The Political Economy of Environmental Policy with Overlapping Generations*

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Abstract

A two-sector OLG model illuminates the intergenerational effects of a tax that protects an environmental stock. A traded asset capitalizes the economic returns to future tax-induced environmental improvements, benefiting the current asset owners, the old generation. Absent a transfer, the tax harms the young generation by decreasing their real wage. Future generations benefit from the tax-induced improvement in environmental stock. The principal intergenerational conflict arising from the tax is between generations alive at the time society imposes the policy, not between generations alive at different times. A Pareto-improving tax can be implemented under various political economy settings.

Keywords: Open-access resource, two-sector overlapping generations, resource tax, generational conflict, environmental policy, dynamic bargaining, Markov perfection

JEL, classification numbers: E24, H23, Q20, Q52, Q54

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1 Introduction

Most analyses of stock-related environmental problems use assumptions implying that people alive today must sacrifice to preserve consumption opportunities for those alive in the future. The focus on the conflict between agents who live at different points in time, obscures the conflict between different types of agents alive when the policy is first implemented. An overlapping generations (OLG) model with endogenous asset prices shows that suitable transfers amongst those currently alive causes all generations to benefit from environmental policy.

Ramsey models that contain an environmental stock and endogenous capital, e.g. climate models such as DICE (Nordhaus 2008), typically use a one-commodity setting in which output can be either consumed or invested. When investment is positive, the normalization that sets the commodity price to 1 implies that the asset price is fixed, also at 1. In these models, environmental policy affects the trajectory of an environmental stock, which affects the future productivity of capital, thereby altering current investment. In this setting, the trajectory of capital is endogenous but the price of capital is fixed. The fixed asset price means that these models exclude a channel through which policy-induced changes in future productivity effect the level and distribution of welfare.

To examine the role of asset prices, we reverse these assumptions: there is a fixed or exogenously changing stock of capital and no depreciation, forcing the price of capital to be endogenous and responsive to policy-induced changes in future productivity. For given environmental stocks, stricter environmental policy reduces current real aggregate income, exactly as in previous models. Stricter policy also lowers the current *real* wage and rental rate of capital; in that respect the welfare effect of policy is symmetric across factors of production. However, by increasing future rental rates via improved environmental stocks (relative to Business as Usual, or BAU), the stricter policy increases the *price* (as distinct from *rental rate*) of the asset. In our model, environmental policy increases the welfare of current owners of capital, because the higher asset price more than offsets the lower current rental rate. Despite the policy's symmetric effects on real returns to the two factors, capital and labor are fundamentally asymmetric: the price of capital reflects future productivity, whereas the price of the labor depends only on current productivity. This difference drives the welfare comparisons.

In our OLG setting, agents live for two periods and can use an environmental tax. The current old generation owns capital, which it sells to young agents. Because a tax lowers current aggregate real income and increases the welfare of old agents, it necessarily decreases the first period utility of young agents. Young agents benefit from the policy-induced environmental improvement in the second period of their life, but under circumstances relevant to most environmental problems, this second period utility benefit does not compensate for the first period loss. Therefore, absent transfers, environmental policy increases lifetime welfare of the (first period) old asset-rich and lowers the lifetime welfare of the young asset-poor. However, the first-period old generation can retain all of the benefits of the higher asset value and compensate the young using only revenue from the environmental tax. In this way, the old rich pay the young poor to accept stricter environmental policy. They make this transfer not because of a moral imperative, but because it is in their interest to do so: absent the transfer, the young have no reason to agree to implement the policy.

We consider two types of policy settings. In the first, we obtain analytic results, summarized above, for arbitrary perturbations from BAU. We then use numerical methods to study the Markov perfect equilibrium (MPE) in a dynamic political economy setting. These numerical results support and illustrate the analytic results. In each period of this game, the current old and young generations pick a current tax to maximize their aggregate lifetime welfare. They understand that this tax affects the evolution of the environmental stock and future equilibrium taxes, which affect future rental rates. The current tax thus affects the current asset price, and current generations' welfare. Recent papers use similar dynamic settings to study MPE in political economy games involving redistribution and/or the provision of a public good (Hassler et al., 2003, 2005 and 2007; Conde-Ruiz and Galaso, 2005; Klein et al., 2008; Bassetto, 2008). Although environmental considerations motivate our research, we rely heavily on elements more commonly used in macroeconomics, particularly the focus on asset prices, the OLG structure, and the emphasis on endogenous policy determination.

The literature examining environmental policy in OLG models neglects the role of asset prices that we emphasize. Kemp and van Long (1979) and Mourmouras (1991) are among the first to use an OLG framework with renewable resources. Later contributions study welfare when a social planner corrects an environmental externality (Mourmouras, 1993; Howarth, 1991, 1996, 1998; Howarth and Norgaard, 1990, 1992; Krautkraemer and Batina, 1999; Rasmussen, 2003). John et al. (1995) discuss the steady state inefficiencies under environmental externalities, and John and Pecchenino (1994) consider the transitional dynamics. Marini and Scaramozzino (1995) study time-consistent fiscal policy under environmental externalities. Laurent-Lucchetti and Leach (2011) note that current owners of capital capture the benefits of policy-induced innovation. Bovenberg and Heijdra (1998, 2002) and Heijdra et al. (2006) note that the issuance of public debt achieves intergenerational transfers, thus providing a means of compensating the current generation for sacrifices that benefit future generations. Public debt ameliorates the missing market arising from the inability of agents living in different periods to trade directly with each other. We show that asset price endogeneity facilitates Pareto-improving policy even if the government cannot use bonds to distribute income across generations.

Lucas (1978) models the equilibrium pricing of productive assets. Several papers recognize that asset prices depend on adjustment costs, without, however, developing the idea that the endogenous asset price might provide an incentive for current generations to improve the welfare of future generations (Huberman, 1984; Huffman, 1985, 1986; and Labadie, 1986). Our dynamic general equilibrium model is similar to that of Copeland and Taylor (2009). However, we have rents in the manufacturing sector, leading to the asset price, which is central to our analysis. In addition, we emphasize the transitional dynamics under selfish agents, whereas they consider exclusively the steady state under a social planner. Our model is close to that of Koskela et al. (2002) in its OLG structure, but differs by separating conventional capital and the renewable resource into different sectors and by allowing for open-access in the latter; see also Galor (1992) and Farmer and Wendner (2003).[1]

There are previous challenges to the conventional view that environmental policy requires sacrifices by those alive today. Correcting multiple market failures jointly might be a "win-win" opportunity; e.g., a reduction of greenhouse gas emissions, benefiting future generations, might also reduce transient local pollutants, benefiting those currently living. Rebalancing society's investment portfolio, reducing saving of man-made capital and increasing saving of environmental capital, might also benefit all generations (Foley, 2009; Rezai et al., 2012). We exclude these possibilities.

[1] Guesnerie (2004), Hoel and Sterner (2007) and Traeger (2012) examine the importance, to dynamic environmental policy, of imperfect substitutability between goods that rely (primarily) on either environmental or on man-made capital. In a two-good (general equilibrium) model, this imperfect substitutability alters the relative price of commodities.

2 Model

There is a single endogenously changing stock, the environment. Agents living in different periods cannot trade directly with each other, and we rule out the use of public debt to achieve intergenerational transfers. Agents live for two periods and they care only about their own lifetime welfare; their only means of influencing the future is to change their current use of the environmental stock. These assumptions bias the model against Pareto-improving environmental policy. However, an environmental tax with appropriate allocation of tax revenues creates a Pareto improvement and can be implemented in a political economy equilibrium.

One sector, "manufacturing", produces a good M using mobile labor and a sector-specific input, capital. The stock of capital is fixed, $K \equiv 1$. (Appendix B1 considers the case where the stock or productivity of capital changes exogenously.) The other sector produces a good F using labor and an endogenously changing resource stock, x. We suppress time indices where no confusion results. There are perfect property rights for the stock of manufacturing capital, and no property rights for the resource stock. Absent environmental policy, mobile labor competes away all rent in the resource-intensive sector.

Young agents receive a wage, income from the resource sector, and possibly a share of tax revenues. They divide their income between consumption and savings, in the form of shares in the firm. The old generation earns the profits from the manufacturing firm, the proceeds from selling the firm, and its share of the tax revenue. The non-altruistic old generation consumes all of its income. The size of each cohort is fixed and normalized to 1.

Competitive labor and commodity markets clear in each period. Employment in the resource sector equals L, and free movement of labor between the sectors ensures that the return to labor there equals the manufacturing wage. Manufacturing is the numeraire good, and the relative price of the resource-intensive good is P. Output in the resource-intensive sector is $F = L\gamma x$, with the constant $\gamma > 0$. Manufacturing output is $M = (1 - L)^{\beta}$ with $0 < \beta < 1$, so that there are profits (rent) in this sector; β is labor's share of the value of manufacturing output. The old generation owns the only asset, K, which it sells to the young generation after production occurs.

The open access of the resource sector means that too much labor moves to this sector. An ad-valorem tax, T, on production of the resource-intensive good reduces this misallocation. The revenue accruing to workers in the resource sector, under the tax, equals $P(1-T)L\gamma x$. Society returns the tax revenue, $R = PTL\gamma x$, in a lump sum, but possibly different shares, to the young and old generations.

Agents have intertemporal additive utility, with the single period utility function $u\left(c_{F,t}, c_{M,t}\right)$, where $c_{i,j}$ is the consumption level of good i at time j. Their pure rate of time preference is ρ. Agents take as given, or have rational point expectations of the wage (w_t), relative commodity price (P_t), asset price (σ_t), share of tax receipts (R_t), and profits (π_t). A tilde over a next-period variable signifies a point expectation. The lifetime decision problem of the agent who is young in period t, is

$$\max_{c^y_{F,t}, c^y_{M,t}, c^o_{F,t+1}, c^o_{M,t+1}, s_t} u(c^y_{F,t}, c^y_{M,t}) + \frac{1}{1+\rho} u(c^o_{F,t+1}, c^o_{M,t+1}) \tag{1}$$

subject to the budget constraints in the first and second periods of their life:

$$w_t + \chi_t R_t \geq P_t \; c^y_{F,t} + c^y_{M,t} + \sigma_t s_t, \text{ and} \tag{2}$$

$$s_t(\tilde{\pi}_{t+1} + \tilde{\sigma}_{t+1}\tilde{s}_{t+1}) + (1 - \tilde{\chi}_{t+1})\tilde{R}_{t+1} \geq \tilde{P}_{t+1} c^o_{F,t+1} + c^o_{M,t+1}, \tag{3}$$

where s_t is the fraction of shares that the young agent purchases. In equilibrium, supply of shares equals demand, i.e. $s_t \equiv 1 \; \forall t$; hereafter we suppress s_t. The superscripts o and y on consumption variables indicate whether the agent is old or young at a point in time. We suppress those superscripts if the meaning is clear from the context. The young agent spends all of her time to working and the old agent manages the manufacturing firm.

The utility function is Cobb-Douglas: $u(\cdot) = c^{\alpha}_{F,t} c^{1-\alpha}_{M,t}$, with α the constant budget share for the resource-intensive good. With e equal to expenditures and $\mu \equiv \alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}$, an agent's single period indirect utility is

$$v(e, P) = \left(\frac{\alpha e}{P}\right)^{\alpha} \left(\frac{(1-\alpha)\, e}{1}\right)^{1-\alpha} = \mu P^{-\alpha} e. \tag{4}$$

With identical homothetic preferences, the share of income devoted to each good, and the equilibrium commodity price, are independent of both the level and distribution of income. The ratio of demand for both goods is a function of this price. The requirements that workers are indifferent between working in either sector, $P(1-T)\gamma x = w$, and that manufacturing firms maximize profits, determine the wage, the allocation of labor, and supply of both goods. The relative price, P, causes product markets to clear. These equilibrium conditions for the labor and product markets lead to the following expressions for the values of L, w, and P:

$$\begin{gathered}
L = \frac{1-T}{\frac{1-\alpha}{\alpha}\beta + 1 - T}, \qquad w = \beta\left(1 + \frac{1-T}{\frac{1-\alpha}{\alpha}\beta}\right)^{1-\beta} \\
P = \frac{w}{(1-T)\gamma x} = \frac{\beta\left(1+\frac{1-T}{\frac{1-\alpha}{\alpha}\beta}\right)^{1-\beta}}{(1-T)\gamma x} \equiv p(x,T).
\end{gathered} \tag{5}$$

The equilibrium allocation of labor and the wage depend on the tax T but not on the resource stock, x. However, the equilibrium commodity price depends on x because the stock affects the supply of the resource-intensive good. Firms' profits, π, the tax revenue, R, and the sectoral values of output, PF and M, also depend only on T and model parameters:

$$\begin{gathered}
\pi = \frac{1-\beta}{\beta} w(1-L), \qquad R = \frac{T}{1-T} Lw, \\
M = (1-L)^{\beta}, \qquad PF = \frac{\alpha}{1-\alpha}(1-L)^{\beta}.
\end{gathered} \tag{6}$$

Systems (5) and (6) determine the static equilibrium of the economy.

2.1 The asset price

The young buy manufacturing firms from the old; the asset price affects welfare through expenditure. Systems (5) and (6) enable us to express the young and old generation's expenditure levels, e^y and e^o, as functions of current tax T and the asset price, $\sigma(x, \mathbf{T})$, where $\mathbf{T}$ is the tax trajectory:

$$e^y = w(T) + \chi R(T) - \sigma(x,\mathbf{T}) \quad \text{and} \quad e^o = \pi(T) + (1-\chi)R(T) + \sigma(x,\mathbf{T}). \tag{7}$$

Under BAU, $\mathbf{T} = \mathbf{0}$. The standard first order condition for optimal saving behavior requires that the young's marginal loss in utility from purchasing a unit of the asset in the current period equals her discounted marginal gain in utility from having that asset in the next period. This condition determines the demand for the asset as a function of its current price and the expectation of next period rental rate and price. The demand function and the fixed (or exogenously changing) supply of capital, determine the current asset price as a function of expected next period rental rate and price, leading to:

Proposition 1 *The price of a unit of capital equals the stream of future dividends, arising from the firm's profits, discounted at the equilibrium rates of intertemporal substitution:* $\sigma_t = P_t^{\alpha} \sum_{i=1}^{\infty} (1+\rho)^{-i} P_{t+i}^{-\alpha} \pi_{t+i}$.

(The appendix contains proofs not found in the text.) Due to the assumption of Cobb Douglas utility, the indirect utility function is linear in expenditures, so the elasticity of intertemporal substitution is infinite. As in Lucas (1978), the equilibrium rate of intertemporal substitution is constant and equal to the pure rate of time preference, ρ.

Current asset owners, the old, benefit from a policy that increases the asset price. The changed asset price has no effect on the welfare of asset purchasers, the young. Given that saving behavior is optimal, the young pay for the asset exactly what is worth to them in terms of discounted utility. Although the change in asset price changes their current expenditures, the offsetting change in future receipts leads to a zero change in their welfare:

Corollary 1 *(i) An unanticipated change in the asset price does not affect the lifetime utility of current and future young generations. (ii) Unanticipated changes in the asset price affect only the current old generation.*

2.2 Resource dynamics

We assume that the resource stock obeys a logistic growth function:

$$\begin{aligned} x_{t+1} &= x_t + r x_t \left(1 - \tfrac{x_t}{C}\right) - L(T_t)\gamma x_t = \left(1 + r\left(1 - \tfrac{x_t}{C}\right) - L(T_t)\gamma\right) x_t \\ &= \left(1 + \bar{r}_t(T_t, x_t)\right) x_t; \text{ with } \ \bar{r}_t \equiv \bar{r}\left(T_t, x_t\right) = \left(r\left(1 - \tfrac{x_t}{C}\right) - L(T_t)\gamma\right), \end{aligned} \tag{8}$$

with r the intrinsic growth rate, C the carrying capacity of the resource, and $\bar{r}\left(T, x\right)$ the endogenous growth rate of the resource. A higher tax conserves the resource because it causes labor to seek employment in the manufacturing sector: $\frac{dL_t}{dT_t} < 0 \Rightarrow \frac{d\bar{r}_t}{dT_t} > 0 \Rightarrow \frac{dx_{t+1}}{dT_t} > 0$. An output tax is an efficient means of reducing current resource use.

The steady state, x_∞, is defined as the solution to $\bar{r}\left(T_\infty, x_\infty\right) = 0$; we restrict parameter values to ensure that under BAU there exists an interior steady state to which trajectories beginning near that steady state converge monotonically. The necessary and sufficient conditions for this are $0 < \frac{d(1+\bar{r})x}{dx} < 1$, evaluated at $T = 0$, $x = x_\infty$. These inequalities are equivalent to

$$1 < \varsigma < 2 \text{ with } \varsigma \equiv r + \frac{\beta\left(1-\alpha\right) + \alpha\left(1-\gamma\right)}{\beta\left(1-\alpha\right) + \alpha}. \tag{9}$$

Denote $\hat{x}$ as the solution to $\bar{r}(0, \hat{x}) = -1$ and define the interval $\hat{X} = (0, \hat{x})$, the set of initial conditions for which subsequent stocks are positive under BAU. We assume that $x_0 \in \hat{X}$. The BAU growth function, $\bar{r}(0, x_t) x_t$ is single-peaked, but not monotonic; therefore, the BAU equilibrium trajectory need not be monotonic over time. However, for any $x_0 \in \hat{X}$, all subsequent BAU stocks, x_1, x_2, ... are in the interval for which $(1 + \bar{r}(0, x)) x$ is an increasing function of x; the trajectory after the initial period is therefore monotonic with respect to time.

2.3 Relation to one-commodity Ramsey models

The real wage, $\mu P^{-\alpha} w$, and the real rental rate, $\mu P^{-\alpha} \pi$, equal the amount of utility that an agent obtains by renting out one unit of labor or one unit of capital, respectively. These real factor returns depend on both the tax and the stock of the resource.

Our model has two features in common with familiar one-commodity Ramsey models: the current tax reduces current aggregate utility and reduces both the current real wage and the real rental rate. We emphasize this similarity, lest the reader mistakenly think that our main results depend on incidental features of the model.

Proposition 2 *(i) An increase in the tax at time t reduces aggregate period-t utility. (ii) For a predetermined level of the environmental stock, a higher tax decreases both the real wage and the real rental rate. (iii) A higher environmental stock increases aggregate utility and both the real wage and the real rental rate.*

The tax causes labor to leave the resource sector, reducing the nominal wage and increasing nominal profits. The tax also reduces the supply from the resource sector, thereby increasing the relative commodity price, P. Therefore, the tax unambiguously decreases the real wage. In a more general model, the effect of the tax on the real return to capital is ambiguous. In our model, the tax-induced commodity price increase dominates the increase in nominal return, so the tax reduces the real return to capital. The higher stock has no direct effect on nominal returns, but it lowers the commodity price and therefore increases both real returns. Thus, a change in the stock or the tax have the same qualitative effect on the two real returns. In this important respect, the model treats the two factors symmetrically. The general equilibrium framework shows how changes in the environmental stock

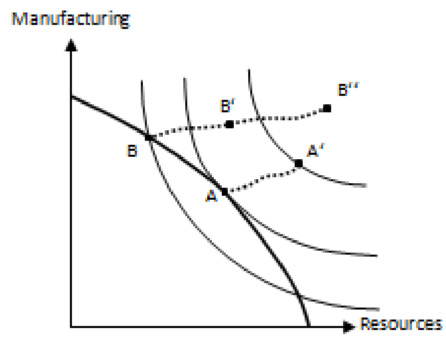

Figure 1: Consumption expansion paths under BAU ($A - A'$) and under an environmental policy ($B - B' - B''$)

affect the real return to factors that do not directly depend on this stock – manufacturing capital in our model.

Figure 1 uses a production possibility frontier to illustrate the welfare effect of environmental policy both in standard Ramsey models and in our OLG model. Under BAU, current aggregate consumption is at point A, a level that maximizes current aggregate utility, ignoring the environmental externality. The tax moves consumption to point B, where current aggregate utility is lower. Therefore, someone must have lower *current* utility at B than at A.

Figure 1 illustrates the conventional view that environmental policy creates a conflict between those alive today and those alive in the future. The aggregate (single period) consumption path under BAU moves along the curve from A to A', a trajectory that incorporates changes in both environmental and man-made capital stocks, including technological change (introduced in Appendix B1). The aggregate consumption trajectory under the environmental policy moves along the curve from point B to B''. Agents alive at the initial time have higher *current* aggregate utility under trajectory AA', and those alive later have higher (single period) aggregate utility under trajectory BB'', so dynastic (rather than OLG) models lead to the conventional view that a welfare comparison depends on the social discount rate.

Previous challenges to the conventional view, including the existence of win-win opportunities or the possibility of rebalancing society's investment portfolio, suggest that society might move from trajectory AA' to trajectory $B'B''$, thus increasing aggregate utility in each period. Our model excludes those possibilities. Environmental policy lowers aggregate *current* utility in the first period, just as in the standard Ramsey framework. The current old live for a single period, so the tax increases their lifetime welfare if and only if it increases their utility in the current period. The current young are also alive in the next period. Their lifetime welfare depends on utility in both periods of their life.

Although a higher tax and a higher environmental stock have the same qualitative effects on the real wage and the real rental rate, the two factors are fundamentally asymmetric: the *price* of capital depends on future rental rates, whereas the price of labor depends on only its current value of marginal productivity. Current owners of capital benefit from the future increases in productivity created by the environmental policy, even though they are not alive to enjoy them directly. Absent transfers, current owners of labor benefit from these future productivity increases only to the extent that they are alive to enjoy them.

3 Welfare Effects of a Tax

Under BAU, the environmental tax is identically 0. Consider an arbitrary non-negative tax trajectory, the vector $\bar{\mathbf{T}}$, with element $\bar{T}_i \geq 0$. The index i denotes the number of periods in the future, so $i = 0$ denotes the current period; $\bar{T}_i > 0$ for at least one i, including $i = 0$ (because consideration of delayed policies yields only obvious results). A non-negative perturbation of the zero tax BAU policy is $\mathbf{T} = \varepsilon \bar{\mathbf{T}}$, with $\varepsilon \geq 0$ the perturbation parameter. A larger ε therefore is equivalent to a higher tax policy. We assume that the fraction of tax revenue given to the young, χ, is constant, and establish:

Proposition 3 *For all* $\chi \in [0, 1]$*, a small tax increase* ($\varepsilon > 0$) *benefits the current old generation. This benefit increases in its tax share,* $(1 - \chi)$*.*

The intuition for Proposition 3 follows from the fact that a small tax has only a second order effect on the real rental rate (as can be seen by inspection of equation (A.16) in the Appendix). However, a small tax leads to a first order positive effect on the next period resource stock. By Proposition 2 (iii), these higher future stocks increase the future real rental rate. As noted above, that increase arises because a higher stock leads to a higher equilibrium supply and lower equilibrium price of the resource good. Given that the interest rate is fixed at the discount rate, these higher real rental rates translate unambiguously into a higher first period asset price (by Proposition 1), which increases the (first period) old generation's real income. We have:

Corollary 2 *The tax decreases first-period utility of the present young generation.*

Proof. Propositions 2 and 3 state that the tax decreases aggregate current utility and increases the old generation's utility. Therefore, first-period utility of the current young must fall. ■

The OLG framework shows that a policy that discourages over-use of a resource benefits asset holders and in the first period harms the young agents. The policy also changes the consumption of the current-young in the next period, thereby creating the possibility of higher *lifetime* welfare. To avoid uninteresting complications, we assume for the rest of this section that $\bar{T}_1 = \bar{T}_0 > 0$.

Proposition 4 *For a constant $\chi \in [0, 1]$, a small increase in tax rates (larger ε) increases lifetime welfare of the present young generation if and only if: (a) it receives less than the entire tax revenue while young ($\chi < 1$), and (b) the resource is below its BAU steady state and the agent is sufficiently patient (ρ is sufficiently small).*

From Corollary 1, the young generation's life-time expenditure equals its wage income plus its share of tax revenue when young and old. The proof of Proposition 4 shows that the tax has a zero first order effect on the utility value of the sum of the wage and the tax revenue. Therefore, if the young generation obtains all of the tax revenue ($\chi = 1$), the tax has a zero first order effect on its lifetime welfare. If the young generation receives some tax revenue in the next period ($\chi < 1$), it might benefit or be harmed by the tax policy. It benefits if the discounted gain due to the larger environmental stock in the next period exceeds the loss in current utility due to the tax. Our assumptions on functional forms and of constant tax rates and sharing rules in the first two periods, imply that the net benefit is positive if and only if $\frac{(1+\bar{r}(0,x_t))^{\alpha}}{1+\rho} > 1$. This inequality implies that the present value increase in next period's tax revenue due to the policy-induced change in the environmental stock exceeds 1.

The young generation cares about the distribution of tax revenues, regardless of whether it benefits from the tax policy. Given the constraint that $\chi \in [0, 1]$ is constant, we have

Proposition 5 *The young generation prefers a constant* $\chi = 0$ *(i.e. receipt of all tax revenue when its old) if and only if it benefits from a tax introduction. If the policy lowers their welfare, they prefer to receive all of the tax revenue while young.*

In summary, if the environmental problem is that the resource is below its 0-tax steady state and therefore recovering, but just not recovering sufficiently quickly, then sufficiently patient young would support a tax that speeds recovery. In that circumstance, both the young and the old generations want all of the tax revenue to go to the old, under the constraint that the share is constant. In the more relevant circumstance where the environmental objective is to keep the resource from degrading excessively (i.e. the stock lies above its BAU steady state, so $\bar{r} < 0$), the young would oppose a tax that helps to solve the problem. If such a tax were forced upon them, and the tax share χ were constant, they would prefer to receive all of the tax revenue while young. Thus, in the case that is relevant to most problems involving environmental stocks, this OLG model shows that there is a conflict between generations alive at the time society imposes the tax. The old generation favors the environmental policy because some of the future benefits of that policy are capitalized into the asset value. The current young obtain none of those capitalized benefits, and they do not live long enough to reap significant benefits from the improved environment.

A small tax creates only a second order loss in "static efficiency", the efficiency calculation that holds the trajectory of the resource stock fixed. However, the tax has a positive first order effect on the trajectory of the resource, and that increased stock creates a first order welfare gain. Therefore, there always exists a tax trajectory that benefits all generations born after the initial period. Absent transfers, a constant tax is more likely to benefit future generations compared to the current young generation: the tax-induced higher stock benefits each of the future generations in two periods, whereas it benefits the current young generation in only one period. (Appendix B2.)

4 Transfers

Proposition 4 and 5 are based on the assumption that the old in each period receive the same share of tax revenue, i.e. that $1 - \chi$ is constant. That assumption is useful for understanding the distributional effect of environmental policy, but it might be a poor policy prescription. The old in the period when the tax is imposed – unlike the old in any other period – capture the future benefits that are capitalized in the asset price (Corollary 1). In addition, the young in future periods benefit from a higher resource stock (relative to BAU) in both periods of their life; the young in the current period benefit from environmental protection in only the second period of their life. Therefore, it is reasonable to treat the old and the young in the period when the policy is introduced differently than their counterparts in future periods. In particular, the current young should receive a larger share of tax revenues, compared to the young in future periods.

Given an appropriate transfer, both generations can benefit from the tax even when under BAU the resource is degrading, $\bar{r}(0, x_t) < 0$. The proof of Proposition 4 shows that a small tax has only a second order welfare effect on the young if they receive all of the tax revenue while young ($\chi = 1$). We noted above that the old obtain a first order welfare gain even if they receive none of the tax revenue. Given these two results it is not surprising that for a small tax, it is always possible for the old generation to make a transfer to the young, in addition to giving them all of the tax revenue, so that both generations are better off. This compensation requires that the old give the young a portion of the tax-induced increase in the asset value.

An alternative means of compensating the young is to give them a higher share of tax revenue, compared to the future young. One way to do this is to decrease χ (thereby increasing the share that today's young receive in the next period, when they are old) and simultaneously to give today's young the fraction ξ of today's old generation's share of tax revenue. This transfer scheme ($\xi > 0$), which occurs only in the first period, allows the first period old to keep all of the capital gains and the fraction $(1 - \chi)(1 - \xi)$ of tax revenue. In this way, *the future young (rather than the current old) compensate the current young to make the latter willing to accept the tax policy.* We state this formally:

Proposition 6 *For constant $\chi < 1$ there exists a tax transfer rate $0 \leq \xi^{crit} < 1$ from the present old to the present young such that with $\xi > \xi^{crit}$, a small tax policy with $\bar{T}_0 = \bar{T}_1 > 0$ creates a Pareto improvement.*

Because the young gain under this tax and transfer, an argument parallel to that which establishes Proposition 5 implies that for any $\xi > \xi^{crit}$, both generations prefer $\chi = 0$. The fact that a tax and transfer combination creates a Pareto improvement for the generations alive at the time society imposes the policy is noteworthy because it arises in a model that appears biased in favor of finding that an environmental policy harms some generation. Agents alive at the time the policy is imposed do not care about the welfare of future generations. In addition, they have only one means of accumulation: protecting the environment. That protection always requires that aggregate first period utility of consumption falls.

5 Political Economy Equilibria

In each period, both generations can gain from a tax, given proper allocation of tax revenues. Here we numerically compare the equilibria under BAU, a social planner who maximizes the present discounted value of the stream of aggregate utility, and two political economy models. In the political economy models, we consider a Markov Perfect equilibrium (MPE). Agents condition the choice of the current tax, T, and the current sharing rule, χ, on the only directly payoff-relevant state variable, the environmental stock. The MPE consists of a policy function mapping the state variable into the current policy variables. If it is optimal for current agents to set the current policy levels equal to the value returned by that policy function, given their belief that future agents will use that policy function, then we have a MPE.

The first political economy setting uses a probabilistic voting model (Lindbeck and Weibull, 1987; Perrson and Tabellini, 2000; Hassler et al., 2005). Each of two parties presents a platform consisting of the current values of (T, χ) and an exogenous ideological component. Agents in the two generations currently living care about the consumption transfers achieved through the political process (T and χ); their preference for ideology is orthogonal to the transfers. Preferences may differ between two groups, but agents within a group have the same random preference for consumption relative to ideology. The greater is a group's relative preference for ideology, the fewer swing voters there are in that group, and the less incentive the political parties have to select (T, χ) in an effort to woo voters in that group. In equilibrium, the political parties choose the same endogenous platform (T, χ), and each party has equal chance of winning the election.

The equilibrium (T, χ) maximizes a "political preference function", a weighted combination of the groups' non-ideological component of welfare (which depends only on consumption). Because each generation has the same number of voters, the generation with weaker ideological preferences has the highest weight in this function. The current value of χ appears linearly in this preference function, so the equilibrium χ is on the boundary of its feasible set if the generations' relative preference for consumption differs, and is indeterminate if the generations have the same relative preference. The second political economy setting uses a Nash bargaining model.

In every case, successors' equilibrium decision rules affect current decision makers' incentives. In this game, the equilibrium tax is typically not Pareto efficient, just as in Battaglini and Coate (2007). Bargaining between those currently alive does not resolve the conflict across generations that live during different periods. Generations in the future always prefer that previous generations use a larger tax, to generate a larger environmental stock. Our point is simply that starting with a major unsolved environmental problem, here represented by a zero BAU tax, all generations can be made better off when agents alive at each point are able to use a politically determined tax, even when they do not care about the welfare of those who will live in the future.

5.1 The games

Here we provide the details of the social planner benchmark and the two political economy models. The social planner corrects the environmental externality and the distortion arising from the inability of generations living in different periods to trade with each other. This planner maximizes the present value of the stream of single period aggregate utility. The nominal value of national income in period t is $Y(T_t) = P_t F_t + (1-L)^\beta = \pi_t + w_t + R_t$ and the aggregate utility in period t (real national income) is $p^{-\alpha}(x_t, T_t)Y(T_t)$. (Throughout this section we suppress the constant μ, which merely scales utility.) Schneider, Traeger and Winkler (2012) explain the problems with using parameters that describe individual preferences in an OLG setting to calibrate a social discount rate. Nevertheless, we take the social discount rate to be the individual agent's pure rate of time preference, so that the social planner's problem is time-consistent. The social planner has a standard optimal control problem:

$$\begin{gathered}\max_{\{T_t\}_{t=0}^\infty} \sum_{t=0}^\infty (1+\rho)^{-t} p(x_t, T_t)^{-\alpha} Y(T_t) \\ \text{subject to } x_{t+1} = (1 + \bar{r}_t(x_t, T_t))\, x_t \quad \text{with } x_0 \text{ given.}\end{gathered} \tag{10}$$

Given the solution to this problem (and the fact that utility is linear in income), we can use the asset price equation in Proposition 1 to determine the equilibrium asset price, and thereby determine the intergenerational distribution of welfare under the social planner.

The two political economy models induce different endogenous policy functions. The function $T_t = \Upsilon(x_t)$ gives the equilibrium current period tax and it also induces an equilibrium asset price (measured in units of utility), the functional $\bar{\sigma}(x_{t+1}, \Upsilon(x_{t+1})) \equiv P_t^{-\alpha}\sigma_t$. By purchasing the asset from the old in period t, the agent who is young in period t obtains the utility derived from profits and asset sales when she is old. This functional satisfies the recursion:

$$\bar{\sigma}(x_t, T_t) = \frac{1}{1+\rho}\left\{p^{-\alpha}(x_{t+1}, \Upsilon(x_{t+1}))\pi(\Upsilon(x_{t+1})) + \bar{\sigma}(x_{t+1}, \Upsilon(x_{t+1}))\right\}. \tag{11}$$

Equation (11) is the first order condition for optimal saving behavior of the young agent. The function $\chi_t = \Xi(x_t)$ gives the equilibrium share of tax revenues that the time-t young generation receives. Our goal is to find the pair of equilibrium policy functions Υ and Ξ, which differ in the two political economy models. In both cases, an equilibrium requires that given agents' belief that $T_{t+i} = \Upsilon(x_{t+i})$ and $\chi_{t+i} = \Xi(x_{t+i})$ for $i > 0$, the decisions for the agents choosing the current tax and sharing rule are $T_t = \Upsilon(x_t)$ and $\chi_t = \Xi(x_t)$.

In the probabilistic voting model, the political preference function assigns the weight 1 to the old generation's consumption-related welfare, and the weight $1+\delta$, with $\delta \in (-1, \infty)$, to the young generation's lifetime consumption-related welfare. Using $\pi = Y - w - R$ and the old generation's expenditure $e^o = \pi + (1-\chi)R + \sigma$, we write this political preference function as

$$\begin{gathered} W_t \equiv p(x_t, T_t)^{-\alpha} Y(x_t, T_t) + \bar{\sigma}(x_t, T_t) \\ +\delta p_t^{-\alpha}\left[w_t + \chi_t R_t\right] + \tfrac{1+\delta}{1+\rho} p_{t+1}^{-\alpha}\left[\left(1-\chi_{t+1}\right) R_{t+1}\right]. \end{gathered} \tag{12}$$

The current T_t, χ_t maximize this function, given beliefs about p_{t+1}, R_{t+1}, and χ_{t+1}, which depend on x_{t+1} and the next-period policy functions, Υ and Ξ. The preference function W_t is linear in χ_t, so in equilibrium χ_t is on the boundary of its feasible set, $[b_l, b_u]$; these bounds represent institutional constraints that we take as exogenous. We find (numerically) that in equilibrium, for all t, $R_t > 0$, which implies that $\chi_t = b_u$ if $\delta > 0$ and $\chi_t = b_l$ if $\delta < 0$. The current χ_t is indeterminate, and thus the equilibrium T_{t-i} is indeterminate for $i \geq 1$ if $\delta = 0$. The equilibrium value of χ changes discontinuously as δ passes through 0. However, given χ, both W_t and the maximizing T_t are continuous in δ. Hereafter we assume that $\delta \neq 0$, so that the equilibrium in this voting model is determinate, and also that $|\delta| \approx 0$, which enables us to replace the preference function with

$$\tilde{W}_t = p(x_t, T_t)^{-\alpha} Y(x_t, T_t) + \bar{\sigma}(x_t, T_t) + \frac{1}{1+\rho} p_{t+1}^{-\alpha} \left[\left(1 - \chi_{t+1}\right) R_{t+1} \right], \quad (13)$$

with the understanding that χ_{t+1} is a constant, equal to either b_l (if $\delta < 0$) or b_u (if $\delta > 0$). The political preference function is independent of χ_t, a consequence of the assumption that $|\delta|$ is arbitrarily small. However, the function depends on χ_{t+1}, which determines the revenue split between the current young and next-period young. A larger value of χ_{t+t} means that the next-period young receive more of the next-period tax revenue. When the equilibrium tax is increasing in the stock, a higher value of χ_{t+1} decreases current agents' incentive to protect the stock.

The second political economy setting, emphasized in the numerical analysis, uses a Nash bargaining model in which failure to reach an agreement on the current T, χ results in a zero tax. The "disagreement policy" $T = 0$ implies zero tax revenue, making the choice of the disagreement χ irrelevant. The equilibrium χ is bounded here, so we do not impose exogenous constraints on its feasible set. (If a constraint on χ is binding, the numerical problem becomes more complicated; see Appendix B3.) With unconstrained χ, the equilibrium to the Nash bargaining problem can be found in two steps. Given the policy functions Υ and Ξ that current agents expect future agents to use, the current T_t maximizes $\tilde{W}_t$ and the current χ_t splits the surplus equally between the two generations. This surplus equals the difference between the maximized value of $\tilde{W}_t$ and its value when $T_t = 0$.

In summary, for both political economy models, the equilibrium solves

$$\begin{gathered} \max_{T_t} U_t^o + U_t^y = \max_{T_t} \{ p^{-\alpha}(x_t, T_t) Y(T_t) \\ + \bar{\sigma}(x_t, T_t) + \tfrac{1}{1+\rho} p^{-\alpha}(x_{t+1}, \Upsilon(x_{t+1})) \left[1 - \Xi(x_{t+1})\right] R(\Upsilon(x_{t+1})) \} \qquad (14) \\ \text{subject to } x_{t+1} = (1 + \bar{r}_t(T_t, x_t)) x_t \quad \text{with } x_t \text{ given.} \end{gathered}$$

Problem (14) states that the objective is to maximize the sum of the lifetime utility of the current old and the current young generation. This maximand equals the utility value derived by both generations from current national income, and from owning the asset and receiving the tax revenue in the next period. The solution to the problem depends on the functions $\Upsilon(\cdot)$ and $\Xi(\cdot)$ and on the induced function $\bar{\sigma}(\cdot)$. In the voting model, $\Xi(\cdot)$ returns the constant b_u (for $\delta > 0$) or b_l (for $\delta < 0$), and in the Nash bargaining model $\Xi(\cdot)$ splits the surplus. In both models, $\Upsilon(\cdot)$ returns the maximand to the problem in equation (14). Because the function $\Xi(\cdot)$ differs across the two models, so does $\Upsilon(\cdot)$ and $\bar{\sigma}(\cdot)$.

The primitives of the model lead to explicit expressions for the functions $p(x, T)$ and $Y(T)$. Equation (11) recursively determines the function $\bar{\sigma}(x_t, T_t)$. Agents at time t take the functions $\Upsilon(x_{t+1})$, $\Xi(x_{t+1})$ and $\bar{\sigma}(x_{t+1}, \Upsilon(x_{t+1}))$ as given, but they are endogenous to the problem. We obtain a numerical solution using the collocation method and Chebyshev polynomials (Judd, 1998; Miranda and Fackler, 2002); see Appendix B4.[2]

5.2 Calibration

We set the parameter α, the share of the resource-intensive commodity in the consumption basket, equal to 0.2. We set $\beta = 0.6$, the approximate wage share in U.S. manufacturing. We set the annual pure rate of time preference at 2%/year which gives $\rho = 0.41$ assuming one period lasts 35 years.

We model the renewable resource as easily exhaustible and slowly regenerating, in order to capture the idea that the environmental problem is serious. We choose units of the resource stock, x, such that its carrying capacity is normalized to one, $C = 1$, so that x equals the capacity rate. The productivity parameter γ equals the inverse of the amount of labor that would exhaust the resource in a single period, starting from the carrying capacity $x_0 = 1$. We set $\gamma = 3.33$ and $r = 1.37$ which is equivalent to an uncongested growth rate of 2.5%/year. On a 0-tax trajectory the resource continues to degrade to a steady state of $x_\infty = 0.285$. System (15) summarizes the parameter values:

$$\alpha = 0.2;\ \beta = 0.6;\ \rho = 1;\ r = 1.37;\ \gamma = 3.33. \tag{15}$$

[2] Models of this genus typically have multiple equilibria, as a consequence of the infinite horizon. Experiments suggest that our numerical approach always returns a unique equilibrium. An algorithm that iterates over the value function can be interpreted as the limit as the horizon goes to infinity of a finite horizon model. In view of the generic uniqueness of finite horizon models, the uniqueness of the numerical results is not surprising.

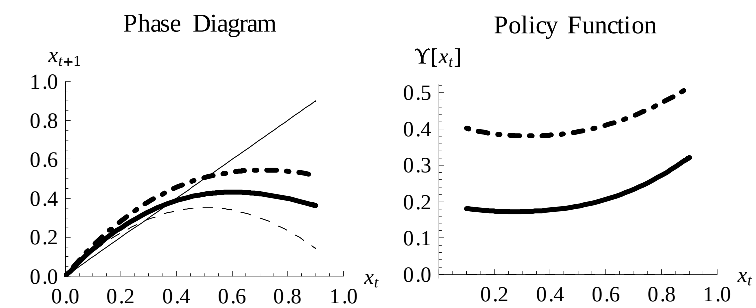

Figure 2: Left panel: the phase diagram for the resource stock under Nash bargaining (solid), BAU (dashed), and the social planner's equilibrium (dot-dash). Right panel: the tax policy under the social planner (dot-dash) and Nash bargaining (solid).

For this parameter set, the old generation has a higher expenditure level than the young under BAU for any stock level. Here, the asset-rich and the asset-poor correspond to the rich and the poor. This calibration also ensures that the young agent can always afford to pay the asset price given her wage income along the equilibrium trajectories. The BAU trajectory is monotonic if and only if $x_0 \leq 0.73$. For larger initial conditions, the BAU trajectory drops below the steady state of 0.28 in the first period and then approaches the steady state monotonically from below.

5.3 A social planner

Figures 2 and 3 contain information on equilibria under the social planner (dot-dash graphs), BAU (dashed graphs), and the MPE under Nash bargaining (solid graphs). The left panel in Figure 2 shows the phase diagrams and the right panel shows the social planner and Nash bargaining tax functions. The equilibrium stock and tax trajectories are higher under the social planner than under the alternatives. The social planner's steady state tax is $T = 0.40$, which induces a steady state level of $x = 0.51$.

Under the social planner, the stock trajectory is a monotonic function of time. In contrast, under BAU, for large initial values of x, the subsequent

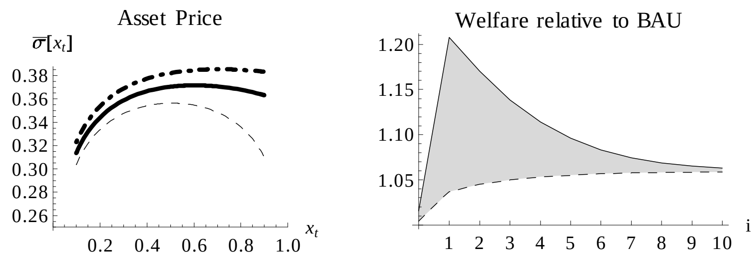

Figure 3: Left panel: The asset price function under Nash bargaining (solid), the social planner (dot-dash), and BAU (dash). Right panel: Welfare effects of Nash bargaining tax policy relative to BAU for initial resource stock $x_0 = 0.45$ (dashed) and $x_0 = 0.9$ (solid).

level of x is below the steady state. (See the dashed curve in left panel of Figure 2.) The BAU trajectory first overshoots the steady state and then approaches the steady state from below. The possibility of overshooting helps to explain why the planner's tax policy is non-monotonic in the stock (right panel in figure 2), and also why the asset value, in units of utility, is (virtually) monotonic in the stock under the social planner, but non-monotonic under BAU (left panel Figure 3). At high values of the resource stock, a high tax prevents the stock from overshooting the steady state, as would occur under BAU. At low values of the resource stock, a high tax helps the resource to regenerate. The planner's tax therefore reaches a minimum for an intermediate value of the stock. Under BAU, the possibility of overshooting causes the asset value to be a non-monotonic function of the stock. A low current resource stock leads to low future stocks and low dividends (the real return to capital), in view of Proposition 2.iii. At a sufficiently high current resource stock, overshooting causes future stocks and real rental rates of capital to be low.

As we noted in discussing Figure 1, a policy intervention always decreases *first period* aggregate utility. Thus, the standard interpretation of results in the Ramsey model implies that policy requires those alive in the present to sacrifice for those alive in the future. Our (standard) choice of the social planner's maximand means that she pays no attention to the asset market.

Nevertheless, her policy sequence changes the asset price sequence; in particular, her intervention creates a capital gain for the first-period asset owner. The resulting transfer from the future to the present means that intervention by the social planner increases the *lifetime* aggregate welfare of those alive in the first period. Ramsey models that either ignore the asset market, or adopt assumptions that make that market trivial, ignore the possibility that all generations can gain from policy intervention.

5.4 The Nash bargaining equilibrium

Figures 2 and 3 also show outcomes under the Nash Bargaining MPE given by the solution to (14), where $\chi = \Xi(x)$ splits the surplus between the generations. Despite the lack of altruism here, this political equilibrium leads to a higher next period stock, compared to BAU, for any current stock: environmental policy protects the resource. The steady state stock level in the political economy equilibrium is 0.38, 33% higher than the BAU level. The MPE equilibrium tax policy, although lower than the social planner's, leads to a utility value of the asset between the BAU and social planner levels. The higher asset values, relative to BAU, compensates generations currently living for the direct loss in utility caused by the tax.

Figure 3, right panel, shows agents' welfare under the MPE tax, relative to BAU levels. For future generations ($i \geq 1$) the figure shows the welfare gain of the young agent, and for the current generation ($i = 0$) it shows the aggregate lifetime welfare gain for the current young and old generations. The dashed curve corresponds to the initial condition $x_0 = 0.45$ and the solid curve corresponds to $x_0 = 0.9$. For intermediate initial conditions, the welfare gain lies between these two curves. If the economy starts out slightly higher than the with-policy steady state, agents gain because under BAU welfare would fall to a low level as the resource degenerates. If the initial resource stock is far above the steady state, future generations additionally benefit because the tax prevents overshooting. For large initial stocks ($x_0 = 0.9$), the gain in moving from BAU to Nash bargaining is large; as the stock falls toward the bargaining steady state, the extent of BAU overshooting also falls, so the welfare gain falls over time. For moderate initial stocks ($x_0 = 0.45$), Nash bargaining keeps the stock above the steady state $x = 0.38$, whereas under BAU the stock would approach $x = 0.28$; in this case, the welfare gain rises over time. Nash bargaining increases the first generations' aggregate welfare by $0.5 - 2\%$, and increases the steady state welfare by 6%.

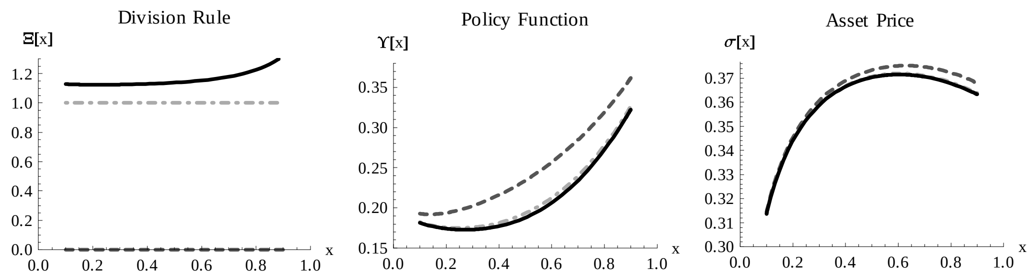

Figure 4: Left panel: The equilibrium sharing rule under Nash bargaining (solid) and in the probabilistic voting model where the young generation has more swing voters ($\chi = 1$: light gray, dot-dash) and where the old generation has more swing voters ($\chi = 0$: dark gray, dash). Middle panel: The equilibrium tax policy corresponding to these three cases. Right panel: the asset price corresponding to the three cases.

5.5 Comparison of political economy models

Here we compare the Nash bargaining and the probabilistic voting equilibria. In the latter, χ is on the boundary of its feasible set if generations have different preference for ideology relative to consumption. We take the feasible set $[b_l, b_u] = [0, 1]$ and consider the two cases where the young have (slightly) more or fewer swing voters than the old, corresponding to $\chi = 1$ and $\chi = 0$. The left panel of Figure 4 shows the equilibrium $\Xi(x)$ in the Nash bargaining game and in the two probabilistic voting models; the middle panel shows the tax policy rules corresponding to these three case, and the right panel shows the asset price. There are three results.

First, the equilibrium χ under Nash bargaining exceeds 1 and increases with the stock. The young receive all of the tax revenue and additional transfers from the old; the transfers increase with the size of the stock. Recall that the current allocation of labor depends only on the current tax, and that the threat point involves a zero current tax. Moreover, at high stock levels there is overshooting under BAU. A zero current tax harms the current young because it leads to a lower stock in the second period of their life. The zero tax harms the current old because it lowers the next period stock, lowering the asset value. At large stocks, where overshooting occurs under

BAU extraction, this decrease in asset price is particularly strong. Both generations lose under a zero tax, but for our calibration, the loss is greater for the old than for the young, especially at higher stock levels. Consequently, the young extract a transfer from the old in addition to all tax revenues. Proposition 6 notes that with a small exogenous tax, the young are better off even if they receive only a fraction of tax revenues. Here, however, the endogenous tax is not small, and we require that the young's surplus equal the old's, not merely that it is positive.

Second, the tax functions are higher, the lower is the function $\Xi(x)$. In all cases the equilibrium tax depends on the next period value of χ, but not on its current value. The current value determines the revenue split between the currently living agents, whereas the next period value of χ determines the fraction of revenue that the next-period young receive. For most of the stock levels, the equilibrium tax increases in the stock. Over this interval, a higher χ_{t+1} gives current agents less incentive to protect the stock, because more of the benefit of conservation accrues to the next-period young. Thus, higher values of χ_{t+1} induce smaller current taxes.

Third, the difference in equilibrium tax rules and asset values under Nash bargaining, where χ is unconstrained, and under probabilistic voting when the young have more swing voters ($\chi = b_u = 1$) is negligible. Therefore, we do not consider sensitivity analysis with respect to b_u. We do, however, consider other types of robustness checks, based on quite different political economy models. Referees' Appendix B5 reports these alternatives; the qualitative results described in the text are robust to the alternatives in the appendix.

6 Discussion

Analysis of stock-related environmental policy often starts from the presumption that this policy requires current sacrifices in order to protect future generations. Existing challenges to this presumption include the existence of win-win situations, or the possibility of making agents in all periods better off by reallocating savings from man-made to natural capital. We provide a different perspective, using a model that excludes these kinds of challenges to the conventional view.

The key to our result is that the policy-induced improvements in the future environmental stock, relative to BAU, increase the value of a traded productive asset, capital. This benefit more than offsets the fall in the current real rental rate, caused by the tax-induced increase in the relative commodity price. The tax therefore increases the welfare of the old generation, who owns the asset. Asset prices are a means of transferring gains in the future to the present period. The young generation bears the policy cost in the first period of their life, without benefiting from the change in asset price. If the environmental stock is decreasing on the BAU trajectory, the tax reduces their welfare. However, if the old give the young a sufficiently larger share of the tax revenue – compared to the share that future young generations will obtain – both generations are better off. Future generations are also better off because of the improved environmental quality. Such Pareto-improving policy can be implemented and sustained in various political economy settings.

We presented these results using a generic renewable resource model, but an important motivation for the research arises from the controversy surrounding climate policy, and in particular the extent to which meaningful policy reduces currently living agents' welfare. The generic renewable resources model has the virtue of simplicity and familiarity, but it is not designed to study climate policy. For that purpose, the following features are useful: an endogenously changing capital stock, leading to a model with at least two state variables, the environmental stock and the capital stock; a source of friction that maintains a non-trivial asset price, i.e. that prevents the price of capital from being identically equal to the price of the composite consumption good; and a richer range of utility functions, e.g., dropping the assumption of infinite elasticity of intertemporal substitution. Ongoing research confirms that the qualitative insights of the simpler model presented here, survive under empirically plausible circumstances in the more complicated setting.

Real economies contain many different types of capital, whereas our model, like most integrated assessment models used to study climate policy, has a single type of capital. In real economies, environmental policy likely decreases the price of some assets and increases the price of others. For example, climate policy would reduce the value of coal mines and coal-fired power plants, but increase the value of assets in the renewable energy sector; stricter fishing quotas would decrease short run fishing profits but might increase the value of the fishing fleet, or some other sector-specific asset. A model with a single type of capital cannot capture this range of possibility. However, if asset owners have a diversified portfolio, the single-asset approximation may be reasonable. Asset owners care about the price of their portfolio, not the price of a specific asset.

In our closed economy general equilibrium model, the privately owned asset competes with the open access resource for mobile labor. A higher resource stock increases the wage and reduces the nominal return to the asset. But because of changes in the relative commodity price, the higher stock increases the asset's real return. The general equilibrium framework emphasizes the link, in a closed economy, between the endogenous relative commodity price and the real rental rate or asset price. This link disappears in a composite-commodity model, where nominal and real returns are equivalent. The general equilibrium model is also suitable for comparing the incentives to protect a resource across closed and small open economies; in the latter, the relative commodity price is fixed, just as in the composite commodity model. Work in progress shows that opening a closed economy to trade significantly alters the incentives to protect a resource.

Our paper considers the distributional effects of a policy that ameliorates a particular market failure, weak property rights to a resource. However, the message of our paper is more general, and likely applies to other market and political failures. Efforts to improve such defects often create transitional costs to society; a regime bent on reform may make matters worse in the short run. The message of our paper is that even if different groups (owners of capital and labor in our model) face qualitatively similar short run effects (lower real returns in our model) the groups' differential ownership of assets may cause them to face qualitatively different welfare effects. People who own assets are able to capture some of the future benefits of reform, via the asset market. Those without assets benefit from the reforms only to the extent that they will be alive long enough to enjoy them, or if they receive transfers from currently living beneficiaries. If the principal benefit of reform occurs after the lifetime of those without assets, as is the case in our model, then transfers are needed to sustain support for the reform. However, even a rather pessimistic political environment, one without altruism, intergenerational borrowing, or commitment devices, can produce such support.

A Proofs

Proof. Proposition 1. The vector $\mathbf{T}_t$ consists of taxes in periods t, $t+1$, $t+2$,... . The price of a firm this period is σ_t and the expectation of the next-period price is $\tilde{\sigma}_{t+1}$. In equilibrium the young generation buys s_t shares of the firm today and sells s_{t+1} of it in the next period. With intertemporally additive, homothetic lifetime utility, the present value of total utility of the young agent is:

$$U_t^y = \mu P_t^{-\alpha} e_t^y + \tfrac{1}{1+\rho}\mu \tilde{P}_{t+1}^{-\alpha} \tilde{e}_{t+1}^o = \mu \times$$
$$\left(P_t^{-\alpha}\left(w_t + \chi_t R_t - s_t \sigma\left(x_t, \mathbf{T}_t\right)\right) + \tfrac{1}{1+\rho}\tilde{P}_{t+1}^{-\alpha}\left[(1-\tilde{\chi}_{t+1})\tilde{R}_{t+1} + s_t\left(\tilde{\pi}_{t+1} + \tilde{s}_{t+1}\tilde{\sigma}(x_{t+1}, \mathbf{T}_{t+1})\right)\right]\right). \tag{A.1}$$

If a young person buys one share of the factory today, costing σ_t, the loss in utility is $\mu P_t^{-\alpha}\sigma_t$ assuming that $\sigma_t < w_t$.[3] Purchase of one share of the factory today increases expenditures next period by $\tilde{\pi}_{t+1} + \tilde{s}_{t+1}\tilde{\sigma}_{t+1}$; the increase in the present value of utility next period due to the purchase of the factory is $\frac{1}{1+\rho}\mu\tilde{P}_{t+1}^{-\alpha}\left(\tilde{\pi}_{t+1} + \tilde{s}_{t+1}\tilde{\sigma}_{t+1}\right)$. The equilibrium price-of-factory sequence requires that excess demand for the asset is 0, which, under rational expectation, requires satisfaction of the first order condition for optimal saving behavior

$$P_t^{-\alpha}\sigma_t = \frac{1}{1+\rho} P_{t+1}^{-\alpha}\left(\pi_{t+1} + \sigma_{t+1}\right). \tag{A.2}$$

Write this first order condition, equation (A.2), as

$$\sigma_t = \frac{1}{1+\rho}\left(\frac{P_t}{P_{t+1}}\right)^{\alpha}\left(\pi_{t+1} + \sigma_{t+1}\right) \tag{A.3}$$

or

$$\sigma_{t+i} = \frac{1}{1+\rho}\left(\frac{P_{t+i}}{P_{t+1+i}}\right)^{\alpha}\left(\pi_{t+i+1} + \sigma_{t+i+1}\right), \tag{A.4}$$

so

$$\sigma_t = \frac{1}{1+\rho}\left(\frac{P_t}{P_{t+1}}\right)^{\alpha}\pi_{t+1} + \frac{1}{1+\rho}\left(\frac{P_t}{P_{t+1}}\right)^{\alpha}\left[\frac{1}{1+\rho}\left(\frac{P_{t+1}}{P_{t+2}}\right)^{\alpha}\left(\pi_{t+2} + \sigma_{t+2}\right)\right]. \tag{A.5}$$

[3]Throughout our derivations, we assume that such a non-negativity constraint is not binding.

By repeated substitution obtain

$$\sigma_t = \sum_{j=1}^{S} \left(\frac{1}{1+\rho}\right)^j \left[\left\{\Pi_{s=0}^{j-1}\left(\frac{P_{t+s}}{P_{t+s+1}}\right)^\alpha\right\}\pi_{t+j}\right] + \left(\frac{1}{1+\rho}\right)^S \left[\left\{\Pi_{s=0}^{S-1}\left(\frac{P_{t+s}}{P_{t+s+1}}\right)^\alpha\right\}\sigma_{t+S}\right] \tag{A.6}$$

If the system converges to a steady state, then the second term goes to 0 as $S \to \infty$ and

$$\sigma_t = \sum_{j=1}^{\infty} \left(\frac{1}{1+\rho}\right)^j \left[\left\{\Pi_{s=0}^{j-1}\left(\frac{P_{t+s}}{P_{t+s+1}}\right)^\alpha\right\}\pi_{t+j}\right]. \tag{A.7}$$

Note that

$$\Pi_{s=0}^{j-1}\left(\frac{P_{t+s}}{P_{t+s+1}}\right)^\alpha = \left(\frac{P_t}{P_{t+j}}\right)^\alpha \tag{A.8}$$

Using this relation we have

$$\sigma_t = P_t^\alpha \sum_{i=1}^{\infty} (1+\rho)^{-i} P_{t+i}^{-\alpha} \pi_{t+i}. \tag{A.9}$$

■

Proof. Corollary 1. (i) The imposition of the first order condition simplifies the lifetime welfare expression of the young, equation (A.1), to:

$$U_t^y = \mu \left[p(T_t, x_t)^{-\alpha}(w(T_t) + \chi_t R(T_t)) + \frac{p(T_{t+1}, x_{t+1})^{-\alpha}}{1+\rho}(1-\chi_{t+1})R(T_{t+1})\right]. \tag{A.10}$$

The first order condition implies that the young generation's utility is independent of the asset price. A loss in utility from the higher asset price in the first period equals the discounted utility gain from increased profits and asset price in the second period. As a consequence, the young generation's expenditure equals wage income in the first period and their share of the tax revenue in the first and second period. Their welfare considerations are limited to these expenditure components and the price effects.

(ii) The same holds for all future generations. Asset prices enter only the welfare expression of the current old generation. The current owners of the asset capture all future benefits reflected in a changed asset price. ■

Proof. Proposition 2. (Sketch) (i) Using systems (5) and (6), the nominal value of national income in period t is

$$Y(T_t) = P_t F_t + (1-L)^\beta. \tag{A.11}$$

We multiply nominal national income by $\mu P^{-\alpha}$ to convert dollars to utils; $P = p(x_t, T_t)$ is a function of both the tax and the environmental stock. Using the equilibrium expressions for $Y(T)$ and $p(x, T)$, The single period aggregate utility is

$$U(x_t, T_t) \equiv \mu p(x_t, T_t)^{-\alpha} Y(T_t) = x_t^{\alpha} \phi(T_t), \tag{A.12}$$

$$\text{with } \phi(T_t) \equiv \mu \left(\frac{\beta \left(1 + \frac{1-T}{\frac{1-\alpha}{\alpha}\beta}\right)^{1-\beta}}{(1-T)\gamma} \right)^{-\alpha}. \tag{A.13}$$

Differentiating with respect to T and simplifying gives, for $T \neq 0$,

$$\frac{dU}{dT} = -\mu P^{-\alpha} \frac{(1-\alpha)\beta LT}{(1-T)^2} Y < 0. \tag{A.14}$$

(ii) The tax decreases the nominal wage, w, and increases the equilibrium relative price, P, and therefore decreases the real wage.

The real rental rate is $\mu P^{-\alpha}\pi$. The tax lowers the equilibrium nominal wage, increasing nominal profits, π, but it also increases the commodity price. Using the fact that preferences are homothetic and that the wage share is constant, we have

$$\mu P^{-\alpha}\pi = \mu P^{-\alpha}(1-\beta)\frac{Y}{1-\alpha}. \tag{A.15}$$

Differentiating this with respect to T gives, for $T \neq 0$,

$$\frac{d\mu P^{-\alpha}\pi}{dT} = \mu \frac{1-\beta}{1-\alpha}\frac{dP^{-\alpha}Y}{dT} = -\mu P^{-\alpha}\frac{(1-\beta)\beta LT}{(1-T)^2} Y < 0. \tag{A.16}$$

(iii) A higher stock does not affect the nominal wage but it decreases the equilibrium relative price, so it increases the real wage. A higher stock does not alter nominal profits, but decreases the commodity price, thereby increasing real profits. ■

Proof. Proposition 3. We begin by establishing two inequalities, which we then use to prove the proposition:

$$\text{(i) } \frac{\partial x_{t+i}}{\partial T_{t+i-1}} > 0 \text{ and (ii) } \frac{\partial x_{t+i+1}}{\partial x_{t+i}} > 0 \text{ for } i \geq 1 \text{ and } x_t \in \hat{X} \tag{A.17}$$

The first inequality states that a tax in any period increases the stock in the next period; the second inequality states that for all initial conditions

$x_t \in \hat{X}$, a higher stock in any future period $t+i$ leads to a higher stock in the subsequent period, $t+i+1$. The discussion below equation (8) establishes inequality (A.17.i). To establish inequality (A.17.ii), we use the fact that the BAU mapping from x_t to x_{t+1} is single-peaked and reaches its maximum at $x = \frac{C\varsigma}{2r} = \frac{1}{2}\left(x_\infty + \frac{C}{r}\right) < x_\infty$ (with the inequality following our restriction on ς). Define $X = \left(0, \frac{C\varsigma}{2r}\right)$, the set for which the mapping $(1 + \bar{r}(0, x_t))\, x_t$ is strictly monotonically increasing. Due to monotonicity, the set X is absorbing: for any $x_t \in X$, $x_{t+1} \in X$. Strict monotonicity implies that a small increase in $x_t \in X$ leads to an increase in x_{t+1}. $X \subset \hat{X}$ and, in addition, for any initial condition $x_t \in \hat{X}$, $x_{t+1} \in X$. To verify this claim, consider the two sets X and $\hat{X}/X$. For $x_t \in X$, $x_{t+1} \in X$ because X is absorbing. For $x_t \in \hat{X}/X$, $x_{t+1} \in X$ follows from the definition of the sets X and $\hat{X}$ and the fact that the growth function is single-peaked.

The old generation's remaining lifetime welfare consists of the utility it obtains from current consumption,

$$U_t^o(\varepsilon) = \mu\left(p\left(x_t, T_t\right)^{-\alpha}(1-\chi)R_t + \sum_{i=0}^{\infty}(1+\rho)^{-i} p\left(x_{t+i}, T_{t+i}\right)^{-\alpha}\pi_{t+i}\right). \tag{A.18}$$

We start with the derivative of the second term in U^o, the return to holding the asset. We differentiate each term in the sum by $T_i = \varepsilon \bar{T}_i$, recognizing that T_i has a direct effect on $\pi_{t+i}p_{t+i}^{-\alpha}$ and an indirect effect, via its effect on x_{t+j}, on $\pi_{t+j}p_{t+j}^{-\alpha}$ for $j > i$. We use $T_i = \varepsilon \bar{T}_i$, so $dT_i = \bar{T}_i d\varepsilon$.

$$\begin{aligned}\frac{\mathrm{d}\sum_{i=0}^{\infty}(1+\rho)^{-i}\pi_i p_{t+i}^{-\alpha}}{\mathrm{d}\varepsilon} &= \frac{\partial \pi_t p_t^{-\alpha}}{\partial T_t}\bar{T}_t + (1+\rho)^{-1}\left[\frac{\partial \pi_{t+1}p_{t+1}^{-\alpha}}{\partial T_{t+1}}\bar{T}_{t+1} + \frac{\partial \pi_{t+1}p_{t+1}^{-\alpha}}{\partial x_{t+1}}\frac{\partial x_{t+1}}{\partial T_t}\bar{T}_t\right] \\ &+(1+\rho)^{-2}\left[\frac{\partial \pi_{t+2}p_{t+2}^{-\alpha}}{\partial T_{t+2}}\bar{T}_{t+2} + \frac{\partial \pi_{t+2}p_{t+2}^{-\alpha}}{\partial x_{t+2}}\left(\frac{\partial x_{t+2}}{\partial T_{t+1}}\bar{T}_{t+1} + \frac{\partial x_{t+2}}{\partial x_{t+1}}\frac{\partial x_{t+1}}{\partial T_t}\bar{T}_t\right)\right] \\ &+(1+\rho)^{-3}\left[\frac{\partial \pi_{t+3}p_{t+3}^{-\alpha}}{\partial T_{t+3}}\bar{T}_{t+3} + \frac{\partial \pi_{t+3}p_{t+3}^{-\alpha}}{\partial x_{t+3}}\left(\frac{\partial x_{t+3}}{\partial T_{t+2}}\bar{T}_{t+2} + \frac{\partial x_{t+3}}{\partial x_{t+2}}\frac{\partial x_{t+2}}{\partial T_{t+1}}\bar{T}_{t+1} + \frac{\partial x_{t+3}}{\partial x_{t+2}}\frac{\partial x_{t+2}}{\partial x_{t+1}}\frac{\partial x_{t+1}}{\partial T_t}\bar{T}_t\right)\right] + \ldots.\end{aligned} \tag{A.19}$$

We simplify this expression using the fact that at $\varepsilon = 0$, $T_0 = T_1 = ... = 0$. Evaluating the different expressions along the BAU trajectory, we have

$$\frac{\partial \pi_i p_i^{-\alpha}}{\partial T_i} = 0; \quad \pi_0 = \pi_1 = ... = \pi; \quad \text{and} \quad \frac{\partial \pi_i p_i^{-\alpha}}{\partial x_i} = \eta x_i^{\alpha-1}, \text{ with } \eta \equiv \alpha\pi\left(\frac{w}{\gamma}\right)^{-\alpha} > 0. \tag{A.20}$$

Using the convention that $\prod_{j}^{j-1} z_j = 1$, we write the i'th term in the sum above as $(1+\rho)^{-i}\theta_i$, with

$$\theta_i \equiv \eta x_{t+i}^{\alpha-1}\left[\sum_{j=0}^{i-1}\left\{\frac{\partial x_{t+i-j}}{\partial T_{t+i-j-1}}\bar{T}_{t+i-j-1}\left(\prod_{k=0}^{j-1}\frac{\partial x_{t+i-k}}{\partial x_{t+i-k-1}}\right)\right\}\right] \geq 0. \qquad \text{(A.21)}$$

The inequality $\theta_i \geq 0$ holds strictly for some i; this claim uses inequalities (A.17 i and ii) and the assumption, $\tilde{T}_{t+i-j-1} \geq 0$ with strict inequality for some $i-j-1 \geq 0$.

The old also receive a share of the tax revenue. The effect of a tax increase on current tax revenue is

$$\begin{aligned}\left.\frac{dP^{-\alpha}(1-\chi)R}{d\varepsilon}\right|_{\varepsilon=0} &= \underbrace{(1-\chi)R\frac{dP^{-\alpha}}{d\varepsilon}}_{=0} + (1-\chi)P^{-\alpha}\left.\frac{dR}{d\varepsilon}\right|_{\varepsilon=0} \\ &= (1-\chi)P^{-\alpha}\frac{\alpha\left(1+\frac{\alpha}{\beta(1-\alpha)}\right)^{-\beta}}{1-\alpha}\bar{T}_t > 0 \qquad \text{(A.22)}\end{aligned}$$

Given that the derivatives of both terms in U^o are positive, a small tax trajectory increases the welfare of the old generation.

For a positive current tax, $R_t > 0$, and the old generation's utility strictly increases in its share of the tax revenue. ■

Proof. Proposition 4. The lifetime welfare of the young, from equation (A.10), is

$$U_t^y(\varepsilon) = \mu p(\bar{T}_t\varepsilon, x_t)^{-\alpha}\left(w(\bar{T}_t\varepsilon) + \chi R(\bar{T}_t\varepsilon) + \frac{\left(1+\bar{r}(\bar{T}_t\varepsilon, x_t)\right)^{\alpha}}{1+\rho}(1-\chi)R(\bar{T}_{t+1}\varepsilon)\right). \qquad \text{(A.23)}$$

Differentiating this expression with respect to ε gives

$$\frac{dU_t^y}{d\varepsilon} = \frac{d}{d\varepsilon}\mu P_t^{-\alpha}(w(\bar{T}_t\varepsilon)+\chi R(\bar{T}_t\varepsilon)) + \frac{d}{d\varepsilon}\left[\mu P_t^{-\alpha}\frac{\left(1+\bar{r}(\bar{T}_t\varepsilon, x_t)\right)^{\alpha}}{1+\rho}(1-\chi)R(\bar{T}_{t+1}\varepsilon)\right]. \qquad \text{(A.24)}$$

The pre-tax allocation maximizes current aggregate utility. Therefore, the tax has a zero first order effect on aggregate current utility: $\left.\frac{dP^{-\alpha}w}{d\varepsilon}\right|_{\varepsilon=0} +$

$\left.\frac{dP^{-\alpha}\pi}{d\varepsilon}\right|_{\varepsilon=0} + \left.\frac{dP^{-\alpha}R}{d\varepsilon}\right|_{\varepsilon=0} = 0$. Equation (A.16) shows that $\left.\frac{dP^{-\alpha}\pi}{d\varepsilon}\right|_{\varepsilon=0} = 0$, which implies $\left.\frac{d\left[P^{-\alpha}(w+R)\right]}{d\varepsilon}\right|_{\varepsilon=0} = 0$. Using this relation, and the assumption $\bar{T}_0 = \bar{T}_1$, we write equation (A.24) as

$$\frac{dU_t^y}{d\varepsilon} = \mu\,(1-\chi)\,\frac{d}{d\varepsilon}\left[\left(\frac{\left(1+\bar{r}(\bar{T}_t\varepsilon, x_t)\right)^{\alpha}}{1+\rho} - 1\right) P_t^{-\alpha} R(\bar{T}_t\varepsilon)\right] \tag{A.25}$$

Using the fact that $R(0) = 0$, this equation simplifies to

$$\frac{dU_t^y}{d\varepsilon} = \mu\,(1-\chi)\,P_t^{-\alpha}\left(\frac{\left(1+\bar{r}(0, x_t)\right)^{\alpha}}{1+\rho} - 1\right)\left[\left.\frac{dR(\bar{T}_t\varepsilon)}{d\varepsilon}\right|_{\varepsilon=0}\right]. \tag{A.26}$$

The first-order response of tax revenue to a small tax introduction is positive: $\left.\frac{dR(\bar{T}_t,\varepsilon)}{d\varepsilon}\right|_{\varepsilon=0} = \frac{\alpha}{1-\alpha}\left(1+\frac{\alpha}{\beta(1-\alpha)}\right)^{-\beta}\bar{T}_t > 0$. Consequently, we have

$$\left.\frac{dU_0^y}{d\varepsilon}\right|_{\varepsilon=0} > 0 \Leftrightarrow (1-\chi)\left(\frac{\left(1+\bar{r}(0, x_0)\right)^{\alpha}}{1+\rho} - 1\right)\bar{T}_0 > 0. \tag{A.27}$$

With $\chi \in [0,1]$, a small tax increases the lifetime welfare of the young generation if and only if $\chi < 1$ and $\left(1+\bar{r}(0,x_t)\right)^{\alpha} > (1+\rho)$. A small tax creates a zero first order welfare effect for the young generation that receives all tax revenue ($\chi = 1$). Condition (b) in the Proposition is equivalent to $\bar{r}(0,x_t) > (1+\rho)^{\frac{1}{\alpha}} - 1$. For $\rho > 0$, the expression on the right side of the previous inequality is positive. Thus, a necessary condition for the young to benefit from a tax is that the resource is below its 0-tax steady state, and is in the process of sufficiently strong recovery. ■

Proof. Proposition 5. The last equation in system (5) implies that $p(T_t, x_{t+1})^{-\alpha} = p(T_t, x_t)^{-\alpha}(1+\bar{r}(T_t, x_t))^{\alpha}$. This equality and the fact that the young generation's welfare is linear in χ, from equation (A.10), implies that

$$\frac{dU_t^y}{d\chi} < 0 \Leftrightarrow T_t\left(\frac{\left(1+\bar{r}(T_t, x_t)\right)^{\alpha}}{1+\rho} - 1\right) > 0. \tag{A.28}$$

We also have $\frac{d\bar{r}_t}{dT_t} > 0$. This inequality and inequalities (A.27) and (A.28) imply that if the young benefit from a small tax, then they prefer to receive all of the tax revenue when they are old, i.e. they prefer $\chi = 0$. In contrast, if the young are harmed by a small tax, then provided that the tax is small they prefer to receive all of the tax revenue when young ($\chi = 1$). ■

Proof. Proposition 6. With ξ the share of the old generation's tax revenue transferred to the young in the period when the tax is first imposed, the first period's tax receipts are now $(\chi_0 + (1-\chi_0)\xi)R_0$ for the young and $(1-\chi_0)(1-\xi)R_0$ for the old. Under the assumption that current and next period tax rates are changed by the same small amount and that χ is constant, an argument similar to that in the proof of proposition 4 leads to the following condition for the young to benefit from the combined transfer and tax:

$$\left.\frac{dU_0^y}{d\varepsilon}\right|_{\varepsilon=0} > 0 \Leftrightarrow (1-\chi)\left(\frac{(1+\bar{r}(0,x_0))^\alpha}{1+\rho} - (1-\xi)\right)\bar{T}_0 > 0. \qquad \text{(A.29)}$$

Setting $\xi = 0$, equation (A.29) reproduces equation (A.27). For

$$\xi > \xi^{crit} \equiv 1 - \frac{(1+\bar{r}(0,x_0))^\alpha}{1+\rho} \qquad \text{(A.30)}$$

the young strictly prefer the combined tax and transfer compared to the BAU status quo. Even if the resource is degrading on the BAU trajectory, $\xi^{crit} < 1$. Therefore, by transferring less than their entire share of the tax revenue to the young, the old can make the young better off under a small tax. Because Proposition 3 states that the tax improves the old generation's welfare even if they receive none of the tax revenue, the old are also better off under the combined tax and transfer, compared to the status quo. ■

B Appendix for "The Political Economy of Environmental Policy with Overlapping Generations"

This appendix collects supplementary information alluded to in the journal publication.

B1 Exogenous Productivity Growth

In the context of most environmental problems, the natural resource is degrading on the 0-tax trajectory. In our model of constant productivity and capital, the world becomes poorer and future generations have lower welfare on that trajectory. This Appendix introduces exogenous productivity growth in both sectors. Let $a \geq 0$ be the growth rate of total factor productivity in manufacturing and $b \geq 0$ the growth rate of efficiency in output per unit flow of the resource. Sectoral output is

$$M_t = e^{at}(1 - L_t)^{\beta} \quad \text{and} \quad F_t = e^{bt} L_t \gamma x_t.$$

The inequality $a > 0$ can also be interpreted as exogenous growth in the stock of capital. The extraction of the resource is still $L_t \gamma x_t$ (not $e^{bt} L_t \gamma x_t$). This model of resource productivity growth implies that each extracted unit of the resource increases the supply of the resource-intensive commodity. If we think of the resource as being energy, the assumption means that the economy becomes less energy intensive. The assumption of exponential productivity growth simplifies the discussion, but the next proposition also holds if the productivity parameters a and b decrease over time. The exponential productivity growth implies a growth factor of $e^{(a-b)}\left(1 + \bar{r}(T_t, x_t)\right)$ for the price level and of e^a for all other variables (w_t, R_t, and π_t). For the following proposition we assume that $\chi \in (0, 1)$ is constant and that there is no transfer between generations, i.e. $\xi = 0$.

Proposition 7 *A larger value of $a - b$ increases the stringency of the necessary and sufficient condition under which a small constant tax increases the welfare of the young.*

Proof. Using a derivation parallel to that contained in the proof of proposition 4, we have

$$\left.\frac{dU_0^y}{d\varepsilon}\right|_{\varepsilon=0} > 0 \Leftrightarrow (1-\chi)\left(\frac{e^{-(a-b)\alpha}\left(1+\bar{r}(0,x_0)\right)^{\alpha}}{1+\rho}-1\right)\bar{T}_0 > 0.$$

The second inequality is equivalent to

$$\left(\frac{1+\bar{r}(0,x_0)}{e^{(a-b)}}\right)^{\alpha} > 1+\rho. \tag{B.1}$$

The left side of inequality (B.1) is decreasing in $a-b$, so an increase in $a-b$ decreases the set of parameter values and initial conditions under which the inequality is satisfied, i.e. the circumstances under which the young benefit from the tax. ■

Under proportional growth ($a=b$), the condition for the young to benefit from the tax is the same as when $a=b=0$. The welfare effect of the tax, for the young, depends on the change in the price level. A *ceteris paribus* increase in $a-b$ increases the next period relative supply of the manufacturing good, thereby increasing the future relative price of the resource-intensive good, P_{t+1}. The higher price lowers the marginal utility of next period income, making it "less likely" that the young are willing to forgo income today in order to have higher income in the next period. For $a>b$, the young would require a higher transfer from the old in order to agree to the tax. If, however, the productivity in the resource sector grows much faster than in the manufacturing sector ($b >> a$), the young might support a tax even when the resource is shrinking on the 0-tax trajectory, and in the absence of a transfer.

B2 Future generations

Merely in order to avoid uninteresting complications, we assume that for future generations the tax is constant: $\bar{T}_0=\bar{T}_1=\bar{T}_2$.... The life-time welfare of the next young generation is

$$U_1^y(\varepsilon) = \mu p(\bar{T}_1\varepsilon,x_1)^{-\alpha}\left(w(\bar{T}_1\varepsilon)+\chi R(\bar{T}_1\varepsilon)+\frac{\left(1+\bar{r}(\bar{T}_1\varepsilon,x_1)\right)^{\alpha}}{1+\rho}(1-\chi)R(\bar{T}_2\varepsilon)\right).$$

Differentiating this expression with respect to ε gives

$$\frac{dU_1^y}{d\varepsilon} = \frac{d}{d\varepsilon}\mu P_1^{-\alpha}(w(\bar{T}_1\varepsilon)+\chi R(\bar{T}_1\varepsilon))+\frac{d}{d\varepsilon}\left[\mu P_1^{-\alpha}\frac{\left(1+\bar{r}(\bar{T}_1\varepsilon, x_1)\right)^{\alpha}}{1+\rho}(1-\chi)R(\bar{T}_2\varepsilon)\right]$$

Using the simplifications found in the proof of Proposition 4, (including $R(0)=0$ and $\frac{\partial P_1^{-\alpha}}{\partial x_1} = \alpha P_1^{-\alpha}x_1^{-1}$) the expression simplifies to

$$\begin{aligned}
\left.\frac{dU_1^y}{d\varepsilon}\right|_{\varepsilon=0} > 0 \;\; &\Leftrightarrow \;\; \bar{T}_1 P_1^{-\alpha}(1-\chi)\left(\frac{(1+\bar{r}(0,x_1))^{\alpha}}{1+\rho}-1\right)\left.\frac{dR}{d\varepsilon}\right|_{\varepsilon=0} > -\bar{T}_0 w(0)\frac{\partial P_1^{-\alpha}}{\partial x_1}\frac{\partial x_1}{\partial T_0} \\
&\Leftrightarrow \;\; (1-\chi)\left(\frac{(1+\bar{r}(0,x_1))^{\alpha}}{1+\rho}-1\right)\bar{T}_0 > -\left(w(0)\alpha x_1^{-1}\frac{\partial x_1}{\partial T_0}\right)\left(\frac{1}{dR/d\varepsilon|_{\varepsilon=0}}\right)\bar{T}_0
\end{aligned}$$

Comparing this condition to inequality (A.27), we see that when the stock is degrading (i.e. $\bar{r}(0,x_0)<0$), a small tax is more likely to benefit the next period's young generation compared to today's, which always loses in the absence of transfers. The difference arises for two reasons: A lower stock increases the BAU growth rate, $\frac{d\bar{r}(0,x_t)}{dx_t} = -r < 0$, so that the left side is less negative. The right side of the inequality above is negative. Therefore, the condition here is weaker than the condition in inequality (A.27). In fact, it is satisfied for any initial stock value in the calibration used in Section 5.

B3 Nash Bargaining

Using equations (5) - (7), we define the lifetime welfare of the two agents

$$\begin{gathered}
U^o\left(x_t, \Upsilon\left(x_t\right), \chi_t\right) = P^{-\alpha}e^o \\
= p^{-\alpha}(x_t, \Upsilon\left(x_t\right))\left[\pi(\Upsilon\left(x_t\right)) + (1-\chi_t)R(\Upsilon\left(x_t\right))\right] + \bar{\sigma}(x_t, \Upsilon\left(x_t\right)). \\
U^y\left(x_t, \Upsilon\left(x_t\right), \chi_t\right) = \\
= p^{-\alpha}(x_t, \Upsilon\left(x_t\right))\left[w(\Upsilon\left(x_t\right)) + \chi_t R(\Upsilon\left(x_t\right))\right] \\
+\tfrac{1}{1+\rho}p^{-\alpha}(x_{t+1}, \Upsilon\left(x_{t+1}\right))\left(1-\chi_{t+1}\right)R(\Upsilon\left(x_{t+1}\right).
\end{gathered}$$

Denote

$$\tilde{U}^o\left(x_t\right) = U^o\left(x_t, 0, \chi\right) \text{ and } \tilde{U}^y\left(x_t\right) = U^y\left(x_t, 0, \chi\right),$$

the value of lifetime utility of the two agents when they impose a zero tax in the current period. Under a 0 tax, $R=0$, so the current value of χ

does not affect the agents' lifetime welfare. Therefore $\tilde{U}^o(x_t)$ and $\tilde{U}^y(x_t)$ do not depend on χ. They do, however, depend on the current value of x and of course they depend on the decision rules used in the future; they are functionals. The pair $\left(\tilde{U}^o(x_t), \tilde{U}^y(x_t)\right)$ is the threat-point in the Nash bargaining game. Total surplus equals

$$S(x_t, T_t, \chi_t) \equiv U^o(x_t, T_t, \chi_t) + U^y(x_t, T_t, \chi_t) - \left(\tilde{U}^o(x_t) + \tilde{U}^y(x_t)\right).$$

The Nash bargaining solution maximizes the Nash product,

$$\left(U^o(x_t, T_t, \chi_t) - \tilde{U}^o(x_t)\right)\left(U^y(x_t, T_t, \chi_t) - \tilde{U}^y(x_t)\right).$$

It is well known that when there are lump sum transfers (χ is unconstrained) the bargaining solution maximizes surplus, which is equivalent to maximizing aggregate lifetime welfare, the maximand in equation (14). That maximand does not involve χ_t. The choice of χ enables decisionmakers to make a lump sum transfer between generations. In this case, the transfer is chosen to split the surplus evenly between the two generations, implying:

$$\begin{aligned} U^o(x_t, T_t, \chi_t) - \tilde{U}^o(x_t) &= U^y(x_t, T_t, \chi_t) - \tilde{U}^y(x_t) \Longrightarrow \\ U^o(x_t, T_t, \chi_t) - U^y(x_t, T_t, \chi_t) &= \tilde{U}^o(x_t) - \tilde{U}^y(x_t) \end{aligned}$$

Using the formulae for equilibrium and disagreement payoffs and solving the last equation for χ_t gives

$$\chi_t = \Xi(x_t) = \frac{1}{2} + \frac{\pi(\Upsilon(x_t)) - w(\Upsilon(x_t))}{2R(\Upsilon(x_t))} + \frac{1}{2p^{-\alpha}(x_t, \Upsilon(x_t))R(\Upsilon(x_t))}C \tag{B.2}$$

with the definition

$$C \equiv \bar{\sigma}(x_t, \Upsilon(x_t)) - \frac{1}{1+\rho}p^{-\alpha}(x_{t+1}, \Upsilon(x_{t+1}))(1 - \Xi(x_{t+1}))R(\Upsilon(x_{t+1})) - \left(\tilde{U}^o(x_t) - \tilde{U}^y(x_t)\right).$$

The function Ξ appears in the definition of C, both explicitly and implicitly via the definition of $\tilde{U}^y(x_t)$. Therefore, equation (B.2) is a functional in Ξ. We numerically approximate the fixed point to this equation, and to the functional equation that determines Υ. If χ is constrained, and the constraint is binding, then it is no longer the case that aggregate surplus is split equally between the two generations. In that case, the equilibrium T_t that maximizes the Nash product, does not maximize aggregate surplus.

B4 Numerics

We approximate $\Upsilon(x_{t+1})$ and $\bar{\sigma}(x_{t+1}, \Upsilon(x_{t+1})) \equiv \Phi(x_{t+1})$ as polynomials in x_{t+1}, and find coefficients of those polynomials so that the solution to

$$\max_{T_t} P^{-\alpha}(x_t, T_t) Y(T_t) +$$
$$\left\{\Phi(x_{t+1}) + \tfrac{1}{1+\rho} P^{-\alpha}(x_{t+1}, \Upsilon(x_{t+1})) \left[\pi(\Upsilon(x_{t+1})) + (1 - \Xi(x_{t+1})) R(\Upsilon(x_{t+1}))\right]\right\}$$
$$\text{subject to } x_{t+1} = (1 + \bar{r}_t(x_t, T_t)) x_t \quad \text{with } x_0 \text{ given.}$$

approximately equals $\Upsilon(x_t)$. Appendix B3 explains the functional equation used to approximate Ξ in the Nash bargaining case. In the probabilistic voting model, Ξ is a known constant. We use 13-degree Chebyshev polynomials evaluated at 13 Chebyshev nodes on the $[0.1, 0.9]$ interval. At each node the recursion defining $\bar{\sigma}(x_t, \Upsilon(x_t))$,

$$\Phi(x_t) = \frac{1}{1+\rho} \left\{p^{-\alpha}(x_{t+1}, \Upsilon(x_{t+1}))\pi(\Upsilon(x_{t+1})) + \Phi(x_{t+1})\right\} \tag{B.3}$$

and the optimality condition

$$\tfrac{d}{dT_t}\left[P^{-\alpha}(x_t, T_t) Y(T_t) + \tfrac{1}{1+\rho}\Omega\right] = 0$$
$$\text{with} \quad \Omega \equiv \left\{\Phi(x_{t+1}) + \tfrac{1}{1+\rho} P^{-\alpha}(x_{t+1}, \Upsilon(x_{t+1})) \left[\pi(\Upsilon(x_{t+1})) + (1 - \Xi(x_{t+1})) R(\Upsilon(x_{t+1}))\right]\right\} \tag{B.4}$$

subject to $x_{t+1} = (1 + \bar{r}_t(x_t, T_t)) x_t$ and $T_t = \Upsilon(x_t)$ must be satisfied. If χ is endogenous, we additionally require that $\chi_t = \tilde{\Xi}(x_t) = \Xi(x_t)$ with $\tilde{\Xi}(x_t)$ as explained in Appendix B3.

Starting with an initial guess for the coefficients of the approximations of $\Phi(\cdot)$ and $\Upsilon(\cdot)$ and, possibly, $\Xi(\cdot)$, we evaluate the right side of equation (B.3) for at each node. Using these function values, we obtain new coefficient values for the approximation of $\Phi(\cdot)$. We then use the optimality condition (B.4) to find the values of $\Upsilon(\cdot)$ at the nodes; we use those values to update the coefficients for the approximation of $\Upsilon(\cdot)$. For endogenous χ, the new coefficients for $\Phi(\cdot)$ and $\Upsilon(\cdot)$ also allow the updating of the coefficients for the approximation of $\Xi(\cdot)$. We repeat this iteration until the coefficients' relative difference between iterations falls below 10^{-6}. See chapter 6 of Miranda and Fackler (2002) for details.

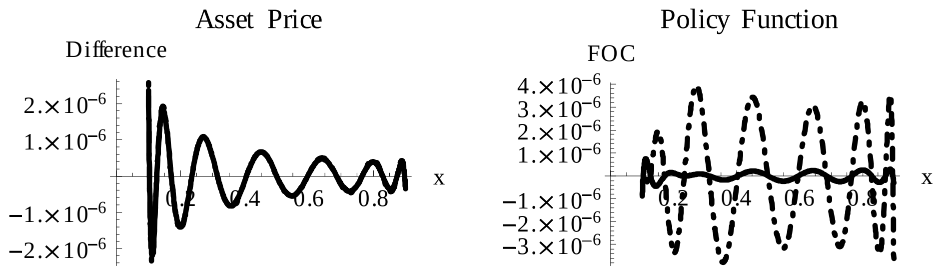

Figure 5: Deviation of asset price (left) and policy function (right) approximation from true value outside of approximation nodes for the efficient bargaining (solid) and the social planner's (dot-dashed) problems

Figures 5 graph the differences (the "residuals") between the right and left sides of equations (B.3) and (B.4), respectively. These residuals equal 0 at the nodes, because we set both the degree of the polynomial and the number of nodes equal to n. We choose $n = 13$ to ensure that residuals are at least 5 orders of magnitudes below the solution values on the $[0.1, 0.9]$ interval.

B5 Robustness checks

We computed two variations as a further robustness check. In the first variation, young agents select the current tax and receive all of the surplus, but have to compensate the old generation to ensure that the latter's welfare does not fall below a default level. This default level equals their welfare under the tax chosen in the previous period. The rationale for this model is that inertia favors the existing tax, and that young agents have to compensate the now-old agents to persuade them to change the tax that the latter chose when they were young. For this experiment we set $\chi = 1$. We find that this variation results in a tax policy very close to, but slightly lower than the policy under the previous formulation with $\chi = 1$. We conclude that our results are not sensitive to changes in χ or to moderate changes in the structure of the political economy model.

In the second variation, motivated by Proposition 5 and the comments following it, $\chi = 0$. Here, the old in the first period to propose a transfer rate ξ. Conditional on this choice, the old and the young each propose a constant tax. Due to inertia, society chooses the smaller of these two taxes. We then confirmed numerically that this tax is time consistent. Future young generations would like to lower the tax and future old generations would like to increase it, but the welfare gain that either achieves is insufficient to compensate the other. Therefore, no proposed change achieves consensus. The belief in the initial period that the tax will be constant is therefore confirmed by the equilibrium. The steady state stock is about 2% higher than in the political economy framework (with $\chi = 0$) and 10% lower than under the social planner.