Intersectoral migration costs and multiple equilibria

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Abstract

In familiar models, a decrease in the friction facing mobile factors (e.g., lowering their adjustment costs) *increases* a coordination problem, leading to more circumstances where there are multiple equilibria. We show that a decrease in friction can *decrease* coordination problems if, for example, a production externality arises from a changing stock of knowledge. In general, the relation between the amount of friction that mobile factors face and the likelihood of multiple equilibria is non-monotonic.

Keywords: C61, J21,O13, R23

JEL classification numbers: Multiple equilibria, Coordination games, Intersectoral Migration, Factor reallocation, learning-by-doing, Costs of adjustment

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1 Introduction

The purpose of this paper is to extend our intuition about intersectoral migration models with increasing returns to scale. Our chief contribution is to explain why multiplicity may be very unlikely to arise in exactly the circumstances where previous models would suggest that it is likely to occur.

In some circumstances, the payoff from migrating is higher if many other agents also migrate. In this situation – i.e. where actions are strategic complements – there may be multiple equilibria. For example, in Matsuyama (1991) and Krugman (1991)'s migration models, agents decide whether to work in the Agricultural or Manufacturing sector. For some range of labor allocations, an externality causes the benefit of working in a particular sector to increase with the number of workers there. Agents' decisions depend on their beliefs about what other agents will do, rather than merely on exogenous economic fundamentals. This model has been used to explain why similar countries might follow completely different development paths. In this setting, a decrease in friction, which makes it possible for workers to change sectors more cheaply, makes the multiplicity of equilibria "more likely".

If it is very costly for agents to change previous decisions – i.e. if the amount of friction is extreme – then there is little scope for their current decisions to depend on their beliefs about what other agents will do. In this case, the multiplicity of equilibria is unlikely. If the cost of changing previous decisions is negligible (and given that actions are strategic complements), it is natural to think that beliefs are a greater factor in the decision-making process, making the multiplicity of equilibria more likely. This type of reasoning leads to the conjecture that the relation between friction and the likelihood of multiplicity is monotonic. We show that this conjecture is not true in general.

We modify Krugman (1991)'s migration model by introducing learning-by-doing in Manufacturing (as in Matsuyama (1992)). Labor productivity in that sector increases as a result of experience and decays in the absence of production. This description is probably more realistic than the assumption that increasing returns to scale depend directly on the current level of employment in the sector.[1] More importantly, this generalization fundamentally changes the

[1] In another interpretation of this model, Manufacturing output creates pollution that damages an environmental stock that determines labor productivity in Agriculture, as in Copeland and Taylor (1999). At a point in time the wage differential depends only on the environmental stock and is therefore independent of the labor allocation. Increased Manufacturing output lowers future labor productivity in Agriculture, via changes in the environmental

intuition from the simpler model. We first describe our research question and results, and then provide a brief literature review. The next two sections present the model and the results.

Research question and results Dynamic models with complementarities can give rise to two or more stable steady states. To each of these steady states there exists a basin of attraction, defined as the set of initial conditions from which there is an equilibrium trajectory that approaches the corresponding steady state. The intersection of two (or more) basins of attraction is the "Region of Multiplicity", or ROM. If the ROM is empty, there exists a unique equilibrium trajectory for all initial conditions. In this case, we regard the equilibrium as unique, despite the existence of multiple steady states. (In our usage, "equilibrium" always refers to a trajectory, not simply to a steady state.) If the ROM has positive measure, then there exists a set of points (with positive measure) such that from any point in this set there are multiple equilibrium trajectories. In this case, the model exhibits multiple equilibria (at least for some initial conditions).

We are interested in a particular comparative statics question: How does a change in friction in inter-sectoral labor adjustment (i.e., migration costs) affect the multiplicity of equilibria? There are two ways to interpret the statement that a parameter change makes multiplicity "more likely":

> Interpretation 1: The change increases the measure of the set of other parameters for which multiplicity occurs.
>
> Interpretation 2: The change increases the measure of the ROM, holding fixed other parameters.

Interpretation 2 means that there are more initial conditions for which there exist multiple equilibria.

Our model has two state variables, the current fraction of labor in Manufacturing, L, and the current stock of knowledge in Manufacturing, K. The wage differential between Manufacturing and Agriculture depends only on K, rather than on L as in previous models. As the speed of learning and knowledge decay is increased, the stock of knowledge tracks L closely, and in the limit as this speed goes to infinity, knowledge (and thus the wage differential) is a function of L. A limiting case of the two-dimensional model reproduces the standard one-dimensional model in which the current wage differential depends on the amount of labor in Manufacturing.

stock, thereby changing the future wage differential.

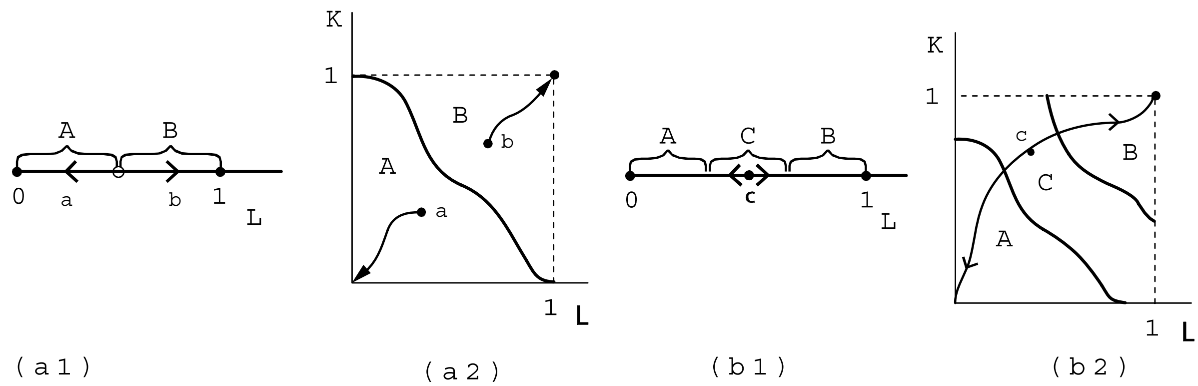

Figure 1: Phase space for different models. (a1) one state, unique equilibrium; (a2) two state, unique equilibrium; (b1) one state, multiple equilibria; (b2) two state, multiple equilibria

Figure 1 illustrates the relation between the models with one and with two state variables, and the distinction between the two interpretations of “more likely”. In the model with one state variable (L), (illustrated by panels (a1) and (b1)) the state space is $[0, 1]$; there are two steady states, $L = 0$ and $L = 1$ where, respectively, all labor is in Agriculture and all labor is in Manufacturing. In the model with two state variables, (L, K) (illustrated by panels (a2) and (b2)) the state space is the unit square $[0, 1]\, x\, [0, 1]$. There are two stable steady states, $(0, 0)$ and $(1, 1)$. In the first, all labor is in Agriculture and there is no knowledge in Manufacturing. In the second, all labor is in Manufacturing and knowledge is at its maximum level, equal to 1.

In Figure 1 (a1) and (a2) the basins of attraction for the steady states (in both the one-state and the two-state models) are the sets A and B. (These sets are intervals in the one-state model and regions in the two-state model.) The sets A and B have no intersection (except for the boundary, which is of measure 0). A typical point $a \in A$ (or $b \in B$) has a unique trajectory to the steady state where all labor is in Agriculture (respectively, Manufacturing). Panels (a1) and (a2) illustrate the situation where there the equilibrium is unique and there is hysteresis (the steady state depends on the initial condition).

In contrast, in Figure 1 (b1) and (b2) the state space consists of three sets, A, B and C. The basin of attraction for the steady state with all labor in Agriculture is $A \cup C$ and the basin of attraction for the steady state with all labor in Manufacturing is $B \cup C$. The intersection of these two sets, C, is the ROM; this set has positive measure. For example, from point $c \in C$ there exist two trajectories, which approach different steady states. The equilibrium is not unique in the models represented by Figure 1 (b1) and (b2), since there is a set of initial conditions, with positive measure, for which there are multiple equilibrium trajectories.

According to Interpretation 1, a decrease in friction makes multiplicity more likely if it increases the measure of the set of parameters for which Figure 1 (b1) or (b2) rather than Figure 1 (a1) or (a2) describes the dynamics. According to Interpretation 2, the decrease in friction makes multiplicity more likely if it increases the measure of the ROM, C

The chief result from our two-dimensional model is that there is a non-monotonic relation between the measure of the ROM and friction in the adjustment for labor. Thus, according Interpretation 2, a decrease in labor adjustment costs may make multiplicity either more or less likely. We also find that a decrease in labor adjustment costs does increase the set of other parameter values for which the ROM is positive. Thus, according to Interpretation I, lower adjustment costs make multiplicity "more likely", just as in the standard one-state models. Taken together, the two results imply that if labor adjustment costs are extremely small, the measure of the ROM is positive for a wide range of parameter values, but the measure is always extremely small. Multiplicity in this case is possible – but not very likely.

To understand the non-monotonic relation, suppose we hold constant the speed of adjustment of the knowledge stock. If labor migration is very costly (i.e., if friction is large), then the labor allocation will change slowly, if at all. In this case, the representative agent does not need to consider the possibility of having very different labor allocations in the near- to mid-term, regardless of the beliefs that other agents have. Because of discounting, the benefit to this worker of migrating is therefore relatively insensitive to the beliefs that other agents have. Since there is little scope for expectations to affect the equilibrium outcome, the ROM has small or zero measure. A decrease in adjustment costs (i.e., lowering friction) means that labor can move rapidly, making the equilibrium outcome sensitive to agents' beliefs. This decrease in labor adjustment cost tends to increase the measure of the ROM, just as in the standard (one-dimensional state variable) setting.

At the other extreme, suppose that labor adjustment costs are extremely small. In this case, a worker bases her decision (whether to change sectors) almost entirely on the predetermined stock of knowledge. *She knows that regardless of what other agents do, it will be cheap for her to change sectors in the future, in order to remain in the high-wage sector.* In this case, the measure of the ROM decreases as migration becomes cheaper. With extremely small adjustment costs, it is rational for agents to behave almost myopically for most initial conditions;

expectations do not matter (much). Thus, the relation between the amount of friction and the measure of the ROM is non-monotonic in the two-state model. This non-monotonicity requires the existence of the second state variable (knowledge). In the one-state model, where the wage differential is a function of the current labor allocation, the identification of the high wage sector depends on agents' collective actions at a point in time. In this situation, an agent cannot be sure of being in the high wage sector unless she knows what other agents are going to do in the same period.

Literature review Previous literature, excepting Krugman (1991) and Fukao and Benabou (1993), neglects the relation between parameters of the model and the measure of the ROM, and concentrates on the relation between parameter values and the existence of a ROM with positive measure. That is, the literature stresses Interpretation 1 and almost ignores Interpretation 2. There are two likely reasons for this emphasis. First, although it is sometimes relatively straightforward to determine conditions under which the ROM has positive measure, the comparative statics of this measure are complicated. Second, intuition (supported by Krugman's model) may have encouraged the idea that the two senses in which a parameter change can make multiplicity "more likely" are essentially the same. Multiplicity of equilibria arises when there are increasing returns to scale, or some other feature that makes the economy non-convex. Greater convexity of adjustment costs (more generally, increased friction) appears to convexify the economy, offsetting the effect of increasing returns to scale. Therefore, a natural (but incorrect) conjecture is that more convex adjustment costs make multiplicity less likely.

There is little empirical evidence regarding the type of multiplicity we described above. Davis and Weinstein (2002) and Davis and Weinstein (2004) find that Japanese data is consistent with increasing returns to scale, but that the data is inconsistent with the existence of multiple stable steady states – a necessary condition for multiplicity of equilibria in this setting. Moro (2003) estimates a multiple equilibrium model of wage inequality; Brock and Durlauf (2001) and Brock and Durlauf (2002) discuss the estimation of discrete choice models with social interactions, a situation that can lead to multiple equilibria.

There is a different type of multiplicity, often referred to as "indeterminacy", that occurs when there exists a continuum of equilibrium trajectories to a single steady state. Indeterminacy is an important topic in macro-economics, where the role of costs of adjustment (friction) is significant. Benhabib and Farmer (1999) review this literature; recent contributions include Cooper and Haltiwanger (1996), Cooper and Johri (1997), Benhabib, Meng, and Nishimura (2000), Nishimura and Shimonura (2002), Wen (1998a), Wen (1998b), and Lubik and Schorfheide (2004). A recurring question concerns the specification for which plausible estimates of adjustment costs and returns to scale are consistent with indeterminacy. In these models, lower costs of adjustment (less friction for the mobile factor) mean that indeterminacy is more likely, using Interpretation 1. Since the analysis of these models (typically) examines behavior in the neighborhood of the steady state, no attention is paid to Interpretation 2.

Recent theoretical papers show that changing an assumption of migration models may eliminate the multiplicity of equilibria. Frankel and Pauzner (2000) show that multiplicity disappears in a variation of Matsuyama's model where the wage differential is subject to Brownian motion and there exist "dominance regions". In another variation of this model, Herrendorf, Valentinyi, and Waldman (2000) show that there is a unique equilibrium if agents are sufficiently heterogeneous. We provide another explanation that might either increase or diminish the importance of coordination problems.

2 The model

We begin by describing a continuous time, infinite horizon model, and then present the discrete time finite horizon approximation of that model. The two types of models (continuous and discrete time) serve different purposes, and we need both of them. We use the continuous time model to establish clearly the link between the models with one and two state variables. We need the (simpler) discrete time model to obtain our analytic results.

In the continuous time setting, our model with two state variables (labor allocation and knowledge) collapses to the familiar model with one state variable (labor allocation) as the speed of adjustment of knowledge approaches infinity. It is therefore clear that the two-state model is a generalization of the one-state model, and not a fundamentally different model. This fact is important, because our central point is that a significant conclusion from the familiar model is not robust to a reasonable "perturbation".

There is an obvious sense in which our discrete time formulation represents an approximation to the continuous time model, justifying our use of the discrete model for analysis. Define a "provisional" steady state of knowledge as the steady state corresponding to a fixed value of labor allocation. In the continuous time setting, adjustment to a provisional steady state can be made arbitrarily rapid, by increasing the speed of adjustment of knowledge. In contrast, in the discrete time setting, adjustment of knowledge to a provisional steady state takes at least one period, i.e. it always occurs with a lag. The two-state discrete time model therefore cannot exactly reproduce the one-state model even if the second state variable adjusts very rapidly – i.e., even if knowledge reaches its provisional steady state in only one period, the minimum amount of time possible. A reader who has not seen the continuous time formulation might doubt that our two-state model is a natural generalization of the familiar model (and not simply a fundamentally different model). In view of the lagged adjustment, the discrete time two-state model is never (exactly) a perturbation of the one-state model.

Karp and Paul (2005) study a version of the continuous time model described in the next subsection; analysis of the continuous time model requires the use of numerical methods. The advantage of the discrete time model studied here is that all results can be obtained analytically, without any loss of intuition. The simulation results for the continuous time model and the analytic results that we obtain below for the discrete time model are qualitatively the same. Appendix A2 sketches a more general continuous time model. The discussion in the appendix provides intuitive support for the claim (established numerically in Karp and Paul (2005)) that the results we obtain in the discrete time setting survive in the continuous time setting.

2.1 The continuous time model

There are two sectors. Agriculture has constant returns to scale, and Manufacturing has increasing returns to scale that are external to the firm. The stock of labor is normalized to 1 and at time t the stock of labor in Manufacturing is L_t. We first review the familiar one state variable model, and then present a two state variable generalization.

In the one state variable model, the constant wage in Agriculture is $\alpha_A > 0$ and the wage in Manufacturing is $\alpha_M + bL_t$, where $b > 0$ determines the extent of increasing returns to scale. The Manufacturing-Agricultural wage differential is $a + bL_t$, with $a \equiv \alpha_M - \alpha_A$. By assumption, $a < 0$ and $a + b > 0$: if all workers are in the same sector, that is the high-wage sector.

The flow of labor into Manufacturing is $u_t = \dot{L} \equiv \frac{dL}{dt}$. The social cost of migration is $\frac{u_t^2}{2\gamma}$. Migration services are competitively supplied, so the price of migration (the amount that a migrant at time t pays in order to switch sectors) is $\frac{|u_t|}{\gamma}$. A higher value of γ, the speed of adjustment parameter, means that adjustment costs are lower: there is less friction. An agent who decides to migrate pays the migration cost in the current instant, in order to be in a different sector at the next instant. The instantaneous discount rate is $r > 0$.

Krugman shows that there are multiple rational expectations (perfect foresight) competitive equilibrium (i.e., the ROM has positive measure) if and only if $\gamma > \frac{r^2}{4b}$. Thus, a decrease in friction (larger γ) increases the range of other parameters (here r and b) for which there may be multiple equilibria. Fukao and Benabou (1993) show how to calculate the ROM, which is non-decreasing in γ (strictly increasing when the measure is between 0 and 1). Thus, for both interpretations given in the Introduction, a decrease in friction makes multiplicity more likely.

In our two state variable generalization of this model, the increasing returns to scale in Manufacturing depend on experience, not on the current size of the Manufacturing labor force. It takes time for learning to be incorporated into greater productivity, and a higher wage. This greater realism comes at the cost of greater complexity. This greater complexity fundamentally changes the insight produced by the one-state variable model.

We assume that the average and marginal product of labor in Manufacturing depend on knowledge, K_t, that resulted from previous industry-wide learning-by-doing. In the short run both sectors operate under constant returns to scale, but there are long-run increasing returns in Manufacturing. Manufacturing output equals bK_tL_t. In order to simplify notation, we hereafter set $b = 1$ so the Manufacturing wage is K_t. The Agricultural wage is a constant, which we set equal to 0.5, so the wage differential is $K_t - 0.5$. Greater activity in Manufacturing (higher L_t) increases the stock or knowledge, and this stock depreciates at a constant rate, g. The change in the stock of knowledge is

$$\dot{K} = g\left(L_t - K_t\right). \tag{1}$$

The steady state stock of knowledge equals the steady state stock of labor in Manufacturing.

As above, the flow of labor into Manufacturing is $\dot{L} = u_t$; the social cost of migration is $\frac{u_t^2}{2\gamma}$, so a person who migrates at time t pays the price $\frac{|u_t|}{\gamma}$. Again, there are two stable steady states. If all labor is in Manufacturing, the knowledge steady state is $K = 1$ and the Manufacturing-Agriculture wage differential is 0.5, so no worker wants to leave the Manufacturing sector. If all labor is in Agriculture, the knowledge steady state is $K = 0$ and the wage differential is -0.5, so no worker wants to leave Agriculture.

As in the one-state model, γ is inversely related to friction; γ is a speed-of-adjustment parameter for labor. The parameter g determines the speed of adjustment of the knowledge stock. When g is orders of magnitude larger than γ, the time scales over which the two state variables change are different: there are “slow-fast dynamics”.

Equation (1) can be rewritten as

$$\frac{\dot{K}}{g} = L_t - K_t. \tag{2}$$

In the limit, as $g \to \infty$, equation (2) implies that $K_t = L_t$, which implies that the Manufacturing-Agricultural wage differential in period t is $L_t - 0.5$. This formula for the wage differential is the same as in Krugman's setting, with $a = -.5$ and $b = 1$. (We adopt these two parameter restrictions only to simplify the exposition.) More generally, when g is large (for fixed γ), K adjusts rapidly, relative to the speed of adjustment of L. Therefore, for large g, the stock of knowledge closely tracks L: $K \approx L$ for g large. Thus, it would appear that the one state variable model should provide a good approximation to the two state variable model when the second state adjusts rapidly. This conjecture is false.

When g is finite, current migration affects the second time-derivative of the wage differential,

$$\frac{d^2\,(K - 0.5)}{dt^2} = g\,(u - g(L - K)).$$

In contrast, in Krugman's model ($g = \infty$), current migration affects the first time-derivative of the wage differential. The presence of the second state variable mediates migration's effect on the wage differential. In a discrete time setting, the presence of the second state variable causes migration to affect the wage differential with a lag.

2.2 The discrete time model

Most of the insight from Krugman's model can be obtained from a two-period model, in which all migration occurs in the first period. The model with two state variables requires three time periods; all migration occurs during the first two periods.

2.2.1 The two-period (one state variable) model

The initial stock of labor in Manufacturing is L and the measure of entrants into Manufacturing is u, so the amount of labor in Manufacturing in the next period is $L + u$. Using the same notation as in the previous subsection, the present value (in the current period) of the Manufacturing-Agriculture wage differential in the next period is $\beta\,(a + b(L + u))$; the discount factor is $\beta = e^{-r}$. As before, the price that an individual pays to move sectors depends on the total number of agents who move; this price is $\frac{|u|}{\gamma}$. Agents who migrate incur this cost in the current period. The present value of the benefit minus the cost of migrating to Manufacture ($u > 0$) or to Agriculture ($u < 0$) is

$$n(L,u) \equiv \frac{1}{\gamma}\left(\beta\gamma\left(a + b\left(L + u\right)\right) - u\right). \tag{3}$$

2.2.2 The three-period (two state variable) model

As in section 2.1, the wage in Manufacturing equals K_t and the wage in Agriculture equals 0.5. (Again, we set $b = 1$, $a = -0.5$.) In period $t + 1$ the stock of knowledge is

$$K_{t+1} = K_t + G\left(L_t - K_t\right), \tag{4}$$

the discrete time analog of equation (1).

Migration decisions are made at $t = 0$ and at $t = 1$. At the beginning of period t, L_t and K_t are predetermined, so the current wage is pre-determined. Current migration affects the stock of labor only in the next period. In view of equation (4), knowledge in the next period (and therefore the next period wage) is also pre-determined in the current period. Agents base their migration decisions on their beliefs about wages in periods $t = 1$ and $t = 2$ and migration costs in periods $t = 0$ and $t = 1$.

In the initial period, ($t = 0$) the state variables L_0, K_0 are given. Denote the amount of migration to Manufacturing in period 0 as u (as in the one-period model) and the amount of migration to Manufacturing in period 1 as v. Negative values mean that migration is into Agriculture. The social cost of migration is quadratic in migration, and the price that an agent (who migrates) pays is $\frac{|u|}{\gamma}$ in period 0 and $\frac{|v|}{\gamma}$ in period 1.

The (unstable) interior steady state is $L = 0.5$ and $K = 0.5$. In order to simplify notation, we define the state variables as deviations from these values: $l_t \equiv L_t - 0.5$ and $k_t \equiv K_t - 0.5$. By construction, k_t equals the Manufacturing-Agricultural wage differential in period t. The state space for the model is the square

$$-0.5 \leq l_t \leq 0.5, \quad -0.5 \leq k_t \leq 0.5, \tag{5}$$

and the equation of motion for the transformed knowledge stock (equal to the wage differential) is

$$k_{t+1} = k_t + G\left(l_t - k_t\right). \tag{6}$$

A parameter restriction Equations (1) and (4) both involve a single parameter, either g or G. Suppose that we hold L_t in equation (1) constant for one unit of time and solve that equation, to rewrite it in the same form as equation (4). Comparison of this solution and equation (4) shows that the relation between the two parameters is $G = 1 - e^{-g}$. As $g \to \infty$ the environmental stock adjusts instantaneously. Rapid adjustment of the environmental stock appears to lead to a model that is "similar" to the one-state variable model. Rapid adjustment of the environmental stock is an obvious justification for using the one-state model as an approximation. Therefore, this case is of special interest.

Instantaneous adjustment in the continuous time model ($g = \infty$) corresponds, in the discrete time model, to complete adjustment within a single period ($G = 1$). This observation (and the assumption that $g > 0$) suggests that the following parameter restriction is economically reasonable

$$0 < G \leq 1. \tag{7}$$

Since our objective is to demonstrate and explain a counter-intuitive result, it is sufficient to show that this result holds for reasonable – not for all – parameterizations.[2] The special case where $G = 1$ gives particularly sharp results, and we emphasize it below.

3 Results

We begin by showing that our two-period (one state variable) model reproduces most of the results in Krugman's continuous time model. We then present our chief results, using the three-period (two state variable) model. Next, we compare the one and two state variable models.

3.1 Results for the one state variable model

The function $n(\cdot)$ defined in equation (3) is the present value of the benefit minus the cost of migrating to Manufacture ($u > 0$) or to Agriculture ($u < 0$). For $b\gamma\beta < 1$ actions are strategic substitutes, since the net benefit of migration decreases with the number of other agents migrating; here the equilibrium is unique. Agents play a coordination game (i.e. actions are

[2] If we did not impose the upper limit in equation (7), our proof of Proposition 1 requires distinguishing between the two cases where $G \gtrless 1 + \frac{\sqrt{\gamma\beta+1}}{\gamma\beta}$. The restriction $G \leq 1$ means that we need consider only one of these two cases.

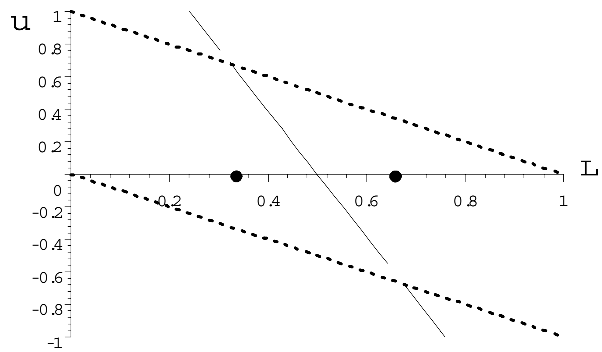

Figure 2: The ROM in a one period model with $\beta\gamma G > 1$

strategic complements), and there are multiple equilibria, if and only if $b\gamma\beta > 1$, i.e. for $\gamma > \frac{e^r}{b}$. This inequality has the same characteristics as the condition for multiplicity in the infinite horizon continuous time model, $\gamma > \frac{r^2}{4b}$.

Figure 2 graphs the migration constraints $0 \leq L + u \leq 1$ (the dotted lines) and the solution to $n = 0$ for $b\gamma\beta > 1$ (the solid line). The L coordinates of the points of intersection between the graph of $n = 0$ and the migration constraints (shown as heavy dots in Figure 2) define the interval $[1 - (a+b)\,\beta\gamma, -\beta a\gamma]$. (This interval corresponds to the interval C in Figure 1 (b1).) In general, the ROM consists of the intersection of this interval and the set of feasible initial conditions, $[0, 1]$. For an initial condition (a value of L) in the ROM, the value of u that satisfies $n(L, u) = 0$ (i.e., a point on the solid line) is an unstable equilibrium.[3] At initial conditions inside the ROM there are two stable equilibria: all labor moves to Manufacturing or to Agriculture. For example, at $u = 1 - L$, for $L \in ROM$ the benefit of moving to Manufacturing exceeds the cost ($n(L, 1-L) > 0$) so it is a stable equilibrium for all labor to move to Manufacturing. The length of the ROM is

$$\text{Length of ROM } = \max\{0, \min\{1, -\beta a\gamma\}) - \max\{0, 1 - (a+b)\beta\gamma\}\}. \quad (8)$$

For both the infinite horizon continuous time and for the two-period version of the model, the existence of multiplicity requires a combination of patience, a large externality, and low adjustment costs (large values of β, b, γ). An increase in any of these factors increases the

[3]We use the standard notion of stability. At an interior equilibrium $n = 0$ and $\frac{dn}{du} > 0$. If a small measure of agents "deviate" (e.g., they migrate to Manufacturing when their equilibrium action is to remain in Agriculture), then other agents in would want to follow the deviation. The interior equilibrium is therefore unstable.

length of the ROM when this is positive and less than 1. Thus, lower adjustment costs makes multiplicity "more likely" in both senses described in the Introduction. This conclusion is independent of the particular measure used to assess the likelihood of multiplicity; that is, it is independent of the priors on the initial condition and on b and β. For example, an increase in γ can cause initial conditions to enter the ROM but never cause initial conditions to leave the ROM. Therefore, there is no loss in generality in using the length of the ROM to measure the likelihood of multiplicity. This measure corresponds to a uniform prior over initial conditions.

3.2 Results for the two state variable model

Our principal results use the following definitions:

$$\begin{array}{c} X\left(k_0\right) = -\frac{\phi}{\chi}k_0 - 0.5\frac{\gamma\beta^2G-1}{\chi} \\ Y\left(k_0\right) = -\frac{\phi}{\chi}k_0 + 0.5\frac{\gamma\beta^2G-1}{\chi} \end{array} \tag{9}$$

where

$$\begin{aligned} \chi &= \gamma\beta G\left(\gamma G\beta + 3 + \beta - \beta G - G\right) + 1 \\ \phi &= \gamma\beta\left(\gamma\beta G + 2.0 - 1.0\gamma\beta G^2 - 3.0G + G^2 - 2.0\beta G + \beta + \beta G^2\right). \end{aligned}$$

The following Proposition summarizes our main results; the Appendix contains the proof.

Proposition 1 *Suppose that inequality* (7) *holds in our two-period model. (i)* $\beta^2 G\gamma > 1$ *is necessary and sufficient for the ROM to have positive measure. Thus, an increase in* γ *(i.e., a decrease in friction) increases the range of other parameter values (*β *and* G*) for which the ROM has positive measure. (ii) The ROM is defined by the following set.*

$$ROM = \left\{(k,l) : -.5 \leq k \leq .5 \cap -.5 \leq l \leq .5 \cap Y(k) \leq l \leq X(k)\right\}.$$

(iii) For $\beta^2 G\gamma > 1$*, the area of the ROM is non-monotonic in* γ.

The condition for the ROM to have positive measure, $\beta^2 G\gamma > 1$, is essentially the same as for the one-period model (with G playing a role analogous to b) except that the condition involves β^2 rather than β. This difference is due to the fact that migration in period t affects the wage differential in period $t+2$ rather than in period $t+1$, as was the case in the one-period model.

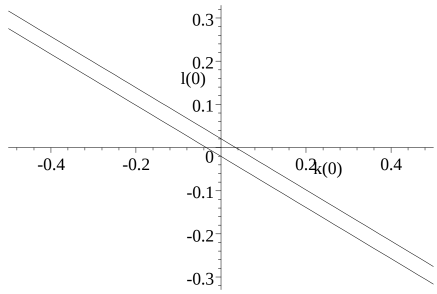

Figure 3: The ROM (area between lines) for $\beta = 0.8, G = 0.5, \gamma = 2.5$

Figure 3 shows an example of the ROM. (Compare to figure 1 (b2).) The lower line is the graph of X and the upper line is the graph of Y, defined in equation (9). The area between these two lines is the ROM. Restriction (7) and $\beta^2 G\gamma > 1$ imply that $\chi > 0$. (Details available on request.) This fact, equation (9), and the assumption that $\gamma\beta^2 G > 1$ imply that $X < Y$. For any k_0, the vertical distance between Y and X is

$$M \equiv Y - X = \frac{\gamma\beta^2 G - 1}{\chi} > 0. \tag{10}$$

In general, we do not have a simple closed form expression for the area of the ROM. However, if $Y(.5) < -0.5$ or $X(.5) > -0.5$, then the ROM does not include the NW or SE corners of state space, and the ROM is a parallelogram. If $X(.5) > -0.5$, the area of the ROM is simply M.[4]

For general parameter values, the area of the ROM is equivalent to the likelihood that an initial condition is in the ROM only under the assumption of uniform priors. For general parameter values, a change in γ causes the ROM to rotate as its area changes. Therefore, for two values of γ, γ_1 and γ_2 the inequality $area(ROM(\gamma_1)) > area(ROM(\gamma_2)$ does not imply that $ROM(\gamma_2) \subset ROM\,(\gamma_1)$ When $G = 1$, the future wage differential is independent of k_0, from equation (6). Therefore, when $G = 1$ the ROM is flat; its boundaries are independent of k_0. For $G = 1$, the magnitude of γ affects only the vertical distance between the two lines in Figure

[4]If $Y(.5) < -0.5$ the measure of the ROM is the horizontal rather than the vertical distance between the lines X and Y. If the ROM includes the corner of state space (i.e., if $X(.5) < -0.5 < Y(.5)$) then in computing its area we need to account for the "missing triangles" at the corners, and the formula for the measure becomes more complicated. It is easy to confirm any of these three configurations are possible, depending on parameter values.

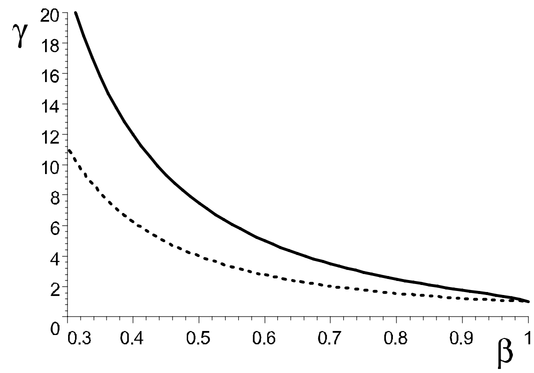

Figure 4: For $G = 1$, measure of ROM is 0 below the dotted curve, increasing in γ between curves, and decreasing in γ above solid curve.

3. In this case, $area(ROM(\gamma_1)) > area(ROM(\gamma_2)$ *does* imply that $Rom(\gamma_2) \subset Rom\,(\gamma_1)$. Thus, for $G = 1$, a larger area of the ROM means that multiplicity is "more likely", *regardless of the priors on the initial condition.*

3.3 Comparison of one state and two state models

We noted that the presence of a second state variable (the stock of knowledge) changes the relation between the amount of friction in labor adjustment and the likelihood of multiplicity of equilibria, as captured by the measure of the ROM. There are two reasons why the special case $G = 1$ is particularly useful for illustrating this difference. First, as noted above, in this situation there is no loss in generality in using a uniform prior for initial conditions. Second, the case $G = 1$ is of special interest because it leads to a model that appears to approximate the one-state variable model. Recall that $G = 1$ corresponds to complete adjustment of the knowledge stock, following changes in L, within a single period. That is, when $G = 1$ the stock of knowledge adjusts very rapidly – precisely the situation where we might expect that a one-dimensional model provides a good approximation to the two-dimensional model.

When $G = 1$ the area of the ROM is $\max\{0, M\}$, with $M = \frac{\gamma\beta^2-1}{\gamma^2\beta^2-1}$; $\gamma > \beta^{-2}$ is necessary and sufficient for the ROM to have positive measure when $G = 1$. For $\beta = 1$, $M = \frac{1}{\gamma+1}$ which is strictly *decreasing* in γ. For $\beta < 1$, M is first increasing in γ (in the neighborhood β^{-2}) and then decreasing. The measure reaches its maximum at $\gamma^m \equiv \frac{1}{2\beta^2}\left(2 + 2\sqrt{1-\beta^2}\right)$ and thereafter decreases. The maximum point γ^m converges to β^{-2} as $\beta \to 1$. Figure 4 shows the

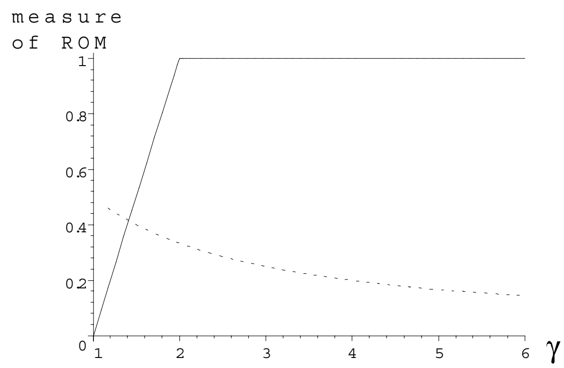

Figure 5: Meausure of ROM, $\beta = G = b = 1$. Solid curve for model with one state variable; dotted curve for model with two state variables.

graph of γ^m (the solid curve) and of β^{-2} (the dotted curve). For values of γ below the dotted curve, the measure of the ROM is 0; for values between the two curves, the measure of the ROM is increasing in γ; and for values of γ above the solid curve, the measure is decreasing in γ.

In presenting the model with two state variables we set $b = 1$ and $a = -0.5$. Therefore, to compare the two models we need to use these values in the model with one state variable. Here we also set $\beta = 1$, so that the necessary and sufficient condition for a positive measure of the ROM is $\gamma > 1$ in both models. With these restrictions, Figure 5 graphs the measure of the ROM in the two models, for $\gamma > 1$. This figure uses equations (8) and (10). The figure shows that the friction parameter γ has the opposite effect on the measure of the ROM in the two models.

4 Conclusion and discussion

Many dynamic models can be viewed as extensions of static models, obtained by introducing a payoff-relevant state variable that adjusts slowly. There is usually more than a single way of making a model dynamic. If we think that the friction in the adjustment of the mobile factor is the single most important source of dynamics, it is essential to include the labor allocation as a state variable. The additional complexity that comes from including a second state variable is a powerful argument in favor of the one state variable model.

In addition, we may think that the single state variable model is adequate. For example, we might agree that increasing returns to scale is not literally associated with the *current* labor allocation, and that instead it is associated with knowledge gained from experience in production. If the stock of knowledge tracks the current labor allocation very closely, because both learning and decay of knowledge occur rapidly, it might appear that there would be little loss of economic insight in treating the stock of knowledge as equivalent to the current labor allocation, i.e. in using a single state variable model. In a standard optimal control setting (i.e., a game against nature), when the speed of adjustment of different state variables differs by orders of magnitude (i.e., there are "slow-fast" dynamics), a higher dimensional problem can often be well-approximated using a lower dimensional state space. In contrast, in an equilibrium problem involving agents with rational expectations, that kind of approximation may be misleading. We used a simple model with two state variables to illustrate and explain this possibility.

When we ask how an increase in labor adjustment costs (friction) affects the likelihood of multiplicity, we might have in mind two quite different relations. Friction affects the measure of the set of (other) parameter values under which multiplicity is a possibility, and it affects the measure of the set of initial conditions for which multiplicity actually occurs. In Krugman's model and in our two-period simplification of that model, the answer to the comparative statics question is the same, regardless which of these two interpretations we have in mind. The theoretical and empirical literature on indeterminacy emphasizes the first interpretation. There is little discussion in the literature involving the second interpretation, which is arguably as important as the first.

We showed that when the labor allocation affects the wage differential with a lag, either because of learning-by-doing or some other (e.g. environmental) externality, then the answer to the comparative statics question may differ, depending on which interpretation one adopts. Under the first interpretation, a decrease in friction always makes multiplicity more likely in our model, just as in the one state variable setting. Under the second interpretation the relation is non-monotonic, unlike in the one state variable setting.

When it is harder for an agent to take an action, such as moving to a new sector, it seems that there would be fewer initial conditions under at her decision would depend on beliefs about what other agents will do. This conclusion is (by now) so well-established that it seems obvious. However, when the current wage differential is sluggish, as occurs in our two state variable model, the relation is reversed for low cost of labor adjustment. With low adjustment costs, for most initial states the agent's migration decision does not depend on what others do;

whatever their actions, it is cheap for an agent to move in future periods in order to remain in the high wage sector. That is, for most initial conditions, agents have a dominant strategy when adjustment costs are low.

This theoretical point shows the danger of drawing conclusions about the importance of multiplicity based on estimates of structural parameters of the model (i.e., based on Interpretation 1). Parameter estimates might suggest, for example, that there are significant increasing returns to scale (or some other source of non-convexity); that factor adjustment costs are very low; and that a one state variable model apparently provides a good approximation to the economy (because other state variables adjust rapidly). The conventional wisdom is that in these circumstances the ROM is likely to have positive measure. Our results agree with this conclusion, but also suggest that the measure of the ROM is likely to be very small, and therefore the economy is unlikely to have multiple equilibria.

An economy that has multiple steady states but a unique equilibrium might evolve in very different ways, depending on the initial condition. However, exogenous shocks or changes in policies have predictable effects, given knowledge of the economic fundamentals. In contrast, if the economy has multiple equilibria, the effect of policy changes depends on agents' beliefs as well as economic fundamentals. The policy problems in these two cases are qualitatively different.

References

BENHABIB, J., AND R. FARMER (1999): "Indeterminacy and sunspots in Macroeconomics," in *Handbook of Macroeconomics*, ed. by J. B. Taylor, and M. Woodford. Amsterdaam: North Holland.

BENHABIB, J., Q. MENG, AND K. NISHIMURA (2000): "Indeterminacy under constant returns to scale in multisector economies," *Econometrica*, 68, 1541–48.

BROCK, W., AND S. DURLAUF (2001): "Discrete Choice with Social Interactions," *Review of Economic Studies*, 68, 235–260.

——— (2002): "Market Implications of Peer and Neighborhood Effects," *American Economic Review*, 92, 298–309.

COOPER, R., AND J. HALTIWANGER (1996): "Evidence on Macroeconomic Complementarities," *Review of Economics and Statistics*, (78), 78–94.

COOPER, R., AND A. JOHRI (1997): "Dynamic Complementarities," *Journal of Monetary Economics*, (40), 97–119.

COPELAND, B., AND M. TAYLOR (1999): "Trade, spatial separation, and the environment," *Journal of International Economics*, 47(1), 137–168.

DAVIS, D., AND D. E. WEINSTEIN (2002): "Bones, Bombs and Break Points: the geography of economic activity," *American Economic Review*, 92, 1269–89.

——— (2004): "A Search for multiple equilibria in urban industrial structure," NBER Working Paper 10252.

FRANKEL, D., AND A. PAUZNER (2000): "Resolving Indeterminacy in Dynamic Settings: the Role of Shocks," *Quarterly Journal of Economics*, CXV, 285–305.

FUKAO, K., AND R. BENABOU (1993): "History versus Expectations: a comment," *Quarterly Journal of Economics*, pp. 535–542.

HERRENDORF, B., A. VALENTINYI, AND R. WALDMAN (2000): "Ruling out Multiplicity and Indeterminancy: the Role of Heterogeneity," *Review of Economic Studies*, 67(2), 295–307.

KARP, L., AND T. PAUL (2005): "Indeterminacy with Environmental and Labor Dynamics," Unpublished working paper; http://are.Berkeley.EDU/ karp/.

KRUGMAN, P. (1991): "History versus Expectations," *Quarterly Journal of Economics*, (106), 651–67.

LUBIK, T. A., AND R. SCHORFHEIDE (2004): "Testing for Indeterminacy: an Application to U.S. Monetary Policy," *American Economic Review*, 94(1), 190– 217.

MATSUYAMA, K. (1991): "Increasing Returns, Industrialization, and Indeterminancy of Equilibrium," *Quarterly Journal of Economics*, 106, 587–597.

——— (1992): "Agricultural Productivity, Comparative Advantage, and Economic Growth," *Journal of Economic Theory*, 58(2), 317–34.

MORO, A. (2003): "The effect of statistical discrimination on Black-White wage inequality: Estimating a model with multiple equilibria," *International Economic Review*, 44, 467–500.

NISHIMURA, K., AND K. SHIMONURA (2002): "Indeterminacy in a dynamic small open economy," *Journal of Economic Dynamics and Control*, 27, 271–81.

WEN, Y. (1998a): "Capacity utilization under increasing returns to scale," *Journal of Economic Theory*, 81, 7–36.

——— (1998b): "Indeterminacy, dynamic adjustment costs and cyclces," *Economics Letters*, 59, 213–216.

A Appendix:

The Appendix contains the proof of Proposition 1 and the sketch of a general model in which the measure of the ROM is non-monotonic in the amount of friction.

A.1 Proof of Proposition 1

We construct the equilibrium by working backwards, beginning with the agents' problem in period 1 (the last period during which they can migrate).

Using equation (6) and $l_1 = l_0 + u$, we write the present value at $t = 1$ of being in Manufacturing in period 2, as

$$\beta k_2 = \beta \left(k_1 + G(l_1 - k_1)\right) = \beta(G(2-G)l_0 + (G-1)^2 k_0 + Gu) \equiv f(u; k_0, l_0).$$

Our timing conventions imply that this value is predetermined at period 1. The equilibrium for the subgame beginning in period 1 is therefore unique. Agents are indifferent between migrating and staying in their current sector if and only if $\beta k_2 = \frac{v}{\gamma}$, i.e. if $v = \gamma f(u; l_0, k_0)$. The speed of adjustment parameter affects the magnitude but not the sign of the quantity $\gamma f(\cdot)$, and $f(\cdot)$ is increasing in u for all $G > 0$.

Taking into account the labor supply constraint, the equilibrium value of v is

$$v(u) = \left\{ \begin{array}{ccc} 0.5 - l_0 - u & \text{if} & \gamma f > 0.5 - l_0 - u \\ \gamma f & \text{if} & -0.5 - l_0 - u \leq \gamma f \leq 0.5 - l_0 - u \\ -0.5 - l_0 - u & \text{if} & \gamma f < -0.5 - l_0 - u \end{array} \right\}. \tag{11}$$

Figure 6 shows an example of the graph of $v(u)$, given particular values $l_0 = 0$ and $k_0 > 0$.

The u coordinate of the left and the right kink in this graph are, respectively

$$\text{left kink:} \qquad p \equiv \rho(l_0, k_0) - \frac{.5}{1 + \gamma\beta G}$$

$$\text{right kink:} \qquad q \equiv \rho(l_0, k_0) + \frac{.5}{1 + \gamma\beta G},$$

using the definition

$$\rho(l_0, e_0) \equiv \frac{1}{1 + \gamma\beta G}(\gamma\beta G(G-2) - 1)l_0 - \gamma\beta(G-1)^2 k_0).$$

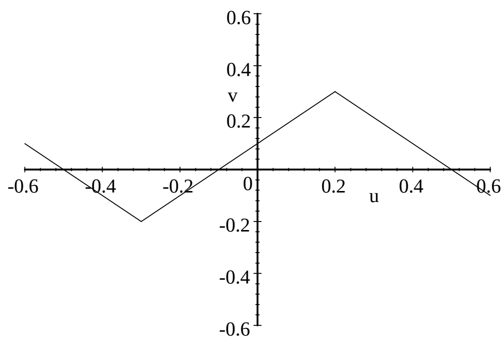

Figure 6: Equilibrium second period migration (v) as a function of first period migration (u) for $\beta = 0.8$, $\gamma = 2.5$, $G = 0.5$, $l_0 = 0$ and $k_0 = 0.2$

For all l_0 and k_0, it is always the case that $p < q$. Inequality (7) implies that ρ is a decreasing function of l_0, so p and q are decreasing functions of l_0 – a fact that we use below.

Using these definitions and equation (11) implies

$$\frac{dv(u)}{du} = \left\{ \begin{array}{ccc} -1 & \text{if} & u > q \\ \beta G \gamma & \text{if} & p < u < q \\ -1 & \text{if} & u < p \end{array} \right\}. \tag{12}$$

Thus, an increase in u increases the equilibrium v, provided that v is interior. In contrast, an increase in u decreases the equilibrium v when this variable is on the boundary of the labor supply constraint, as Figure 6 illustrates.

In period 1 an agent is either indifferent between migrating and staying in her current sector (at an interior equilibrium) or she strictly prefers to migrate (at a boundary equilibrium). Agents with rational expectations understand this fact in period 0. Therefore, the benefit of migrating to Manufacturing in period 0 is the present value of the wage differential in period 1 (βk_1), plus the present value of migration costs in period 1 ($\beta \frac{v(u)}{\gamma}$).[5] The present value of migrating to Manufacturing in period 0 is therefore

$$\beta \left(k_1 + \frac{v\left(u\right)}{\gamma} \right) = \beta \left((1 - G)k_0 + Gl_0 + \frac{v\left(u\right)}{\gamma} \right).$$

[5]The agent who migrates in period 0 avoids paying the period 1 migration costs. If migration in period 1 is at an interior level, period 1 migration costs equal the present value of the wage differential in period 2.

If the value of this expression is negative, it's absolute value is the value of migrating to Agriculture. For $u > 0$ the cost of moving to Manufacturing in period 0 is $\frac{u}{\gamma}$; for $u < 0$, the cost of moving to Agriculture is $\frac{-u}{\gamma}$.

Define the difference between benefits and costs of moving to Manufacturing in the first period as

$$h(u; l_0, k_0) \equiv \beta\left((1-G)k_0 + Gl_0 + \frac{v(u)}{\gamma}\right) - \frac{u}{\gamma}. \tag{13}$$

(Again, if $h < 0$, then $-h$ is the value of moving to Agriculture.) Using equation (12), we have

$$\frac{dh}{du} = \left\{ \begin{array}{ccc} \frac{-\beta-1}{\gamma} & \text{if} & u > q \\ \frac{\beta^2 G\gamma - 1}{\gamma} & \text{if} & p < u < q \\ \frac{-\beta-1}{\gamma} & \text{if} & u < p \end{array} \right\}. \tag{14}$$

Period 0 actions are always strategic substitutes for $u < p$ and for $u > q$. For $q < u < p$ period 0 actions are strategic complements if and only if $\beta^2 G\gamma > 1$. When actions are strategic substitutes (for all values of the state variable) the equilibrium is generically unique; $\beta^2 G\gamma > 1$ is therefore necessary for the ROM to have positive measure, as Part (i) of the Proposition states.

Since we are interested in the measure of the ROM as a function of γ, we hereafter assume that $\beta^2 G\gamma > 1$. Given this condition, we want to characterize the ROM, i.e. the region of the (k, l) plane such that if (k_0, l_0) is in this region, there are multiple equilibria in period 0.

An interior equilibrium requires that $h = 0$ and a *stable interior* equilibrium requires in addition that $\frac{dh}{du} < 0$, evaluated at the equilibrium. (See footnote 3.) Since we are interested only in stable equilibria, equation (14) means that we can rule out the possibility of interior equilibria where $p < u < q$. We are left with three possibilities: (i) The equilibrium is interior with $0.5 - l_0 > u > q$, (ii) The equilibrium is interior with $-0.5 - l_0 < u < p$, and (iii) The equilibrium is on the boundary, i.e. $u = -0.5 - l_0$ or $u = 0.5 - l_0$.

In order to construct the equilibrium, we determine the values of u for which $h(u; l_0, k_0) = 0$ at a stable equilibrium. We first consider the case where $u \geq q$; here, by equation (11), $v = 0.5 - l_0 - u$. We substitute $v = 0.5 - l_0 - u$ into the function $h(\cdot)$ defined in equation (13), and solve $h(\cdot) = 0$ to obtain an expression for u as a function of l_0, k_0. Denote this function as $x(l_0, k_0)$. Next, we consider the case $u \leq p$, where $v = -0.5 - l_0 - u$. We use this relation in

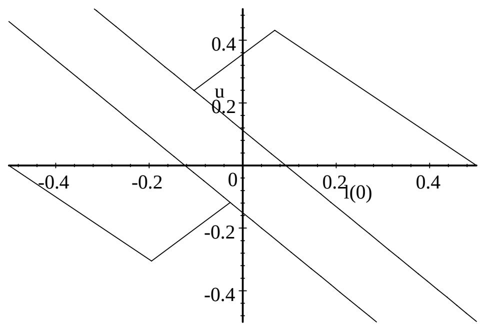

Figure 7: The equilibrium first period migration (u) as a function of l_0 for $\beta = 0.8$, $\gamma = 5$, $G = 0.7$ and $k_0 = 0.2$

the equation $h(\cdot) = 0$ and solve for u to obtain a function that we denote as $y(l_0, k_0)$. These functions $x(\cdot)$ and $y(\cdot)$ are

$$\begin{array}{c} x(l_0, k_0) \equiv \alpha + \frac{0.5\beta}{\beta+1} \\ y(l_0, k_0) \equiv \alpha - \frac{0.5\beta}{\beta+1} \end{array}, \tag{15}$$

using the definition

$$\alpha \equiv \frac{\beta}{1+\beta}\left(\left(\gamma G - 1\right) l_0 - \gamma\left(G - 1\right) k_0\right).$$

With this notation, we write the equilibrium correspondence:

$$u(l_0, k_0) = \left\{ \begin{array}{ll} \min\{x, 0.5 - l_0\} & \text{if} \quad x \geq q \\ \max\{y, -0.5 - l_0\} & \text{if} \quad y \leq p \end{array} \right\}. \tag{16}$$

The first line states that if $x \geq q$, then a stable equilibrium is $u = x$, provided that this value is less than the upper limit of migration, $0.5 - l_0$; otherwise the labor supply constraint is binding, and all labor moves to Manufacturing. The second line has a similar interpretation. Thus, there are two equilibria if the initial condition satisfies both $q \leq x$ and $y \leq p$. Using previous definitions, these two inequalities can be rewritten as

$$0.5\frac{1 - \gamma\beta^2 G}{(1.0 + \gamma\beta G)(\beta + 1.0)} \leq \alpha - \rho \leq 0.5\frac{\gamma\beta^2 G - 1}{(1.0 + \gamma\beta G)(\beta + 1.0)} \tag{17}$$

This inequality defines the ROM.

Figure 7 shows the graph of the equilibrium migration correspondence for $k_0 = 0.2, G = 0.7, \beta = 0.8$, and $\gamma = 5$. The top kinked line is the graph of $\min\{x, .5 - l_0\}$ over the interval where $x \geq q$. The kink occurs where $x = .5 - l_0$. The top straight line is the graph of q. The bottom kinked line and the bottom straight line are the graphs of $\max\{y, -.5 - l_0\}$ and of p, respectively. The overlap of the two kinked lines defines the ROM, given $k_0 = 0.2$. If, for example, $l_0 = -0.05$, the two equilibrium values of migration are $u = -0.144$ (a movement to Agriculture) and $u = 0.3$ (a movement to Manufacturing).

Our assumptions $\beta^2 G\gamma > 1$ and $\beta \leq 1$ imply that $G\gamma > 1$, so the slope x and y (as functions of l_0) are always positive, as shown. We noted above that inequality (7) implies that the slope of p and q (graphed as functions of l_0) is negative. Therefore, if $x \geq p$ is satisfied, it holds for large values of l_0; if $y \leq q$ is satisfied, it holds for small l_0.

The boundaries of the overlap are determined by the solution to $x = q$ and $y = p$. Denote $X(k_0)$ as the value of l_0 that satisfies $x = q$, and denote $Y(k_0)$ as the value of l_0 that satisfies $y = p$. Some calculation yields the formulae in equation (9) of the text. This step establishes Part (ii) of the Proposition.

The vertical distance between the boundaries of the ROM is M, defined in equation (10). We noted in the text that inequality (7) and the assumption $\gamma\beta^2 G > 1$ imply that $\chi > 0$. Therefore, when these two inequalities hold, the ROM has positive measure. This fact establishes sufficiency in Part (i) of the Proposition. The denominator of M is quadratic in γ and the numerator is linear, so $M \to 0$ as $\gamma \to \infty$. Thus, the measure of the ROM approaches 0 as $\gamma \to \infty$. Since the measure is 0 for $\gamma\beta^2 G < 1$, positive for $\gamma\beta^2 G > 1$ and approaches 0 as $\gamma \to \infty$, it is nonmonotonic in γ, as Part (iii) of the Proposition states.

A.2 Sketch of a general model

The fact that the "likelihood" of multiplicity of equilibria is nonmonotonically related to the friction associated with a mobile factor, is very simple and general. However, demonstrating this point requires a model with two state variables. Unfortunately, it is difficult to obtain analytic results using a two-state rational expectations model; therefore, our results in the text use a simplification of an already simple model. This procedure leads to clear results, but the special model has two disadvantages. First, it may leave the reader with the impression that the conclusions require this sort of special setting, and therefore are not robust. Second, the analysis of the simple model requires some tedious calculation, which obscures intuition. To offset these disadvantages, we sketch here a general model that, under mild assumptions, reproduces the non-monotonicity result shown formally for the special model.

In the interests of brevity, we do not describe all of the assumptions that lead to the model presented here, or all of its implications. However, it is worth pointing out that here we assume that the steady states are interior and are approached asymptotically.

There are two state variables: L_t is the fraction of labor in Manufacturing; K_t, the stock of knowledge in Manufacturing. There are constant returns to scale in Agriculture. The Manufacturing-Agriculture wage differential, $\omega(K, L)$, is an increasing function of K (because more knowledge increases productivity) and a decreasing function of L (because of short-run decreasing returns to scale in the sector). Denote Ω_t as the trajectory over $(t, \infty]$ of $\omega_\tau \equiv \omega(K_\tau, L_\tau)$ and denote $\vec{0}_t$ as the trajectory where $\omega(K_\tau, L_\tau) = 0$ for $\tau \geq t$. The dynamics of the state variables are given by

$$\dot{L} = \frac{dL}{dt} = \gamma h(\Omega_t), \text{ with } h(\Omega_t) = 0 \text{ iff } \Omega_t = \vec{0}_t, \tag{18}$$

$$\frac{dK}{dt} = gf(K, L), \tag{19}$$

with f increasing in L and decreasing in K.

Agents' intersectoral migration decisions depend on their beliefs about future wage differentials, Ω_t. In a deterministic rational expectations equilibrium, agents' beliefs are correct in equilibrium. The functional $h(\cdot)$ is determined by the equilibrium condition to agents' problems.[6] The parameter $\gamma > 0$ is inversely related to the amount of friction (e.g. the costs of migration). The restriction on $h(\cdot)$ states that migration stops if and only if the future trajectory of the wage differential is identically 0. The function $f(\cdot)$ is given exogenously, and $g > 0$.

Suppose that the solution to $f(L, 0) = 0$ is less than the solution to $\omega(L, 0) = 0$, so that the graph of $f(K, L) = 0$ intersects the graph of $\omega(K, L) = 0$ from below. Since both graphs of increasing, there are an odd number of steady states. Suppose, for concreteness, that there are exactly three steady states; two of these are stable and the intermediate steady state is unstable. By construction, all of the steady states are independent of the speed of adjustment parameters γ and g. There may or may not be multiple equilibria; that is, the ROM may have positive or 0 measure.

[6] For example, $h(\cdot)$ may be a function of the present discounted value of the future stream of wage differentials, denoted q_t. Let $p\left(\dot{L}_t\right)$ be the price that an individual pays to migrate at time t. The equilibrium condition is $p\left(\dot{L}_t\right) = q$ or $\dot{L}_t = p^{-1}(q) \equiv \gamma h(q)$. The complete dynamical system of the model consists of equations (18) and (19) and $\dot{q} = rq_t - \omega_t$, where r is the constant discount rate.

In the limit, as $\gamma \to \infty$ and $g \to \infty$, we obtain a static model for which the two stable steady states of the dynamic model are stable equilibria. In this static model there is certainly a coordination problem (multiple equilibria). The ROM here is (trivially) the entire "state space", since there are equilibria at either steady state that are independent of initial values of K and L.

For finite γ with $g = \infty$ we obtain a model that has many of the same features as the one-state models discussed in the text. We adopt

Assumption 1 *For the one-state model (with $g = \infty$), the ROM is non-empty if and only if γ is sufficiently large.*

This assumption can be shown to hold if, for example, the model is closed using the equilibrium condition discussed in footnote 6.

The interesting situation arises if agents' expectations are sufficiently smooth in model parameters so that the following is satisfied:

Assumption 2 *The equilibrium correspondence (mapping initial conditions and parameter values into trajectories) is continuous in γ and g for all positive values.*

If Assumption 2 was not satisfied, then the comparative statics question addressed in this paper would be rather artificial. The two assumptions may appear to suggest that the one-state model should provide a good approximation to the two-state model if the omitted state adjusts rapidly. In an important respect, however, the one-state model can be misleading precisely when the omitted state adjusts rapidly. In order to understand why, consider two limiting cases, in each of which the state is one-dimensional.

> Case i) $\gamma = \infty$ and $g < \infty$, so that the single state variable is K. In this case, unless K begins at the unstable steady state, all labor moves immediately to the high wage sector and the system then moves toward one of the two steady states. The equilibrium is unique; here the measure of the ROM is 0.
>
> Case ii) $g = \infty$ and $\gamma < \infty$, so that the single state variable is L. In this case, by Assumption 1, the ROM has positive measure if and only if γ is sufficiently large.

The more interesting case occurs where g is large but finite and $\gamma < \infty$. If γ is large, the ROM has positive measure, by virtue of the two Assumptions. For large γ it is difficult for agents to predict what other agents will do in the future, because migration is cheap; this inability is important because the wage differential adjusts quickly to migration (g is large). Therefore the measure of the ROM is positive. However, as γ approaches ∞, we move toward Case i, where the measure of the ROM is 0. Given Assumption 2, the measure of the ROM must be decreasing in γ for γ large. For small γ, migration is slow in any equilibrium, so the value of being in a particular sector depends mostly on the predetermined variable K. For sufficiently small γ, expectations have negligible effect on the equilibrium, so the measure of the ROM is 0. For this model, and for $g < \infty$ but large, the measure of the ROM is therefore non-monotonic in γ.

It is worth emphasizing that this non-monotonicity arises in the situation where the state variable K adjusts quickly, precisely the situation where it might seem that little insight is lost by treating it as adjusting instantaneously.