The choice of exchange rate regime and speculative attacks*

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Abstract

We develop a framework for studying the choice of exchange rate regime in an open economy where the local currency is vulnerable to speculative attacks. The optimal regime is determined by a policymaker who trades off the loss from nominal exchange rate uncertainty, against the cost of maintaining a given regime. This cost is affected in turn by the likelihood of a speculative attack. Searching for the optimal regime within the class of exchange rate bands, we show that the optimal regime is either a peg (a zero-width band), a free float (an infinite-width band), or a non degenerate finite width band. In the latter case, the exchange rate is allowed to move freely only within a band set around some center rate. We examine the determinants of the optimal band width and show, among other things, that, ceteris paribus, lower costs of moving across currencies induce policymakers to set more flexible exchange rate systems. This lowers, in turn, the likelihood of financial crises. More generally the framework of the paper can be used to shed new light on the recent world wide trend towards a bipolar system of exchange rate arrangements.

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1 Introduction

Except for some key currencies like the US Dollar, the British Pound, the Japanese Yen, the German Mark, and nowadays the Euro, most currencies are not allowed to freely float against other currencies. Policymakers in many countries and especially in small open economies engage either explicitly or implicitly in policies whose objective is to limit nominal exchange rate volatility. However such policies are not maintained under all circumstances. In the presence of sufficiently large runs on the domestic currency, policymakers suspend their intervention policy and allow the exchange rate to float. They often reinstate their intervention policy at a new level of the exchange rate, after the turbulence in currency markets has subsided. A recent example of such a pattern is the 1997/8 East Asian crisis (see e.g., Calvo and Reinhart, 2000, and McKinnon, 2001).

This paper develops a framework for analyzing some of the factors that affect the choice of exchange rate regime by a policymaker who wishes to limit uncertainty about the nominal exchange rate but faces speculators who might attack the exchange rate when it deviates substantially from its laissez faire level. The framework makes it possible to study the interaction between the choice of exchange rate regime and speculative attacks. We search for the optimal regime within the class of explicit exchange rate bands. In this class of regimes, the policymaker allows the exchange rate to move freely within a band around a prespecified center rate and commits to intervene in the market and prevent the exchange rate from moving outside the band. Although it does not include all the possible varieties of exchange rate regimes the class of explicit exchange rate bands is rather broad and includes as special cases two of the most commonly analyzed regimes, namely free floats (an infinitely wide band) and pegs (a zero-width band).

In order to focus on the strategic interaction between the choice of exchange rate regime and speculators we follow Obstfeld (1996) and Morris and Shin (1998) and model some of the underlying macroeconomic structure in a reduced form manner. A basic premise of our framework is that exporters, importers, as well as borrowers and lenders in foreign currency denominated financial assets dislike uncertainty about the level of the nominal exchange rate and that policymakers internalize at least part of this aversion. Recent empirical findings by Calvo and Reinhart (2000) are consistent with this premise. To reduce uncertainty in the nominal exchange rate, the policymaker may commit to an exchange rate band. However since, in order to maintain the currency within the band, the policymaker needs to use up foreign exchange reserves or deviate from the level of the interest rate that is consistent with other domestic objectives, such a commitment is costly. The cost of either option rises if the exchange rate comes under a speculative attack. If the policymaker decides to abandon the band and avoid the costs of defending it, he losses credibility. The optimal exchange rate regime reflects, therefore, a trade-off between reducing exchange rate uncertainty (and thereby reducing risk and promoting economic activity) and incurring the cost of committing to an exchange rate band.

An essential feature of our framework is that the probability of speculative attack is affected by the exchange rate regime that the policymaker chooses. In turn, the policymaker takes this probability into account when he sets the exchange rate regime. To the best of our knowledge, this paper is the first to simultaneously analyze the choice of exchange rate regime and the probability of a currency attack.[1] Thus far, the literature on optimal exchange rate regimes did not consider the possibility of speculative attacks (e.g., Helpman and Razin, 1982, and Devereux and Engel, 1999), while the literature on currency attacks took the exchange rate regime as exogenous (e.g., Krugman, 1979; Obstfeld, 1996; and Morris and Shin, 1998). Our paper shows that explicit recognition of the interaction between the choice of exchange rate regimes and the decision of speculators on whether or not to attack, yields a number of novel predictions about the optimal exchange rate regime and the likelihood of a currency crisis. In particular, for cases in which the optimal regime is a band, our analysis highlights some of the policy trade-offs involved in the choice of band width.[2]

The analysis implies that, depending on the policymaker's aversion to nominal exchange rates uncertainty, the optimal regime can either be a free float, a peg, or a band. In the latter case, the optimal width of the band rises (and may even become a free float) as the policymaker becomes less concerned with exchange rate uncertainty. Conversely, the stronger their aversion to nominal exchange rate uncertainty, the narrower the band chosen by policymakers. But tighter bands are associated with higher probabilities of speculative attacks and therefore more susceptible to currency crises.

Some of the factors that give rise to cross country variations in the degree of aversion to

[1]In a recent paper, Guembel and Sussman (2001) study the choice of exchange rate regime in the presence of speculative trading. But their model is quite different from ours in its focus and in the formulation of speculation. In particular, they do not consider the endogenous choice of exchange rate band and do not allow for the possibility of realignments by assuming that the policymaker is always fully committed to the existing exchange rate regime.

[2]This is in contrast to most of the literature on target zones from the early 90's that takes the existence of bands and their width as exogenous (see for instance the surveys in Ch. 1 and 2 in Krugman and Miller, 1992, and Svensson, 1992). Recent exceptions are Cukierman, Kiguel, Leiderman, and Spiegel (1999), Cukierman and Spiegel (1999), and Koren (2000). However, neither of those papers explicitly considers the simultaneous interaction between the choice of band width and the behavior of speculators.

exchange rate uncertainty are the fraction of financial assets and liabilities and of capital flows that is denominated in foreign exchange, the importance of foreign trade, and the fraction of trade that is invoiced in foreign exchange (McKinnon, 2000; Gylfason, 2000; and Wagner, 2000). In conjunction with this observation, our model predicts that relatively small and open economies are likely to have less flexible exchange rate regimes and more frequent currency attacks. Since policymakers of key currency countries are usually less sensitive to nominal exchange rate uncertainty than policymakers in small open economies, the model predicts that large economies with key currencies like the US, Japan, and the Euro area will float, while small open economies like Hong-Kong, Panama, Estonia, Lithuania, Bulgaria, and Argentina will adopt pegs, currency boards, or even full dollarization. This indeed happens to be the case.

Interestingly, the model predicts that if the policymaker commits to a peg or to a non-degenerate band, actual volatility in the exchange rate is higher than the volatility that is implied by the announced regime. This is due to the fact that, following speculative attacks, the policymaker occasionally decides to abandon the band, and lets the exchange rate float beyond the preannounced bounds.

Following are highlights of other results. First, as the distribution of stochastic shocks to the freely floating exchange rate becomes more spread out, countries with pegs move to narrow bands and countries with bands widen them and may even adopt free floats.

Second, financial liberalization that lowers the transaction costs of switching between currencies induces policymakers to adopt a more flexible exchange rate regime. The reason is that, all else equal, speculative attacks become more likely when it is easier to switch between currencies. As a result, it is more costly to intervene in the exchange rate market. Therefore, policymakers in countries with pegs move to narrow bands while policymakers in countries with exchange rate bands increase the width of their bands. The actual changes that occurred in the exchange rate systems of many countries during the last two decades are broadly consistent with this prediction.

Third, the analysis implies that even though financial liberalization may lower the cost of mounting a speculative attack, it does not necessarily increase the probability of currency attacks. In fact, in our model exactly the opposite is true. This counterintuitive result arises because a decrease in transaction costs due to liberalization produces two opposing effects. On one hand, holding the width of the exchange rate band fixed, it increases the likelihood of a crisis. But on the other hand, liberalization also induces policymakers to adopt wider bands, and this reduces the probability of currency attacks. This implies that, contrary to conventional wisdom, a so-called

"Tobin tax" on currency transfers (proposed by Tobin (1978) as a way to reduce the likelihood of financial crisis) may actually be counterproductive.

Fourth, our analytical framework implies that the decision of speculators on whether or not to attack the band depends on the policymaker's reputation about his resolve to intervene in the exchange rate market. We show that a good reputation weakens the incentive of speculators to attack the exchange rate and thereby allows policymakers to set a less flexible regime (a narrow band or even a peg). Hong-Kong's currency board is a good example. Since it has never abandoned its currency board in the past, Hong-Kong's currency board enjoys a good reputation. There is thus a "virtuous circle" between good reputation and the performance of the currency board.

The paper is organized as follows. Section 2 presents the basic framework. Section 3 derives the equilibrium behavior of speculators and of policymakers. Section 4 characterizes the equilibrium properties of the exchange rate regime and identifies conditions under which it is a peg, a free float, or a band, and in the later case, examines the determinants of the band's width and of its symmetry. Section 5 analyses the effects of capital account liberalization. Section 6 extends the analysis to the case in which speculators are uncertain about the policymaker's resolve to maintain the band (i.e., the policymaker's reputation). The concluding section reflects on the link between our model and the recent evidence on convergence to a bipolar system of exchange rate arrangements (pegs or free floats). All proofs are in the Appendix.

2 The model

Consider an open economy in which the initial level of the exchange rate (defined as the number of units of domestic currency per one unit of foreign currency) is denoted by e_{-1}. Absent policy interventions, the nominal exchange rate, e, fluctuates stochastically in the exchange rate markets. The fluctuations of e reflect various shocks to the current account and to the capital account of the balance of payments excluding the behavior of speculators which is modeled separately. For the purpose of this paper, it turns out that it is more convenient to work with the laissez faire rate of change in e, $x \equiv (e - e_{-1})/e_{-1}$, rather than with its level, e. We assume that the value of x is drawn from a distribution function $f(x)$ on $\Re$ with c.d.f. $F(x)$ and that, once it realizes, x persists for some time.[3] We make the following assumption on $f(x)$:

[3] We assume that the distribution of x has an unbounded support mainly for convenience. This assumption is not essential and can be relaxed although this would require some additional assumptions to ensure that various parameters of the model are not too large relative to the bounds of the support of x.

Assumption 1: $f(x)$ is unimodal with a mode at $x = 0$. That is, $f(x)$ is increasing for all $x < 0$ and decreasing for all $x > 0$.

Assumption 1 states that large rates of change in the exchange rate (i.e., large devaluations when $x > 0$ and large appreciations when $x < 0$) are less likely than small changes. This is a realistic assumption and, as we shall see later, it is responsible for some of the results of the paper.

2.1 The exchange rate band

A basic premise of the paper is that policymakers dislike nominal exchange rate uncertainty. This is because exporters, importers, as well as lenders and borrowers in foreign currency face higher exchange rate risks when there is more uncertainty about the nominal exchange rate. By raising the foreign exchange risk premium, an increase in exchange rate uncertainty reduces international flows of goods and of financial capital. Policymakers, who wish to promote economic activity, internalize at least part of this aversion to uncertainty and therefore have an incentive to limit it.[4]

In general, there are various conceivable institutional arrangements for limiting exchange rate uncertainty. In this paper we search for an optimal institutional arrangement within the class of bands. This class is quite broad and includes pegs (bands of zero width) and free floats (bands of infinite width) as special cases. Under this class of arrangements, the policymaker sets an exchange rate band $[\underline{e}, \overline{e}]$ around the preexisting nominal exchange rate, e_{-1}. The nominal exchange rate, e, is then allowed to move freely within the band in accordance with market forces, but once it reaches the boundaries of the band, the policymaker is committed to intervene and keep it from moving outside the band.[5] The exchange rate band induces a permissible range of rates of change in the exchange rate, $[\underline{\pi}, \overline{\pi}]$, where $\underline{\pi} = (\underline{e} - e_{-1})/e_{-1} < 0$ and $\overline{\pi} = (\overline{e} - e_{-1})/e_{-1} > 0$. Within this range, the domestic currency is allowed to appreciate if $x \in [\underline{\pi}, 0)$, and depreciate if $x \in [0, \overline{\pi})$. In other words, $\underline{\pi}$ is the maximal rate of appreciation and $\overline{\pi}$ is the maximal rate of depreciation that the exchange rate band permits.[6]

[4] Admittedly, some of those risks may be insured by means of future currency markets. However, except perhaps for some of the major key currencies, such markets are largely non-existent, and when they do exist the insurance premia are likely to be prohibitively high.

Rose (2000) presents evidence suggesting that countries with the same currency trade substantially more than comparable countries with their own currencies. Lee (1999) presents evidence from US import markets that is consistent with the view that exchange rate volatility depresses demand for imported consumer durables.

[5] This intervention can be operationalized by buying or selling foreign currency in the market, by changing the domestic interest rate, or by doing some of both.

[6] Note that when $\underline{\pi} = \overline{\pi} = 0$ the band reduces to a peg and when $\underline{\pi} = -\infty$, and $\overline{\pi} = \infty$ it becomes a free float.

But leaning against the trends of free exchange rate markets is costly. To defend a currency under attack, policymakers have to deplete their foreign exchange reserves (Krugman, 1979) or put up with substantially higher domestic interest rates (Obstfeld, 1996). If they decide to avoid those costs by exiting the band, they lose some credibility. For example, breaking a commitment can make it harder for the policymaker to achieve other goals in the same period or in the future (e.g., commit to a low rate of inflation, commit to low rates of taxation, accomplish structural reforms, etc.). We denote the present value of this loss by δ.

Even when they ultimately exit the band, policymakers initially try to maintain it. We allow for such behavior by postulating that following realizations of x that are outside the boundaries of the band, there are two phases. In the first (possibly short) phase, the policymaker always defends the band.[7] During this phase, speculators might attack the band. In the second phase, after observing the fraction of speculators that decided to attack the band, the policymaker can decide to either continue to maintain the band or abandon it. Following Obstfeld (1996) and Morris and Shin (1998), we assume that the cost of maintaining the band in the second phase increases with the size of the disequilibrium that the policymaker tries to maintain (either $x - \overline{\pi}$ or $\underline{\pi} - x$, depending on whether x is positive or negative) and with the number of speculators that have attacked the band in the first phase. Specifically, normalizing the mass of speculators to 1, and using α to denote the fraction of speculators that have attacked the band in the first phase, we assume that the cost of intervention in the exchange rate market in the second phase is given by

$$C(x,\alpha) = \begin{cases} x - \overline{\pi} + \alpha, & x \geq \overline{\pi}, \\ 0, & \underline{\pi} \leq x \leq \overline{\pi}, \\ \underline{\pi} - x + \alpha, & x \leq \underline{\pi}. \end{cases} \tag{2.1}$$

The assumption that $C(x,\alpha)$ increases with α reflects the idea that as more speculators attack the band in the first phase, the policymaker has less resources to continue to defend it, and that it becomes, therefore, more costly to maintain it. For simplicity we assume that α enters the cost function additively. The middle line in equation (2.1) states that when the exchange rate is inside the band, the policymaker does not intervene in the market and bears no cost. Obviously, the policymaker will continue and maintain the band only when $C(x,\alpha)$ is less than δ. Otherwise, maintaining the band is too costly so the policymaker will exit the band and incur the cost of realignment.

[7] This may be due say to short lags in the arrival of information and in decision making.

We formalize the trade-off between uncertainty about the nominal exchange rate and the cost of adopting a band by postulating that the policymaker's objective function is

$$V = -AE\,|\pi| - E\,[Min\{C(x,\alpha),\ \delta\}]\,, \qquad A > 0, \tag{2.2}$$

where π is the actual rate of change in the nominal exchange rate (under laissez faire, $\pi = x$). We think of this objective function mostly as a positive description of how a rational policymaker might approach the problem of choosing the band width. The first component of V represents the policymaker's aversion to uncertain fluctuations in the nominal exchange rate. The latter is measured in terms of the absolute value of π (i.e., by the magnitude of nominal depreciations or appreciations). The parameter A represents the relative importance that the policymaker assigns to exchange rate stability. As A increases, the policymaker becomes more concerned with exchange rate uncertainty and is more willing to incur costs in order to limit it.[8] The parameter A is likely to vary substantially across economies depending on factors like the degree of openess of the economy, its size, the fraction of financial assets and liabilities owned by domestic producers and consumers that is denominated in foreign exchange, and the fraction of foreign trade that is invoiced in foreign exchange. All else equal, residents of small open economies are more averse to nominal exchange rate uncertainty than residents of large, relatively closed, economies like the US or the Euro area.[9] Hence the parameter A is larger in small open economies than in large, relatively closed, economies.

The second component of V represents the policymaker's cost of adopting an exchange rate band. This cost is either equal to $C(x, \alpha)$ if the policymaker defends the band in the second phase or to δ when defending the band is too costly.[10] In the latter case the exchanged rate is realigned. It is important to note that in the case of realignment, the policymaker does not incur an additional cost as a result of the actual change in the exchange rate provided it is perfectly anticipated. This is because in our framework, only uncertainty has negative effects on economic activity, implying that the only reason for intervention in the foreign exchange rate market is to limit the negative effect of this uncertainty on economic activity.[11]

[8] Note that we assume that the policymaker is equally averse to expected depreciations and appreciations. This assumption can be easily relaxed at the cost of more notation.

[9] One important reason for this relative preferences structure is that the pass-through from changes in the exchange rate to domestic prices is swifter and stronger in small open economies than in large and relatively closed economies.

[10] For simplicity, we assume that the first phase is very short relative to the second phase, so that the cost of defending the band in the first phase is negligible. Most of the results in the paper go through when this assumption is relaxed.

[11] More generally, variability and uncertainty do not always coincide (see Cukierman and Wachtel, 1982).

2.2 Speculators

There is a continuum of speculators with a total mass of one. When the exchange rate reaches the upper or lower boundaries of the band, each speculator independently observes a noisy signal on the exchange rate that would prevail under laissez faire. Specifically, we assume that the signal obtained by speculator i is given by

$$\theta_i = x + \varepsilon_i, \tag{2.3}$$

where ε_i is a white noise, independent across speculators, and distributed uniformly on the interval $[-\varepsilon, \varepsilon]$. The conditional density of x given a signal θ_i is given by:

$$f(x \mid \theta_i) = \frac{f(x)}{F(\theta_i + \varepsilon) - F(\theta_i - \varepsilon)}. \tag{2.4}$$

In what follows, we focus on the case where ε is small so that the signals that speculators observe are "almost perfect.". In spite of the fact that the decision of each individual speculator about whether to attack the currency depends on what he believes about the beliefs of other speculators,.this information structure leads to a unique Bayesian equilibrium.[12]

Based on θ_i, each speculator i decides whether or not to attack the currency in the first phase. If the current exchange rate is at the lower bound of the band, speculator i can shortsell the foreign currency at the current (high) price of $\underline{e}$ and then buy the same amount on the market to clear his position. If the policymaker fails to defend the band and the exchange rate falls to $e < \underline{e}$, speculator i's profit from shortselling is $\underline{e} - e$. Denoting by t the nominal transaction cost associated with switching between currencies, the speculator's net payoff is $\underline{e} - e - t$. On the other hand, if the exchange rate stays at $\underline{e}$, the net payoff of the speculator is simply $-t$. Likewise, if the current exchange rate is at the upper bound of the band, speculator i can buy the foreign currency at the current (low) price of $\overline{e}$. If the policymaker exits the band and the exchange rate jumps to $e > \overline{e}$, the speculator's net payoff is $e - \overline{e} - t$. Again, if the policymaker successfully defends the band, the speculative attack fails, and the net payoff of the speculator is $-t$. If the speculator does not attack the band, his payoff is 0.[13]

[12] The fact that very small amounts of noise can lead to a unique equilibrium in models of self-fulfilling beliefs was first observed by Carlsson and van Damme (1993). Recently a few papers applied this approach in the context of financial crises (e.g., Morris and Shin, 1998, and Goldstein and Pauzner, 2000). For a survey of related literature, see Morris and Shin (2001).

[13] In order to focus on speculation against the band we abstract from speculative trading within the band. Thus, the well-known 'honeymoon effect' (Krugman 1991) is absent from the model. Speculation within the band could be added at the cost of additional machinery.

Note that since $x = (e - e_{-1})/e_{-1}$ and since $\underline{e} = (1+\underline{\pi})e_{-1}$ and $\overline{e} = (1+\overline{\pi})e_{-1}$, the net payoffs from attacking the lower and upper bounds of the band can be written as $(1+\underline{\pi})e_{-1} - (1+x)e_{-1} - t = (\underline{\pi} - x)e_{-1} - t$, and $(1+x)e_{-1} - (1+\overline{\pi})e_{-1} - t = (x - \overline{\pi})e_{-1} - t$, respectively. Since x has an unbounded support, then so long as $\underline{\pi} > -\infty$ and $\overline{\pi} < \infty$ (i.e., the exchange rate regime is not a free float), there are sufficiently low realizations of x for which $(\underline{\pi} - x)e_{-1} - t$ is positive and sufficiently high realizations of x for which $(x - \overline{\pi})e_{-1} - t$ is positive. We now make the following assumption on t:

Assumption 2: The real transaction cost, $\frac{t}{e_{-1}}$, is small relative to δ (the future credibility loss from exiting the band) in the sense that $\frac{t}{e_{-1}} < \delta$.

As will become clear below, Assumption 2 ensures that speculators will always attack the band if they believe that x is such that the policymaker will exit the band. This rules out the (uninteresting) possibility that speculators do not attack the band even if they know that the policymaker is not going to defend it.

2.3 The sequence of events and the structure of information

The sequence of events unfolds as follows:

- Stage 1: The policymaker announces a band around the existing nominal exchange rate and commits to intervene in the exchange rate market when $x < \underline{\pi}$ or $x > \overline{\pi}$.

- Stage 2: The "free float" random shock, x, realizes and persists over the remaining stages of the game. There are now two possible cases:

  (i) If $\underline{\pi} \leq x \leq \overline{\pi}$, the nominal exchange rate is allowed to freely move within the boundaries of the band and is, therefore, determined by market forces. Since x persists, the exchange rate immediately adjusts to $(1+x)e_{-1}$ and remains at this value for the remainder of the game.

  (ii) If $x < \underline{\pi}$ or $x > \overline{\pi}$, the policymaker initially intervenes in the exchange rate market and keeps the exchange rate from moving outside the band. As a result, the exchange rate is either at the upper or at the lower bound of the band. Simultaneously each speculator gets the signal, θ_i, on x and decides whether or not to attack the band. Those decisions determine the fraction, α, of speculators who decide to attack the band.

- Stage 3: The policymaker observes x and α, and evaluates the total cost of continuing to defend the band. If the policymaker decides to continue to defend the band, the exchange rate stays at the boundary of the band and the policymaker incurs the cost $C(x, \alpha)$. If the policymaker exits the band and there is realignment, the exchange rate moves to its freely floating rate so the induced rate of change in the exchange rate is x and the policymaker incurs a future credibility loss whose present value is δ.

3 The equilibrium

In this section we characterize the perfect Bayesian equilibrium of the model. To this end, we solve the model backwards. First, whenever $x < \underline{\pi}$ or $x > \overline{\pi}$, then given α, the policymaker decides in stage 3 whether or not to continue to maintain the band. Second, given the signals that they observe in stage 2, speculators decide whether or not to attack the band. When $x \in [\underline{\pi}, \overline{\pi}]$, the policymaker does not intervene in the exchange rate market and the exchange rate moves freely within the band. Finally, in stage 1, prior to the realization of x, the policymaker sets the exchange rate regime.

3.1 The choice between defending the currency and realigning

Suppose that $x < \underline{\pi}$ or $x > \overline{\pi}$. Since the cost of maintaining the band after the initial phase is $C(x, \alpha)$, whereas the loss from exiting the band is δ, the policymaker will continue to maintain the band if and only if $C(x, \alpha) \leq \delta$. Using equation (2.1), this inequality implies that the set of states for which the policymaker will defend the band is

$$\underline{\pi} + \alpha - \delta \leq x \leq \underline{\pi}, \quad \text{and} \quad \overline{\pi} \leq x \leq \overline{\pi} - \alpha + \delta. \tag{3.1}$$

When $x < \underline{\pi} + \alpha - \delta$ or $x > \overline{\pi} - \alpha + \delta$, the cost of defending the band is too large, so the policymaker exists the band and allows the exchange rate to be realigned. Note that as α increases (more speculators attack the band in stage 2), the set of realizations of x for which the exchange rate is realigned expands.

3.2 Speculative attacks

When $x \in [\underline{\pi}, \overline{\pi}]$, the exchange rate is determined solely by market forces. Hence, speculators cannot gain from attacking the currency and their payoff is 0. In contrast, when $x < \underline{\pi}$ or $x > \overline{\pi}$, speculators know that the policymaker will, at least initially, intervene in the foreign exchange market and prevent the exchange rate from moving outside the band. In such cases, the exchange rate no longer reflects market forces and speculators may choose to attack the band in the hope of making a profit in case there is a realignment.

Suppose that the exchange rate reaches the upper bound of the band and recall that the net payoff from attacking the band in this case is $(x-\overline{\pi})e_{-1}-t$ if the policymaker eventually exits the band and $-t$ if he does not. Since the policymaker exits the band if and only if $x>\overline{\pi}-\alpha+\delta$, the net payoff from attacking the upper bound of the band is $(x-\overline{\pi})e_{-1}-t$ if $x>\overline{\pi}-\alpha+\delta$, and $-t$ if $\overline{\pi}\leq x\leq\overline{\pi}-\alpha+\delta$. Analogously, if the exchange rate reaches the lower bound of the band, the net payoff of a speculator who attacks the lower bound of the band is $(\underline{\pi}-x)e_{-1}-t$ if $x<\underline{\pi}+\alpha-\delta$, and $-t$ if $\underline{\pi}+\alpha-\delta\leq x\leq\underline{\pi}$.

Speculators do not observe x directly and need to use the signal θ_i in order to estimate it. Moreover, in order to assess whether the policymaker will or will not defend the band, each speculator needs to estimate α which is the fraction of other speculators that decide to attack the band. Hence, each speculator needs to form a belief about the behavior of other speculators.

The following lemma, establishes that, in the limit as $\varepsilon\to 0$, the equilibrium behavior of each speculator is to attack the upper (lower) bound of the band if and only if the signal that he observes is above (below) some threshold. This threshold is common to all speculators. The proof of the lemma, as well as the proofs of all other results, are in the Appendix.

Lemma 1 *Suppose that speculators have almost perfect information, i.e., $\varepsilon\to 0$. Then,*

(i) when the exchange rate reaches the upper (lower) bound of the band, there exists a unique perfect Bayesian equilibrium, such that each speculator attacks the band if and only if the signal that he observes is above some threshold $\overline{\theta}^*$ (below some threshold $\underline{\theta}^*$).

(ii) The thresholds $\overline{\theta}^*$ and $\underline{\theta}^*$ are given by $\overline{\theta}^*=\overline{\pi}+r$ and $\underline{\theta}^*=\underline{\pi}-r$, where

$$r=\sqrt{\frac{t}{e_{-1}}+\frac{(\delta-1)^2}{4}}+\frac{\delta-1}{2}.$$

(iii) In equilibrium, all speculators attack the upper (lower) bound of the band and the policymaker realigns it if and only if $x>\overline{\theta}^*=\overline{\pi}+r$ $(x<\underline{\theta}^*=\underline{\pi}-r)$. Whenever $\underline{\theta}^*\leq x\leq\overline{\theta}^*$ speculators do not attack the band and the band is not realigned. The probability of a speculative attack is therefore

$$P=F(\underline{\pi}-r)+(1-F(\overline{\pi}+r)).$$

The proof of part (i) of Lemma 1 is based on an iterative elimination of dominated strategies. The idea is as follows: the signal θ_i that speculator i observes is distributed on the interval $[x-\varepsilon, x+\varepsilon]$. If θ_i is above $\overline{\theta} \equiv \overline{\pi}+\delta+\varepsilon$, the speculator correctly infers that $x > \overline{\pi}+\delta$. At this level, the cost of defending the upper bound of the band exceeds the associated benefit even if no speculators attack the band. Hence, the speculator correctly infers that the policymaker is surely going to exit the band, so the net payoff from attacking it is $(x-\overline{\pi})e_{-1}-t$. But since $x > \overline{\pi}+\delta$, the net payoff is at least $\delta e_{-1}-t$, which is strictly positive by Assumption 2. Therefore when $\theta_i \geq \overline{\theta}$, it is a dominant strategy for speculator i to attack the upper bound of the band.[14] But now, if θ_i is just slightly below $\overline{\theta}$, speculator i realizes that a large fraction of other speculators must have observed signals above $\overline{\theta}$ and will surely attack the band. From that, speculator i concludes that the policymaker will exit the band so it is again optimal to attack it. This chain of reasoning can proceed further where each time we lower the critical signal above which speculator i will attack the upper bound of the band, given that speculators who observe even higher signals surely attack it.

When θ_i is below $\underline{\theta} \equiv \overline{\pi}+\frac{t}{e_{-1}}-\varepsilon$, speculator i correctly infers that $x < \overline{\pi}+\frac{t}{e_{-1}}$. Consequently, even if the policymaker will surely exit the band, the payoff from attacking it is negative as $(x-\overline{\pi})e_{-1}-t < \left((\overline{\pi}+\frac{t}{e_{-1}})-\overline{\pi}\right)e_{-1}-t = 0$. Hence, it is a dominant strategy for speculator i not to attack the band when $\theta_i < \underline{\theta}$. But then, if θ_i is just slightly above $\underline{\theta}$, speculator i will infer that a large fraction of other speculators must have observed signals below $\underline{\theta}$ and will surely not attack the band. Hence, speculator i concludes that the policymaker will successfully defend the band so it is optimal not to attack it. Again, this chain of reasoning can proceed further where each time the critical signal below which speculator i will not attack the upper bound of the band is raised.

In the limit as $\varepsilon \longrightarrow 0$, the critical signal above which speculators attack the band coincides with the critical signal below which they do not attack it. This establishes the existence of a unique threshold signal, $\overline{\theta}^*$, such that all speculators attack the upper bound of the band if and only if they observe signals above $\overline{\theta}^*$. Similar arguments establish the existence of a unique threshold signal, $\underline{\theta}^*$, such that all speculators will attack the lower bound of the band if and only if they observe signals below $\underline{\theta}^*$.

The unique equilibrium obtained in Lemma 1 contrasts with second generation models of

[14] The strategy to attack the upper bound of the band if $\theta_i > \overline{\theta}$ is dominant because it is optimal no matter what other speculators are going to do. The existence of a region in which speculators have dominant strategies is crucial for deriving a unique equilibrium (see Chan and Chiu, 2002).

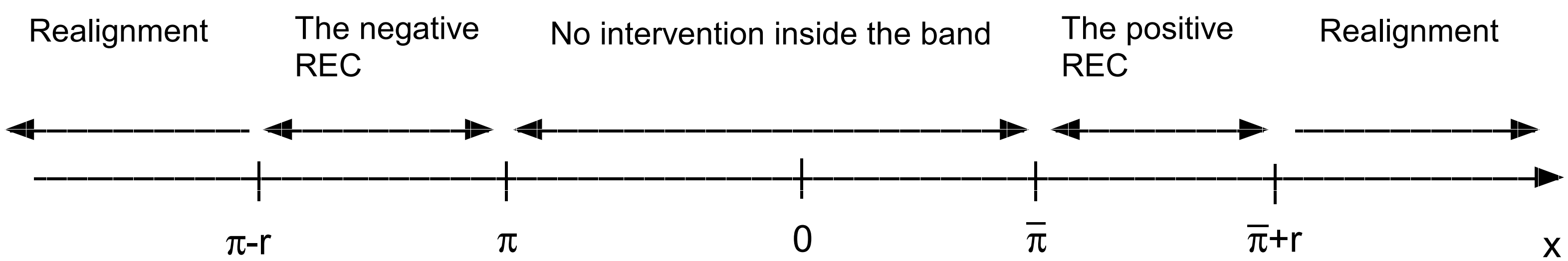
currency crises (e.g., Obstfeld 1996 and 1997) in which multiple equilibria are possible. The reason for the difference is that in that framework the only constraint on expectations is that they must be consistent with the resulting equilibrium. By contrast, in our framework, it is **also** required that each speculator estimates the information available to other speculators based on his personal signal. As a consequence, there is less freedom in choosing the range of possible beliefs that each speculator may entertain about the beliefs of others (and therefore about the probability that each speculator assigns to the event that the policymaker will defend the currency). Since the beliefs of speculators about x and about other speculators' beliefs are tied together by the realization of x, all speculators are led to believe that **either** all of them believe that the policymaker will exit the band, **or** that all of them believe that the policymaker will defend the band. Given the realization of x one of those beliefs about the beliefs of others is inconsistent with a conditional rational inference about the beliefs of others, which rules out multiple equilibria.[15]

Having solved for the behavior of speculators, we turn next to the implications of this behavior for the exchange rate band. Recall that we are interested in cases where $\varepsilon \to 0$. Part (iii) of Lemma 1 implies that the exchange rate band gives rise to two *Ranges of Effective Commitment* (REC) such that the policymaker defends the band only if x falls inside one of these ranges. The positive REC is equal to $[\overline{\pi}, \overline{\theta}^*]$ or $[\overline{\pi}, \overline{\pi} + r]$; when $x \in [\overline{\pi}, \overline{\pi} + r]$, the policymaker keeps the rate of depreciation from being above $\overline{\pi}$. The negative REC is equal to $[\underline{\theta}^*, \underline{\pi}]$ or $[\underline{\pi} - r, \underline{\pi}]$; when $x \in [\underline{\pi} - r, \underline{\pi}]$, the policymaker keeps the rate of appreciation from being above the absolute value of $\underline{\pi}$. When $x < \underline{\pi}$ or when $x > \overline{\pi}$, the policymaker exits the band and despite his earlier announcement, tolerates a realignment. On the other hand, when $x \in [\underline{\pi}, \overline{\pi}]$, the policymaker allows the exchange rate to move freely in accordance with market forces. These five ranges of x are illustrated in Figure 1.

Part (ii) of Lemma 1 indicates that r is independent of $\underline{\pi}$ and $\overline{\pi}$. This means that the actual size of the two ranges of x at which the policymaker intervenes in the exchange rate market does

[15] By contrast, in the absence of this additional constraint, multiple equilibria do arise for some ranges of x. To see that consider, for example, the case where $\bar{\pi} - \delta - 1 < x < \bar{\pi} + \delta$. If all speculators believe that the policymaker will defend the currency so that none attacks it (i.e., $\alpha = 0$), then it indeed pays the policymaker to defend it since the cost of doing that is $C(x, \alpha) = x - \bar{\pi} < (\bar{\pi} - \delta) - \bar{\pi} = \delta$, whereas the cost of exiting the band is δ. Hence, this strategy and the postulated speculators' beliefs constitute an equilibrium. By the same token if all speculators believe that the policymaker is not going to defend the band and so all of them attack it (i.e., $\alpha = 1$), then $C(x, \alpha) = x - \bar{\pi} + 1 > (\bar{\pi} - \delta - 1) - \bar{\pi} + 1 = \delta$, so indeed, it does not pay the policymaker to defend the band. Hence, having a speculative attack and a realignement is also an equilibrium for the same value of x.

Figure 1: Illustrating the exchange rate band



not depend on how wide the band is. What the policymaker can do is to shift the two RECs either closer to or away from 0 by choosing $\underline{\pi}$ and $\overline{\pi}$ appropriately. Part (ii) of Lemma 1 also shows that r increases with t, which is the transaction cost associated with speculative attacks, and with δ which is the policymaker's credibility loss from realignment. These properties are intuitive since they imply that a realignment is less likely when it is more costly for speculators to attack the band and when the policymaker is more averse to realignments.

The discussion is now summarized in the following proposition:

Proposition 1 *The exchange rate band gives rise to a positive range of effective commitment (REC), $[\overline{\pi}, \overline{\pi}+r]$, and a negative REC, $[\underline{\pi}-r, \underline{\pi}]$, where r is defined in Lemma 1.*

- *When x falls inside the positive REC, the policymaker defends the currency and ensure that the rate of depreciation is not larger than $\overline{\pi}$.*
- *When x falls inside the negative REC, the policymaker defends the currency and ensures that the rate of appreciation is not larger in absolute value than $\underline{\pi}$.*
- *When x falls below the negative REC, above the positive REC, or inside the band, the policymaker lets the exchange rate move freely in accordance with market forces.*
- *The width of the two RECs, r, increases with t and with δ but is independent of the boundaries of the band, $\underline{\pi}$ and $\overline{\pi}$.*

3.3 The choice of band width

We now turn to the policymaker's objective function. Note first that the expected variability of the exchange rate around the current level can be written as follows:

$$\begin{aligned} E\left|\pi\right| &= -\int_{-\infty}^{\underline{\pi}-r} xf(x)dx - \int_{\underline{\pi}-r}^{\underline{\pi}} \underline{\pi}f(x)dx - \int_{\underline{\pi}}^{0} xf(x)dx \\ &\quad + \int_{0}^{\overline{\pi}} xf(x)dx + \int_{\overline{\pi}}^{\overline{\pi}+r} \overline{\pi}f(x)dx + \int_{\overline{\pi}+r}^{\infty} xf(x)dx. \end{aligned} \tag{3.2}$$

Equation (3.2) shows that the existence of a band has a moderating effect on $|\pi|$ only inside the two RECs. Over these two regions, the policymaker is expected to keep the exchange rate from moving outside the boundaries of the band.

Second, using equation (2.1) and Lemma 1, the cost of intervention in the exchange rate market is $\underline{\pi}-x$ if $x \in [\underline{\pi}-r, \underline{\pi}]$, and $x-\overline{\pi}$ if $x \in [\overline{\pi}, \overline{\pi}+r]$. When either $x < \underline{\pi}-r$ or $x > \overline{\pi}+r$, there are realignments so the policymaker incurs a credibility loss, δ. Hence, using equations (2.2) and (3.2), the expected payoff of the policymaker, given $\underline{\pi}$ and $\overline{\pi}$, is

$$\begin{aligned} V = {} & A\Bigg[\int_{-\infty}^{\underline{\pi}-r} xf(x)dx + \int_{\underline{\pi}-r}^{\underline{\pi}} \underline{\pi} f(x)dx + \int_{\underline{\pi}}^{0} xf(x)dx \\ & - \int_{0}^{\overline{\pi}} xf(x)dx - \int_{\overline{\pi}}^{\overline{\pi}+r} \overline{\pi} f(x)dx - \int_{\overline{\pi}+r}^{\infty} xf(x)dx\Bigg] \\ & - \int_{-\infty}^{\underline{\pi}-r} \delta f(x)dx - \int_{\underline{\pi}-r}^{\underline{\pi}} (\underline{\pi}-x) f(x)dx - \int_{\overline{\pi}}^{\overline{\pi}+r} (x-\overline{\pi}) f(x)dx - \int_{\overline{\pi}+r}^{\infty} \delta f(x)dx. \end{aligned} \tag{3.3}$$

The first two lines in equation (3.3) represent the policymaker's loss from exchange rate uncertainty while the last line represents the expected cost of adopting a band.

The policymaker chooses the boundaries of the band, $\underline{\pi}$ and $\overline{\pi}$, so as to maximize his expected payoff. The first order conditions for an interior solution to the policymaker's problem (i.e., for $-\infty < \underline{\pi} < 0 < \overline{\pi} < \infty$) are:

$$\begin{aligned} \frac{\partial V}{\partial \underline{\pi}} &= -\left[r(A-1)+\delta\right] f(\underline{\pi}-r) + (A-1)\int_{\underline{\pi}-r}^{\underline{\pi}} f(x)dx \\ &= A\int_{\underline{\pi}-r}^{\underline{\pi}} \left[f(x) - f(\underline{\pi}-r)\right] dx - \left[\int_{\underline{\pi}-r}^{\underline{\pi}} \left[f(x)-f(\underline{\pi}-r)\right] dx + \delta f(\underline{\pi}-r)\right] = 0, \end{aligned} \tag{3.4}$$

and,

$$\begin{aligned} \frac{\partial V}{\partial \overline{\pi}} &= \left[r(A-1)+\delta\right] f(\overline{\pi}+r) - (A-1)\int_{\overline{\pi}}^{\overline{\pi}+r} f(x)dx \\ &= -A\int_{\overline{\pi}}^{\overline{\pi}+r} \left[f(x) - f(\overline{\pi}+r)\right] dx + \left[\int_{\overline{\pi}}^{\overline{\pi}+r} \left[f(x)-f(\overline{\pi}+r)\right] dx + \delta f(\overline{\pi}+r)\right] = 0. \end{aligned} \tag{3.5}$$

We prove in the Appendix that $f''(x) \leq 0$ and $A > 1$, along with Assumption 1, are sufficient (but not necessary) conditions for V to be globally concave in $\underline{\pi}$ and $\overline{\pi}$, in which case equations (3.4) and (3.5) are sufficient for a unique maximum. Equations (3.4) and (3.5) show that by altering the bounds of the band, policymakers move along a trade-off between reduction of exchange rate uncertainty and minimization of the costs of maintaining a band. The first term in the second line of (3.5) is the marginal effect of $\overline{\pi}$ on exchange rate uncertainty. This term is multiplied by A which is the weight that the policymaker assigns to exchange rate stability. Since by Assumption 1, $f(x) - f(\overline{\pi}+r) < 0$ for all $x \in [\overline{\pi}, \overline{\pi}+r]$, this term is negative and represents the marginal cost from raising $\overline{\pi}$. This marginal cost arises because when $\overline{\pi}$ is raised, the positive REC over which the exchange rate is kept constant, shifts further away from the center rate. Assumption 1 implies that large changes in the exchange rate are less likely than small ones. Therefore, the shift of the positive REC away from 0 makes the band less effective in reducing exchange rate uncertainty. The second term in equation (3.5) represents the marginal effect of raising $\overline{\pi}$ on the cost of defending the band. By Assumption 1, the integral term in the brackets is positive, implying that raising $\overline{\pi}$ makes it less costly to defend the band. This is because now it is less likely that the policymaker will actually have to defend the band. The δ term is also positive since increasing $\overline{\pi}$ slightly lowers the likelihood that the exchange rate will move outside the positive REC and lead to a realignment. The interpretation of equation (3.4) is analogous except that here, the signs of the various terms are exactly opposite since raising $\underline{\pi}$ slightly shifts the lower bound of the band closer to 0, whereas raising $\overline{\pi}$ slightly shifts the upper bound of the band away from 0.

4 When is the exchange rate a peg, a float, or a band - and if a band, of what width?

We begin this section by studying the properties of the equilibrium exchange rate regime using equations (3.4) and (3.5).

Proposition 2 *In equilibrium, the exchange rate band has the following properties:*

(i) ***Free float:*** *If $A \leq 1$, then $\underline{\pi} = -\infty$ and $\overline{\pi} = \infty$, so the optimal regime is a free float.*

(ii) ***A nondegenerate band:*** *If*

$$1 < A < \underline{A}(-r) \equiv 1 + \frac{\delta}{\int_{-r}^{0} \left[\frac{f(x)}{f(-r)} - 1\right] dx}, \tag{3.6}$$

then $-\infty < \underline{\pi} < 0$. Likewise, if

$$1 < A < \overline{A}(r) \equiv 1 + \frac{\delta}{\int_{0}^{r} \left[\frac{f(x)}{f(r)} - 1\right] dx}, \tag{3.7}$$

then $0 < \overline{\pi} < \infty$. Hence, the optimal regime is a nondegenerate band.

(iii) ***A peg:*** *If V is concave in $\underline{\pi}$ and in $\overline{\pi}$, and $A > Max\{\underline{A}(r), \overline{A}(r)\}$, then $\underline{\pi} = \overline{\pi} = 0$, so the optimal regime is a peg.*

(iv) ***Symmetry:*** *If $f(x)$ is symmetric around 0, so that $f(-x) = f(x)$ for all x, the band will be symmetric around 0 in the sense that $-\underline{\pi} = \overline{\pi}$.*

Proposition 2 shows that depending on the policymaker's aversion to nominal exchange rates variability, the optimal regime can either be a peg, a free float, or a band. When the policymaker is not too concerned with nominal exchange rate variability, i.e., $A \leq 1$, he sets a free float and avoids the cost of maintaining a band. On the other hand, if the policymaker is sufficiently concerned with nominal exchange rate variability, i.e., $A > Max\{\underline{A}(r), \overline{A}(r)\}$, he minimizes it by adopting a peg.[16] In intermediate cases, the policymaker balances the two objectives, namely limiting exchange rate uncertainty and minimizing the cost of intervention by setting a nondegenerate band. Intervention occurs only when x falls inside the negative or the positive RECs. Part (iv) of the proposition states that a sufficient condition for the band to be symmetric is that the distribution of shocks, $f(x)$, is symmetric around 0 (i.e., under laissez faire, depreciations and appreciations are equally likely).[17]

We turn next to comparative statics analysis for the case in which the policymaker's problem has a unique interior solution, i.e., $-\infty < \underline{\pi} < 0 < \overline{\pi} < \infty$. As Proposition 2 indicates, this requires A to be above 1 but not by "too much."

Proposition 3 *Suppose that the policymaker's problem has a unique interior solution. Then as A increases:*

(i) $\overline{\pi}$ and $\underline{\pi}$ shift closer to 0 so the band becomes tighter, and

(ii) the probability that a speculative attack will occur increases.

Proposition 3 says that the policymaker sets a tighter band and thereby allows the exchange rate to move freely within a narrower range around the center rate, as he becomes more concerned with exchange rate stability (i.e., as A increases). Part (ii) of the proposition shows that this tightening of the band raises the likelihood of a speculative attack. This implies that when A increases, the policymaker is willing to set a tighter band despite the fact that this raises the likelihood of speculative attacks.

[16] Note that a peg does not mean that the exchange rate is fixed under all circumstances. When the absolute value of x exceeds r, the policymaker abandons the peg and the exchange rate is realigned. Nonetheless, under a peg, the policymaker maintains stability over the range of "small" shocks where $x \in [-r, r]$. Given Assumption 1, such small shocks are more likely than big ones, so when A is large, it is optimal for the policymaker to eliminate these shocks by adopting a peg.

[17] Note that the symmetry of the band is in terms of the permissible rates of revaluations and devaluations of the exchange rate rather than in terms of the gap of the upper and lower bounds of the band from the center rate (i.e., the symmetry is in terms of x rather than e).

Note that as Proposition 2 shows, when A goes above $\underline{A}(r)$ and $\overline{A}(r)$, the optimal band width becomes 0 so the optimal regime is a peg. On the other hand, when A falls below 1, the optimal band width becomes infinite implying that the optimal regime is a free float. Given that a substantial part of international trade is invoiced in US Dollar (McKinnon, 1979), it is likely that policymakers of a key currency country like the US are going to be less sensitive to nominal exchange rate uncertainty and therefore have a smaller A than policymakers in small open economies. Therefore, our model predicts that the US, Japan, and the Euro area should be floating, while Hong-Kong, Panama, Estonia, Lithuania, Bulgaria, and Argentina should be on either pegs, currency boards, or even full dollarization. This indeed happens to be the case.

Next, we examine how the exchange rate band changes when more extreme realizations of x become more likely. This comparative statics exercise involves shifting probability mass from realizations of x that are either inside the band or inside the two RECs (and therefore do not lead to realignments) to realizations that are either below the negative REC or above the positive REC and therefore lead to realignments.

Proposition 4 *Suppose that $f(x)$ and $g(x)$ are two density functions with a mode at 0 and equal expected values. Moreover, suppose that $g(x)$ lies above $f(x)$ for all $x < \underline{\pi} - r$ and all $x > \bar{\pi} + r$, where $\underline{\pi}$ and $\bar{\pi}$ are the solutions to the policymaker's problem under the original density function $f(x)$ (that is, $g(x)$ has fatter tails than $f(x)$). Then, the policymaker adopts a wider band under $g(x)$ than under $f(x)$.*

Intuitively, when more extreme realizations of x become more likely (the density of x is $g(x)$ rather than $f(x)$), the policymaker is more likely to incur the loss of future credibility associated with realignments. Therefore, the policymaker widens the band to lower the probability that a costly realignment will take place. Moreover, as larger shocks become more likely, the policymaker finds it optimal to shift the two REC's away from 0 in order to shift his commitment to intervene in the market to a range of shocks that are now more probable. This move benefits the policymaker by counteracting part of the increased uncertainty about the free float value of the exchange rate.

5 The effects of reductions in the cost of switching between currencies

During the last three decades there has been a world-wide gradual lifting of restrictions on currency flows and on related capital account transactions. One consequence of this trend, in terms of the model, is a reduction in the nominal transaction cost, t. The following proposition examines the effect of a reduction in t on the optimal exchange rate regime and on the likelihood of currency crises.

Proposition 5 *Suppose that following a lifting of restrictions on currency flows and capital account transactions, the transaction cost of switching between currencies, t, decreases. Then:*

(i) The bounds $\underline{A}(-r)$ and $\overline{A}(r)$ above which the policymaker adopts a peg decrease, implying that policymakers adopts pegs for a narrower range of values of A.

(ii) When the policymaker's problem has a unique interior solution, $\bar{\pi}$ and $\underline{\pi}$ shift away from 0, so the band becomes wider. Moreover, the probability that a speculative attack will occur decreases.

Part (i) of Proposition 5 predicts that liberalization of the capital account, as characterized by a reduction in t, should lead to a narrowing of the set of countries that maintain pegs. It also implies that, in spite of this trend, countries with strong preference for stability of the exchange rate (e.g., small open economies with relatively large shares of foreign denominated trade and capital flows as well as emerging markets) will continue to peg even in the face of capital market liberalization. By contrast, countries with intermediate preference for exchange rate stability (e.g., more financially mature economies with a larger fraction of domestically denominated debt and capital flows) will move from pegs to bands. These predictions seem to be consistent with casual evidence. Two years following the 1997/8 currency crises in East Asia, most emerging market countries in that region are back on pegs (Calvo and Reinhart, 2000, and McKinnon, 2001). And, in spite of the convertibility that was established in 1991, Argentina has maintained a peg to the US Dollar till the beginning of 2002. On the other hand, following the EMS currency crisis at the beginning of the 90's, the system of cooperative pegs that had existed prior to the crisis was replaced by wide bands until the formation of the EMU at the beginning of 1999.

To see the intuition for part (ii) of Proposition 5, recall from Proposition 1 that a decrease in t (which makes it less costly to attack the band) leads to a decrease in r. Hence, the two

RECs shrink and the band becomes less effective in guaranteeing exchange-rate stability. The policymaker's reaction to this is to set a wider band and allow the exchange rate to move freely within a broader range around the center rate.

Part (ii) of Proposition 5 also implies that on balance, lifting restrictions of free capital markets lowers the likelihood of a currency crisis. This counterintuitive result is the outcome of two opposing effects. First, holding the band width constant, the two RECs shrink when t decreases and this raises the probability of speculative attacks. This effect already appears in the recent literature on international financial crises (e.g., Morris and Shin, 1998). But when t decreases, the policymaker pursues less ambitious stabilization objectives and adopts a wider band. This lowers, in turn, the probability of speculative attacks. Although a priori it is not clear which effect is stronger, part (ii) of Proposition 5 shows that when $f(x)$ is unimodal and $A > 1$, the second effect always dominates.

The more general lesson from part (ii) of Proposition 5 is that when the endogeneity of the exchange rate regime is taken into consideration, strong restrictions on capital flows (that lead to a higher t) may actually increase rather than decrease the likelihood of currency crises. One implication of this result is that the so-called "Tobin tax" (Tobin 1978) may actually raise rather than lower the likelihood of financial crises.

Since an increase in t induces the policymaker to pursue more ambitious stabilization policies but also raises the likelihood of financial crisis it might be thought that overall it has an ambiguous effect on the policymaker's objective function. The next proposition shows that this is not so.

Proposition 6 *Policymakers achieve higher equilibrium values of objectives, V, when t is higher.*

Proposition 6 implies that policymakers have an incentive to impose capital account restrictions and thereby raise the transactions costs t, despite the fact that, at least within the framework of this model, this increases the likelihood of currency crises. The reason is that speculative attacks impose a constraint on the policymaker that needs to be taken into account in choosing the optimal exchange rate regime. By raising t, the policymaker lowers the incentive to mount a speculative attack and thereby relaxes this constraint somewhat. It should be noted however that Proposition 6 does not provide a complete account of the desirability of imposing capital account restrictions, since the benefits of free capital mobility are not recognized explicitly by the model.

6 The exchange rate band and the policymaker's reputation

There is typically considerable uncertainty about the commitment ability of policymakers. In this section, we examine how this uncertainty affects the optimal exchange rate regime. To this end, we assume that there are two possible types of policymakers. The first type, to which we refer as dependable, is identical to the policymaker that we considered so far. The second type, to which we refer as opportunistic, differs from the dependable type in that he does not bear the cost δ if the exchange rate is realigned. Although both policymaker types defend the currency initially, an opportunistic policymaker has no incentive to continue to defend it after the initial defense stage.[18] Thus, under an opportunistic policymaker, there is always a realignment when x falls outside the band. We assume that speculators assign a probability β to the policymaker's type being dependable and interpret β as a measure of the policymaker's "reputation." We now examine how the optimal exchange rate regime is affected by changes in β. As a point of reference, it should be noted that the analysis so far referred to the case where $\beta = 1$.

Before beginning the analysis, we modify Assumption 2 as follows:

Assumption 3: The real transaction cost, $\frac{t}{e_{-1}}$, is small relative to δ but not too small in the sense that $\delta(1-\beta) < \frac{t}{e_{-1}} < \delta$.

Assumption 3 ensures that speculators will always attack the band if they believe that x is such that a dependable policymaker will exit the band, but never attack it if they believe that x is such that the dependable policymaker will defend the band.

With Assumption 3 in place, we examine how the presence of an opportunistic policymaker affects the decisions of speculators on when to attack the band.

Lemma 2 *Suppose that $\varepsilon \to 0$. Then,*

(i) speculators will attack the upper bound of the band if and only if they observe signals above $\overline{\theta}^*_\beta$. They will attack the lower bound of the band if and only if they observe signals below $\underline{\theta}^*_\beta$. The two thresholds, $\overline{\theta}^*_\beta$ and $\underline{\theta}^*_\beta$, are given by $\overline{\theta}^*_\beta = \overline{\pi} + r^\beta$, and $\underline{\theta}^*_\beta = \underline{\pi} - r^\beta$, where

$$r^\beta = \sqrt{\frac{t}{\beta e_{-1}} + \frac{(\delta - \frac{1}{\beta})^2}{4}} + \frac{\delta - \frac{1}{\beta}}{2}. \tag{4.1}$$

[18] Even an opportunistic policymaker that does not intend to persist in defending the band needs to put the initial (short) defense stage on "automatic pilot" in order to prevent the public from separating him, already in the first stage (when the band is announced), from his dependable counterpart.

(ii) r^{β} increases with β.

The speculators' behavior implies that the positive REC is now given by $[\overline{\pi}, \overline{\pi} + r^{\beta}]$ while the negative REC becomes $[\underline{\pi} - r^{\beta}, \underline{\pi}]$. Since r^{β} increases with β, it follows that when the policymaker's reputation is imperfect (i.e., $\beta < 1$), the two RECs become narrower relative to the perfect reputation case (i.e., when $\beta = 1$).[19] The intuition underlying this result is that when $\beta < 1$, speculators believe that the policymaker will exit the band with a positive probability. Hence, the expected gain from attacking the band is now larger so a dependable policymaker finds it more difficult to defend the band.

Next, we examine the impact of the policymaker's reputation on the choice of exchange rate regime. We begin with the expected variability in the exchange rate. When the policymaker's reputation is imperfect, it is anticipated that with probability β the policymaker is dependable and will defend the band against speculative attacks whenever x falls inside the two RECs, and with probability $1-\beta$ the policymaker is opportunistic and never defends the band. Hence, the expected variability in the exchange rate around the current level is:

$$\begin{aligned} E^{\beta}|\pi| \; = \; & -\int_{-\infty}^{\underline{\pi}-r^{\beta}} xf(x)dx - \int_{\underline{\pi}-r^{\beta}}^{\underline{\pi}} [\beta\underline{\pi} + (1-\beta)x] f(x)dx - \int_{\underline{\pi}}^{0} xf(x)dx \\ & + \int_{0}^{\overline{\pi}} xf(x)dx + \int_{\overline{\pi}}^{\overline{\pi}+r^{\beta}} [\beta\overline{\pi} + (1-\beta)x] f(x)dx + \int_{\overline{\pi}+r^{\beta}}^{\infty} xf(x)dx. \end{aligned} \tag{4.2}$$

Note that β affects $E^{\beta}|\pi|$ both through its effect on the width of the two RECs and through its effect on the expected change in the exchange rate inside the two RECs which are now linear combinations of $\underline{\pi}$ and of x inside the negative REC and of $\overline{\pi}$ and of x inside the positive REC.

Given $E^{\beta}|\pi|$, the expected payoff of a dependable policymaker becomes,

$$\begin{aligned} V^{\beta} \; = \; & A\left[\int_{-\infty}^{\underline{\pi}-r^{\beta}} xf(x)dx + \int_{\underline{\pi}-r^{\beta}}^{\underline{\pi}} [\beta\underline{\pi} + (1-\beta)x] f(x)dx + \int_{\underline{\pi}}^{0} xf(x)dx \right. \\ & \left. - \int_{0}^{\overline{\pi}} xf(x)dx - \int_{\overline{\pi}}^{\overline{\pi}+r^{\beta}} [\beta\overline{\pi} + (1-\beta)x] f(x)dx - \int_{\overline{\pi}+r^{\beta}}^{\infty} xf(x)dx \right] \\ & - \int_{-\infty}^{\underline{\pi}-r^{\beta}} \delta f(x)dx - \int_{\underline{\pi}-r^{\beta}}^{\underline{\pi}} (\underline{\pi} - x) f(x)dx - \int_{\overline{\pi}}^{\overline{\pi}+r^{\beta}} (x - \overline{\pi}) f(x)dx - \int_{\overline{\pi}+r^{\beta}}^{\infty} \delta f(x)dx. \end{aligned} \tag{4.3}$$

We do not need to specify the expected payoff of the opportunistic policymaker because the decision problem of such a policymaker is simple. Given that an opportunistic policymaker does not intend

[19] This is analogous to a result in Cukierman and Liviatan (1991) in the context of a Barro-Gordon (1983) inflation bias equilibrium in which the public is uncertain about the dependability of policymakers. Cukierman and Liviatan show that the lower the reputation of a (dependable) policymaker the less ambitious is his inflation target.

to defend the band, he always wishes to announce the same band as his dependable counterpart in order to reduce exchange rate uncertainty ex ante. A dependable policymaker in contrast chooses the boundaries of the band, $\underline{\pi}$ and $\overline{\pi}$, so as to maximize his expected payoff. The next proposition characterizes the optimal exchange rate regime when the policymaker has imperfect reputation.

Proposition 7 *In equilibrium, the exchange rate band has the following properties:*

(i) ***Free float:*** *If $A\beta \leq 1$, then $\underline{\pi} = -\infty$ and $\overline{\pi} = \infty$, so the optimal regime is a free float.*

(ii) ***A nondegenerate band:*** *If*

$$1 < A\beta < \underline{A}(-r^{\beta}) \equiv 1 + \frac{\delta}{\int_{-r^{\beta}}^{0} \left[\frac{f(x)}{f(-r^{\beta})} - 1\right] dx}, \tag{4.6}$$

then $-\infty < \underline{\pi} < 0$. Likewise, if

$$1 < A\beta < \overline{A}(r^{\beta}) \equiv 1 + \frac{\delta}{\int_{0}^{r^{\beta}} \left[1 - \frac{f(x)}{f(r^{\beta})}\right] dx}, \tag{4.7}$$

then $0 < \bar{\pi} < \infty$. Hence, the optimal regime is a nondegenerate band.

(iii) ***A peg:*** *If V^{β} is concave in $\underline{\pi}$ and in $\bar{\pi}$ and $A\beta > Max\{\underline{A}^{\beta}, \bar{A}^{\beta}\}$ then $\underline{\pi} = \bar{\pi} = 0$, so the optimal regime is a peg.*

(iv) ***The width of the band and the likelihood of speculative attacks:*** *Suppose that the policymaker's problem has a unique interior solution. Then, $\bar{\pi}$ and $\underline{\pi}$ shift closer to 0 as β increases towards 1, implying that as the policymaker's reputation improves, he adopts a tighter band. Hence, $\frac{\partial P^{\beta}}{\partial \beta} > 0$, implying that as the policymaker's reputation improves, there is a greater likelihood of speculative attacks. At the same time, the likelihood of speculative attacks increases.*

Parts (i)-(iii) of Proposition 7 modify the corresponding parts of Proposition 2 for the case where the policymaker's reputation is imperfect. Part (iv) of Proposition 7 says that as the policymaker's reputation improves, the exchange rate band becomes tighter. This implies that a good reputation induces the policymaker to adopt a more ambitious goal. The reason for that is twofold. First, when the policymaker's reputation improves, a tighter band has a greater moderating effect on the expected variability of the nominal exchange rate. Second, holding the width of the band constant, improved reputation lowers the likelihood of speculative attacks, and therefore makes it less costly for the policymaker to set a tighter band. Hong-Kong's currency board fits into this "box" of the model. Since the peg has never been abandoned in the past, Hong-Kong's currency board has good reputation, which induces the authorities to defend the peg under a wider set of circumstances than is the case under a lower reputation level. Part (iv) of Proposition 7 also says that when a policymaker has a better reputation there is an overall increase in the likelihood of speculative attacks. Although better reputation leads to a wider RECs (i.e., ranges of x for which the policymaker defends the band), it also induces the policymaker to adopt tighter bands and hence makes the exchange rate regime more susceptible to speculative attacks. Under Assumption 1, the second effect is stronger so overall there is an increase in the likelihood that a speculative attack will actually take place.

7 Concluding reflections

Drawing on data from the IMF Annual Report 2000, Fischer (2001) reports evidence that supports the bipolar view according to which during the 1990's, there has been a gradual shift away from intermediate exchange rate regimes to either hard pegs or to freely floating regimes.[20] In what follows we discuss the set of restrictions on the exogenous parameters of our framework needed to deliver this trend.

Our framework features three main exogenous parameters: The relative importance attributed by policymakers to limiting nominal exchange rate uncertainty (A), the credibility loss from realignments (δ), and the cost of switching between currencies (t). To these parameters we can also add the policymaker's reputation (β) considered in Section 6.

Proposition 2 implies that countries with relatively large values of A will have pegs and Proposition 1 implies that countries with relatively large values of δ will have wider ranges of effective commitment implying that they will defend the currency under a wider set of circumstances. Thus, our framework predicts that countries characterized by relatively high values of *both* A and δ (e.g., small open economies with large shares of financial assets and liabilities, and of international trade invoicing in foreign exchange) will institute hard pegs. The reason for this is that, to the extent that a current realignment reduces the reputation of policymakers, a country with a high value of A is likely to put a higher value on protecting its reputation since it allows it to limit

[20] Hard pegs include currency boards (like Lithuania and Hong-Kong) and full dollarization (like Panama). In terms of our framework, Fischer's hard pegs are characterized not only by the fact that the width of the band is zero, but also by the fact that the commitment to defend the peg is relatively strong.

exchange rate uncertainty at a lower cost both in the present as well as in the future. Those observations, in conjunction with the analytical results of the paper imply that countries like Panama, Hong-Kong, Lithuania and numerous other small open economies are likely to institute hard pegs. The evidence presented in Fischer (2001) is consistent with this prediction.

Liberalization of world capital markets is likely to affect several of the exogenous parameters of our framework. On one hand, by reducing t, it makes speculation easier. On the other hand, by opening most economies to wider, foreign exchange denominated assets and liabilities, it makes the reduction of nominal exchange rate uncertainty in both the present and the future more valuable. This takes the form of larger values of both A and δ. Proposition 6 implies that the reduction in t induces policymakers to move towards more flexible exchange rate arrangements whereas Propositions 1 and 3 imply that the increase in A and δ induces them to move in the opposite direction. Therefore, in the absence of additional information, the overall effect of liberalization appears to be ambiguous.

But the magnitude of the effect of capital market liberalization, via the increase in A and δ varies across countries. It is likely to be large for small open economies whose currencies are not used much for either capital account or current account transaction in world markets, and to be small or even negligible for large key currency economies. Hence, the first effect is likely to be dominant in large, relatively closed blocks, and the second is likely to be dominant in small open economies. It follows that, all else equal, the liberalization of world capital markets should induce relatively large currency blocks to move towards more flexible exchange rate arrangements while pushing small open economies in the opposite direction. The upshot is that, given an additional hypothesis about the relative magnitude of the effect of capital market liberalization on A and δ in different countries, the analytical results of this paper provide an explanation for the recent trend towards a bipolar system of exchange rate arrangements.

Finally, the analysis in Section 6 implies that countries that established a good reputation for dependability are more likely to maintain hard pegs since (by Lemma 2 and Proposition 7) they are predicted to have both wider ranges of effective commitment and tighter bands. Thus, over time there is a virtuous circle between good reputation and the likelihood that a hard peg will be maintained. Hong-Kong appears to be a case in point.

8 Appendix

Proof of Lemma 1: (i) We analyze the behavior of the policymaker and of the speculators after the exchange rate reaches the upper bound of the band. We show that in the limit as $\varepsilon \longrightarrow 0$, there exists in this case a unique perfect Bayesian equilibrium in which each speculator attacks the band if and only if he observe a signal above a unique threshold signal $\overline{\theta}^*$. The proof for the case where the exchange rate reaches the lower bound of the band is analogous.

We start with some notation. From equation (3.1) we know that the policymaker defends the upper bound of the band if and only if $x \leq \overline{\pi} - \alpha + \delta$. Using this expression, let

$$\alpha^*(x) = \begin{cases} 0, & if \quad \overline{\pi} - x + \delta < 0, \\ \overline{\pi} - x + \delta, & if \quad 0 \leq \overline{\pi} - x + \delta \leq 1, \\ 1 & if \quad \overline{\pi} - x + \delta > 1, \end{cases} \tag{A-1}$$

be the critical measure of speculators below which the policymaker defends the upper bound of the band when the laissez faire rate of change in the exchange rate is x. Using the definition of $\alpha^*(x)$, the net payoff from attacking the upper bound of the band is:

$$v(x, \alpha) = \begin{cases} (x - \overline{\pi}) e_{-1} - t, & if \quad \alpha \geq \alpha^*(x), \\ -t, & if \quad \alpha < \alpha^*(x). \end{cases} \tag{A-2}$$

Equation (A-2) shows that $v(x, \alpha)$ is weakly increasing in α because the assumption that $x \geq \overline{\pi}$ implies that $(x - \overline{\pi}) e_{-1} - t \geq -t$ (hence, the expression in the top line in (A-2) exceeds the one at the bottom line of (A-2)). Moreover, noting from equation (A-1) that $\alpha^*(x)$ is weakly decreasing in x, equation (A-2) shows that $v(x, \alpha)$ is weakly increasing in x, and strictly increasing in x if $v(x, \alpha) \geq 0$.

Let $\alpha_i(x)$ be speculator i's belief about the measure of speculators who will attack the band for each level of x. We will say that the belief $\alpha_i'(x)$ is lower (higher) than $\alpha_i(x)$ if $(\alpha_i'(x) \geq \alpha_i(x))$ for all x with strict inequality for at least one x.

The decision of speculator i on whether or not to attack the band depends on the signal θ_i that the speculator observes and on the speculator's belief, $\alpha_i(x)$. Since $\theta_i = x + \varepsilon_i$, where $\varepsilon_i \sim U[-\varepsilon, \varepsilon]$, and using equation (2.4), the net expected payoffs of speculator i from attacking the upper bound of the band is:

$$h(\theta_i, \alpha_i(x)) = \int_{\theta_i - \varepsilon}^{\theta_i + \varepsilon} v(x, \alpha_i(x)) f(x \mid \theta_i) dx = \frac{\int_{\theta_i - \varepsilon}^{\theta_i + \varepsilon} v(x, \alpha_i(x)) f(x) dx}{F(\theta_i + \varepsilon) - F(\theta_i - \varepsilon)}. \tag{A-3}$$

We establish three properties of $h(\theta_i, \alpha_i(x))$. First, since θ_i affects the numerator of $h(\theta_i, \alpha_i(x))$ only through the boundaries of integration and since $F(\cdot)$ is a continuous function, it follows that $h(\theta_i, \alpha_i(x))$ is continuous in θ_i. Second, recalling that $v(x, \alpha)$ is weakly increasing in α, it follows that if $\alpha_i'(x) \leq \alpha_i(x)$ ($\alpha_i'(x) \geq \alpha_i(x)$), then $h(\theta_i, \alpha_i'(x)) \leq h(\theta_i, \alpha_i(x))$ ($h(\theta_i, \alpha_i'(x)) \geq h(\theta_i, \alpha_i(x))$) for all θ_i. Third, note that:

$$\begin{aligned}\frac{\partial h(\theta_i, \alpha_i(x))}{\partial \theta_i} &= \frac{\int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon} f(x)dx \left[v\left(\theta_i+\varepsilon, \alpha_i(\theta_i+\varepsilon)\right) f(\theta_i+\varepsilon) - v\left(\theta_i-\varepsilon, \alpha_i(\theta_i-\varepsilon)\right) f(\theta_i-\varepsilon)\right]}{\left(F\left(\theta_i+\varepsilon\right) - F\left(\theta_i-\varepsilon\right)\right)^2} \\ &\quad - \frac{\int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon} v(x, \alpha_i(x)) f(x)dx \left[f(\theta_i+\varepsilon) - f(\theta_i-\varepsilon)\right]}{\left(F\left(\theta_i+\varepsilon\right) - F\left(\theta_i-\varepsilon\right)\right)^2} \\ &= \frac{f(\theta_i+\varepsilon) \int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon} \left[v\left(\theta_i+\varepsilon, \alpha_i(\theta_i+\varepsilon)\right) - v(x, \alpha_i(x))\right] f(x)dx}{\left(F\left(\theta_i+\varepsilon\right) - F\left(\theta_i-\varepsilon\right)\right)^2} \\ &\quad + \frac{f(\theta_i-\varepsilon) \int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon} \left[v(x, \alpha_i(x)) - v\left(\theta_i-\varepsilon, \alpha_i(\theta_i-\varepsilon)\right)\right] f(x)dx}{\left(F\left(\theta_i+\varepsilon\right) - F\left(\theta_i-\varepsilon\right)\right)^2}.\end{aligned} \tag{A-4}$$

Recalling from above that $v(x, \alpha)$ is weakly increasing in both x and α, it follows that if $\alpha_i(x)$ is non-decreasing in x, then $h(\theta_i, \alpha_i(x))$ is weakly increasing in θ_i. Next, we show that whenever it is nonnegative, $h(\theta_i, \alpha_i(x))$ must be strictly increasing in θ_i. To this end, note that since $h(\theta_i, \alpha_i(x))$ is the expected value of $v(x, \alpha_i(x))$ when $x \in [\theta_i - \varepsilon, \theta_i + \varepsilon]$, then $h(\theta_i, \alpha_i(x)) \geq 0$ implies that there exists at least one value of x in the interval $[\theta_i - \varepsilon, \theta_i + \varepsilon]$ for which $v(x, \alpha_i(x)) > 0$ (otherwise $h(\theta_i, \alpha_i(x)) \leq 0$). Since we showed above that $v(x, \alpha)$ is strictly increasing in x if $v(x, \alpha) \geq 0$, it follows that $v(x, \alpha)$ is strictly increasing in x for at least one value of $x \in [\theta_i - \varepsilon, \theta_i + \varepsilon]$. But since $v(x, \alpha)$ is weakly increasing in x and strictly increasing in x for at least one value of x, it follows that $v\left(\theta_i + \varepsilon, \alpha_i(\theta_i + \varepsilon)\right) > v\left(\theta_i - \varepsilon, \alpha_i(\theta_i - \varepsilon)\right)$. Consequently, $h(\theta_i, \alpha_i(x))$ is strictly increasing in θ_i whenever $h(\theta_i, \alpha_i(x)) \geq 0$.

In equilibrium, the strategy of speculator i is to attack the upper bound of the band if $h(\theta_i, \alpha_i(x)) > 0$ and not attack it if $h(\theta_i, \alpha_i(x)) < 0$. Moreover, the equilibrium belief of speculator i, $\alpha_i(x)$, must be consistent with the equilibrium strategies of all other speculators (for short we will simply say that in equilibrium, the belief of speculator i is consistent). To characterize the equilibrium strategies of speculators, we first show that there exists a range of sufficiently large signals for which speculators have a dominant strategy to attack the band and likewise, there exists a range of sufficiently small signals for which speculators have a dominant strategy not to attack the band. Then, we use an iterative process of elimination of dominated strategies to establish the existence of a unique signal, $\overline{\theta}^*$, such that speculator i attacks the upper bound of the band if and only if $\theta_i > \overline{\theta}^*$.

Suppose that speculator i observes a signal $\theta_i > \overline{\theta} \equiv \overline{\pi} + \delta + \varepsilon$. Then speculator i realizes that $x > \overline{\pi} + \delta$. Using equation (A-1), this means that $\alpha^*(x) = 0$ so the policymaker is surely going to exit the band. By equation (A-2), the net payoff from attacking the band is therefore $v(x, \alpha) = (x - \overline{\pi})e_{-1} - t$, for all α. But since $x > \overline{\pi} + \delta$, it follows that $v(x, \alpha) > \delta e_{-1} - t$ for all α, which is strictly positive by Assumption 2. Hence, the net expected payoff of the speculator is such that $h(\theta_i, \alpha_i(x)) > 0$ for all $\theta_i > \overline{\theta}$ and all $\alpha_i(x)$, implying that it is a dominant strategy for a speculator who observes a signal above $\overline{\theta}$ to attack the upper bound of the band. Similarly, suppose that speculator i observes a signal $\theta_i < \underline{\theta} \equiv \overline{\pi} + \frac{t}{e_{-1}} - \varepsilon$ (since we focus on the case where $\varepsilon \to 0$ and since $t > 0$, such signals are observed with a positive probability whenever $x > \overline{\pi}$), the speculator realizes that $x < \overline{\pi} + \frac{t}{e_{-1}}$. Consequently, even if the policymaker surely exits the band, the payoff from attacking it is negative as $v(x, \alpha) = (x - \overline{\pi})e_{-1} - t < \left((\overline{\pi} + \frac{t}{e_{-1}}) - \overline{\pi}\right) e_{-1} - t = 0$. This implies in turn that $h(\theta_i, \alpha_i(x)) < 0$ for all $\theta_i < \underline{\theta}$ and all $\alpha_i(x)$, so it is a dominant strategy for speculator i not to attack the band after observing a signal below $\underline{\theta}$.

Now, we start an iterative process of elimination of dominated strategies from $\overline{\theta}$, in order to expand the range of signals for which speculators will surely attack the band. To this end, let $\alpha(x, \theta)$ represent a speculator's belief regarding the measure of speculators who will attack the band for each level of x, when the speculator believes that all speculators will attack the upper bound of the band if and only if they observe signals above some level θ. Since $\varepsilon_i \sim U[-\varepsilon, \varepsilon]$, it follows that

$$\alpha(x, \theta) = \begin{cases} 0, & if \quad x < \theta - \varepsilon, \\ \frac{x - (\theta - \varepsilon)}{2\varepsilon}, & if \quad \theta - \varepsilon \le x \le \theta + \varepsilon, \\ 1, & if \quad x > \theta + \varepsilon. \end{cases} \tag{A-5}$$

The iterative process of elimination of dominated strategies works as follows. Above, we already established that $h(\theta_i, \alpha_i(x)) > 0$ for all $\theta_i > \overline{\theta}$ and all $\alpha_i(x)$. But since $h(\theta_i, \alpha_i(x))$ is continuous in θ_i, it follows that $h(\overline{\theta}, \alpha_i(x)) \ge 0$ for all $\alpha_i(x)$, and in particular for $\alpha_i(x) = \alpha(x, \overline{\theta})$. Thus, $h(\overline{\theta}, \alpha(x, \overline{\theta})) \ge 0$. Note that since in equilibrium, the beliefs of speculators are consistent, only beliefs that are higher than or equal to $\alpha(x, \overline{\theta})$ can hold in equilibrium (because all speculators attack the band when they observe signals above $\overline{\theta}$). Thus, we say that $\alpha(x, \overline{\theta})$ is the "lowest" consistent belief on α.

Let $\overline{\theta}^1$ be the value of θ_i for which $h(\theta_i, \alpha(x, \overline{\theta})) = 0$. That is, $h(\overline{\theta}^1, \alpha(x, \overline{\theta})) \equiv 0$. Note that $\overline{\theta}^1 \le \overline{\theta}$, and that $\overline{\theta}^1$ is defined uniquely because we showed above that $h(\theta_i, \alpha_i(x))$ is strictly increasing in θ_i whenever $h(\theta_i, \alpha_i(x)) \ge 0$. Using the second and third properties of $h(\theta_i, \alpha_i(x))$

and recalling that $\alpha(x,\overline{\theta})$ is the lowest consistent belief on α, it follows that $h(\theta_i,\alpha_i(x))>0$ for any $\theta_i>\overline{\theta}^1$ and any consistent belief $\alpha_i(x)$. Thus, in equilibrium, speculators must attack the band if they observe signals above $\overline{\theta}^1$. As a result, $\alpha(x,\overline{\theta}^1)$ becomes the lowest consistent belief on $\alpha_i(x)$.

Starting from $\overline{\theta}^1$, we can now repeat the process along the following steps (these steps are similar to the ones that were used in order to establish $\overline{\theta}^1$). First, note that since $h(\overline{\theta}^1,\alpha(x,\overline{\theta}))\equiv 0$ and since $\alpha(x,\theta)$ is weakly decreasing with θ and $h(\theta_i,\alpha_i(x))$ is weakly increasing with $\alpha_i(x)$, it follows that $h(\overline{\theta}^1,\alpha(x,\overline{\theta}^1))\geq 0$. Second, find a $\theta_i\leq\overline{\theta}^1$ for which $h(\theta_i,\alpha(x,\overline{\theta}^1))=0$, and denote it by $\overline{\theta}^2$. Using the same arguments as above, $\overline{\theta}^2$ is defined uniquely. Third, since $\alpha(x,\overline{\theta}^1)$ is the lowest consistent belief on $\alpha_i(x)$ and using the second and third properties of $h(\theta_i,\alpha_i(x)))$, it follows that speculators must attack the band if they observe signals above $\overline{\theta}^2$. The lowest possible belief on $\alpha_i(x)$ becomes $\alpha(x,\overline{\theta}^2)$.

We repeat this process over and over again (each time lowering the value of θ above which speculators will attack the upper bound of the band), until we reach a step n such that $\overline{\theta}^{n+1}=\overline{\theta}^n$, implying that the process cannot continue further. Let $\overline{\theta}^\infty$ denote the value of θ at which the process stops. (Clearly, $\overline{\theta}^\infty\leq\overline{\theta}$.) By definition, speculators will attack the band if they observe signals above $\overline{\theta}^\infty$. Since $\overline{\theta}^\infty$ is the point where the process stops, it must be the case that $h(\overline{\theta}^\infty,\alpha(x,\overline{\theta}^\infty))=0$ (otherwise, we can find some $\theta_i<\overline{\theta}^\infty$ for which $h(\theta_i,\alpha(x,\overline{\theta}^\infty))=0$, and the iterative process could have been continued further).

Starting a similar iterative process from $\underline{\theta}$ and following the exact same steps, we also obtain a signal $\underline{\theta}^\infty(\geq\underline{\theta})$ such that speculators will never attack the band if they observe signals below $\underline{\theta}^\infty$. At this signal, it must be the case that $h(\underline{\theta}^\infty,\alpha(x,\underline{\theta}^\infty))=0$. Since we proved that in equilibrium speculators attack the upper bound of the band if they observe signals above $\overline{\theta}^\infty$ and do not attack it if they observe signals below $\underline{\theta}^\infty$, it must be the case that $\overline{\theta}^\infty\geq\underline{\theta}^\infty$.

The last stage of the proof involves showing that $\overline{\theta}^\infty=\underline{\theta}^\infty$. We do that for the case where $\varepsilon\to 0$. First, note that $\overline{\theta}^\infty$ is defined by the equation $h(\overline{\theta}^\infty,\alpha(x,\overline{\theta}^\infty))=0$. Using equations (A-3) and (A-5), this equality can be written as

$$\frac{\int_{\overline{\theta}^\infty-\varepsilon}^{\overline{\theta}^\infty+\varepsilon}v(x,\frac{x-(\overline{\theta}^\infty-\varepsilon)}{2\varepsilon})f(x)dx}{F\left(\overline{\theta}^\infty+\varepsilon\right)-F\left(\overline{\theta}^\infty-\varepsilon\right)}=0. \tag{A-6}$$

Using the equality $\alpha=\frac{x-(\overline{\theta}^\infty-\varepsilon)}{2\varepsilon}$ to change variables in the integration, equation (A-6) can be written as:

$$\frac{2\varepsilon \int_0^1 v(\overline{\theta}^{\infty} + 2\varepsilon\alpha - \varepsilon, \alpha) f(\overline{\theta}^{\infty} + 2\varepsilon\alpha - \varepsilon) d\alpha}{F\left(\overline{\theta}^{\infty} + \varepsilon\right) - F\left(\overline{\theta}^{\infty} - \varepsilon\right)} \tag{A-7}$$

$$= \frac{\int_0^1 v(\overline{\theta}^{\infty} + 2\varepsilon\alpha - \varepsilon, \alpha) f(\overline{\theta}^{\infty} + 2\varepsilon\alpha - \varepsilon) d\alpha}{\frac{F(\overline{\theta}^{\infty} + \varepsilon) - F(\overline{\theta}^{\infty} - \varepsilon)}{2\varepsilon}} = 0.$$

At the limit as $\varepsilon \longrightarrow 0$, this equation becomes $\int_0^1 v(\overline{\theta}^{\infty}, \alpha) d\alpha = 0$ (by L'Hôpital's rule, the denominator approaches $f\left(\overline{\theta}^{\infty}\right)$ as $\varepsilon \longrightarrow 0$). Similarly, note that $\underline{\theta}^{\infty}$ is defined by the equation $h(\underline{\theta}^{\infty}, \alpha(x, \underline{\theta}^{\infty})) = 0$, which at the limit as $\varepsilon \longrightarrow 0$, can be written as $\int_0^1 v(\underline{\theta}^{\infty}, \alpha) d\alpha = 0$. Now, assume by way of negation that $\overline{\theta}^{\infty} > \underline{\theta}^{\infty}$. Since $v(x, \alpha)$ is weakly increasing in x and strictly increasing in x when $v(x, \alpha) \geq 0$, it follows that $\int_0^1 v(\overline{\theta}^{\infty}, \alpha) d\alpha > \int_0^1 v(\underline{\theta}^{\infty}, \alpha) d\alpha$ (the strict inequality follows because the equation $\int_0^1 v(\overline{\theta}^{\infty}, \alpha) d\alpha = 0$ implies that $v(\overline{\theta}^{\infty}, \alpha) > 0$ for at least some values of α). This inequality contradicts the fact that $\int_0^1 v(\overline{\theta}^{\infty}, \alpha) d\alpha = 0$ and $\int_0^1 v(\underline{\theta}^{\infty}, \alpha) d\alpha = 0$.

Using the notation $\overline{\theta}^* \equiv \overline{\theta}^{\infty} = \underline{\theta}^{\infty}$, we proved that at the limit as $\varepsilon \longrightarrow 0$, there exists a unique threshold signal, $\overline{\theta}^*$, such that all speculators will attack the upper bound of the band if and only if they observe signals above $\overline{\theta}^*$.

(ii) We now characterize $\overline{\theta}^*$. The characterization of $\underline{\theta}^*$ is then completely analogous. Suppose that the exchange rate is at the upper bound of the band and suppose that absent intervention, the rate of change in the exchange rate is $x < \overline{\theta}^* - \varepsilon$. Recalling that the signals that speculators observe are drawn from the interval $[x - \varepsilon, x + \varepsilon]$, it is clear that the highest signal that a speculator can observe in this case is less than $\overline{\theta}^*$. Hence, no speculator will attack the band so $\alpha = 0$. On the other hand, if $x > \overline{\theta}^* + \varepsilon$, then the lowest signal that a speculator can observe is above $\overline{\theta}^*$. Hence, all speculators will attack the band and $\alpha = 1$. In intermediate cases where $\overline{\theta}^* - \varepsilon \leq x \leq \overline{\theta}^* + \varepsilon$, some speculators will observe signals above $\overline{\theta}^*$ and will attack the band while others will observe signals below $\overline{\theta}^*$ and will not attack the band. Given that ε_i is distributed uniformly on the interval $[-\varepsilon, \varepsilon]$, the density of speculators who observe signals above $\overline{\theta}^*$ and attack the band is $\frac{x - (\overline{\theta}^* - \varepsilon)}{2\varepsilon}$. In sum, given x and given $\overline{\theta}^*$, the fraction of speculators who choose to attack the upper bound of the band is given by:

$$\alpha(x, \overline{\theta}^*) = \begin{cases} 0, & x < \overline{\theta}^* - \varepsilon, \\ \frac{x - (\overline{\theta}^* - \varepsilon)}{2\varepsilon}, & \overline{\theta}^* - \varepsilon \leq x \leq \overline{\theta}^* + \varepsilon, \\ 1, & x > \overline{\theta}^* + \varepsilon. \end{cases} \tag{A-8}$$

Given $\alpha(x, \overline{\theta}^*)$, the cost that the policymaker incurs when defending the upper bound of the band against a speculative attack is $C(x, \alpha(x, \overline{\theta}^*))$, where $C(x, \cdot)$ is given by equation (2.1). Since the policymaker incurs a future credibility loss, δ, if a realignment takes place, it follows that his optimal policy is to defend the band so long as $C(x, \alpha(x, \overline{\theta}^*)) \leq \delta$, and exit it if $C(x, \alpha(x, \overline{\theta}^*)) > \delta$. Since $\alpha(x, \overline{\theta}^*)$ is weakly increasing in x, a realignment will occur if and only if x is above some threshold level $\overline{x}(\overline{\theta}^*)$. Note that in a perfect Bayesian equilibrium, $\overline{x}(\overline{\theta}^*)$ cannot be below $\overline{\theta}^* - \varepsilon$, because then, the number of speculators that attack the band at $x = \overline{x}(\overline{\theta}^*)$ is zero, and the policymaker should strictly prefer not to exit the band (unless $\overline{x}(\overline{\theta}^*) \geq \overline{\pi} + \delta$, but this means that $\overline{\theta}^*$ is above $\overline{\pi} + \delta + \varepsilon$, which is a contradiction to the fact that speculators have a dominant strategy to attack the band when they observe signals above $\overline{\pi} + \delta + \varepsilon$). Similarly, one can show that $\overline{x}(\overline{\theta}^*)$ cannot be above $\overline{\theta}^* + \varepsilon$. Then, using equations (3.1) and (A-8), we get that when the exchange rate reaches the upper bound of the band, a realignment takes place if and only if

$$x > \overline{x}(\overline{\theta}^*) \equiv \frac{\varepsilon(2\overline{\pi} + 2\delta - 1) + \overline{\theta}^*}{2\varepsilon + 1}. \tag{A-9}$$

Next, consider the decision problem that speculator i faces after observing the signal θ_i. Given that θ_i is drawn from the interval $[x - \varepsilon, x + \varepsilon]$, the speculator realizes that x is distributed on the interval $[\theta_i - \varepsilon, \theta_i + \varepsilon]$, and its conditional density is $f(x \mid \theta_i)$ as given by equation (2.4). But, since the speculator anticipates that the policymaker will defend the band whenever $x < \overline{x}(\overline{\theta}^*)$, it follows that he expects a net payoff of $(x - \overline{\pi})e_{-1} - t$ if $x > \overline{x}(\overline{\theta}^*)$ and $-t$ if $x < \overline{x}(\overline{\theta}^*)$. Part (i) of the lemma implies that, in equilibrium, speculators attack the band if and only if they observe signals above $\overline{\theta}^*$. A speculator that observes exactly $\overline{\theta}^*$ is indifferent between attacking the band and not attacking the band. Using this indifference condition, we get the following equation:

$$\int_{\overline{x}(\overline{\theta}^*)}^{\overline{\theta}^* + \varepsilon} (x - \overline{\pi})e_{-1} f(x \mid \overline{\theta}^*)dx - t = 0. \tag{A-10}$$

Substituting the expression for $f(x \mid \theta_i)$ from equation (2.4) into equation (A-10) and letting $\varepsilon \to 0$, we get:[21]

$$\lim_{\varepsilon \to 0} \frac{\int_{\overline{x}(\overline{\theta}^*)}^{\overline{\theta}^* + \varepsilon} (x - \overline{\pi})e_{-1} f(x)dx}{F\left(\overline{\theta}^* + \varepsilon\right) - F\left(\overline{\theta}^* - \varepsilon\right)} = t. \tag{A-11}$$

Substituting the expression for $\overline{x}(\overline{\theta}^*)$ from equation (A-9), using L'Hôpital's rule, and recalling from equation (A-9) that $\overline{x}(\overline{\theta}^*) \to \overline{\theta}^*$ as $\varepsilon \to 0$, we obtain:

$$(\overline{\theta}^* - \overline{\pi})\left[1 - \delta + (\overline{\theta}^* - \overline{\pi})\right] = \frac{t}{e_{-1}}. \tag{A-12}$$

[21] Note that when ε tends to zero both numerator and denominator in equation (3.5) go to zero.

Solving this equation for $\overline{\theta}^*$ yields the expression in the statement of the proposition.

(iii): Suppose that $\varepsilon \to 0$. Then, equation (A-9) shows that $\overline{x}(\overline{\theta}^*) \to \overline{\theta}^*$ and equation (A-5) shows that $\alpha = 0$ if $x \leq \overline{\theta}^*$ and $\alpha = 1$ if $x > \overline{\theta}^*$. Applying the same logic to the lower bound of the band, it follows that $\underline{x}(\overline{\theta}^*) \to \underline{\theta}^*$ and $\alpha = 0$ if $x \geq \underline{\theta}^*$ and $\alpha = 1$ if $x < \underline{\theta}^*$. **Q.E.D.**

Sufficient conditions for the policymakers' problem to be globally concave in $\underline{\pi} \leq x \leq \pi$:

Using equation (3.4), we get

$$\begin{aligned} \frac{\partial^2 V}{\partial \underline{\pi}^2} &= -\left[r(A-1)+\delta\right] f'(\underline{\pi}-r) + (A-1)\left[f(\underline{\pi}) - f(\underline{\pi}-r)\right] \qquad \text{(A-13)} \\ &= -\delta f'(\underline{\pi}-r) - r(A-1)\left[f'(\underline{\pi}-r) - \frac{f(\underline{\pi}) - f(\underline{\pi}-r)}{r}\right]. \end{aligned}$$

By Assumption 1, the first term on the second line of (A-4) is negative. Now, if $A > 1$ and $f"(\cdot) \leq 0$, the second term on the second line of (A-4) is nonpositive so $\frac{\partial^2 V}{\partial \underline{\pi}^2} < 0$, implying that V is concave in $\underline{\pi}$.

Likewise, using equation (3.5), we get

$$\begin{aligned} \frac{\partial^2 V}{\partial \overline{\pi}^2} &= \left[r(A-1)+\delta\right] f'(\overline{\pi}+r) - (A-1)\left[f(\overline{\pi}+r) - f(\overline{\pi})\right] \qquad \text{(A-14)} \\ &= \delta f'(\overline{\pi}+r) + r(A-1)\left[f'(\overline{\pi}+r) - \frac{f(\overline{\pi}+r) - f(\overline{\pi})}{r}\right]. \end{aligned}$$

By Assumption 1, the first term on the second line of (A-14) is negative. Now, if $A > 1$ and $f"(\cdot) \leq 0$, the second term on the second line of (A-14) is nonpositive so $\frac{\partial^2 V}{\partial \overline{\pi}^2} < 0$, implying that V is concave in $\overline{\pi}$. **Q.E.D.**

Proof of Proposition 2 : (i) By Assumption 1, $f(x) > f(\underline{\pi} - r)$ for all $x \in [\underline{\pi} - r, \underline{\pi}]$. Hence, if $A \leq 1$, $\frac{\partial V}{\partial \underline{\pi}} < 0$ for all $\underline{\pi} < 0$, implying that the policymaker will push $\underline{\pi}$ all the way to $-\infty$. Likewise, by Assumption 1, $f(x) > f(\overline{\pi} + r)$ for all $x \in [\overline{\pi}, \overline{\pi} + r]$; if $A \leq 1$, then $\frac{\partial V}{\partial \overline{\pi}} > 0$ for all $\overline{\pi} > 0$, implying that the policymaker will push $\overline{\pi}$ all the way to ∞.

(ii) To establish that $\underline{\pi} < 0$, it is sufficient to show that evaluated at $\underline{\pi} = 0$, $\frac{\partial V}{\partial \underline{\pi}} < 0$ (the policymaker will not push $\underline{\pi}$ all the way up to 0). Using equation (3.4) we obtain that

$$\begin{aligned} \left.\frac{\partial V}{\partial \underline{\pi}}\right|_{\underline{\pi}=0} &= -\delta f(-r) + (A-1)\int_{-r}^{0} \left[f(x) - f(-r)\right] dx \qquad \text{(A-15)} \\ &= -\int_{-r}^{0} \left[f(x) - f(-r)\right] dx \left[\frac{\delta}{\int_{-r}^{0}\left[\frac{f(x)}{f(-r)} - 1\right] dx} + 1 - A\right]. \end{aligned}$$

By Assumption 1, the integral term outside the square brackets on the second line of equation (A-15) is negative. If $A < \underline{A}(-r)$ then the term in square brackets is positive, so it is optimal to set $\underline{\pi} < 0$. To show that $\overline{\pi} > 0$, it is sufficient to show that evaluated at $\overline{\pi} = 0$, $\frac{\partial V}{\partial \overline{\pi}} > 0$ (the policymaker will increases $\overline{\pi}$ above 0). Using equation (3.5) we obtain that,

$$\left.\frac{\partial V}{\partial \overline{\pi}}\right|_{\overline{\pi}=0} = \delta f(r) - (A-1)\int_0^r [f(r) - f(x)]\,dx \tag{A-16}$$
$$= \int_0^r [f(r) - f(x)]\,dx \left[\frac{\delta}{\int_0^r \left[\frac{f(x)}{f(r)} - 1\right] dx} + 1 - A\right].$$

By Assumption 1, the integral term outside the square brackets on the second line of equation (A-16) is positive. If $A < \overline{A}(r)$ then the square bracketed term is positive. Hence, it is optimal to set $\overline{\pi} > 0$.

(iii) If V is concave in $\underline{\pi}$, then a sufficient condition for $\underline{\pi} = 0$ is that,evaluated at $\underline{\pi} = 0$, $\frac{\partial V}{\partial \underline{\pi}} \geq 0$ (the policymaker would like to push $\underline{\pi}$ all the way up to 0). From part (ii) of the proposition it is obvious that this occurs when $A > \underline{A}(-r)$. Likewise, if V is concave in $\overline{\pi}$, then a sufficient condition for $\overline{\pi} = 0$ is that, evaluated at $\overline{\pi} = 0$, $\frac{\partial V}{\partial \overline{\pi}} \leq 0$ (the policymaker would not like to increase $\overline{\pi}$ above 0). From part (ii) of the proposition it is obvious that this is the case when $A > \overline{A}(r)$.

(iv) If $f(x)$ is symmetric around 0, then for $0 < a < b$,

$$f(x) = f(-x), \quad \textit{and} \quad \int_{-b}^{-a} f(x)dx = \int_a^b f(x)dx. \tag{A-17}$$

Using these properties, equation (3.4) can be written as:

$$\delta f(-\underline{\pi} + r) - (A-1)\int_{-\underline{\pi}}^{-\underline{\pi}+r} [f(x) - f(-\underline{\pi} + r)]\,dx = 0. \tag{A-18}$$

Replacing $\underline{\pi}$ with $-\bar{\pi}$ in the last expression we obtain equation (3.14), implying that $\underline{\pi} = -\bar{\pi}$. **Q.E.D**.

Proof of Proposition 3: **(i)** The proof follows by straightforward differentiation of equations (3.4) and (3.5) and by using Assumption 1.

(ii) Recall from Lemma 1 that the probability of a speculative attack is $P = F(\underline{\pi} - r) + (1 - F(\overline{\pi} + r))$. Straightforward differentiation of this expression along with part (i) of the proposition establish the result. **Q.E.D**.

Proof of Proposition 4: Let $\underline{\pi}^f$ and $\bar{\pi}^f$ be the solutions to the policymaker's maximization problem when the density function is $f(x)$ and let $\underline{\pi}^g$ and $\bar{\pi}^g$ be the corresponding solutions when the density function is $g(x)$. $\underline{\pi}^g$ is defined by equation (3.4) with $g(x)$ replacing $f(x)$. Now, let's evaluate $\frac{\partial V}{\partial \underline{\pi}}$ when the density is $g(x)$ at $\underline{\pi}^f$:

$$\begin{aligned}
\left.\frac{\partial V}{\partial \underline{\pi}}\right|_{\underline{\pi}=\underline{\pi}^f} &= -\delta g(\underline{\pi}^f - r) + (A-1)\int_{\underline{\pi}^f - r}^{\underline{\pi}^f}\Big[g(x) - g(\underline{\pi}^f - r)\Big]\,dx \\
&= -\delta f(\underline{\pi}^f - r) + (A-1)\int_{\underline{\pi}^f - r}^{\underline{\pi}^f}\Big[g(x) - f(\underline{\pi}^f - r)\Big]\,dx \qquad \text{(A-19)} \\
&< -\delta f(\underline{\pi}^f - r) + (A-1)\int_{\underline{\pi}^f - r}^{\underline{\pi}^f}\Big[f(x) - f(\underline{\pi}^f - r)\Big]\,dx = 0,
\end{aligned}$$

where the first equality follows because by assumption, $g(\underline{\pi}^f - r) = f(\underline{\pi}^f - r)$. The inequality follows because $f(x)$ lies above $g(x)$ whenever $x > \underline{\pi}^f - r$, and the second equality follows from equation (3.4). Since $\left.\frac{\partial V}{\partial \underline{\pi}}\right|_{\underline{\pi}=\underline{\pi}^f} < 0$, it follows that $\underline{\pi}^f > \underline{\pi}^g$. The proof that $\bar{\pi}^f < \bar{\pi}^g$ is analogous. Hence, $\underline{\pi}^g < \underline{\pi}^f < \bar{\pi}^f < \bar{\pi}^g$, so the band becomes wider under $g(x)$. **Q.E.D.**

Proof of Proposition 5: **(i)** $\underline{A}(-r)$ and $\overline{A}(r)$ are given respectively by equations (3.6) and (3.7). Differentiating these expressions with respect to t yields:

$$\frac{\partial \underline{A}(-r)}{\partial t} = -\frac{\delta f'(-r)\int_{-r}^{0}\frac{f(x)}{f(-r)^2}dx}{\left(\int_{-r}^{0}\left[\frac{f(x)}{f(-r)} - 1\right]dx\right)^2}\frac{\partial r}{\partial t} < 0,$$

and

$$\frac{\partial \overline{A}(r)}{\partial t} = \frac{\delta f'(r)\int_{0}^{r}\frac{f(x)}{f(r)^2}dx}{\left(\int_{0}^{r}\left[1 - \frac{f(x)}{f(r)}\right]dx\right)^2}\frac{\partial r}{\partial t} < 0.$$

The signs of those expressions follow from the fact that, by Proposition 1, $\frac{\partial r}{\partial t} > 0$ and since by Assumption 1, $f'(r) < 0$ and $f'(-r) > 0$. Part (iii) of Proposition 2 implies that the policymaker prefers to set a peg when $A > Max\{\underline{A}(r), \overline{A}(r)\}$. Since both $\underline{A}(-r)$ and $\overline{A}(r)$ fall with t, this condition is more likely to hold when t is larger.

(ii) Proposition 1 implies that r decreases when t decreases. Straightforward differentiation of equations (3.4) and (3.5) and use of Assumption 1 show that $\underline{\pi}$ decreases and $\bar{\pi}$ increases when r decreases. Hence a decrease in t leads to a decrease in $\underline{\pi}$ and to an increase in $\bar{\pi}$.

Second, recall from Lemma 1 that the probability of a speculative attack is $P = F(\underline{\pi} - r) + (1 - F(\overline{\pi} + r))$. Differentiating this expression with respect to t yields:

$$\begin{aligned}
\frac{\partial P}{\partial t} &= f(\underline{\pi} - r)\left[\frac{\partial \underline{\pi}}{\partial r}\frac{\partial r}{\partial t} - \frac{\partial r}{\partial t}\right] - f(\overline{\pi} + r)\left[\frac{\partial \overline{\pi}}{\partial r}\frac{\partial r}{\partial t} + \frac{\partial r}{\partial t}\right] \\
&= \left[f(\underline{\pi} - r)\left[\frac{\partial \underline{\pi}}{\partial r} - 1\right] - f(\overline{\pi} + r)\left[\frac{\partial \overline{\pi}}{\partial r} + 1\right]\right]\frac{\partial r}{\partial t}.
\end{aligned}$$

By Proposition 1, $\frac{\partial r}{\partial t} > 0$. Hence it is sufficient to establish that $\frac{\partial \underline{\pi}}{\partial r} > 1$ and $\frac{\partial \overline{\pi}}{\partial r} < -1$. Using equation (3.4), it follows that

$$\frac{\partial \underline{\pi}}{\partial r} = \frac{\frac{\partial^2 V}{\partial r \partial \underline{\pi}}}{\frac{\partial^2 V}{\partial \underline{\pi}^2}} = \frac{-\left(\left(A-1\right)r+\delta\right)f'(\underline{\pi}-r)}{-\left[r(A-1)+\delta\right]f'(\underline{\pi}-r)+(A-1)\left[f(\underline{\pi})-f(\underline{\pi}-r)\right]}, \tag{A-20}$$

which exceeds 1 because of Assumption 1 and because $A > 1$. Likewise, using equation (3.5), it follows that,

$$\frac{\partial \overline{\pi}}{\partial r} = \frac{\frac{\partial^2 V}{\partial r \partial \overline{\pi}}}{\frac{\partial^2 V}{\partial \overline{\pi}^2}} = \frac{-\left(\left(A-1\right)r+\delta\right)f'\left(\overline{\pi}+r\right)}{-\left(\left(A-1\right)r+\delta\right)f'\left(\overline{\pi}+r\right)\,+\,\left(A-1\right)\left(f\left(\overline{\pi}+r\right)-f\left(\overline{\pi}\right)\right)}, \tag{A-21}$$

which is less than -1 due to Assumption 1 and because $A > 1$. **Q.E.D**

Proof of proposition 6: Using equation (3.3) and the envelope theorem it follows that:

$$\frac{\partial V}{\partial t} = \left(r\left(A-1\right)+\delta\right)\left[f(\underline{\pi}-r)+f(\overline{\pi}+r)\right]\frac{\partial r}{\partial t} > 0, \tag{A-22}$$

where the inequality follows from the restriction $A > 1$ and because by Proposition 1, $\frac{\partial r}{\partial t} > 0$. **Q.E.D**

Proof of Lemma 2: (i) Suppose the exchange rate reaches the upper bound of the band. Given x, a dependable policymaker will defend the upper bound of the band if and only if $\alpha < \alpha^*(x)$, where $\alpha^*(x)$ is given by equation (A-1). Hence, the net payoff from attacking the upper bound of the band is:

$$v^{\beta}(x,\alpha) = \begin{cases} \left(x-\overline{\pi}\right)e_{-1}-t, & if \quad \alpha \geq \alpha^*\left(x\right), \\ \left(1-\beta\right)\left(x-\overline{\pi}\right)e_{-1}-t, & if \quad \alpha < \alpha^*\left(x\right). \end{cases}$$

Since $v^{\beta}(x,\alpha)$ has the same properties as $v(x,\alpha)$ in equation (A-2), the equilibrium analysis here is exactly as in the proof of part (i) of Lemma 1. Hence, once again we have a unique equilibrium in which speculators attack the upper bound of the band if and only if they observe a signal above a unique threshold, $\overline{\theta}^*_{\beta}$.

We turn next to a characterization of the behavior of the dependable policymaker in equilibrium. Since in equilibrium, $\alpha(x)$ is increasing in x ($\alpha(x)$ is given by (A-8), where $\overline{\theta}^*_{\beta}$ replaces $\overline{\theta}^*$), and since $C(x,\alpha(x))$ is increasing in both x and $\alpha(x)$, then the dependable policymaker will exit the band if and only if x is above some threshold level, $\overline{x}_{\beta}(\overline{\theta}^*_{\beta})$.

In order to establish that $\overline{x}_{\beta}(\overline{\theta}^*_{\beta}) \to \overline{\theta}^*_{\beta}$ as $\varepsilon \to 0$, we now show that $\overline{x}_{\beta}(\overline{\theta}^*_{\beta})$ must be in the interval $\left[\overline{\theta}^*_{\beta}-\varepsilon, \overline{\theta}^*_{\beta}+\varepsilon\right]$. In order to see this, suppose by way of negation that $\overline{x}_{\beta}(\overline{\theta}^*_{\beta}) > \overline{\theta}^*_{\beta}+\varepsilon$.

Then, speculators who observe $\overline{\theta}_\beta^*$ know that a dependable policymaker will defend the band. Thus, the payoff they expect to get from attacking the band is lower than $(1-\beta)\left(\overline{\theta}_\beta^* + \varepsilon - \overline{\pi}\right) e_{-1} - t$. By equilibrium conditions and continuity, we know that speculators who observe $\overline{\theta}_\beta^*$ must be indifferent between attacking the band and not attacking it. This means that $(1-\beta)\left(\overline{\theta}_\beta^* + \varepsilon - \overline{\pi}\right) e_{-1} - t > 0$. However, using Assumption 3, this condition will hold only if $\overline{\theta}_\beta^* + \varepsilon > \overline{\pi} + \delta$. Since by assumption, $\overline{x}_\beta(\overline{\theta}_\beta^*) > \overline{\theta}_\beta^* + \varepsilon$, this implies in turn that $\overline{x}_\beta(\overline{\theta}_\beta^*) > \overline{\pi} + \delta$, thereby contradicting the fact that a dependable policymaker always exits the band when $x > \overline{\pi} + \delta$. Thus, $\overline{x}_\beta(\overline{\theta}_\beta^*)$ cannot be above $\overline{\theta}_\beta^* + \varepsilon$. Next, suppose by way of negation that $\overline{x}_\beta(\overline{\theta}_\beta^*) < \overline{\theta}_\beta^* - \varepsilon$. Then, at $\overline{x}_\beta(\overline{\theta}_\beta^*)$, a dependable policymaker knows that no speculator attacks the band. By equilibrium conditions and continuity, at $\overline{x}_\beta(\overline{\theta}_\beta^*)$, a dependable policymaker must be indifferent between existing the band and maintaining it, that is, $\overline{x}_\beta(\overline{\theta}_\beta^*) = \overline{\pi} + \delta$. Since by assumption, $\overline{x}_\beta(\overline{\theta}_\beta^*) < \overline{\theta}_\beta^* - \varepsilon$, this means that $\overline{\theta}_\beta^* > \overline{\pi} + \delta + \varepsilon$, which contradicts the fact that speculators have a dominant strategy to attack the band when they observe signals above $\overline{\pi} + \delta + \varepsilon$. Thus, $\overline{x}_\beta(\overline{\theta}_\beta^*)$ cannot be below $\overline{\theta}_\beta^* - \varepsilon$

Given the fact that $\overline{x}_\beta(\overline{\theta}_\beta^*)$ is in the interval $\left[\overline{\theta}_\beta^* - \varepsilon, \overline{\theta}_\beta^* + \varepsilon\right]$ and using Equation (A-8), it follows that:

$$\overline{x}_\beta(\overline{\theta}_\beta^*) = \frac{\varepsilon(2\overline{\pi} + 2\delta - 1) + \overline{\theta}_\beta^*}{2\varepsilon + 1}.$$

Since a speculator that observes $\overline{\theta}_\beta^*$ is indifferent between attacking the band and not attacking it, the equation that defines $\overline{\theta}_\beta^*$ is given by:

$$\beta \int_{\overline{x}_\beta(\overline{\theta}_\beta^*)}^{\overline{\theta}_\beta^* + \varepsilon} (x - \overline{\pi}) e_{-1} f(x \mid \overline{\theta}_\beta^*) dx + (1-\beta) \int_{\overline{\theta}_\beta^* - \varepsilon}^{\overline{\theta}_\beta^* + \varepsilon} (x - \overline{\pi}) e_{-1} f(x \mid \overline{\theta}_\beta^*) dx = t, \tag{A-23}$$

where $f(x \mid \overline{\theta}_\beta^*)$ is defined by equation (2.4). This equation coincides with equation (A-8) if $\beta = 1$. Substituting from equation (2.4) for $f(x \mid \overline{\theta}_\beta^*)$ into (A-23), and taking the limit as $\varepsilon \to 0$, yields:

$$\lim_{\varepsilon \to 0} \frac{\beta \int_{\overline{x}_\beta(\overline{\theta}_\beta^*)}^{\overline{\theta}_\beta^* + \varepsilon} (x - \overline{\pi}) e_{-1} f(x) dx + (1-\beta) \int_{\overline{\theta}_\beta^* - \varepsilon}^{\overline{\theta}_\beta^* + \varepsilon} (x - \overline{\pi}) e_{-1} f(x) dx}{F\left(\overline{\theta}_\beta^* + \varepsilon) - F(\overline{\theta}_\beta^* - \varepsilon\right)} = t. \tag{A-24}$$

Using L'Hôpital's rule, and the expression for $\overline{x}_\beta(\overline{\theta}_\beta^*)$, and recalling that $\overline{x}_\beta(\overline{\theta}_\beta^*) \to \overline{\theta}_\beta^*$ as $\varepsilon \to 0$, we obtain:

$$(\overline{\theta}_\beta^* - \overline{\pi})(1 - \beta\delta + \beta(\overline{\theta}_\beta^* - \overline{\pi})) e_{-1} = t. \tag{A-25}$$

Solving this equation for $\overline{\theta}_\beta^*$ reveals that $\overline{\theta}_\beta^* = \overline{\pi} + r^\beta$, where r^β is defined by equation (4.1). Using similar arguments, it follows that $\underline{\theta}_\beta^* = \underline{\pi} - r^\beta$.

(ii) Differentiating r^β with respect to β and we obtain:

$$\frac{\partial r^\beta}{\partial \beta} = \frac{1}{2\beta^2}\left[1 - \frac{\frac{t}{e_{-1}} - \frac{\delta - \frac{1}{\beta}}{2}}{\sqrt{\frac{t}{\beta e_{-1}} + \frac{(\delta - \frac{1}{\beta})^2}{4}}}\right]. \tag{A-26}$$

This derivative is positive if and only if the expression inside the brackets is positive. This is the case, in turn, if and only if

$$\frac{t}{\beta e_{-1}} + \frac{(\delta - \frac{1}{\beta})^2}{4} > \left[\frac{t}{e_{-1}} - \frac{\delta - \frac{1}{\beta}}{2}\right]^2. \tag{A-27}$$

Further rearrangement of the last inequality shows that it is equivalent to Assumption 2. Hence, r^β increases with β. **Q.E.D**.

Proof of Proposition 7: The first order conditions for an interior solution for the problem of a dependable policymaker are:

$$\begin{aligned}\frac{\partial V^\beta}{\partial \underline{\pi}} &= -\left[r^\beta(A\beta - 1) + \delta\right] f(\underline{\pi} - r^\beta) + (A\beta - 1)\int_{\underline{\pi}-r^\beta}^{\underline{\pi}} f(x)dx \qquad \text{(A-28)}\\ &= A\beta \int_{\underline{\pi}-r^\beta}^{\underline{\pi}} \left[f(x) - f(\underline{\pi} - r^\beta)\right] dx - \left[\int_{\underline{\pi}-r^\beta}^{\underline{\pi}} \left[f(x) - f(\underline{\pi} - r^\beta)\right] dx + \delta f(\underline{\pi} - r^\beta)\right] = 0,\end{aligned}$$

and,

$$\begin{aligned}\frac{\partial V^\beta}{\partial \overline{\pi}} &= \left[r^\beta(A\beta - 1) + \delta\right] f(\overline{\pi} + r^\beta) - (A\beta - 1)\int_{\overline{\pi}}^{\overline{\pi}+r^\beta} f(x)dx \qquad \text{(A-29)}\\ &= -A\beta \int_{\overline{\pi}}^{\overline{\pi}+r^\beta} \left[f(x) - f(\overline{\pi} + r^\beta)\right] dx + \left[\int_{\overline{\pi}}^{\overline{\pi}+r^\beta} \left[f(x) - f(\overline{\pi} + r^\beta)\right] dx + \delta f(\overline{\pi} + r^\beta)\right] = 0.\end{aligned}$$

As in the case where $\beta < 1$, it can be shown that $f''(x) \leq 0$ and $A\beta > 1$, along with Assumption 1, are sufficient for V^β to be globally concave in $\underline{\pi}$ and $\overline{\pi}$ so equations (A-28) and (A-29) are sufficient for a unique maximum.

(i)-(iii) The proofs follow the corresponding proofs in Proposition 2 with r replaced by r^β.

(iv) Differentiating $\frac{\partial V^\beta}{\partial \overline{\pi}}$ with respect to β and $\overline{\pi}$, and using the implicit function theorem,yields:

$$\frac{\partial \overline{\pi}}{\partial \beta} = -\frac{A\int_{\overline{\pi}}^{\overline{\pi}+r^\beta}\left[f(\overline{\pi} + r^\beta) - f(x)\right] dx + \left[r^\beta(A\beta - 1) + \delta\right] f'(\overline{\pi} + r^\beta)\frac{\partial r^\beta}{\partial \beta}}{-(A\beta - 1)\left[f(\overline{\pi} + r^\beta) - f(\overline{\pi})\right] + \left[r^\beta(A\beta - 1) + \delta\right] f'(\overline{\pi} + r^\beta)}. \tag{A-30}$$

Assumption 1 ensures that the integral term in the numerator is negative and it also ensures that $f'(\overline{\pi} + r^\beta) < 0$. Since by Lemma 2, $\frac{\partial r^\beta}{\partial \beta} > 0$, it follows that the numerator is negative. By the second order conditions for maximization, the denominator is negative so $\overline{\pi}$ decreases towards 0. Similarly, it can be shown that as β increases, $\underline{\pi}$ increases towards 0. Hence, an increase in β leads to a tighter band.

Finally, the probability of a speculative attack is now $P^{\beta} = F(\underline{\pi} - r^{\beta}) + (1 - F(\overline{\pi} + r^{\beta}))$. Differentiating this expression with respect to β yields:

$$\frac{\partial P^{\beta}}{\partial \beta} = f(\underline{\pi} - r)\left[\frac{\partial \underline{\pi}}{\partial \beta} - \frac{\partial r^{\beta}}{\partial \beta}\right] - f(\overline{\pi} + r)\left[\frac{\partial \overline{\pi}}{\partial \beta} + \frac{\partial r^{\beta}}{\partial \beta}\right].$$

To determine the sign of this expression, note that using (A-30) we obtain

$$\frac{\partial \overline{\pi}}{\partial \beta} + \frac{\partial r^{\beta}}{\partial \beta} = -\frac{A\int_{\overline{\pi}}^{\overline{\pi}+r^{\beta}}\left[f(\overline{\pi}+r^{\beta}) - f(x)\right]dx + (A\beta - 1)\left[f(\overline{\pi}+r^{\beta}) - f(\overline{\pi})\right]\frac{\partial r^{\beta}}{\partial \beta}}{-(A\beta - 1)\left[f(\overline{\pi}+r^{\beta}) - f(\overline{\pi})\right] + \left[r^{\beta}(A\beta - 1) + \delta\right]f'(\overline{\pi}+r^{\beta})}. \tag{A-31}$$

Assumption 1 ensures that the integral term in the numerator as well as $f(\overline{\pi} + r^{\beta}) - f(\overline{\pi})$ are both negative. Since by assumption, $A\beta > 1$ and since by Lemma 2, $\frac{\partial r^{\beta}}{\partial \beta} > 0$, it follows that the numerator is negative. The denominator is also negative by the second order conditions for maximization. Hence, $\frac{\partial \overline{\pi}}{\partial \beta} + \frac{\partial r^{\beta}}{\partial \beta} < 0$. Similar calculations establish that $\frac{\partial \underline{\pi}}{\partial \beta} - \frac{\partial r^{\beta}}{\partial \beta} > 0$. Hence, $\frac{\partial P^{\beta}}{\partial \beta} > 0$, implying that as the policymaker's reputation improves, there is a greater likelihood of speculative attacks. **Q.E.D.**

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