect on social welfare. Scalpers can increase social welfare by allocating tickets, and they can decrease social welfare if the transactions cost is high. If scalpers transactions cost is high and the excess demand for tickets is low (so that the social benefit of allocating tickets is small) scalpers reduce social welfare. With low transactions costs and large excess demand, they increase social welfare.

3   The model without common knowledge

Here we drop the assumption that high types have common knowledge about the value of a ticket. The absence of common knowledge removes the indeterminacy and nonexistence of equilibria described in the previous section. The change also makes the predictions of the model more plausible. The first subsection describes how information changes over time. The next two subsections analyze the buyers' first period problem with and without scalpers, taking as given the first period price.

In the absence of common knowledge about payoffs, the model without scalpers is a standard "global game"; Morris and Shin (2003) explain the techniques and the intuition of this sort of game. Its key features are that agents receive private signals about payoffs, there exist values of the unknown payoff-relevant parameter for which taking either action is a dominant strategy, and actions are global strategic complements. The last assumption means that the advantage of taking a particular action (e.g. buying a ticket in the first period) increases with the measure of other agents who take the same action. These features (together with some technical assumptions) imply that there is a unique equilibrium in which agents take a particular action if and only if their signal exceeds a certain level – a "threshold equilibrium".

The model with scalpers does not satisfy the global complementarity assumption; in this model we have a unique threshold equilibrium, but we cannot rule out the existence of other kinds of equilibria. Two recent papers, Goldstein and Pauzner (2003) and Karp, Lee, and Mason (2003), also relax the global complementarity assumption in a global games setting. Goldstein and Pauzner (2003) show that in a model of bank runs there is a unique equilibrium – a threshold equilibrium; Karp, Lee, and Mason (2003) show that when strategic substitutability of actions is sufficiently strong there exists no equilibrium that is monotonic in the signal, thus excluding threshold equilibria. These two papers and our model with scalpers thus provide three examples of the effect of relaxing the assumption of global complementarity in a

global games setting. An appendix, available on request, compares these models and explains why they reach different conclusions concerning the existence and uniqueness of threshold strategies. This explanation is based on a comparison of the graphs of the payoff of taking a particular action, as a function of the number of other agents who take this action. In none of the models is this graph monotonic (i.e., global complementarity does not hold). A single crossing property holds in Goldstein and Pauzner (2003) but not in the other two models.

3.1 The flow of information

Buyers do not know the true value of the ticket in the first period; each buyer has a private assessment of the value. For example, buyers assess the quality of the performance based on knowledge of the identity of the participants (the director, performers, team members). Buyers do not know the exact assessment that others have made, but they know something about the distribution of those assessments. In the second period, an important piece of information becomes common knowledge. For example, potential customers read critics' reviews, find out which actors are in the cast, or learn the identities of playoff teams. When this information becomes available, all buyers know the value of the ticket; they all know that all other buyers know the value, and they know that others know that others know, *ad infinitum.*

Formally, the true quality of the performance is given by a parameter γ. The true value of a ticket for type i is γp^i, $i = h, l$. Before sales in the first period, and before observing a private signal, high-type buyers have non-informative priors (i.e. a diffuse prior) on γ. Therefore, before observing their private signal, these buyers think that the value of γ might be either so high (or so low) that it is a dominant strategy for them to buy (or to wait) in the first period, regardless of the actions of other buyers. These ranges of very high or very low signals are called dominance regions.

Each high-type buyer receives a signal η about the quality of the performance (and hence the value of the ticket); η is drawn from a uniform distribution:

$$\eta \sim U[\gamma - \epsilon, \gamma + \epsilon],$$

where ϵ is a measure of the amount of uncertainty or heterogeneity of information. The distribution of η is common knowledge, but the value of the individual's signal is private information, and γ is unknown to all buyers. Whether the individual decides to buy or to wait and attempt to obtain a cheaper ticket in the second period depends on the first-period price, the individual's

private signal, and this person's beliefs about what other agents will do. Buyers do not regard p_1 as a signal about quality.

At the beginning of the second period, the true value of γ is revealed, and the fraction of high types who waited, α, is also public information. Thus, the analysis of the second period is the same as in the complete information game, except that γp^i replaces p^i, $i = h, l$. The outcome in the second period depends on α and on whether scalpers can enter.

3.2 The buyers' problem without scalpers

If scalpers cannot enter in the second period, buyers know that the second-period price will be $\gamma p^l = \gamma$. In period 1, high-type buyers receive a signal, observe the first-period price, and decide whether to buy.

Let $\pi(\eta)$ be a decision rule; $\pi(\eta)$ is the probability that a high type who receives the signal η decides to wait (i.e., not buy in the first period). This class of decision rules includes mixed strategies, but we show that the unique *equilibrium decision rule* is a pure strategy. If the equilibrium decision rule is $\pi(\eta)$, the fraction of high types who do not buy when the value of the unknown parameter is γ is[3]

$$\alpha = \alpha\left(\gamma; \pi\right) = \frac{1}{2\epsilon} \int_{\gamma-\epsilon}^{\gamma+\epsilon} \pi(\eta) d\eta. \tag{8}$$

If the value of the unknown parameter is γ, then the fraction $\alpha(\gamma; \pi)$ of high types wait, and the payoff in the second period is $\gamma B^0(a(\gamma; \pi)) \geq 0$. The posterior distribution of γ, conditional on the signal η, is uniform over $[\eta - \epsilon, \eta + \epsilon]$. Given a signal η and given that the buyer expects other high types to use the equilibrium strategy π, the expected value of waiting is

$$V^0(\eta; \pi) = \frac{1}{2\epsilon} \int_{\eta-\epsilon}^{\eta+\epsilon} \gamma B^0(a(\gamma; \pi)) d\gamma. \tag{9}$$

The expected value of buying in the first period is $\eta p^h - p_1$ The *advantage of waiting* is the difference in the expected benefit of waiting and the expected value of buying the first period:

$$A^0(\eta, p_1; \pi) = V^0(\eta; \pi) - \left(\eta p^h - p_1\right). \tag{10}$$

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[3]This expression is valid for values of γ more than ϵ distance from the boundaries of the support of the prior for γ. Although this qualification applies to the subsequent integrals in the text, we do not repeat it. However, we are careful about these limits of integration in the proofs.

The function $\pi(\eta)$ represents a general strategy. We now consider a particular type of strategy – a threshold strategy – that we denote $I_k(\eta)$. This threshold strategy is a step function:

$$I_k(\eta) = \left\{ \begin{array}{l} 1 \text{ if } \eta < k \\ 0 \text{ if } \eta \geq k \end{array} \right\}. \tag{11}$$

The parameter k is the threshold signal. If $\pi(\eta) = I_k(\eta)$, then high types buy if and only if they receive a signal greater than or equal to k, and the fraction of high types who wait is (using Equation (8))

$$\alpha\left(\gamma; I_k\right) = \left\{ \begin{array}{c} 1 \text{ if } \gamma + \epsilon < k \\ \frac{k - \gamma + \epsilon}{2\epsilon} \text{ if } \gamma - \epsilon \leq k \leq \gamma + \epsilon \\ 0 \text{ if } k < \gamma - \epsilon \end{array} \right\}. \tag{12}$$

We use a variation of a proof in Morris and Shin (1998) to show that the unique equilibrium decision rule $\pi(\eta)$ is a step function $I_k(\eta)$, and that the threshold k is unique. We characterize the equilibrium value of k as a function of an arbitrary first-period price, p_1, the amount of uncertainty, ϵ, and the other parameters of the model. We state the result in terms of functions ρ^0 and ϖ^0, which depend only on exogenous parameters. The Appendix defines these functions.

Proposition 3 *If scalpers are unable to enter in the second period and Assumption 1 holds, there is a unique equilibrium to the game with uncertainty. In this equilibrium, high types buy in the first period if and only if they receive a sufficiently high signal. That is, $\pi^*(\eta) = I_k$. The critical signal k^{0*} is given by the formula*

$$k^{0*} = -\frac{\varpi^0 \epsilon + p_1}{\rho^0}; \quad \rho^0 < -1, \quad \varpi^0 < 0. \tag{13}$$

This threshold is an increasing function of the first-period price and a decreasing function of the uncertainty parameter, ϵ.

3.3 The buyers' problem with scalpers

We now turn to the equilibrium where scalpers are allowed to enter in the second period. In the second period, when the quality, γ, and the measure of high types who do not yet have tickets, αH, are common knowledge, the equilibrium is the same as in Section 2. The monopoly wants to sell to scalpers if and only if $\alpha > \hat{\alpha}$, defined in Equation (3). Our assumption that types' willingness to pay is γp^h and γp^l implies that $\hat{\alpha}$ is independent of γ (see equation (3)).

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For an arbitrary strategy $\pi(\eta)$, Equation (8) gives the fraction of high types who wait. If a high type receives a signal η and other high types use the strategy π, the expected value of waiting is

$$V(\eta; \pi) = \frac{1}{2\epsilon} \int_{\eta-\epsilon}^{\eta+\epsilon} \gamma B(a(\gamma; \pi)) d\gamma. \tag{14}$$

Equation (6) defines the function $B(\cdot)$. The expected advantage of waiting is

$$A(\eta, p_1; \pi) = V(\eta; \pi) - (\eta p^h - p_1). \tag{15}$$

Because $B(\alpha) \leq B^0(\alpha)$ and the inequality is strict for $\alpha > \hat{\alpha}$, it follows that $A(\eta, p_1; \pi) \leq A^0(\eta, p_1; \pi)$ and that the inequality holds strictly for some values of η. High types wait in the first period if and only if the value of waiting is positive. Therefore, *for a given strategy π and a given price p_1, the potential entry of scalpers in the second period may increase first-period sales.* Thus, it is reasonable to expect that the ability of scalpers to enter causes the first-period demand function to shift out. However, the potential presence of scalpers changes the equilibrium decision rule π^*, so the comparison is not straightforward.

If $\pi = I_k$ then α is given by Equation (12). In this case, using Equation (3) we have

$$\alpha \leq \hat{\alpha} \iff \gamma \geq k + \epsilon(1 - 2\hat{\alpha}) \equiv \hat{\gamma}. \tag{16}$$

When agents use the strategy I_k, high types who do not buy in the first period obtain positive surplus in the second period (scalpers do not enter) if and only if the true quality exceeds a threshold level $\hat{\gamma}$. If γ is less than this threshold, many agents wait because they receive low signals in the first period. Consequently, a large number of high types want to buy in the second period. Therefore, the monopoly sells to scalpers so as to capture the potential surplus of the remaining high types (less transaction costs).

Proposition 3 showed that when scalpers are prohibited from entering the market, the unique equilibrium strategy is a threshold strategy. The threshold strategy has two reasonable characteristics. First, it is *monotonic* in the signal: $[\pi(\eta) - \pi(\eta')]/[\eta - \eta']$ has the same sign for all $\eta \neq \eta'$. For example, if agents are willing to buy when they receive a particular signal, they will also buy when they receive a higher signal. Second, the candidate is a *symmetric pure strategy*: Agents who receive the same signal behave in the same way and they do not randomize. The only other monotonic symmetric pure strategy reverses the inequalities, so that $\pi(\eta) = 0$ for $\eta \leq k$ and $\pi(\eta) = 1$ for $\eta > k$. However, a simple calculation confirms

that this alternative cannot be an equilibrium. Thus, I_k is the only candidate within the class of monotonic symmetric pure strategies.

We have the following description of the equilibrium when scalpers are allowed to enter. This description uses ϖ and ρ, functions of exogenous parameters defined in the Appendix.

Proposition 4 *Suppose that scalpers are able to enter in the second period, Assumption 2 holds, and we restrict the equilibrium to be a member of the class of monotonic symmetric pure strategies. That is, we assume that high types use threshold strategies. Under these assumptions, there exists a unique equilibrium threshold, k^* (conditional on p_1). In this equilibrium, high types buy in the first period if and only if they receive a signal $\eta \geq k^*$. The equilibrium threshold k^* is linear in p_1 and ϵ.*

$$k^* = -\frac{\varpi\epsilon + p_1}{\rho}; \quad \rho < -1. \tag{17}$$

When scalpers are allowed to enter the market, $B(\alpha)$ is nonmonotonic. This non-monotonicity means that the proof used for Proposition 3 does not carry over to the case where scalpers are permitted. We cannot rule out the possibility that mixed strategies exist, and therefore we cannot show that the equilibrium is unique in this case.[4] In contrast, when scalpers are prohibited the equilibrium is unique (Proposition 3). When scalpers are permitted, we have a weaker result: uniqueness within the class of threshold strategies.

3.4 The demand function

When the payoffs are not common knowledge ($\epsilon > 0$), the first-period demand function is piecewise linear. For prices above the choke price, $\alpha = 1$, so demand is 0. For prices below the limit price, $\alpha = 0$, so demand is H.[5] For intermediate prices, demand equals $[1 - \alpha(\gamma; I_k)] H$. The function $\alpha(\gamma; I_k)$, given by Equation (12), is linear in k. The equilibrium value of k, given by

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[4]The claim contained in Step b.i of the proof of Proposition 3 does not follow if the payoff function $B(\alpha)$ is nonmonotonic. Thus, although the ability of scalpers to enter in the second period leads to a unique equilibrium under common knowledge about payoffs, it may lead to nonuniqueness of equilibria without common knowledge.

[5]In this context, the choke price is higher than the limit price, as is always true for a standard demand function – i.e. one with a negative slope. Note that this order is reversed when there is complete information, as in Section 2. (See the discussion of Figure 1b.) Our use of "choke price" and "limit price" is consistent, however. In all versions of the model, there exists an equilibrium where all high types buy if the price is less than the limit price, and there exists an equilibrium where no high types buy if the price exceeds the choke price.

either Equation (13) or (17) depending on whether scalpers can enter, is linear in p_1. Therefore, the inverse demand is linear in p_1 at intermediate prices. Solving for p_1 in the equations $\alpha = 0$ and $\alpha = 1$ gives the limit price p^{L0} and the choke price p^{C0} when scalpers are not allowed to enter:

$$p^{L0} = \left(\rho^0 - \varpi^0\right)\epsilon - \gamma\rho^0; \quad p^{C0} = -\left(\varpi^0 + \rho^0\right)\epsilon - \gamma\rho^0. \tag{18}$$

(We obtain the limit and choke price when scalpers are allowed to enter by replacing ρ^0 and ϖ^0 by ρ and ϖ.) As $\epsilon \to 0$, $p^{L0} \to p^{C0}$. That is, as the amount of uncertainty becomes small, the first-period demand curve becomes flatter at prices above the limit price. In the limit as $\epsilon \to 0$, demand is perfectly elastic at the limit price, which is $-\gamma\rho^0$ when scalpers cannot enter the market and $-\gamma\rho$ when scalpers are allowed to enter.

We can compare these results to those in the model where high types have common knowledge. Equation (5) gives the values of the limit and choke prices ($\bar{p}$ and $\tilde{p}$) under certainty when scalpers are prohibited from entering the market. For any price below $\bar{p}$, there is an equilibrium in which all high types buy in the first period. For any price below $\tilde{p}$, this equilibrium is unique. Thus, $\bar{p}H + (1 - H)$ is the monopoly's supremum of the set of equilibrium payoffs under certainty, when scalpers are prohibited. By charging $\tilde{p}$, the monopoly is guaranteed the reservation level of profit $\tilde{p}H + (1 - H)$.

In order to compare the outcomes with and without common knowledge, we set $\gamma = 1$. We have

Proposition 5 *Suppose that scalpers cannot enter the market and that the monopoly knows the true quality (γ). (i) Given a small amount of buyer uncertainty ($\epsilon \approx 0$), the profit of a monopoly that charges the limit price is higher than its reservation level under common knowledge ($-\rho^0 > \tilde{p}$), but lower than the supremum under common knowledge ($-\rho^0 < \bar{p}$). (ii) Greater uncertainty (larger ϵ) lowers the limit price, ($\rho^0 - \varpi^0 < 0$).*

Simulations suggest that the first part of Proposition 5 also holds when scalpers can enter, but we have not been able to show this result analytically.

We also compare the limit prices in the models with and without scalpers:

Proposition 6 *For a small level of uncertainty ($\epsilon \approx 0$), the limit price when scalpers are allowed to enter is higher than the limit price when scalpers are prohibited from entering: $-\rho > -\rho^0$.*

Under common knowledge, scalpers have no effect on the limit price, but they remove the indeterminacy of the equilibrium (Proposition 2). The potential for scalpers eliminates the possibility that sales will be low in the first period, but has no effect on the maximum equilibrium level of profits unless scalpers actually enter in equilibrium. With an arbitrarily small amount of uncertainty, the demand functions with or without scalpers are well defined, so there is a unique equilibrium for a given price; the monopoly that knows γ faces no uncertainty about the equilibrium. However, scalpers increase the limit price, thereby increasing profits under limit pricing, at least for small levels of uncertainty (Proposition 6).

4 The monopoly's problem with buyer uncertainty

For any first-period price, the buyers' lack of common knowledge about the value of the ticket induces a unique equilibrium if there are no scalpers, and a unique equilibrium within a restricted class if there are scalpers. We continue to assume that buyers do not treat p_1 as a signal of quality.[6] Proposition 7, below, shows that the monopoly's optimal first period price is a piece-wise linear function of γ. A monopoly that knows the value of γ can use the optimal price; a monopoly who merely has beliefs about γ can use its subjective distribution to calculate the optimal first period price. We first study the monopoly's problem without scalpers, and then with scalpers.

When scalpers are not permitted to enter, an increase in the number of high types who do not buy in the first period increases the probability that high type will obtain a ticket in the next period – regardless of whether there is uncertainty about payoffs. In this sense, the "supply effect" of high types decisions always dominates the "demand effect", in the absence of scalpers. When scalpers are permitted to enter, the monopoly's first period price determines whether they actually enter in the second period. If they do not enter, high types' probability of obtaining a ticket in the second period is again an increasing function of the number of high

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[6]In an alternative model, buyers think that the first period price contains information about quality (over and above the information contained in the private signal), and the monopoly understands this belief. With this alternative, the monopoly has an additional incentive in choosing the first period price, a desire to manipulate information. This alternative is much more complicated than our model, and it would very likely lead to a multiplicity of equilibria. In general, the requirement that the buyers have consistent beliefs about the monopoly's information is not strong enough to result in a unique equilibrium. Angeletos and Pavan (2003) study this kind of a model.

types who wait in the first period. If scalpers do enter, then high types obtain a ticket (from scalpers) with probability 1, but they receive no surplus. With scalpers, and uncertainty about payoffs, the effect of "waiting" therefore depends on the monopoly's first period price. We show how this price is related to the parameters of the model.

4.1 The monopoly's problem without scalpers

Monopoly revenue is

$$R^0 = (1 - \alpha) H p_1 + \gamma [1 - (1 - \alpha) H] = (1 - \alpha) H (p_1 - \gamma) + \gamma \qquad (19)$$

where α is given by (12) and k is given by (13). We need only consider first-period prices between the limit and the choke price, i.e. prices that satisfy $p^{L0} \leq p_1 \leq p^{C0}$. Since all high types buy at $p_1 < p^{L0}$, revenue increases over that range of prices. For $p_1 > p^{C0}$ first-period demand is 0, so revenue is constant over that range.

For first-period prices strictly between the limit and the choke price, the monopoly revenue is a concave quadratic function. Define $\breve{p}(\gamma)$, a function of γ, as the price that maximizes R^0 *ignoring the constraints* $0 \leq \alpha \leq 1$:

$$\breve{p}(\gamma) = 0.5 \left((1 - \rho) \gamma - (\rho + \varpi) \epsilon \right). \qquad (20)$$

Because revenue is increasing at prices below the limit price and revenue is constant at prices above the choke price, the monopoly that knows the value of γ sets $p_1 = p^{C0}$ if and only if $\breve{p} \geq p^{C0}$. Using the definitions of $\breve{p}$ and p^{C0}, we can rewrite this last inequality as $-[\varpi + \rho]/[1 + \rho] > \gamma/\epsilon$. This equality is never satisfied, since $\varpi < 0$ and $\rho < -1$. Therefore, the monopoly always makes some sales in the first period: It never sets the choke price.

In other words, the monopoly that knows the value of γ sets $p_1 = p^{L0}$ if and only if $\breve{p} \leq p^{L0}$, and it sets $p_1 = \breve{p}$ if and only if $\breve{p} > p^{L0}$. We can determine the optimal first period price for the monopoly that has subjective beliefs about γ by using the fact that the fully informed monopoly's first order condition is piecewise linear in γ.

Using the definitions above, we summarize the summarize the monopoly's optimal first period price in the following proposition.

Proposition 7 *Define the function*

$$\zeta \equiv \frac{1.5 \rho^0 - .5 \varpi^0}{.5 (\rho^0 + 1)} > 0.$$

This function is independent of ϵ and γ, but it depends on all of the other parameters of the model. The monopoly that knows γ uses the limit price (sells to all high types in the first period) if the amount of uncertainty is small. It sets the price $\check{p}$ (sells to some, but not to all high types in the first period) if the amount of uncertainty is sufficiently large. The optimal first-period price is

$$p_1^* = \left\{ \begin{array}{l} p^{L0} \text{ if } \zeta \leq \frac{\gamma}{\epsilon} \\ \check{p}(\gamma) \text{ if } \frac{\gamma}{\epsilon} < \zeta \end{array} \right\}, \tag{21}$$

For the monopoly with subjective beliefs about γ, denote E as the monopoly's subjective expectation operator conditional on the belief that $\gamma \leq \epsilon\zeta$, and denote P as the subjective probability that $\gamma \leq \epsilon\zeta$. With these beliefs, the monopoly's optimal first period price is

$$p_1^{**} = \check{p}(E\gamma)P + (1 - P)p^{L0}. \tag{22}$$

To illustrate that under reasonable circumstances the fully informed monopoly wants to choose a price greater than the limit price, which induces some but not but not all high types buy in the first period, we calculate the equilibrium value of α for the following:

Example 1 *Let $D = 1 = \gamma$ and $E = .5$ (i.e. the total number of potential buyers is 50 percent greater than the number of seats). The value $D = 1$ and Assumption 1 imply that $H < .5$. Figure 3 shows the equilibrium value of α for $0.1 < H < 0.5$ and $0.1 < \epsilon < 0.4$.*
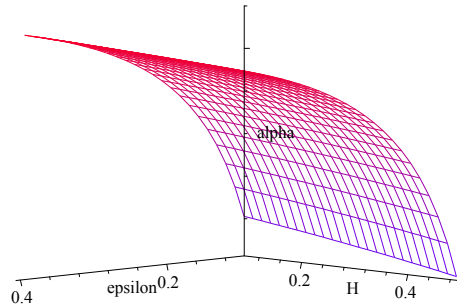

Figure 3: Equilibrium α without scalpers for $D = 1 = \gamma$, $E = .5$

The only possible source of inefficiency in this model (for $\gamma > 0$) is that some high types might end up without tickets. The fraction of high types who do not buy tickets in either the

first or the second period is $\alpha(1-\theta)$, where θ is given by Equation (1) with $s = 0$ and $L^* = L$. The number of high types who do not obtain tickets is $\alpha(1-\theta)H$, and the expected social loss for each ticket that goes to a low type rather than to a high type is γD. Therefore, the expected social loss (the amount of inefficiency) is

$$\Omega \equiv \alpha(1-\theta)HD\gamma = \alpha\left(1 - \frac{1-(1-\alpha)H}{E+1-(1-\alpha)H}\right)HD\gamma.$$

The social loss Ω is increasing in α.

For parameter values such that the monopoly limit prices, $\alpha = 0$, the equilibrium is efficient, so $d\Omega/d\epsilon = 0$. However, a change in ϵ changes the limit price and hence monopoly profit and the welfare of high types. When the monopoly does not limit price, a change in ϵ changes α, leading to a change in efficiency:

Proposition 8 *(i) For values of ϵ small enough that they satisfy $\zeta \leq \gamma/\epsilon$, monopoly profit is decreasing in ϵ and welfare of high types is increasing in ϵ. (ii) For values of ϵ large enough that they satisfy $\zeta > \gamma/\epsilon$, the level of efficiency (social welfare) is decreasing in ϵ.*

4.2 The monopoly's problem with scalpers

With scalpers, the monopoly's revenue is[7]

$$
\begin{aligned}
R(p_1) &= p_1(1-\alpha)H + \gamma[1-(1-\alpha)H]\max\{p^s(\alpha), 1\} &(23)\\
&= p_1(1-\alpha)H + \gamma\max\left[1-(1-\alpha)H, \phi\left(1-\left(1-\alpha p^h\right)H\right)\right].
\end{aligned}
$$

The first term is the revenue from sales to high types in period 1, and the second term is the revenue from sales to scalpers or to high and low types in period 2. The second line of the equality uses the definition of p^s in Equation (2). The equilibrium value of α, a function of p_1, is obtained using Equations (12) and (17).

Proposition 7 shows that the monopoly wants to limit price in the absence of scalpers when ϵ is small, and Proposition 6 shows that scalpers increase the limit price when ϵ is small. With scalpers, the monopoly is able to limit price, but it is not necessarily optimal to do so. Its profit under the optimal policy is at least as great as it would be if it were to limit price. Consequently, we have

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[7]The definition of the equilibrium second-period price under scalpers, Equation (4), shows that if both p^h and p^l are scaled up by γ, then p^s is also scaled up by γ. Therefore, we can factor out γ in the expression for second-period profit.

Corollary 1 *For small ϵ, the ability of scalpers to enter increases monopoly profit and reduces welfare of high types.*

The fully informed monopoly could use four types of strategies in the first period. It could use the limit price or the choke price, p^L or p^C, in which case it either sells to all high types or no high types. We define the entry price p^E as the price that results in the number of first-period sales that leaves the monopoly indifferent between selling to scalpers or directly to final buyers in the second period. That is, p^E is the price that induces $\alpha = \hat{\alpha}$, as defined in Equation (3). For $p^L < p_1 < p^E$, there are some high types remaining in the second period, but too few for the monopoly to want to use scalpers. For $p^E < p_1 < p^C$, some high types buy in the first period, but enough remain for the monopoly to want to use scalpers in the second period.

The complexity of the second-period revenue function makes it difficult to obtain analytic results. However, the following example illustrates that when ϵ is large it is optimal to sell to some but not all high types in the first period, and to induce scalpers to enter in the second period.

Example 2 *Let $D = L = 1$, $H = .4$, and $\phi = .9$. The high type's willingness to pay is twice the low type's willingness, there are enough low types to exactly fill the venue, enough high types to fill 40% of the venue, and the scalper captures 90% of the rent from price discrimination. The monopoly uses the limit price if $\epsilon < 0.12\gamma$ and sets p_1 such that $p^E < p_1 < p^C$ if $\epsilon > 0.12\gamma$. It is never optimal to use either the choke price or a price between p^L and p^E. For $\epsilon = 0.182\gamma$, the monopoly payoff at the local maximum without scalpers is 1.21γ and the payoff at the local maximum with scalpers is 1.28γ, a 6% increase in profit, so the monopoly induces scalpers to enter. The fraction of high types who wait is $\alpha = 0.7$.*

5   Summary and conclusions

The presence of resellers may aid the monopoly provider of a nondurable good in several ways. In a one-period market with full information, scalpers, touts, bucket shops, or other resellers may enable a monopoly that cannot price discriminate to earn nearly the price-discrimination level of profit. Scalpers allow the monopoly to effectively bundle the tickets for both types of customers.

In a two-period model with common knowledge, the monopoly's cannot solve its maximization problem because it does not know a portion of its expected demand function. This

indeterminacy arises with or without scalpers, but the source of the problem differs. In the absence of scalpers, there are two possible equilibrium outcomes for prices between the choke and limit prices. Here, we cannot determine the monopoly's optimal first-period price. With scalpers, the equilibrium conditional on the price is unique if it exists. However, the equilibrium does not exist for a range of prices where monopoly profit is not defined.

With almost common knowledge of payoffs, there is a unique equilibrium for a given price when scalpers cannot enter the market. If the amount of uncertainty is small but not zero (and scalpers cannot enter), the monopoly wants to sell to all customers with a high willingness to pay in the first period (i.e. to limit price). The resulting level of monopoly profit is greater than its reservation level under certainty, but less than the highest possible profit under certainty. Greater uncertainty lowers the monopoly's limit price and benefits consumers. The ability of scalpers to enter the market increases the limit price, increasing monopoly profits and reducing consumer welfare, at least for small levels of uncertainty.

We identified a plausible circumstance (a small degree of heterogeneity of information) where these laws benefit consumers without reducing efficiency. However, the model is sufficiently complicated that other possibilities also arise. For example, if the amount of uncertainty is sufficiently large and scalpers are prohibited, the monopoly sells to only a fraction of people with high willingness to pay in the first period. This outcome is not socially efficient. If scalpers are permitted to enter and their transactions cost is negligible, they are a near-perfect agent for price discrimination so the outcome is approximately efficient. In this case, prohibiting scalpers lowers social welfare.

In our model, the ability of scalpers to enter does not benefit consumers. This negative result is a consequence of our assumption that scalpers capture all of the surplus when dealing with buyers. If this surplus were shared, the potential entry of scalpers into the second period market might benefit some consumers with high reservation prices.

The model with almost common knowledge provides richer and more plausible predictions of monopoly behavior. The model also illustrates the manner in which the equilibrium outcome changes when the assumption of global strategic substitutes is relaxed in a global games setting.

6   Appendix:Proofs

Proof. (Lemma 1) In view of Assumption 1, $p_2 \geq p^h$ is never optimal, so we need only consider second-period prices less than p^h. At $p_2 < p^h$ all high types try to obtain tickets from the monopoly. If the monopoly sets $p_2 = p^l$ its revenue in the second period is $p^l [T - (1 - \alpha) H]$ and $L^* = L$.

If $p_2 > p^l$, then $L^* = 0$. If it is optimal to set $p_2 > p^l$ then the monopoly must sell some tickets to scalpers: $s > 0$. (Otherwise the monopoly revenues are less than $p^h \alpha H$, which by Assumption 1 is less than $p^l(T - (1 - \alpha) H)$, the level of revenue obtained by setting $p_2 = p^l$.)

If $p_2 > p^l$ and $s > 0$, it must be the case that $s > (1 - \theta)\alpha H$. If the last inequality did not hold, second period monopoly revenue is again less than $p^h \alpha H$. Therefore, if it is optimal for the monopoly to set $p_2 > p^l$, scalpers have tickets left over after selling to high types. In other words, scalpers sell $(1 - \theta)\alpha H$ tickets to the high types who did not buy in the first period and who were unable to obtain discounted tickets from the monopoly in the second period. Scalpers sell their remaining tickets to low types.

The zero-profit condition for scalpers is therefore

$$\phi \left[p^h(1 - \theta)\alpha H + p^l \left(s - (1 - \theta)\alpha H \right) \right] - p_2 s = 0. \tag{24}$$

Substituting Equation (1) and $L^* = 0$ into (24) and rearranging, we have

$$p_2 = \phi p^h - \frac{\phi D (T - H)}{s} \tag{25}$$

Equation (25) shows that the price that is consistent with 0 profits for scalpers is an increasing function of sales to scalpers, s. When the monopoly sells more tickets to scalpers and fewer to high types, it increases the likelihood that scalpers will be able to sell to high types. Consequently, scalpers are willing to pay more for a ticket. If the monopoly wants to sell to scalpers, it would like to sell them all the remaining tickets. That is, if it is optimal to set $p_2 > p^l$, then $s = T - (1 - \alpha)H$ is the optimal level of sales to scalpers. Substituting this value of s into Equation (25) and rearranging gives equation (2).

We now need to show that the monopoly can achieve $s = T - (1 - \alpha)H$ by setting $p_2 = p^s(\alpha)$. When $L^* = 0$ and S scalpers and αH high types compete for tickets, the probability that any individual obtains a ticket is $[T - (1 - \alpha) H]/[S + \alpha H]$, so the number of tickets that scalpers obtain is

$$s = \frac{T - (1 - \alpha) H}{S + \alpha H} S.$$

When $p_2 = p^s(\alpha)$, $S = \infty$ is consistent with 0 profits for scalpers, and $S = \infty$ implies that $s = T - (1 - \alpha)H$.

By setting $p_2 = p^s(\alpha)$ the monopolist is able to insure that scalpers capture all remaining tickets. Obviously, if the monopoly is able to discriminate by selling directly to scalpers, it can achieve the same outcome. ∎

Proof. (Proposition 3) We prove the proposition in two parts. In part (a) we show that there exists a unique equilibrium within the class of step functions and that k^* is increasing in the first-period price. In part (b) we follow an argument in Morris and Shin (1998)

Part (a): Within the class of equilibrium candidates of the form of Equation (11), there exists a unique equilibrium.

Step (i): Substituting Equation (12) in Equation (10), with $\pi = I_k$, we learn that $A^0 > 0$ for sufficiently small η and $A^0 < 0$ for sufficiently large η.

Step (ii): Next we establish that

$$\frac{\partial A^0}{\partial \eta} < 0. \tag{26}$$

To obtain this inequality, we consider the three cases where $\eta < k - \epsilon$, $k - \epsilon \leq \eta \leq k + \epsilon$, and $k + \epsilon < \eta$. For each of these cases, a straightforward calculation (details omitted) establishes that $\partial A^0 / \partial \eta < 0$. Since a buyer waits if and only if $A^0 > 0$, Equation (26) implies that the optimal decision rule of a high type is to buy if and only if that person receives a sufficiently high signal, given that the person believes that other high types are buying if and only if they receive a sufficiently high signal.

Step (iii): Next we show that there is a unique solution to the equation $A^0(k, p_1; I_k) = V^0(k; I_k) - (kp^h - p_1) = 0$. That is, there is a unique value of k such that I_k is the optimal decision rule for a high type, when all other high types use I_k. Integrating the expression in Equation (9) we obtain

$$
\begin{aligned}
V^0(k; I_k) &= Z^0 k + \varpi^0 \epsilon & (27)\\
Z^0 &= D\left(1 - \ln(H + L) + \frac{1 - L}{H}\ln(H + L) - \frac{\ln L}{H}(1 - L - H)\right)\\
\varpi^0 &= \frac{D}{H}[2H + (H + 3L - 1)(\ln L - \ln(L + H))\\
&\quad + \frac{2}{H}(1 - L)L(\ln(L + H) - \ln L) - H]. & (28)
\end{aligned}
$$

Because this function is linear in k, A^0 is linear in k, and there exists a unique solution to $A^0(k, p_1; I_k) = 0$.

Step (iv) We use Equation (27) to write

$$A^0(k, p_1; I_k) = \rho^0 k + \varpi^0 \epsilon + p_1, \tag{29}$$

$$\rho^0 \equiv \frac{(1 - L - H)\left[\ln(L + H) - \ln(L)\right]}{H}D - 1 < -1$$

Since $1 < L + H$ (there are more high and low type buyers than there are seats), $\rho < 0$ so A^0 is strictly decreasing in k. Therefore, k^*, the unique solution to $A^0(k, p_1; I_{k^*}) = 0$ is increasing in p_1.

(Step v) We want to show that $\varpi^0 < 0$. The formula above for ϖ^0 implies that $d\varpi^0/dD = -E\omega/H^2$, where (using $L = E + 1 - H$)

$$\omega \equiv (2 + 2E - H)\left(\ln(E + 1) - \ln(E + 1 - H)\right) - 2H.$$

$d\varpi^0/dD$ has the opposite sign as ω and $\varpi^0 = 0$ when $D = 0$. Therefore, in order to show that $\varpi^0 < 0$ for $D > 0$, it is sufficient to show that $\omega > 0$. Using the definition of ω, we see that $\omega = 0$ for $H = 0$. Therefore, it is sufficient to show that ω is increasing in H. We have

$$\frac{d\omega}{dH} = \frac{\varphi}{-E - 1 + H}$$
$$\varphi \equiv E\ln(E + 1) - E\ln(E + 1 - H) + \ln(E + 1) - \ln(E + 1 - H)$$
$$- H\ln(E + 1) + H\ln(E + 1 - H) - H.$$

Since $-E - 1 + H < 0$, it is sufficient to show that φ is negative for $H > 0$. Because $\varphi = 0$ when evaluated at $H = 0$, it is sufficient to show that φ is decreasing in H. We have

$$\frac{d\varphi}{dH} = \ln(E + 1 - H) - \ln(E + 1) < 0.$$

Part (b) Here we summarize the argument in Morris and Shin (Morris and Shin 1998) that shows that I_k is the unique equilibrium.

Step (i) (This is Lemma 1 in (Morris and Shin 1998).) For any two strategies $\pi(\eta)$ and $\pi'(\eta)$ such that $\pi(\eta) \geq \pi'(\eta)$, we have $\alpha(\gamma; \pi) \geq \alpha(\gamma; \pi')$ from Equation (8). Since $B(\alpha)$ is an increasing function, we conclude that $V(\eta; \pi) \geq V(\eta; \pi')$.

Step (ii) (This argument is the last part of Lemma 3 in (Morris and Shin 1998).) Given any equilibrium $\pi(\eta)$, define the numbers $\bar\eta$ and $\tilde\eta$ as

$$\bar\eta = \sup\{\eta \mid \pi(\eta) > 0\}$$
$$\tilde\eta = \inf\{\eta \mid \pi(\eta) < 0\}.$$

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By these definitions,

$$\tilde{\eta} \leq \bar{\eta}, \tag{30}$$

which is Equation 6 in (Morris and Shin 1998). By continuity, when the signal $\tilde{\eta}$ is received, the advantage of buying must be at least as high as the advantage of waiting, so $A(\tilde{\eta}; \pi) \leq 0$ (Equation 7 in (Morris and Shin 1998)). Since $I_{\tilde{\eta}} \leq \pi$, Step (b.i) implies that $A(\tilde{\eta}; I_L) \leq A(\tilde{\eta}; \pi) \leq 0$. From Step (a.iv) we know that $A^0(k, p_1; I_k)$ is decreasing in k and that k^* is the unique solution to $A^0(k, p_1; I_k) = 0$. Thus, $\tilde{\eta} \geq k^*$ (Equation 8 in (Morris and Shin 1998)). A symmetric argument shows that $\bar{\eta} \leq k^*$. Thus, $\bar{\eta} \leq \tilde{\eta}$. Given this inequality and Equation (30), $\bar{\eta} = k^* = \tilde{\eta}$. Thus, I_k is the unique equilibrium. ∎

Proof. (**Proposition 4**) Step (i) (Existence of threshold equilibrium.) By assumption, we have restricted the set of possible equilibria to be in the set of threshold strategies. We explained in the text why it cannot be an equilibrium threshold strategy to wait when the signal is high. Consequently, we need only to show that there exists a threshold strategy in which agents wait if the signal is low. That is, if a particular high type believes that all other high types are using strategy I_k, the agent wants to buy if and only if that person receives a sufficiently high signal.

To demonstrate this claim, we study the function $A(\eta, p_1; I_k)$ over three intervals: $\eta < \hat{\gamma} - \epsilon$, $\eta > \hat{\gamma} + \epsilon$, and $\hat{\gamma} - \epsilon \leq \eta \leq \hat{\gamma} + \epsilon$. Although the function B is discontinuous, the function A involves the integral of B and is therefore continuous. If $\eta < \hat{\gamma} - \epsilon$, then $V(\eta; I_k) = 0$ so $A(\eta, p_1; I_k) = -(\eta p^h - p_1)$, which is positive for sufficiently low η and is strictly decreasing in η. In the second region, where $\eta > \hat{\gamma} + \epsilon$, $B(\alpha(\gamma; I_k)) = B^0(\alpha(\gamma; I_k))$ for all possible values of γ. Therefore over this region $A(\eta, p_1; I_k) = A^0(\eta, p_1; I_k)$. In the proof of Proposition 3.a.i (see Equation (26)) we showed that this function is decreasing in η and is negative for sufficiently large η. Finally, in the third case where $\hat{\gamma} - \epsilon \leq \eta \leq \hat{\gamma} + \epsilon$, we know that

$$A(\eta, p_1; I_k) = \frac{1}{2\epsilon} \int_{\hat{\gamma}}^{\eta+\epsilon} B^0(\alpha(\gamma; I_k)) d\gamma - (\eta p^h - p_1),$$

where the lower limit of integration is $\hat{\gamma}$ rather than $\eta - \epsilon$ because $B = 0$ for $\gamma < \hat{\gamma}$. We conclude that

$$\frac{\partial A}{\partial \eta} = B^0(\alpha(\eta + \epsilon; I_k)) - p^h < B^0(1) - p^h < 0,$$

where the first inequality follows from the monotonicity of B^0 in α, and the monotonicity of α in γ. The second inequality follows from the definition of $B(1)$. Thus, we have shown that the optimal response to the strategy I_k is to buy if and only if the signal is sufficiently large.

Step (ii) (Uniqueness of threshold equilibrium.) Now we show that there is a unique solution to the equation $A(k, p_1; I_k) = V(k; I_k) - (kp^h - p_1) = 0$. From Equation (16), we know that

$$1 > \hat{\alpha} > 0 \iff k - \epsilon < \hat{\gamma} < k + \epsilon.$$

Since the first pair of inequalities is true by assumption, we know that $k - \epsilon < \hat{\gamma} < k + \epsilon$. Therefore

$$A(k, p_1; I_k) = \frac{1}{2\epsilon} \int_{\hat{\gamma}}^{k+\epsilon} B^0(\alpha(\gamma; I_k))d\gamma - (kp^h - p_1). \tag{31}$$

For the purpose of comparing the results in the case with and without scalpers, we replace the lower limit of integration with

$$\hat{\gamma} = k + \epsilon - 2\epsilon\beta; \quad \beta \equiv (1 - \phi)\frac{1 - H}{H(\phi D + \phi - 1)} \tag{32}$$

and we rewrite the integral in Equation (31) as

$$\frac{1}{2\epsilon} \int_{k+\epsilon(1-2\beta)}^{k+\epsilon} B^0(\alpha(\gamma; I_k))d\gamma = \varpi\epsilon + Zk \tag{33}$$

where

$$\begin{aligned}
\varpi &= \frac{-D}{H^2}[H \ln L + H^2 \ln(H\beta + L) - 2L \ln(H\beta + L) - H \ln(H\beta + L) - 2L^2 \ln L \\
&\quad -3LH \ln L + 2H\beta - H^2 \ln L + 2L^2 \ln(H\beta + L) + 2L \ln L - 3H^2\beta + H^2\beta^2 \\
&\quad +3HL \ln(H\beta + L) - 2HL\beta] \\
Z &= D\left(\frac{1}{H}(-1 + L + H)\ln L + \frac{1}{H}(-L - H + 1)\ln(H\beta + L) + \beta\right)
\end{aligned}$$

Since $\hat{\gamma} > k - \epsilon$, it must be the case that $\beta < 1$. Also, if we set $\beta = 1$ rather than the value given in Equation (32), Equation (33) gives the expected benefit of waiting in the absence of scalpers. We use these facts below.

Substituting Equation (33) into (31) and noting that $p^h = D + 1$, we find that the advantage of waiting is

$$\begin{aligned}
A(k, p_1; I_k) &= \varpi\epsilon + \rho k + p_1 \tag{34} \\
\rho &= Z - D - 1
\end{aligned}$$

Because A varies linearly with ρ, there is a unique value of k that solves $A(k, p_1; I_k) = 0$.

(iii) Finally, we need to show that $\rho < -1$. We know that ρ is a function of β and that $\rho_{|\beta=1} = \rho^0 < -1$ by Proposition 2. We know that $d\rho/d\beta = D[1 - H + H\beta]/[H\beta + L] > 0$ because $H < T = 1$. Since $\beta < 1$, we conclude that $\rho < \rho^0 < 1$. $\blacksquare$

Proof. (Proposition 5): (i) We first show that $-\rho^0 > \tilde{p}$, where $\tilde{p} = D + 1 - D/[H + L]$ (using Equation (5)). We know that

$$
\begin{aligned}
-\rho^0 - \tilde{p} &= \left(-(1 - L - H)\frac{\ln(H + L) - \ln L}{H} - 1 + \frac{1}{H + L} \right) D \\
&= D\frac{L + H - 1}{H}\psi; \\
\psi &\equiv \left(\ln\frac{H + L}{L} - \frac{H}{H + L} \right).
\end{aligned}
$$

Next, we treat ψ as a function of H, and use the facts that $\psi(0) = 0$ and $d\psi/dH = H/(H + L)^2 > 0$ to conclude that ψ (and thus $-\rho^0 - \tilde{p}$) is positive for all $H > 0$.

We now show that $-\rho^0 - \bar{p} < 0$, where $\bar{p} = D(L + H - 1)/L + 1$ (using Equation (5)). We note that

$$
\begin{aligned}
-\rho^0 - \bar{p} &= \frac{DE}{HL}\upsilon \\
\upsilon &\equiv L\ln(H + L) - L\ln L - H.
\end{aligned}
$$

Again, we treat υ as a function of H and use the facts that $\upsilon(0) = 0$ and $d\upsilon/dH = -H/(H + L) < 0$ to conclude that υ (and thus $-\rho^0 - \bar{p}$) is negative for all $H > 0$.

(ii) Finally, we need to show that $\rho^0 - \varpi^0 < 0$. Using the formulae for ρ^0 and ϖ^0 we have

$$
\begin{aligned}
\rho^0 - \varpi^0 &= \frac{\tau}{H^2} \\
\tau &\equiv 2DE(-H + 1 + E)(\ln(E + 1) - \ln(E + 1 - H)) - H^2 - 2DEH.
\end{aligned}
$$

Thus it is sufficient to show $\tau < 0$. At $D = 0$ it is obvious that $\tau < 0$, so it is sufficient to show that $d\tau/dD < 0$. We have

$$
\begin{aligned}
\frac{d\tau}{dD} &= 2E\varsigma \\
\varsigma &\equiv (1 - H + E)(\ln(E + 1) - \ln(E + 1 - H)) - H.
\end{aligned}
$$

Thus, it is sufficient to show that $\varsigma < 0$. Since $\varsigma = 0$ when $H = 0$, it is sufficient to show that ς is decreasing in H. We have

$$
\frac{d\varsigma}{dH} = \ln(E + 1 - H) - \ln(E + 1) < 0.
$$

∎

Proof. (Proposition 6): We already established the inequality $\rho < \rho^0$ in the proof of Proposition 4. ∎

Proof. (Proposition 7): Equation (21) merely restates the result in the paragraph preceding the proposition. Thus, we need only confirm that the critical ratio $(1.5\rho^0 - .5\varpi^0)/[.5(\rho^0 + 1)] > 0$. The denominator is negative because $\rho^0 < -1$ from Proposition 3.a.iv. Since $\varpi^0 < 0$, we know that $1.5\rho^0 - .5\varpi^0 < 1.5\rho^0 - \varpi^0 < \rho^0 - \varpi^0$. In the proof of Proposition 5.ii, we confirmed that $\rho^0 - \varpi^0 < 0$. Thus, both the numerator and the denominator or the critical ratio are negative, so the ratio is positive. ∎

Proof. (Proposition 8): (i) Because $\rho^0 - \varpi^0 < 0$ and given the definition of the limit price (Equation (18), the limit price decreases with ϵ. Combining this fact and Proposition 6, we confirm part (i).

(ii) For small values of ϵ such that $[1.5\rho - .5\varpi]/[.5(\rho + 1)] > \gamma/\epsilon$, we know from Proposition 6 that the monopoly uses the price $\breve{p} = 0.5[(1 - \rho)\gamma - (\rho + \varpi)\epsilon]$. Substituting this price into Equation (13) and then substituting the result into Equation (12), we determine that the equilibrium level of α is

$$\alpha^* = -\frac{1}{4}\frac{\varpi\epsilon + \gamma + \gamma\rho - 3\epsilon\rho}{\epsilon\rho}. \tag{35}$$

Using this equation and previous definitions, we find that

$$\frac{d\alpha^*}{d\epsilon} = \frac{1}{4}\gamma\frac{1 + \rho}{\epsilon^2\rho} > 0,$$

where the inequality follows because $\rho < -1$. Since an increase in ϵ leads to an increase in the equilibrium number of high types who wait (α^*), efficiency falls. ∎

Proof. (Corollary 1): For small ϵ, the monopoly limit prices in the absence of scalpers (Proposition 6). With scalpers, it is feasible to limit price, and scalpers increase the limit price (for small ϵ) by Proposition 5. Therefore , scalpers strictly increase monopoly profits when ϵ is small. ∎

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