

High Unemployment Yet Few Small Firms:

Model Appendix

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1 Appendix: Model

For all groups of firms, assume that total demand at price p is given by $D_t(p)$ in town t^1 . The wage that would be set by a bargaining council, w^{BC} , is in between the equilibrium wage, w^* , and the privately negotiated union wage, w^U . It is useful to separate firms by category, that is, into self employment, small firms, large unionized firms, and large non-unionized firms. For modelling purposes, small firms differ from large firms in that large firms have capital \bar{k} , and can choose to hire L laborers to maximize profits, while small firms hire a single worker and have no capital. In the model below, I abstract from the potential of single worker or small firms to avoid labor arbitration; in the empirical section, I will consider these groups separately.

1.1 Small Firms

The good is produced by both small and large firms, where small firms produce quantity $S_t^s(p)$ and large firms produce $S_t^L(p)$. Small firms are heterogeneous with respect to produc-

¹All variables (i.e. demand, production technologies, etc.) are presumed to vary across industries; this subscript is omitted for notational simplicity.

tion technologies. Each small firm hires a single worker, and firm f produces quantity q_f . Therefore, for a given wage w , and price p , small firms in industry i demand $n_t(w, p)$ units of labor where $n_t(w, p)$ represents the number of firms for whom q_f is greater than w/p . Suppose wages are set by the intersection of labor supply and labor demand for small firms, resulting in a wage $w_t^*(p)$. If we order small firms in terms of productivity, the total output of small firms is thus given by

$$S_t^s(p) = \sum_{\{f: q_f \geq w_t^*(p)/p\}} q_f \quad (1)$$

using $n_t(w^*, p)$ units of labor.

If a bargaining council mandates wages, then all small firms are required to pay wage w^{BC2} . Bargaining council wages are binding, and small firms produce

$$S_{tBC}^s(p) = \sum_{\{f: q_f \geq w^{BC}/p\}} q_f \quad (2)$$

where $S_{tBC}^s(p) < S_t^s(p)$, and $n_t^{BC}(p) < n_t^*(p)$ firms producing for all p . Note that small firms employ fewer workers in the presence of a bargaining council if

$$w^{BC}/p_t^{BC} > w_t^*/p_t^{nBC}$$

where p_t^{BC} and p_t^{nBC} denote prices in the presence or absence of a bargaining council agreement.

²In a more general and realistic model, only some fraction of small firms would be obliged to pay w^{BC} as enforcement would not be perfect. I abstract from this; however, if enforcement capacity varies it also may enter into the decision to pursue a bargaining council agreement.

1.2 Large Firms

There are two types of large firms: unionized large firms and non-unionized large firms. Both types of firms have \bar{k} units of capital and access to a concave production technology, $f(L, \bar{k})$.

In the absence of bargaining councils, unionized large firms pay w^U which is mandated by the local union, and hires $L^U(p)$ units of labor by setting $w^U = p \frac{\partial f(L, \bar{k})}{\partial L}$. Each unionized large firm thus produces $f(L^U(p), \bar{k})$ units of output. In contrast, non-unionized large firms pay wage w_t^* and set $w_t^* = p \frac{\partial f(L, \bar{k})}{\partial L}$, to hire $L_t^*(p)$ units of labor. If there are Q_t large firms, fraction λ of whom are unionized, large firm output is given by

$$Q_t [\lambda f(L^U(p), \bar{k}) + (1 - \lambda) f(L_t^*(p), \bar{k})] \quad (3)$$

If a bargaining council agreement is passed, all large firms are forced to pay wages w^{BC} . Therefore, large firms hire $L^{BC}(p)$ units of labor by setting $w^{BC} = p \frac{\partial f(L, \bar{k})}{\partial L}$, and each produce $f(L^{BC}(p), \bar{k})$, so that large firm output is given by $Q_t f(L^{BC}(p), \bar{k})$. Nonunionized large firms will employ fewer workers (and produce less) in the presence of a bargaining council agreement if $w^{BC}/p^{BC} > w_t^*/p_t^{nBC}$, while unionized large firms employ fewer workers if $w^{BC}/p^{BC} > w^U/p_t^{nBC}$ by the concavity of the production function.

1.3 Equilibrium

In equilibrium, prices adjust until supply equals demand, so that, in the absence of a bargaining council agreement,

$$Q_t [\lambda f(L^U(p_t^{nBC}), \bar{k}) + (1 - \lambda) f(L_t^*(p_t^{nBC}), \bar{k})] + n_t^*(p_t^{nBC}) = D_t(p_t^{nBC})$$

Unionized large firms earn (short-run) profit $\pi_{it}^U = p_t^U f(L^U(p_t^{nBC}), \bar{k}) - w^U L^U(p_t^{nBC}) - r\bar{k}$, where r is the rental rate of capital. In the long run, firms would adjust their capital stock; this will not help in motivating the empirical analysis below so I abstract from it. Non-unionized large firms earn $\pi_t^* = p_t^{nBC} f(L^*(p_t^{nBC}), \bar{k}) - w_t^* L_t^*(p_t^{nBC}) - r\bar{k}$. Since $w_t^* < w^U$, non-unionized large firms both hire more labor and are more profitable.

Under a bargaining council agreement,

$$Q_t f(L^{BC}(p^{BC}), \bar{k}) + n_t^{BC}(p^{BC}) = D_t(p^{BC})$$

and each large firm earns profit $p^{BC} f(L^{BC}(p^{BC}), \bar{k}) - w^{BC} L^{BC}(p^{BC}) - r\bar{k}$. Equilibrium in this model has several predictions. First, employment in small firms and in non-unionized large firms will fall in the presence of a bargaining council agreement. This happens as, if prices stay the same or fall, then $w^{BC}/p^{BC} > w_t^*/p_t^{nBC}$ since $w^{BC} > w_t^*$ by assumption. However, if prices rise, then at least one type of firm must be producing less (and, hence, employing fewer workers). Since $w_t^*/p_t^{nBC} < w^U/p_t^{nBC}$, small firms and large non-unionized firms will cut employment whenever large unionized firms do, and hence if anyone is employing fewer workers, both small firms and large non-unionized firms are. A similar argument reveals that large, unionized firms must be increasing output in response to the lower, bargaining council wages³. Therefore, small firms will overall employ fewer workers under the bargaining council regime, while the direction of the large firm employment effect will depend on λ and the production function. Overall employment also has ambiguous predictions in

³If prices lower, than at least one type of firm must be increasing output; since we know that small firms and non-unionized large firms are not, it must be the case that large unionized firms are. In contrast, if prices are higher, than $w^U/p^{nBC} > w^{BC}/p^{BC}$ and large firms increase employment.

principle. However, since small firms are more labor intensive than large firms, we may anticipate that the growth in employment in unionized large firms is overbalanced by the shrinking employment in small firms.