A Coasian Model of International Production Chains

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Abstract

International supply chains require coordination of numerous activities across multiple countries and firms. We develop a theoretical model of supply chains in which the measure of tasks completed within a firm is determined by parameters that define transaction costs and the cost of coordinating more activities within the firm. The structural parameters that govern these costs explain variation in supply chain length as well as cross-country variation in gross-output-to-value-added ratios. The structural parameters are linked to comparative advantage along and across supply chains. We provide an analytical treatment of trade and welfare responses to trade cost change in a simple two-country model. To explore the model's implications in a richer setting we calibrate the model to match key observables in East Asia, and evaluate implications of changes in model parameters for trade, welfare, the length of supply chains and countries' relative position within them.

Keywords: Fragmentation of production, Transaction costs, Trade in intermediate goods, Boundary of the firm
JEL Classification: F10, L23

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1 Introduction

The nature of international trade changed dramatically in recent decades, as vertically integrated production processes spread across international borders, increasing trade in parts and components along the way.[1] This phenomenon raises a number of important questions for economic policy: How large are the gains from international fragmentation, and how are they distributed across countries? How do changes in trade costs affect trade flows and the distribution of value added across countries? How has China's entry into the world trading system affected the international fragmentation of production? Answers to these questions require quantitative models that can represent the complexities of international production chains in a tractable form.

Recent evidence has documented substantial variation in supply chain length.[2] Even within chains, firms vary in their contribution to value added.[3] The length of supply chains and the degree to which they are internationalized are difficult to separate from decisions that determine firm scope. The Ford Model T, for example, was produced in a single plant, while the production of modern day automobiles can involve a myriad of heterogeneous suppliers scattered across multiple countries. International fragmentation is limited, ultimately, by the extent of fragmentation at the firm level. Yet the literature lacks a unified treatment that can explain endogenous firm boundaries within chains, formalize endogenous chain lengths, and determine comparative advantage within and across chains.

We offer a framework that accomplishes these goals. In the model, an optimal allocation of tasks determines jointly the scope of sequentially-arranged firms of varying size, the length of chains and the sequence of countries in production.[4] We calibrate the model using key moments from input-output tables on East Asia and the United States.[5] Our focus on East Asia reflects the importance of international fragmentation in that region. Based on our calibration, we are able to quantify the impact on intermediate and final goods trade, fragmentation and welfare, of changes in: 1) international trade costs, 2) productivity in China, 3) transaction costs in China, and 4) a reduction in bilateral trade costs between the US and China.

[1]Baldwin (2012) surveys these developments and provides insights into how they should affect our thinking about the economics of international trade and trade policy.

[2]See Antràs et al. (2012) and Fally (2012).

[3]See Kraemer et al. (2011), who illustrate the distribution of value added for Apple iPhones and iPads.

[4]Our framework is perfectly competitive and therefore silent about the distinction between firms and plants. We shall use the term 'firms' throughout, but one could reinterpret everything in terms of 'plants' without affecting the results.

[5]Such tables have been used to quantify the extent of fragmentation and the allocation of value across countries. Johnson (2014) surveys a series of papers that use such tables to calculate value added trade. Koopman et al. (2010) use Chinese tables to calculate the domestic content of China's exports. Antras et al. (2012) derive indices of supply chain length from input-output tables.

A central theoretical contribution is the development of a tractable framework in which supply chain length is endogenous. Supply chains vary in the number of firms involved because of endogenous differences in firm scope. Following Coase (1937), firm scope is determined by a marginal tradeoff between the cost of coordinating activities inside the firm and the costs of conducting market transactions. Going further, we formalize the marginal tradeoffs facing a chain of firms in a shadow market for tasks rather than focusing on the boundaries of a single firm.[6] Within countries, outcomes are driven by two key parameters: one that governs coordination costs within the firm and one that summarizes domestic transaction costs. Our continuous representation of a firm allows us to derive strong and transparent links between structural parameters and observables in the data. Specifically, the gross-output-to-value-added ratio at any point in the chain is equal to the ratio of the Coasian parameters that govern coordination costs and transaction costs, respectively.

We allow the Coasian parameters to vary across countries and develop implications for international trade. Vertical specialization in our model is tightly related to firm scope. In equilibrium, firm scope decreases as we move upstream along the chain. In turn, this pattern affects the sorting of countries along the chain. Within a given chain, the most upstream countries are those in which it is most difficult to coordinate multiple tasks within the firm. Transaction costs affect absolute, not comparative, advantage within the chain, but transaction costs have an indirect effect on countries' average position in chains. Countries with high transaction costs are more likely to participate in chains for which the country has low coordination costs, which means that such countries tend to be positioned downstream.

We examine the effect of trade costs on trade and fragmentation in this setting. As expected, we find that opening to trade tends to increase the extent of fragmentation along several dimensions. The channels, however, are not trivial. Trade affects fragmentation at all stages and decreases firm scope even for firms that do not directly offshore production but are related to firms that do. The reduction in firm scope along the chain is associated with a decrease in average costs, especially downstream, and contributes to the reduction in final goods prices. We derive analytical results that reveal transparent links between the impact of fragmentation on firm scope, final goods prices and a shadow cost of tasks that governs firm scope along the chain.

As trade costs decrease, countries tend to move downstream along chains and to enter new chains. We illustrate our finding in a partial-equilibrium setting, holding the set of participating countries and their labor cost constant, and in a two-country general equilibrium setting. In

[6] Our framework builds on that of Kikuchi et al. (2014) who model a Coasian supply chain in a one-country partial equilibrium setting, assuming discrete firms. We develop a continuous firm treatment that facilitates tractable analytical solutions and calibration and extend it to a multi-country general equilibrium setting.

the latter, we also use our framework to examine the response of trade flows to trade costs, both in gross flows and value added content, and the welfare gains from trade. We compare our results to a single-stage Eaton and Kortum (2002) model: the gains from trade are relatively larger in countries that tend to specialize downstream and smaller in countries that tend to specialize upstream.

In order to explore the quantitative implications of our framework, we calibrate a numerical version of our model to match key features of input-output relationships in East Asia. This exercise relies on international input-output tables produced by IDE-JETRO. The data cover the US and nine East Asian countries. This region is interesting because production fragmentation there has grown quickly and is highly prevalent. The IDE-JETRO data are unique in that they track flows in four dimensions: from the making industry in the origin country to the using industry in the destination country.[7] To illustrate our findings, we adapt recently developed quantitative measures of firm position (i.e. upstreamness) so that they track the average number of international borders crossed, rather than plant boundaries as in the original. Our calculations indicate increasing international fragmentation over time, especially in key industries like electronics.

While the model has rich implications for trade and the fragmentation of production, its relative parsimony is useful for the purpose of calibration. We calibrate our model by targeting key moments such as GDP per capita, value added, countries' average position in international supply chains and gross-output-to-value-added ratios. All these moments imply large cross-country differences in productivity, transaction costs and coordination costs.

We use the calibrated model to conduct counterfactual exercises regarding changes in key structural parameters. We first examine what happens when cross-border trade costs decrease by 10%. In this counter-factual simulation, we find larger gains from trade than predicted by Arkolakis, Costinot and Rodrigues-Clare (2012)'s formula (based on imported final goods), especially in downstream countries. We also examine the response in terms of the VAX ratio (the value-added content of exports) as defined by Johnson and Noguera (2012). We find that a decrease in trade costs leads to a decrease in the VAX ratio for most countries, which can be interpreted as an increase in cross-border fragmentation. In subsequent counterfactual exercises, we simulate a 10% increase in productivity in China and a 10% decrease in Chinese transaction costs. Both the productivity and transaction cost shocks produce similar changes in Chinese and other countries' welfare, but the shocks have different implications for the organization of international production. The productivity shock causes China to move downstream while the rest of the world moves upstream. The reduction in Chinese transaction costs causes a relative

[7] Other data that report such figures, like the World Input-Output Database (WIOD), impute these values assuming proportional treatments.

move upstream by China. Reducing transaction costs in China also lengthens global supply chains. This is entirely consistent with the model's qualitative predictions, but the calibration teaches us that the shock to Chinese transaction costs has quantitative effects outside of China. Finally, we simulate a 10% reduction in trade costs between China and the US to investigate the degree to which fragmentation magnifies trade responses. At our benchmark level of trade costs there is little evidence of magnification, but it does appear when we calibrate to lower levels of trade costs.

Relationship to the literature: The paper contributes to the literature in two broad ways: 1) we develop a model that formalizes a role for firms (as distinct from tasks) in a sequential, multi-country general equilibrium with endogenous chain length, 2) we calibrate our model and provide quantitative implications using input-output tables for East-Asian production. We discuss the literature surrounding each of these contributions in turn.

1. Models of production chains. An important question in the literature on international production chains is the spatial organization of production across countries. We contribute to this literature in two ways by endogenizing the extent of fragmentation across firms and by endogenizing the relative position of countries along the chain.

In recent work, Costinot et al (2012) derive an explicitly sequential multi-country model in which mistakes can occur with given probability and these mistakes destroy all accumulated value. They show that countries with relatively high probabilities of mistakes are situated upstream. The intuition for this result broadly follows Kremer (1993), that higher rates of mistakes do less damage if they occur upstream. The Costinot et al (2012) framework has no implications for the extent of fragmentation across firms and the allocation of tasks across firms.

Instead, we formalize the firm's internalization decision and endogenize the range of firms involved in the chain. The motivation for this follows Coase (1937), and our mathematical framework is inspired by Kikuchi et al (2014), who show how Coase's insights can be applied to production chains. Kikuchi et al (2014) solve their model in a sequential partial equilibrium setting, and employ discrete firms. We adapt their framework to a continuum of firms in a multi-country setting where countries differ in key parameters governing transaction costs and diseconomies of scope.

As in Costinot et al (2012), we examine how countries specialize along the chain, but the patterns of specialization are now driven by interactions between firm scope, transaction costs and ad-valorem trade costs affecting cross-border transactions. In addition, we offer explicit links between the Coasian structural parameters in our model and empirical objects that can be observed or constructed from input-output tables. These links make calibration of the model relatively straightforward compared to other models in the literature.

Several models fix the number of production stages by assumption (Krugman and Venables 1996, Hillberry and Hummels 2002, Yi 2003, 2010, Johnson and Moxnes 2013). The focus of this literature is often the geographic location of each production stage, relative to the other(s), and so a finite and countable number of stages is useful for analytical purposes.[8] Relative to our work, these models avoid the question of the allocation of activities or tasks across stages, and focus on the extensive margin of completing a specific stage in a certain location.

These models are also silent about why some countries specialize upstream while others are downstream. In Yi (2010) and in Johnson and Moxnes (2013), for example, the specialization of countries along the chain is driven by exogenous productivity shocks and trade costs. This literature makes important insights about non-linear responses of trade to trade costs and differences between gross and VA trade. Our model also contains these forces, but we introduce intra-firm coordination costs and inter-firm transaction costs as additional sources of cross-country heterogeneity. One goal of our paper is to understand the robustness of these insights to the richer theoretical structure we offer, where the extent of fragmentation and the specialization of countries along the chain are endogenous.

2. Quantitative implications. We contribute to a recent quantitative literature on value chains by using new indexes to calibrate our model of cross-border fragmentation and examine the effect of trade costs on the organization of production chains, trade and welfare.

Our quantification exercise relies on input-output matrices that we exploit in a new way. Input-output matrices and direct requirement coefficients are traditionally taken as an exogenous recipe that is essentially determined by technology. Instead, we argue that input-output matrices reflect transactions in intermediate goods between firms that are themselves endogenous economic outcomes. We show that these tables can be informative about the position of firms within supply chains that link firms both within national borders and across them. Unlike previous papers, our theory determines the allocation of tasks across firms and the length of production chains endogenously, and can thus shed some light on equilibrium input-output relationships when fragmentation is endogenous, both across and within countries.

Under the assumption that IO tables effectively summarize plant-to-plant movements for a representative firm in each industry, matrix algebra can be used to calculate, for each industry in the table, two numerical values: i) a measure of the industry's "distance" from final demand (where distance is a count of the number of plant boundaries that will be crossed prior to final

[8] Antràs and Chor (2013) offer a different perspective by taking the location and length of production chains as exogenous but examining the optimal allocation of ownership along the chain.

consumption) and ii) the average number of stages embodied in an industry's production.[9] We also examine more traditional indexes of fragmentation such as gross-output-to-value-added ratios and the share of intermediate goods in trade. We show that, within our framework, we can map each of these indexes to structural parameters and key summary statistics of our model. These mappings are useful when we calibrate the model to data on interregional input-output relationships in East Asia.

A key purpose of this exercise is to offer a model comparison vis-a-vis other papers in the literature. A prominent literature has emphasized that intermediate goods trade magnifies the effect of trade costs on trade. Yi (2010) and Johnson and Moxnes (2013) focus on the response of trade to trade cost shocks, whereas Krugman and Venables (1996), Hillberry and Hummels (2002), Yi (2010) and Johnson and Noguera (2014) link the spatial clustering of activities to trade costs and intermediate goods trade.[10] Clustering also occurs in our model, with sequential activities locating so as to avoid trade costs. Our calibrated model can be used to investigate the response of trade to trade cost shocks, as in Johnson and Moxnes (2013) or Yi (2010).

We also contribute to the recent literature on the welfare implications of trade cost change. Arkolakis, Costinot and Rodriguez-Clare (2012) show that a broad class of models imply the same response of welfare to trade costs, provided that the models are calibrated to generate the same trade response to trade cost change. Costinot and Rodriguez-Clare (2014) and Melitz and Redding (2014) show that welfare effects are magnified when intermediate goods trade is involved. Like other papers in the literature, these presume an explicit input-output relationship that governs supply chain length, in contrast to the endogenous length in our model. The Armington framework used in these papers also precludes movement along the extensive margin (in terms of countries involved in supply chains), while our theory allows this. Our calibrated model implies larger gains than in standard trade models like those described by Arkolakis et al (2012), especially for countries that tend to be downstream, but smaller gains than Costinot and Rodriguez-Clare (2014) or Melitz and Redding (2014).

[9]The first is described as "distance to final demand" in Fally (2012) and "upstreamness" in Antras et al (2012). The second is developed in Fally (2012) and computed using the BEA input-output tables for the US. Using the highly disaggregated US table, Fally (2012) finds that the two indicators are not correlated, and that the indicators do not appear to be especially sensitive to aggregation concerns. Note also that Fally (2012) and Antras et al (2012) compute these indexes using input-output table for a single country. Here we extend these indicators to multi-rational input-output tables.

[10]More recently, Kee and Tang (2013), Bernard et al. (2014) and Antràs et al. (2014) have used firm-level data to examine both intensive and extensive margins in import decisions. Firm-level data, however, do not allow a full consideration of supply chains over several countries. Multi-country input-output tables are more suitable for exercises like ours.

2 Model setup

We develop a model where the production of each variety of final good requires a continuum of tasks and firms organized across countries. We describe, in turn, consumers' preferences in final goods, tasks and firms involved in the production of each good, the forces shaping firm scope and firm entry along the chain, differences between varieties and the labor market.

Preferences: Consumers have identical Cobb-Douglas preferences over varieties of final goods indexed by ω:

$$U = \int_{\omega} \log q^F(\omega) d\omega \tag{1}$$

where $q^F(\omega)$ denotes quantities of final goods. As in Eaton and Kortum (2002), all countries have access to the same set of product varieties ω but at different prices.

Tasks and firms along the chain: In order to produce the final good of variety ω, a range $[0,1]$ of tasks must be performed sequentially. These tasks may be performed across different firms and different countries.

Firms are arranged sequentially along the chain to produce each good ω. A chain is specific to each variety ω of the final good and the location of final producers. On each chain, we assume that there is a continuum of firms indexed by f. Firms may be located in different countries. For each chain, we rank countries along the chain and index by $i(n,\omega)$ the n^{th} country, $i(1,\omega)$ being the most downstream country and $i(N,\omega)$ being the most upstream country along the chain.

We denote by $F_n(\omega)$ the range of firms involved in the chain in the n^{th} country $i(n)$. An elementary firm df performs a range $s_{nf}(\omega)$ of tasks. Both the range of firms $F_n(\omega)$ and firm scope $s_{nf}(\omega)$ are endogenous, but the range of tasks performed across all firms must sum up to one to obtain a final good:

$$\sum_n \int_{f=0}^{F_n(\omega)} s_{nf}(\omega) df = 1 \tag{2}$$

Denoting $S_n(\omega) = \int_{f=0}^{F_n(\omega)} s_{nf}(\omega) df$ the total range of tasks to be performed in country n, the last constraint can be rewritten:

$$\sum_n S_n(\omega) = 1$$

for all chains ω.

Coordination costs: There are costs and benefits to fragmenting production across firms and countries. Fragmentation across firms reduces total costs because of diseconomies of scope. As firms need to manage employees across different tasks and perform tasks that are away from their core competencies, unit costs increase with the scope of the firm. We will refer to these costs as "coordination costs" that occur within the firm.

Formally, we assume that an elementary firm df in country i requires one unit of intermediate goods and $c_i(s,\omega)df$ units of labor which is a function of firm scope s. The cost of labor is w_i in country i and labor is the only production input besides intermediate goods. We assume that c_i is convex in firm scope s, thus generating gains from fragmentation across firms.

In particular, we specify the following labor requirements:

$$c_i(s,\omega) = a_i(\omega) \int_{t=0}^{s} t^{\theta_i(\omega)}\, dt$$

where t is the distance from the first task and $a_i(\omega)$ and $\theta_i(\omega)$ are specific parameters for each country i for variety ω.[11] The marginal cost of performing additional tasks within the firm increases with t. This follows recent work on the division of labor (Chaney and Ossa, 2013), and in this context represents the productivity loss associated with movement away from the firm's core competencies. $\theta_i(\omega)$ parameterizes "coordination costs" and governs the convexity of the cost function. The higher is $\theta_i(\omega)$, the greater the increase in costs when firms need to manage a larger range of tasks.[12] After integrating and multiplying by the cost of a unit of labor in country i, the cost function appears as:

$$w_i c_i(s,\omega) = w_i a_i(\omega) \frac{s^{\theta_i(\omega)+1}}{\theta_i(\omega)+1}. \tag{3}$$

Transaction costs: Fragmenting production across firms incurs transaction costs. We model transaction costs like iceberg transport costs in standard trade models. More specifically, a transaction in country i with an elementary firm df involves losing a fraction $\gamma_i df$ of the good.

$$q_{i,f+df}(\omega) \;=\; q_{i,f}(\omega)\,(1 +\; \gamma_i\, df) \tag{4}$$

Within each country, quantities thus follow a simple evolution depending on transaction costs γ_i and the position on the chain f. As we go upstream, quantities increase exponentially with the number of firms f to cross along the chain:

$$q_{i,f}(\omega) = \mathrm{e}^{\gamma_i f} q_{i,0}(\omega) \tag{5}$$

[11] $a_i(\omega)$ and $\theta_i(\omega)$ are constant along the chain (for a given country).

[12] Note that we assume diseconomies of scope but constant returns to scale in production. This differs from Chaney and Ossa (2013) and more closely follows Kikuchi et al (2014). In keeping with Kikuchi et al (2014), this framework allows us to examine patterns of fragmentation across firms while keeping a perfectly-competitive framework where the competitive allocation of tasks across firms is optimal.

Since part of the production is lost when transactions occur, upstream firms must produce larger quantities. The necessary increase in quantities is starker when transaction costs are high and when the chain is more fragmented.

In a similar fashion, a *cross-border* transaction between two consecutive countries $i = i(n)$ and $j = i(n+1)$ along the chain involves an iceberg trade cost $\tau > 1$ such that:

$$q_{j,0}(\omega) \;=\; \tau\, q_{i,F}(\omega) \tag{6}$$

where $q_{j,0}(\omega)$ denotes the quantities produced by the most downstream plant in the upstream country j and $q_{i,F}(\omega)$ denotes quantities produced by the next plant, i.e. the most upstream plant in the next country along the chain going downstream. For simplicity, we assume away geographical elements other than borders and impose a common border cost. Here, quantities also increase exponentially as we cross borders along the chain.

Market structure: In addition to assuming constant returns to scale in production, we assume perfect competition as in typical Ricardian frameworks. In this setup, the market equilibrium and the optimal allocation correspond to the social optimum.[13]

Prices along the chain: The price of intermediate goods is thus equal to their unit cost of production. Here, this cost accounts for all transaction costs incurred along the chain (going upstream) and the labor costs incurred by each firm. Within country borders, the price of intermediate goods satisfies the following differential equation which describes its evolution along the chain:

$$p_{fi}(\omega) = w_i c_i(s_{fi})df + (1 + \,\gamma_i\, df)\, p_{i,f+df}(\omega) \tag{7}$$

where $c_i(s_{fi})$ denotes the cost of performing a range s_{fi} of tasks at stage f in country i as specified above. This equation is close to Costinot, et al. (2012) and also features increasing intermediate goods prices as we go downstream. A key difference, however, is that the labor share is endogenous since s_{nf} is endogenous and thus not simply driven by differences in input prices along the chain. In particular, the cost of inputs per unit of labor is no longer necessarily larger for downstream firms. Many of the results in Costinot et al (2012) are driven by this feature and thus no longer hold in our framework.

Across borders, the price is simply multiplied by the international trade cost τ:

$$p_{j,F}(\omega) = \tau\, p_{i,0}(\omega) \tag{8}$$

[13]While there are decreasing return to scale in terms of firm scope, there are constant returns to scale in production in terms of quantities. The equilibrium under perfect competition corresponds to the social optimum. This insight follows Kikuchi et al (2014) generalized to a multi-country setting with heterogenous costs and a continuum of firms.

for cross-border transactions from the most downstream plant in j to the most upstream plant in i.

Industry heterogeneity: While the previous assumptions are sufficient to generate interesting patterns of specialization along a particular chain, we still need to specify how chains vary across varieties. Following Eaton and Kortum (2002), we assume that labor efficiency is a random variable drawn independently across varieties and countries. Specifically, we assume that the labor cost parameter $a_i(\omega)$ is drawn from a Frechet distribution as in Eaton and Kortum (2002). For each country i, the cumulative distribution function for a_i is:

$$Proba(a_i < a) = 1 - \mathrm{e}^{-T_i a^{\xi}} \tag{9}$$

where T_i parameterizes the country average productivity and where ξ is inversely related to productivity dispersion.[14] Note that $a_i(\omega)$ is thus constant along the chain for a specific country and variety ω. Unlike Yi (2003, 2010), Rodriguez-Clare (2010) and Johnson and Moxnes (2013), our framework does not require $a_i(\omega)$ to differ across tasks along the chain to generate trade in intermediate goods. Another component of the cost function is $\theta_i(\omega)$. We will explore different settings. In section 4.2 we simply consider two countries U and D: one where $\theta_U(\omega) = \theta_U$ across all varieties, and another country with $\theta_D(\omega) = \theta_D < \theta_U$ across all varieties. In sections 4.3 and 5 (the calibration exercise), we allow $\theta_i(\omega)$ to vary across countries and varieties. Specifically, we assume that $\theta_i(\omega)$ is log-normally distributed with a country-level shifter $\bar{\theta}_i$ and a common standard deviation.

Labor supply: Finally, to close the model, we assume that workers are homogenous and perfectly mobile within each country, with an inelastic supply of labor L_i in country i. By Walras' law, trade is balanced.

3 Partial equilibrium: optimal organization of chains

In this subsection, we take wages w_i as given and focus on the optimal fragmentation and location of production for a specific chain corresponding to a final good variety ω. For the sake of presentation, we drop the index ω. The reader should keep in mind, however, that the optimal fragmentation and allocation of value across firms, as well as costs parameters a_i and θ_i, are all specific to each variety of final good ω.

For a given chain, we can formulate the equilibrium as the solution to a social planning problem. Given our assumption of perfect competition and constant returns to scale, prices

[14] Parameter ξ corresponds to the notation θ in Eaton and Kortum (2002).

equal unit costs and the competitive equilibrium corresponds to the social optimum.

Let us denote by $i(n)$ the ranking of countries along the chain, with $i(1)$ being the most downstream country and $i(N)$ the most upstream country, assuming that N countries are involved in the chain. One should keep in mind that the ranking of countries is an equilibrium outcome that we will characterize subsequently. In partial equilibrium (i.e. for a given variety, taking wages as given), the optimal organization of chains minimizes the price of the final good. Hence equilibrium can be summarized by the following optimization problem:

$$\min P_1 \tag{10}$$

$$\begin{aligned} \textit{over:} \quad & i(n)\ ,\ s_{nf}\ ,\ F_n\ ,\ S_n\ ,\ P_n \\ \textit{under the constraints:} \quad & P_n = \left[\int_{f=0}^{F_n} \mathrm{e}^{\gamma_{i(n)} f} c_{i(n)}(s_{nf}) df \ + \ \mathrm{e}^{\gamma_{i(n)} F_n} \tau P_{n+1} \right] \\ & S_n = \int_{f=0}^{F_n} s_{nf} df \\ & \sum_{i=1}^{N} S_n = 1 \end{aligned}$$

where N is the optimal number of countries involved in the chain and $P_n \equiv p_{0,n}$ denotes the price at the most downstream stage in country $i(n)$ at the n^{th} position. Recall that exponential terms $\mathrm{e}^{\gamma_{i(n)} f}$ reflect the evolution of quantity requirements along the chain as described in equation (5). The transaction cost parameter $\gamma_{i(n)}$ and the cost function $c_{i(n)}(s)$ are indexed by $i(n)$ because they depend on which country $i(n)$ is at the n^{th} position upstream. As an abuse of notation, P_{N+1} refers to the price of the most upstream good and is set to zero.[15]

Chains are optimized along several dimensions. Choice variables include firm scope $s_{i,f}$, the range of firms F_i in the chain in each country and the range of tasks S_i to be performed in each country i. The ranking of countries along the chain, from most downstream $i(1)$ to most upstream $i(N)$, is itself an endogenous outcome described below.[16]

3.1 Fragmentation of production within countries

Before turning to the cross-border organization of chains, we focus on optimal fragmentation within each country. The optimization problem described in (10) can be formulated as a nested

[15] Alternatively, we could set an exogenous price $P_{N+1} = \bar{p}$ of the most upstream good reflecting the price of primary commodity such as oil and minerals available from an outside economy in exchange for final goods.

[16] We can either interpret this optimization from the point of view of the most downstream country, in which case $i(1)$ is fixed, or we can optimize from the point of view of the consumer, in which case the final good price needs to be multiplied by trade costs τ if the optimum location $i(1)$ is different from the location of the final consumers.

optimization problem. In the inner nest, firms in a specific country i are organized to minimize the price of goods exported by country i conditional on the overall range of tasks S_i to be performed in i and P_i^M the price of intermediate goods imported by country i. The outer nest allocates stages across countries to minimize the cost of producing the final good. Here we focus on the inner nest that optimizes within country i.

The price P_i can be expressed as the solution of the following optimization:

$$\widetilde{P}_i(S_i, P_i^M) = \min_{s_{fi}, F_i} \left[\int_{f=0}^{F_i} \mathrm{e}^{\gamma_i f}\, w_i c_i(s_{fi}) df \;+\; \mathrm{e}^{\gamma_i F_i} P_i^M \right] \tag{11}$$

under the constraint:

$$\int_{f=0}^{F_i} s_{fi} df = S_i \tag{12}$$

To examine the optimal allocation of tasks across firms and the optimal range of firms, it is useful to introduce the Lagrange multiplier λ_i associated with the constraint $\int_0^{F_i} s_f df = S_i$.

The first-order conditions of this planning program are:

$$\text{For } s_{fi}: \quad e^{\gamma_i f} w_i c_i'(s_{if}) \;=\; \lambda_i \tag{13}$$

$$\text{For } F_i: \quad e^{\gamma_i F_i} w_i c_i(s_{i,F_i}) \;+\; e^{\gamma_i F_i} P_i^M \gamma_i \;=\; s_{i,F_i}\, \lambda_i \tag{14}$$

These conditions help us solve for firm scope (s_{fi}) and the number of firms involved in the chain (F_i). Both s_{fi} and F_i depend on λ_i, the shadow cost of a task.

Equation (13) defines a shadow market for tasks. All firms in the chain provide a measure of tasks s_{if} such that their marginal cost of tasks equals the shadow price of a task, λ_i. In this way, the conditions that determine the scope of individual firms also define the allocation of tasks across firms that minimizes the cost of producing a measure of tasks S_i in country i.[17]

Condition (13) offers an additional insight about the relationship between firm heterogeneity and relative position along the chain. A move upstream (i.e. towards higher index f) increases required quantities $e^{\gamma_i f}$, which must be balanced by a reduction in the marginal cost $c_i'(s_{if})$. Hence, with convex costs, condition (13) implies that more upstream firms have smaller firm scope s_{if} and provide less value added. We can be more explicit about this using our parameterization: $c_i' = a_i s_{if}^{\theta_i}$, which implies that firm scope is log-linear in upstreamness f:

$$\frac{\partial \log s_{fi}}{\partial f} = -\frac{\gamma_i}{\theta_i} \;<\; 0 \tag{15}$$

[17] Our Lagrangian formulation in (13) generalizes the condition $\delta c'(s_{f+1}) = c'(s_f)$ in Kikuchi et al (2014) that links the marginal costs of tasks between (discrete) firms f that neighbor one another in the chain.

From a broader perspective, λ_i also links firm scope decisions across countries, a relationship we develop further in the following section of the paper. For those relationships it is helpful to recognize that $\lambda_i = \frac{\partial \widetilde{P}_i}{\partial S_i}$.

In an appendix we solve for s_{if} and F_i as a function of λ_i. We apply these in turn to the constraint $\int_0^{F_i} s_{if} df = S_i$ and derive an explicit solution for the shadow cost of fragmentation.

$$\lambda_i = w_i a_i \left[\frac{\gamma_i S_i}{\theta_i} + \left(\frac{(\theta_i+1)\gamma_i}{\theta_i} \frac{P_i^M}{a_i w_i} \right)^{\frac{1}{\theta_i+1}} \right]^{\theta_i} \tag{16}$$

λ_i increases with all cost parameters a_i, θ_i and γ_i, with the price of intermediate goods P_i^M and with the range of tasks to be performed S_i.

Having solved for the shadow cost of fragmentation, we can now solve for the price of the last-stage goods P_i, the extent of fragmentation F_i in country i and firm scope s_{if} across all firms f within the country. We also examine the (endogenous) intensity in intermediate goods at each stage.

Firm scope: The model is tractable enough to solve for firm scope s_{if} all along the chain. Firm scope s_{i,F_i} for the most upstream firm is:

$$s_{i,F_i} = \left[\frac{(\theta_i+1)\gamma_i}{\theta_i} \frac{P_i^M}{a_i w_i} \right]^{\frac{1}{\theta_i+1}} \tag{17}$$

while the most downstream firm has scope :

$$s_{i,0} = \frac{\gamma_i S_i}{\theta_i} \ + \ s_{i,F_i} \tag{18}$$

Using expression (15), scope at intermediate positions corresponds to: $\log s_{if} = -\frac{\gamma_i}{\theta_i} f + \log s_{i,0}$.

Note again that firms are *ex ante* homogenous but end up with different firm scope due to their position on the chain. The difference $\frac{\gamma_i}{\theta_i} S_i$ between the scope of the most downstream and upstream firms in country i is illustrative of this within-country heterogeneity in firm scope. Heterogeneity is rising in S_i because more tasks produced in country i implies more firms, and thus more room for heterogeneity, conditional on θ_i and γ_i. Larger values of transaction costs γ_i imply more heterogeneity in firm scope because upstream firms must reduce s_{if} relatively more to satisfy equation (13). Larger values of θ_i imply that scope remains more uniform across firms.

Of further interest is the relationship between firm scope and the price of intermediate goods relative to labor costs $\frac{P_i^M}{a_i w_i}$. The scope of both the most upstream and downstream firms are rising in this ratio. The intuition is that when the price of intermediates is relatively high, the cost of outsourcing is relatively higher and firms will choose to add more value in-house. Conversely, when labor costs are high, firms will produce relatively few stages before outsourcing to upstream firms.

Length of the chain: The number (mass) of firms involved sequentially in production is a key measure of fragmentation of the chain. Here, since the range of tasks performed by each firm is endogenous, the length of the chain also becomes endogenous and is no longer proportional to S_i. For a given price P_i^M of imported intermediate goods and range S_i of tasks to be performed, the mass of sequential suppliers is:

$$F_i = \frac{\theta_i}{\gamma_i} \log \left[1 + \frac{S_i}{\theta_i + 1} \left(\frac{A_i w_i}{P_i^M} \right)^{\frac{1}{\theta_i+1}} \right] \tag{19}$$

The mass of suppliers depends negatively on the price of intermediate goods because more expensive components make transactions more costly and lead to more fragmentation. The number of suppliers also depends negatively on transaction costs and positively on θ_i, the parameter for diseconomies of scope.

Aggregate price: After solving for firm scope s_{if} and the number of firms F_i (see appendix), we find that the price of the most downstream good in country i, i.e. the solution of the minimization program (11), is:

$$P_i = \widetilde{P}_i(S_i, P_i^M) = \left[\frac{S_i}{\theta_i + 1} (A_i w_i)^{\frac{1}{\theta_i+1}} + \left(P_i^M \right)^{\frac{1}{\theta_i+1}} \right]^{\theta_i+1} \tag{20}$$

expressed as a function of the synthetic parameter A_i:

$$A_i = a_i \left(\gamma_i \, \tfrac{\theta_i+1}{\theta_i} \right)^{\theta_i} \tag{21}$$

This A_i depends on exogenous country-specific parameters θ_i, a_i and γ_i, and reflects the effective labor productivity in country i. Note that, conditional on A_i, prices no longer depend on transaction costs γ_i. The price mimics a CES cost function with two inputs: imported intermediate goods and labor, where the weight for labor depends on the range of tasks, productivity, transaction costs and coordination costs. The apparent elasticity of substitution is θ_i+1. When coordination costs θ_i are larger, production has to be more fragmented and there is a larger amount of production lost in transaction costs. These costs are larger when the price of intermediate goods P_i^M is high.

Labor vs. imported intermediate goods demand: Each unit of the final-stage good produced in country i also generates a demand $e^{\gamma_i F_i}$ for the most upstream intermediate goods, i.e. intermediate goods imported from the next country in the chain. In terms of value rather than quantities, we obtain that the share of imported inputs in the total cost of production in country i is:

$$\frac{P_i^M q_{i,F_i}}{P_i q_{i,0}} = \frac{\partial \log \widetilde{P}_i}{\partial \log P_i^M} = \frac{\left(P_i^M\right)^{\frac{1}{\theta_i+1}}}{\frac{S_i}{\theta_i+1}(A_i w_i)^{\frac{1}{\theta_i+1}} + (P_i^M)^{\frac{1}{\theta_i+1}}} \tag{22}$$

Using this expression, we can retrieve the demand for local labor in country i. The share of local demand in the production of country i has a simple interpretation: it corresponds to the value-added content of exports for country i in that chain. As with the price of the produced good, this expression mimics a CES cost function. The share of labor (one minus the above expression) depends positively on the range of tasks to be performed as well as the price of intermediate goods. The elasticity of substitution between imported inputs and local labor is in turn endogenously determined by diseconomies of scope at the firm level.

Gross-output-to-value-added ratio: We define gross output as: $GO_i = \int_0^{F_i} p_{fi} e^{\gamma_i f} df$ by integrating the value of all transactions along the chain, while total value added by country i corresponds to: $VA_i = \int_0^{F_i} c_i(s_{fi}) e^{\gamma_i f} df$. The ratio of these two variables has a useful empirical counterpart since it is readily available in typical input-output tables provided by statistical agencies. Here, we find that the GO-VA ratio equals:

$$\frac{GO_i}{VA_i} = \frac{\theta_i}{\gamma_i} \tag{23}$$

Strikingly, this result also holds at the firm level. To be more precise, the ratio of price to cost at each stage is constant and equal to:

$$\frac{p_{if}}{w_i c_i(s_{fi})} = \frac{\theta_i}{\gamma_i} \tag{24}$$

We can interpret this ratio as an index of fragmentation at the firm level. In particular, this ratio reflects the two key forces present in our model: stronger diseconomies of scope (coordination costs) θ_i lead to more fragmentation while larger transaction costs γ_i lead to less fragmentation. As seen in equations (15) and (18), this ratio also dictates the difference in scope between upstream and downstream firms.

The relationship between the structural parameters and summary measures of fragmentation are summarized in the following lemma:

Lemma 1 *Production fragmentation within countries – captured either by the GO/VA ratio or by the range F_i of firms involved in the chain – increases with coordination costs θ_i and decreases with transaction costs γ_i. In particular, the GO/VA ratio equals $\frac{\theta_i}{\gamma_i}$.*

Free entry and cost decomposition: We can also use (24) to better understand the link between our model and perfect competition, as well as the nature of the growth in output prices along the chain. Perfect competition implies that firms' average and marginal costs of producing a stage will be equalized along the chain. If average costs exceed marginal costs firms can reduce costs by expanding their scope. If marginal costs exceed average costs there will be entry and firms will reduce their average scope. Applying (24) and (13), we can equate average and marginal cost for firm f, and link these to the shadow cost (per unit of quantity):

$$\frac{w_i c_i(s_{i,f}) + \gamma_i p_{fi}}{s_{i,f}} = \frac{(1+\theta_i)\, w_i c_i(s_{i,f})}{s_{i,f}} = w_i c_i'(s_{i,f}) = \lambda_i e^{-\gamma_i f}. \tag{25}$$

It is also useful to decompose the sources of costs in the left-hand-side term of (25). Average cost has two components: labor costs associated with producing tasks inside the firm and transaction costs linked to shipments between firms. A decomposition exercise highlights the central role of the coordination cost parameter θ_i, and will be useful in a later discussion of comparative advantage. Using (24), we solve for changes in average cost as we move along the implicit price function.

$$\frac{\gamma_i p_{fi}}{w_i c_i(s_{fi}) + \gamma_i p_{fi}} = \frac{\theta_i}{\theta_i + 1} \tag{26}$$

The contribution of input prices to total cost growth is solely a function of θ. The share of labor costs is, by implication: $\frac{1}{\theta_i+1}$. A notable outcome in this calculation is the absence of a role for γ_i in the decomposition of cost growth, which arises because firms react to higher values of γ_i by bringing more stages inside the firm. We revisit this issue when we describe comparative advantage within the supply chain.

3.2 Cross-border fragmentation

Now that we have described the allocation of tasks along the chain within borders, we turn to the optimal allocation of tasks and firms across borders. In particular, we need to characterize the ordering of countries $i(n)$ on the chain, with $i(1)$ being the most downstream and $i(N)$ the most upstream country.[18]

[18] Recall that we drop for now the variety subscript ω while most parameters vary across varieties.

Given the optimal fragmentation of production across firms in each country $i = i(n)$, summarized by the price function from equation (20), $\widetilde{P}_i(S, P^M)$, the optimal global value chain corresponds to the following minimization program:

$$\min_{\{S_n, P_n\}} P_1 \tag{27}$$

under the constraints:

$$P_n = \widetilde{P}_{i(n)}(S_n, \tau P_{n+1}) \qquad \text{and} \qquad \sum_{i=n}^{N} S_n = 1$$

and where the function $\widetilde{P}_i(S, P^M)$ is the solution of the optimization described in equation (20) in the previous section.

For a given sequence of countries $i(n)$, we can go quite far in characterizing prices, ranges of tasks completed and labor demand along the chain. First, it is useful to explicitly express the Lagrangian:

$$\mathcal{L} = P_1 - \sum_{n=1}^{N} q_n \left[P_n - \widetilde{P}_{i(n)}(S_n, \tau P_{n+1}) \right] - \lambda_G \Big[\sum_{n=1}^{N} S_n - 1 \Big] \tag{28}$$

The Lagrange multipliers associated with price equations correspond to quantities required for each unit of final good. To be more precise, q_n correspond to quantities $q_{i(n),0}/q_{i(1),0}$ required at the most downstream task performed in the n^{th} country $i(n)$ per unit of final good $q_{i(1),0}$. The first-order condition $\frac{\partial \mathcal{L}}{\partial P_{n+1}} = 0$ is equivalent to imposing $q_{n+1} = \tau q_n e^{\gamma_{i(n)} F_n}$ (using the price derivative described in equation 22).

The first-order condition $\frac{\partial \mathcal{L}}{\partial S_n} = 0$ reflects the optimal allocation of tasks across countries. At the optimum, the marginal cost of completing another task should be equalized across all countries on the chain, up to quantities q_n produced by country n: $\lambda_G = q_n \frac{\partial \widetilde{P}_{i(n)}}{\partial S_n} = q_n \lambda_n$. This implies:

$$q_i \lambda_i = q_j \lambda_j \tag{29}$$

for any pair of countries i and j along the chain, where λ_i is the shadow cost of fragmentation within country i (per unit of goods exported by the country). The tight links between the Lagrange multipliers in successive countries serves to link the shadow cost of stages across markets.

Since a move upstream along the chain increases quantities (because of transaction costs and cross-border trade costs), the shadow cost $\lambda_{i(n)} > \lambda_{i(n+1)}$ must decrease. Concretely, a first implication is that firm scope tends to decrease as we go upstream, not just within countries but also across countries. The F.O.C. in S_i implies the following expression which generalizes equation (13) across countries along the chain:

$$q_n e^{\gamma_{i(n)} f} w_{i(n)} c'_{i(n)}(s_{nf}) = \lambda_G \tag{30}$$

where $q_n e^{\gamma_{i(n)} f}$ corresponds to the quantities of intermediate goods required for each unit of final good. Since the latter increases with upstreamness, we obtain that firm scope s_{nf} would be smaller if a country $i = i(n)$ specializes upstream than if it specializes downstream. Therefore, a country with large within-firm coordination costs would have a relatively larger cost downstream than upstream compared to a country with low coordination costs.[19]

This feature has important implications for the sorting of countries along the chain. Because firm scope can be smaller upstream, diseconomies of scope have a smaller impact on upstream stages than downstream stages. Hence, we should then expect countries with high-θ to specialize upstream while low-θ countries tend to specialize downstream. Formally, we can confirm this insight by examining second-order conditions of the optimization problem described in Equation (27), which yields the following Proposition:

Proposition 1 *Let us denote by $i(n)$ the ranking of countries involved in the same production chain, $i(1)$ being the most downstream and $i(N)$ the most upstream country. In equilibrium, the relative position of countries along the chain is fully determined by coordination costs θ_i; countries with smaller coordination costs specialize downstream:*

$$\theta_{i(1)} < \theta_{i(2)} < ... < \theta_{i(N)}$$

Proposition 1 describes comparative advantage within a supply chain, conditional on a country's participation in the chain. Two implications are of primary interest here: the central role of θ_i in determining within-chain comparative advantage, but also the absence of a role for the transaction cost parameter γ_i. The lack of a role for γ_i would seem to run counter to Costinot et al. (2011), where cross-country differences in the rates of mistakes in production drive comparative advantage within the chain. The closest counterpart in our model to the mistakes in Costinot et al. (2011) is the γ_i parameter.[20]

[19] Note, however, that firm scope does not necessarily increase across countries as we go upstream since firm scope also depends on transaction costs. We find that transaction-costs-adjusted firm scope $\frac{s_{if}}{\gamma_i}$ decreases with a move upstream, but firm scope is not necessarily lower in a country specializing upstream than in a country specializing downstream.

[20] Costinot et al. (2011) offer cross-country differences in contract enforcement as a rationale for differences in the rates of mistakes. Here, the parameter most closely related to contract enforcement is clearly γ_i.

What our model shares with Costinot et al. (2011) is the dependence of comparative advantage on the parameter that determines the convexity of the price function as value accumulates along the chain. In both models, prices are increasing at an increasing rate as the measure of completed tasks increases and, in both models, countries with less convex price schedules tend to specialize downstream. In Costinot et al. (2011), convexity arises because the losses due to mistakes increase with the value of the good along the chain, while, in our model, coordination costs are the key source of convexity.

Another way to see this is to exploit the insights in Costinot (2009), who links comparative advantage to the mathematics of log super-modularity. The accumulation of value added along the chain insures that the cost of intermediate goods is rising along the chain. This means that if production costs are log-supermodular in a parameter and input prices, then countries that have low values of that parameter will locate downstream. Using previous results on cost decomposition (equation 26), we find for any country i:

$$\frac{\partial \log \left\{ \frac{w_i c_i(s_{fi}) + \gamma_i p_{fi}}{s_{fi}} \right\}}{d \log p_{fi}} = \frac{\gamma_i p_{fi}}{w_i c_i(s_{fi}) + \gamma_i p_{fi}} = \frac{\theta_i}{1+\theta_i}. \tag{31}$$

In Costinot et al. (2011), s_{fi} is fixed, so average cost is log supermodular in p_{fi} and γ_i. The middle term $\frac{\gamma_i p_{fi}}{w_i c_i(s_{fi}) + \gamma_i p_{fi}}$ would be increasing in γ_i if firm scope were fixed. This implies that transaction costs γ_i have the least impact on average cost when p_{fi} is low (i.e. early in the chain), so that lower transaction costs create a comparative advantage in downstream tasks.

In contrast, γ_i does not appear in (31), and thus does not affect comparative advantage within the chain. The simple explanation is that in our model, firm scope is endogenous to changes in γ_i; in countries with larger transaction costs, firms will endogenously increase firm scope to mitigate the role of higher transaction costs. These endogenous responses nullify the role that transaction cost would otherwise play if firm scope were exogenous. Instead, θ_i plays a singular role in determining countries' positions within the chain. As shown in equation (31), countries with higher coordination costs θ_i should specialize upstream to mitigate the effect of input prices on value added.[21]

Equilibrium allocation of tasks across countries: Given the ranking of countries described in Proposition 1, we now describe the range of tasks performed by each one. Using marginal conditions imposed by the optimization problem, we can also determine prices and firm scope along the chain depending on wages and relative productivity. Specifically, the first-order

[21] A related implication is that there will be no international fragmentation without cross-country variation in θ_i. Also, Proposition 1 implies that there is no back-and-forth trade along a specific chain in equilibrium, except when a final good is shipped back to be consumed in an upstream country.

conditions determine the c.i.f. price between consecutive countries $i(n)$ and $i(n+1)$. First-order conditions between three consecutive countries $i(n-1)$, $i(n)$ and $i(n+1)$ then yield the range of tasks performed in $i(n)$. Denoting A_n, w_n and θ_n the productivity, wages and the coordination cost parameter in the n^{th} country $i(n)$ along the chain, we obtain:

$$\left\{ \begin{array}{rcl} \tau P_{n+1} & = & (A_n w_n)^{\frac{\theta_{n+1}+1}{\theta_{n+1}-\theta_n}} (\tau A_{n+1} w_{n+1})^{-\frac{\theta_n+1}{\theta_{n+1}-\theta_n}} \\ \frac{S_n}{\theta_n+1} & = & \left(\frac{A_{n-1}w_{n-1}}{\tau A_n w_n}\right)^{\frac{1}{\theta_n-\theta_{n-1}}} - \left(\frac{A_n w_n}{\tau A_{n+1} w_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}} \end{array} \right. \tag{32}$$

where τ is the trade cost between any two countries.

Given the range of tasks performed in country $i(n)$, it is interesting to derive the share of local labor in exports to the next country in the chain. Because ours is a single-factor model, this corresponds to the value added locally in the exports of country $i(n)$, a key statistic for economic policy.[22] Here, we find that the demand for labor (in value) in country $i(n)$ per dollar of good exported to the next country $i(n-1)$ in the chain is:

$$\frac{w_n l_n}{P_n} = 1 - \left(\frac{w_n A_n}{\tau w_{n+1} A_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}} \left(\frac{\tau w_n A_n}{w_{n-1} A_{n-1}}\right)^{\frac{1}{\theta_n-\theta_{n-1}}} \tag{33}$$

Intuitively, the share of local value added in exports is higher when the relative labor cost is lower, as lower labor costs allows the country to serve as the low cost location for a larger measure of stages. This operates through margins that are both up- and down-stream. A lower labor cost makes country $i(n)$ more competitive at the margin than the previous upstream country $i(n+1)$ as well as the next country $i(n-1)$ downstream.

The effect of trade costs on this statistic also operates through two channels: higher trade costs reduce the contribution of country $i(n)$ in the downstream country $i(n-1)$ operations, but they also reduce the upstream country's contribution. For a country in the middle of the chain, trade costs have a positive effect on local labor content only if there are stronger complementarities with downstream rather than upstream countries, i.e. when the differences in θ_n are larger with the downstream country than with the upstream country: $\theta_n - \theta_{n-1} > \theta_{n+1} - \theta_n$.

Conditional on the set of countries participating (with θ_n increasing with n along the chain), we can go further and obtain a simple expression for the price of final goods (i.e. price of downstream goods in country 1) as a function of costs parameters A, θ and wages w. Conditional on the set of countries, we can also derive simple expressions for the share of labor costs from a specific country.

[22]For example Koopman et al. (2010) investigate the share of domestic value added in China's exports.

Lemma 2 *Conditional on the set of countries $n = 1, 2, etc.$ participating (with θ_n increasing with n along the chain), the price of the final good is:*

$$P_1 \;=\; \frac{A_1 w_1}{(\theta_1{+}1)^{\theta_1+1}}\;\Theta(\mathbf{wA},\tau) \tag{34}$$

where $\Theta(\boldsymbol{wA},\tau) < 1$ captures the gains from fragmentation for the chain:

$$\Theta(\mathbf{wA},\tau) = \left[1 \;-\sum_{n=1}^{N-1}(\theta_{n+1}{-}\theta_n)\left(\frac{w_n A_n}{\tau w_{n+1}A_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}}\right]^{\theta_1+1}$$

Moreover, country $i(n)$'s contribution to each dollar of final good being produced is equal to:

$$\frac{l_n w_n}{P_1} = \frac{d\log P_1}{d\log w_n} = \frac{d\log\Theta}{d\log w_n} = \frac{\left(\frac{w_{n-1}A_{n-1}}{\tau w_n A_n}\right)^{\frac{1}{\theta_n-\theta_{n-1}}} \;-\; \left(\frac{w_n A_n}{\tau w_{n+1}A_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}}}{\left(\frac{P_1}{A_1 w_1}\right)^{\frac{1}{\theta_1+1}}} \tag{35}$$

In the expression for the final good price above, the first term $\frac{A_1 w_1}{(\theta_1+1)^{\theta_1+1}}$ is the cost of production in country 1 if there is no possibility to fragment production across countries, while the second term $\Theta(\mathbf{wA},\tau)$ is the price reduction obtained from fragmenting production across countries. We can verify that this term increases with trade costs. It also increases with labor requirements A in each upstream country.

Reductions in trade costs allow chains to reorganize some of the tasks abroad, which in turn has an effect on all other firms along the chain. Equation (30) shows that the marginal cost of increasing firm scope has to be equalized across all stages. A decrease in trade costs leading to a decrease in the final good's price also lead to a decrease in firm scope at other stages. The price of the final good is itself tightly linked to the shadow cost of fragmentation:

$$\lambda_G \;=\; \frac{A_1 w_1}{(\theta_1{+}1)^{\theta_1}}\;\Theta(\mathbf{wA},\tau)^{\frac{\theta_1}{\theta_1+1}} \tag{36}$$

As expressed with the Θ term, there is a tight connection between the gains from fragmentation (which reduces the final good's price) and the shadow cost of fragmentation. Any increase in fragmentation and decrease in the final good price follows: $d\log\lambda_G \;=\; \frac{\theta_1}{\theta_1+1}\,d\log\Theta$.

A change in trade costs and wages along the chain has implications for firm scope everywhere on the chain. Each firm equalizes the cost of the marginal task and the shadow cost λ_G of performing the task somewhere else. Hence, a change in the shadow cost of fragmentation λ_G has implications for firm scope everywhere along the chain. In particular, the marginal cost of increasing firm scope in the most downstream firm in the most downstream country,

$w_1 c'(s_{1,f=0})$ is equal to λ_G, which implies:

$$s_{1,f=0} = \frac{\gamma_1}{\theta_1}\, \Theta(\mathbf{wA}, \tau)^{\frac{1}{\theta_1+1}} \tag{37}$$

Hence: $d \log s_{1,f=0} = \frac{1}{\theta_1+1}\, d \log \Theta$, which formalizes how a change in fragmentation and trade costs (changes in Θ) affects firm scope for the last firm in the chain, the one that produces the finished good.

Note, however, that the scope of the average firm in an upstream country $i(n)$ (with $n > 1$) does not decrease with trade costs. As trade costs decrease, a country moves downstream where firms tend to be larger. Upstream firms, which tend to be smaller in scope, exit or relocate. More specifically, we find that both the size of the most downstream and the most upstream firm within a country increase as trade costs decrease:

$$s_{n,0} = \frac{(\theta_n+1)\gamma_n}{\theta_n} \left(\frac{A_{n-1} w_{n-1}}{\tau A_n w_n}\right)^{\frac{1}{\theta_n-\theta_{n-1}}} \qquad \text{and:} \qquad s_{n,F_n} = \frac{(\theta_n+1)\gamma_n}{\theta_n} \left(\frac{A_n w_n}{\tau A_{n+1} w_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}} \tag{38}$$

Proposition 2 below summarizes the effect of trade costs on a chain in partial equilibrium (exogenous wages) for a given set of countries involved in the chain:

Proposition 2 *Holding wages constant, a decrease in cross-border trade costs leads to:*
i) a decrease in the price of the final good;
ii) an increase in the value share of imported inputs at any stage of the chain;
iii) an increase in the range of tasks being offshored;
iv) a decrease in the shadow cost of fragmentation λ_G*;*
v) a decrease in firm scope $s_{n,f}$ *at a given stage* f*;*
vi) an increase in average firm scope in upstream countries $n > 1$*.*

While the ranking of countries along the chain (from downstream to upstream stages) is dictated by the ranking in θ_i (Proposition 1), it is more difficult to characterize the participation of a specific country in the chain. Expression (32) for S_i can be used to obtain a necessary condition for $S_i > 0$, but cannot be used to derive a sufficient condition for country i to participate in the chain. Moreover, the reader should keep in mind that we have dropped the variety subscript ω to simplify the notation, but the costs parameters A_i and θ_i are assumed to be specific to a particular variety of final good ω. Hence, the organization of the chains across firms and countries is specific to each variety and country of final destination.

We address this problem below in two special cases with simple analytical expressions: first, as in Costinot et al (2012), with symmetric chains and frictionless trade; second, in a two-country case with trade costs and heterogeneous chains. We also examine numerically a ten-country case calibrated using input-output data (Section 5). Using expressions (34) and (35) from Lemma 2, we can dramatically reduce the complexity of the numerical problem and reformulate the problem into a simpler linear programming problem that allows us to solve for large economies with a large number of final goods.

4 General equilibrium and aggregation

In this section we investigate the properties of a general equilibrium version of the model. Initially we assume cross country variation in θ_i, but no heterogeneity across chains (ω) and no trade costs. In this setting, we prove the existence and uniqueness of the solution to the general equilibrium system. In order to investigate the implications of trade costs we develop an analytically tractable two-country version of the model in subsection 4.2 and provide analytical solutions to describe the effects of trade costs on trade and on welfare. In subsection 4.3, we discuss the general equilibrium setting that we take to the calibration in section 5. In subsection 4.4, we show that the model can be easily extended to include a distinct assembly sector.

4.1 Equilibrium with frictionless trade and homogenous goods

In the section 3 we derived model implications under the partial equilibrium conditions of fixed wages and incomes. We now examine the existence and uniqueness of equilibrium. We focus here on the case with no trade costs ($\tau = 1$) and homogenous productivity across varieties. Also, a natural assumption is that no country shares the same coordination cost parameter ($\theta_i \neq \theta_j$ if $i \neq j$) to avoid any indeterminacy. In this case, we can prove that the model features a unique equilibrium:

Proposition 3 *With frictionless trade, homogeneous productivity:*

i) the organization of production is symmetrical across all chains, i.e.: all countries participate in all varieties, and their position along the chain and contribution to value added is identical across varieties;

ii) there exists a unique equilibrium such that each country's labor market clears, with a fixed labor supply and labor demand computed as in equation (35).

The proof of this proposition is provided in an appendix. As in the Costinot et al (2011), cross-country differences in the convexity parameter (in our case θ_i) ensures a unique allocation of tasks across countries in the free trade general equilibrium. As discussed above, our model distinguishes itself from Costinot et al (2011) in its ability to explain endogenous chain lengths as measured by F, and in its links to data elements such as the GOVA ratio. Moreover, note that our model offers more flexibility as it does not impose any relationship between wages and the position along supply chains.

However, the free-trade model presented here (and its counterpart in Costinot et al., 2011) retains predictions that make it unsuitable for quantitative analysis. Contrary to what we observe in international trade data, both models predict that every country but one will produce (and export) only intermediate goods. In all but the country producing final goods, both models imply that imports constitute 100% of consumption, and both models predict that two-way trade in goods occurs only within the final two countries in the chain. These free-trade models also predict extremely long chains of production, with every country participating in every chain. This implies, in turn, extremely large measures of upstreamness (or other measures of chain length), which is inconsistent with evidence that we present below. Clearly a richer model structure is needed. We introduce trade costs and heterogeneity into our framework in order to build a richer theory of supply chain trade. We begin by relaxing the free trade assumption in a two-country model that allows analytical links between trade costs, trade flows and welfare.

4.2 A two-country case

In this subsection, we reintroduce trade costs $\tau > 1$ as well as heterogeneity in productivity as in Eaton and Kortum (2002).

Two-country setting: We consider only two countries: country D and country U. Country D has a parameter value θ_D for coordination costs, while country U has a parameter value θ_U. To justify these country names, we assume that $\theta_U > \theta_D$.

As specified in equation (9) in section 2, labor efficiency $a_D(\omega)$ and $a_U(\omega)$ are distributed Frechet with coefficient T_D and T_U respectively for countries D and U. We make no assumption about the relative ranking of T_D and T_U. We also make no assumption about relative transaction costs γ_D and γ_U for countries D and U. Also, we normalize $w_D = 1$.

As demonstrated in the previous sections (equation 21), it is useful to instead define an adjusted labor costs parameter $A_D(\omega) = a_D(\omega)\left(\gamma_D \frac{\theta_D+1}{\theta_D}\right)^{\theta_D}$ and $A_U(\omega) = a_U(\omega)\left(\gamma_U \frac{\theta_U+1}{\theta_U}\right)^{\theta_U}$. The effect of transaction costs is equivalent to a shift in labor productivity. The resulting $A_D(\omega)$

and $A_U(\omega)$ parameters also follow a Frechet distribution with adjusted shift parameters:[23]

$$\begin{aligned}\tilde{T}_D &= T_D \left(\gamma_D \tfrac{\theta_D+1}{\theta_D}\right)^{-\xi\theta_D} \\ \tilde{T}_U &= T_U \left(\gamma_U \tfrac{\theta_U+1}{\theta_U}\right)^{-\xi\theta_U}\end{aligned}$$

Following Dornbusch et al. (1977), we rank varieties ω between 0 and 1 and specify the following relative cost:

$$\frac{A_U(\omega)}{A_D(\omega)} = \left[\frac{\tilde{T}_D}{\tilde{T}_U}\left(\frac{\omega}{1-\omega}\right)\right]^{\frac{1}{\xi}} \equiv A(\omega) \tag{39}$$

where $A(\omega)$ is defined as the relative labor requirement in country U. This ordering implies that U has a comparative advantage in low-ω chains while country D has a comparative advantage in high-ω chains. For the sake of exposition, we normalize $A_D(\omega)$ to unity. It is otherwise equivalent to redefine all prices as relative to $A_D(\omega)$.

Definition of an equilibrium: The optimal organization of chains is described by the optimization problem in (10). In addition, labor demand must equal labor supply in each country.

Sourcing patterns: As shown in Proposition 1, the ranking $\theta_D < \theta_U$ determines relative position on the chain. A chain that involves the two countries necessarily features country U specializing upstream and country D specializing downstream. Some chains may also involve country U only. However, when country D produces the final good, we find that country U is also involved in the chain, at least for some of the most upstream tasks.

When country D produces the final good (with country U involved in upstream tasks), the price of the final good in D is:

$$P_D(\omega) = \frac{1}{(\theta_D+1)^{\theta_D+1}}\left[1-(\theta_U-\theta_D)\left(\tau w_U A(\omega)\right)^{-\frac{1}{\theta_U-\theta_D}}\right]^{\theta_D+1} \tag{40}$$

Consumers in U can also import these goods at a price $\tau P_D(\omega)$. When country U produces the entire range of tasks, the price of final goods in U is:

$$P_U(\omega) = \frac{w_U A(\omega)}{(\theta_U+1)^{\theta_U+1}} \tag{41}$$

while consumers in country D can also import these goods for a price $\tau P_U(\omega)$.

Given the patterns of labor costs across varieties, the ratio of prices $\frac{P_D(\omega)}{P_U(\omega)}$ strictly increases with ω. For each final destination $X \in \{D, U\}$, there is a unique threshold ω_X^* for which the

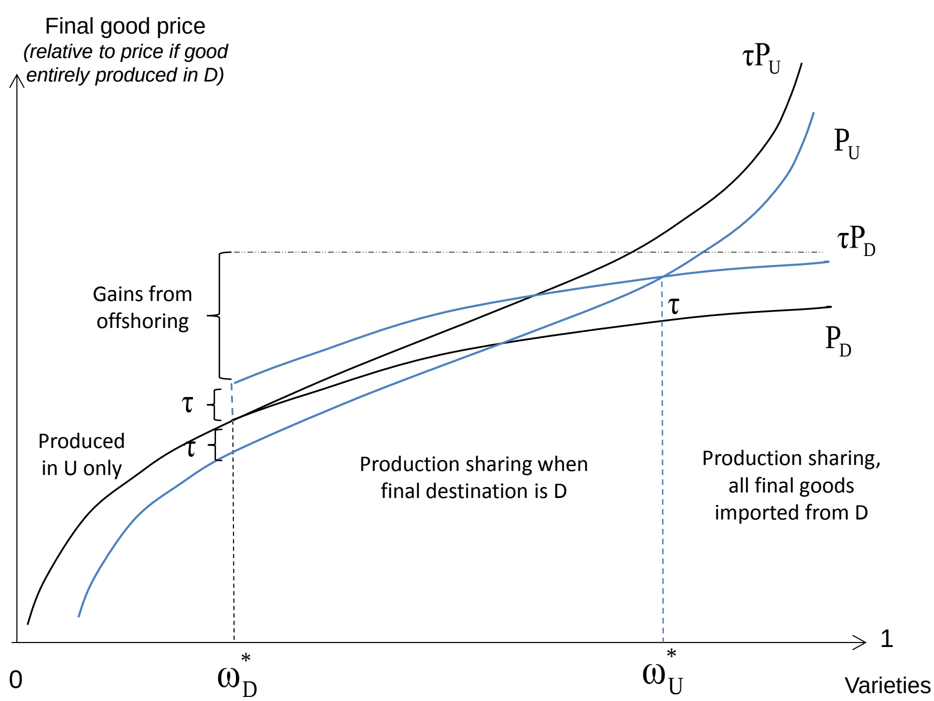
[23] Our parameter ξ is the same as the dispersion parameter θ in Eaton and Kortum (2002).

two prices are equal. These thresholds ω_D^* and ω_U^* are implicitly defined by:[24]

$$P_D(\omega_D^*) = \tau P_U(\omega_D^*) \quad (42)$$
$$\tau P_D(\omega_U^*) = P_U(\omega_U^*) \quad (43)$$

As in Dornbusch, Fisher and Samuelson (1977), these cutoffs ω_D^* and ω_U^* correspond to the goods for which consumers (resp. in D and U) are indifferent between purchasing locally or importing.

Figure 1: Final goods prices depending on source and destination countries



These results are illustrated in Figure 1. Figure 1 plots the price of final goods for each sourcing strategy. The threshold ω_D^* is defined by the intersection between the curves for P_D and τP_U (prices for consumers in D while for final goods purchased from D and U), while ω_U^* is defined by the intersection between the curves for τP_D and P_U. Consumers in D thus purchase goods $\omega \in [\omega_D^*, 1]$ locally (with upstream tasks being offshored to U) and consumers in U purchase goods $\omega \in [0, \omega_U^*]$ in their own country.

The effect of fragmentation on prices is reflected by the upward slope of the price schedule P_D and τP_D (when the final goods is purchased from D) which would have been flat without cross-border fragmentation (i.e. no offshoring of upstream tasks in U). As described below,

[24]The solution for ω_D^* has an analytical expression: $\omega_D^* = \frac{\tilde{T}_U \tau^{-\xi} w_U^{-\xi} (\theta_D+1)^{-(\theta_U-\theta_D)\xi}}{\tilde{T}_D + \tilde{T}_U \tau^{-\xi} w_U^{-\xi} (\theta_D+1)^{-(\theta_U-\theta_D)\xi}}$.

this generates a stronger effect of trade costs on ω_U^* than in a situation with no fragmentation.

Since the production of final goods in D relies on country U to perform upstream tasks, the demand for labor does not only relate to the demand in final goods but also depends on intermediate goods. Using the results from Lemma 2, the demand for labor in U for each dollar of final goods produced in D (at a price P_D) equals:

$$\frac{w_U\, l_U(\omega)}{P_D(\omega)} = \frac{(\theta_D+1)\ (\tau w_U A(\omega))^{-\frac{1}{\theta_U-\theta_D}}}{1-(\theta_U-\theta_D)\ (\tau w_U A(\omega))^{-\frac{1}{\theta_U-\theta_D}}} \tag{44}$$

Wages at equilibrium are determined by a labor market clearing condition or, equivalently, a trade balance condition (see equation 59 in appendix). Based on this, we can show that there is a unique equilibrium in the wage w_U of country U relative to country D, and that τw_U decreases as trade costs τ decrease.[25]

Effect of trade costs on final goods trade: We examine trade in final goods before intermediate goods and the value added content of trade. The share of final goods consumed in D that are imported from U corresponds to the threshold ω_D^*, while the share of goods consumed by U and imported from D equals $1-\omega_U^*$. We find that the elasticity of final good imports with respect to trade costs depends on the final destination of consumption and more specifically on the local labor content of the exporter. For country D, this elasticity is the same as in Eaton and Kortum (2002), i.e. ξ in our notation:

$$\varepsilon_D^F \equiv \frac{d\log\left(\frac{\omega_D^*}{1-\omega_D^*}\right)}{d\log\tau} = -\xi$$

essentially because the exporter (country U) does not rely on imported goods for its exports: from country D's perspective, a 1% increase in trade costs from U is exactly compensated by a 1% decrease in labor costs in U.

Country D, however, relies on imports from U to produce goods that it exports back to U. As in Yi (2010), this back and forth trade generates a higher trade elasticity. There are two reasons for that. When trade costs increase by 1%, it affects the price of imported goods by U from D by more than 1% since the production of these goods in D relies itself on goods imported in U (double penalty). The second reason is that country D would need to decrease its labor cost by more than 1% to offset a 1% increase in the price of its exports since its labor only contributes to a fraction of the value of the good. Combining these two effects, the

[25] While we can get clear-cut results in terms of trade-costs-adjusted wages $w_U\tau$, the effect of trade costs on w_U itself is ambiguous like in a typical Ricardian trade model: the effect of trade on the relatively wage of country U tends to be larger if U is smaller.

elasticity of imports in U to trade costs equals:

$$\varepsilon_U^F \;\equiv\; \frac{d\log\left(\frac{1-\omega_U^*}{\omega_U^*}\right)}{d\log\tau} \;=\; -\xi\,.\,\frac{1+\frac{w_U\,l_U(\omega_U^*)}{P_D(\omega_U^*)}}{1-\frac{w_U\,l_U(\omega_U^*)}{P_D(\omega_U^*)}} \;<\; -\xi$$

where $\frac{w_U\,l_U(\omega_U^*)}{P_D(\omega_U^*)}$ is the foreign labor content of production in country D for the marginal variety ω_U^* imported by country U (see equation 44). The numerator, which is greater than unity, reflects the effect of trade costs on the price after the double penalty. The denominator, which is less than unity, corresponds to the share of D's labor costs in total costs.

The lower the trade costs, the higher the trade elasticity. Because lower trade costs leads to more fragmentation, the foreign labor content for the marginal variety increases. When trade becomes frictionless, the foreign labor content for this marginal variety converges to unity and the trade elasticity ε_U^F goes to infinity.

Vertical specialization and the value-added content of trade: Some of the goods are produced in two locations and some of them are even re-exported to consumers in other countries. There are not many combinations with a two-country model but we can still provide predictions about how the model fits with some of indexes used to describe the extent of fragmentation across countries. We focus here on Johnson and Noguera (2012a)'s "VAX ratio", which is itself a generalization of Hummels, Ishii and Yi (2001)'s vertical specialization index. The VAX ratio for an exporter is the ratio of the value-added content of exports and gross exports. In turn, the value-added content of exports corresponds to the value added to the production of goods eventually consumed by foreign consumers. A decrease in the VAX ratio reflects an increase in fragmentation across borders, characterized by a growing difference between trade measured in gross flows and the value added by each exporter (Johnson 2014).[26]

For country D, the VAX ratio is smaller than unity because country D relies on imported intermediate goods to produce the goods that are then exported:

$$VAX_D = \frac{1}{1-\omega_U^*}\int_{\omega_U^*}^{1}\left(1-\frac{w_U\,l_U(\omega)}{P_D(\omega)}\right)d\omega$$

where $1-\omega_U^*$ is the share of imported goods by consumers in U and where $\frac{w_U\,l_U(\omega)}{P_D(\omega)}$ is the share of foreign labor in the production of variety ω in country D. As trade costs decrease, we show that the foreign labor content increases and therefore that the VAX ratio for country D decreases.

[26] As noted in Fally (2012), the inverse of the VAX ratio for the world can also be interpreted as the embodied number of border crossings.

For country U, all exported goods rely on domestic labor. Hence, the index developed by Hummels et al. (2001) would be equal to one for U. The VAX ratio, however, is again lower than unity and is equal to country D's VAX ratio. For country U, value-added exports only account for the export of intermediate goods that are embodied in final consumption in D, while gross exports also include the exports of intermediate goods that are embodied in the final goods that are re-exported back to U. We show that this back-and-forth trade grows faster than other trade flows as trade costs decrease, which implies that the VAX ratio for country U decreases as trade costs decrease.

We summarize these results on the trade cost elasticity and VAX ratio in the following Proposition:

Proposition 4 *The effect of trade costs on trade is such that:*
i) The elasticity of trade in final goods to trade costs is higher than without fragmentation;
ii) This elasticity is larger when trade costs are smaller;
iii) The VAX ratio (value-added content of trade) decreases as trade costs decrease.

The results of Proposition 4 are intuitive and supported by recent empirical evidence. In particular, Johnson and Noguera (2013, 2012b) use multi-country input-output tables to show that the VAX ratio has decreased over the past decades and that the bilateral VAX ratio depends positively on bilateral trade costs.

Gains from trade: How does trade affect welfare in upstream and downstream countries? A key policy question is whether a country is affected differently depending on its position in international production chains. To examine this question, we derive an exact expression for the price index and the gains from trade relative to autarky and compare it to standard models without cross-border fragmentation of production. In particular, we use the formula developed by Arkolakis et al. (2012) as a benchmark.

For country D, which tends to specialize downstream, the change in the real wage compared to autarky is:

$$\Delta \log\left(\frac{1}{P_D}\right) \;=\; -\frac{1}{\xi}\log\left(1-\omega_D^*\right) \;+\; \frac{1}{\xi}\int_{\omega_D^*}^{1} \frac{w_U\, l_U(\omega)}{P_D(\omega)}\frac{d\omega}{1-\omega} \tag{45}$$

where $P_D = \exp\left[\int_0^1 \log P_D(\omega)d\omega\right]$ is the price index (the nominal wage w_D is normalized to unity in country D). The first term $\frac{1}{\xi}\log\left(1-\omega_D^*\right)$ corresponds to the Arkolakis et al (2012) formula based on final demand trade: the log of the gains from trade are proportional to the log of the domestic content of consumption, where the proportionality coefficient is the inverse of the trade elasticity ξ. For country D, expression (45) indicates that the Arkolakis et al

(2012) formula underestimates the gains from trade, because we must adjust for the foreign labor content in the production of final goods by D: $\frac{w_U\, l_U(\omega)}{P_D(\omega)}$.

Conversely, we find that the Arkolakis et al (2012) formula based on final goods overstates the gains from trade for the upstream country U. Specifically, we find that the difference in real wage compared to autarky equals:

$$\Delta \log\left(\frac{w_U}{P_U}\right) \;=\; \frac{1}{\xi}\int_{\omega_U^*}^{1}\left(\frac{w_U\, l_U(\omega)}{P_D(\omega)}\right) d\log\omega \;<\; -\frac{1}{\xi}\log\left(\omega_U^*\right) \tag{46}$$

While these results are shown here only for a two-country case, our counterfactual simulations in Section 5 suggest that this insight holds more generally. Conditional on final goods trade, our results suggest that gains from trade tend to be underestimated for downstream (and generally richer) countries and overstated for upstream (and often poorer) countries. We calibrate our model and compute gains from trade by using input-output tables and information on domestic and foreign labor content which, as shown above, are crucial to obtain a more adequate measure of the gains from trade when production is fragmented across borders.

Fragmentation and firm scope along the chain: Finally, we examine the effect of trade on fragmentation and firm scope along the chain within each country.

A key determinant of firm scope along the chain is the Lagrange multiplier associated with the total range of tasks, i.e. the shadow cost of fragmentiation $\lambda_G(\omega)$ (which varies across varieties). At optimum, this shadow cost depends negatively on the relative wage in U adjusted for trade costs τ:

$$\lambda_G(\omega) = \frac{A_D(\omega)w_D}{\theta_D+1}\;\left[1-(\theta_U-\theta_D)\;(\tau w_U A(\omega))^{-\frac{1}{\theta_U-\theta_D}}\right]^{\theta_D} \tag{47}$$

(for final goods produced in D). Using Lemma 2, we obtain that the shadow cost of fragmentation decreases with trade cost, which lead to more fragmentation.

We can also express firm scope at all stages along the chain as a function of relative wages. In country D, firm scope at stage f is given by:

$$s_{D,f}(\omega) \;=\; e^{-\frac{\gamma_D}{\theta_D}f}\;.\;\frac{\gamma_D}{\theta_D}\;\left[1-(\theta_U-\theta_D)\;(\tau w_U A(\omega))^{-\frac{1}{\theta_U-\theta_D}}\right] \tag{48}$$

A decrease in trade costs leads to a decrease in trade-cost adjusted wage τw_U and therefore a decrease in firm scope in D at each stage. Moreover, the overall range of tasks $S_D(\omega)$ performed in country D decreases when trade costs decrease:

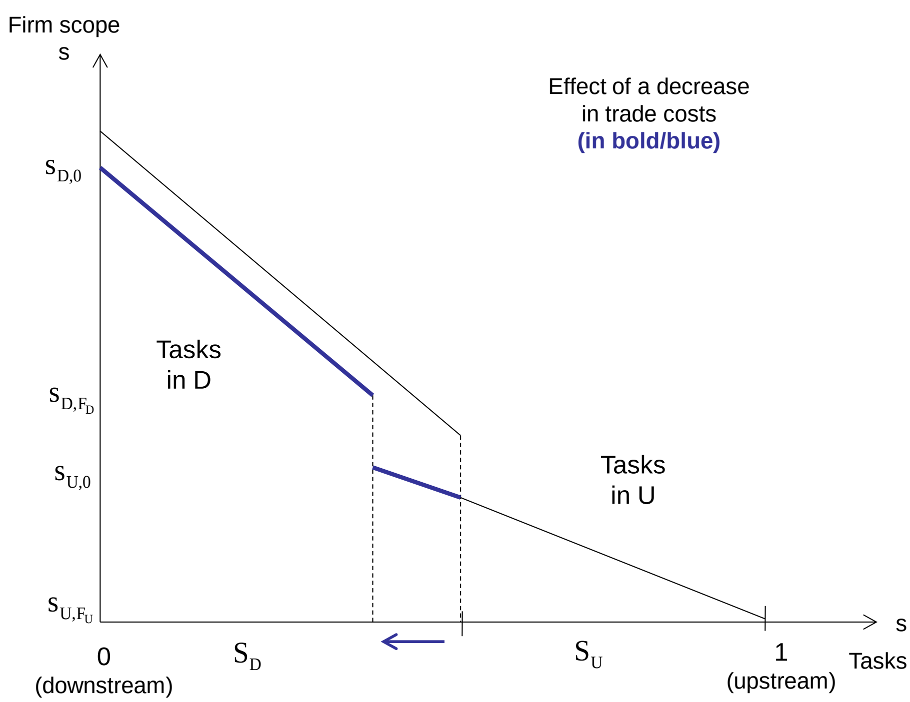
$$S_D(\omega) = 1-(\theta_U+1)\;(\tau w_U A(\omega))^{-\frac{1}{\theta_U-\theta_D}} \tag{49}$$

which also implies that the range of tasks performed by country U: $S_U(\omega) = 1 - S_D(\omega)$ increases when trade costs decrease. Hence both countries tend to perform more downstream tasks as trade costs decrease (i.e. "move up the value chain").

While firm scope decreases for each firm in each country, note that average firm scope in a country does not decrease, due to the reallocation of tasks across countries. Since both countries D and U tend to move downstream, each country only keep its largest firms (downstream firms) when trade costs decrease. Hence, average firm size in each country actually *increases* once we account for this margin of adjustment along the chain (see appendix for mode details). These results are illustrated in Figure 2.[27] Proposition 5 below also summarizes the key results on fragmentation in this two-country setting:

Proposition 5 *In general equilibrium, a decrease in trade costs* τ *leads to:*
i) For each variety, a decrease in the shadow cost of fragmentation λ_G*;*
ii) At any given stage, a smaller firm scope;
iii) A greater share of inputs from U *embodied in final goods produced in* D*;*
iv) A larger share of imports of final goods.

Figure 2: Effect of a decrease in trade costs on firm scope for a given chain



[27] Instead of expressing firm scope as a function of upstreamness f (in terms of firms), we plot firm scope as a function of the position on the range of tasks (ordered from 0 to 1) which yields linear relationships.

4.3 General equilibrium and aggregation: general case

In the quantitative section, section 5, we examine a situation with 10 countries involved in general equilibrium. Several comments are in order to describe the difference between the two-country case above and the calibrated multi-country model.

First, we change our assumptions about $\theta_i(\omega)$. Empirical evidence suggests large heterogeneity in a country's position on supply chains. As shown in the data, all countries export a mix of final goods and intermediate goods. According to Proposition 1, this would not happen if $\theta_i(\omega)$ is constant across all varieties. We extend the approach taken by Eaton and Kortum (2002) and specify that $\theta_i(\omega)$ varies across varieties and is the realization of a random variable. We assume in the calibration section that it is log-normally distributed, with i.i.d. shocks across varieties within countries. There are however systematic differences that can be captured here by differences in countries' average $\bar{\theta}_i$.

Second, this heterogeneity in $\theta_i(\omega)$ opens up an indirect role for cross-country variation in γ_i. As noted above, transaction costs play no direct role for comparative advantage within chains, but now they can play an indirect role. As shown in (21) the effect of γ_i is subsumed in the summary measure of productivity A_i. As such, γ_i affects country i's absolute advantage for a given chain, but not its comparative advantage. But A_i itself is log-supermodular in γ_i and $\theta_i(\omega)$ for each variety ω. The implication is that countries with high transaction costs will tend to have especially high production costs in varieties with idiosyncratically large $\theta_i(\omega)$. As a result, these countries will more often successfully enter chains ω for which they have low values of $\theta_i(\omega)$. Using proposition 1, this implies that high transaction cost countries will tend to participate in relatively downstream stages in those chains in which they do participate. Hence, in the calibrated model, both θ_i and γ_i affect a countries' average position within chains.

4.4 Model extension with assembly

One of the most visible manifestations of production fragmentation in East Asia is pure assembly activity. Often, firms engaged in pure assembly use more than one component, import most of their components, employ relatively low wage labor and add relatively little value in the assembly process. We can extend our model to address these phenomena with a treatment in which the completed continuum of tasks merely produces a component, rather than a final good that is ready for consumption. The component is tradeable, and components are assembled using labor in a capstone stage of production. There is trade in each variety of the final good. In this way we transform our model of 'snakes' into one of 'spiders', using the terminology of Baldwin and Venables (2010). To formalize matters let $P_{1ik}(\omega)$ represent the price in i of the component k used in variety ω, inclusive of trade costs. The cost function used to produce the final good in region j is then:

$$C_i(\omega) = w_i^{\beta} \left[\prod_k P_{1ik}(\omega)^{\eta_k} \right]^{(1-\beta)} \tag{50}$$

where η_k ($\sum_k \eta_k = 1$) represents a constant Cobb-Douglas cost share of component k, β represents the share of labor in assembly, and $1-\beta$ the share of components in the cost of assembly. We assume the technology of assembly is the same across all commodities and regions. The price in region i of the final good, $P_i^F(\omega)$, is the minimum delivered price accounting for trade costs τ when the good is imported. We describe the calibration of this model in section 5.5.

5 Quantitative analysis

5.1 Data

Our main sources of data are the Asian input-output tables developed by IDE-JETRO. These tables provide information on gross output, value-added, and (most importantly) input purchases by product, parent industry (downstream industry), source country and destination country. For instance, the data report the amount of metals purchased from China by the auto industry in Japan. These 4-dimensional input-output tables, are, as far as we know, the only tables that track international transactions directly, rather than imputing them from trade flows. This is an exceptional data set for investigating the organization and evolution of international production fragmentation in a region of the world where fragmentation is an important feature of international trading relationships.

The dataset covers 9 Asian countries and the US.[28] Our analysis mostly focuses on the year 2000 (the most recent year available), but we also compare our results to IDE-JETRO data from 1975 and 1990. This period marks a time in which the region began to emerge as an important location for internationally fragmented production (see Baldwin and Lopez-Gonzalez (2012) for example).

Information on input purchases and production is disaggregated at the 76-sector level in 2000. For the sake of comparison to previous input-output tables (1975 and 1990), we construct a more aggregated 46-sector classification to obtain harmonized product categories across years. The sector classification is far more detailed for manufacturing goods and commodities than services (among the 46 sectors, only 5 of them are service industries). We thus mostly restrict our attention to tradable goods: commodities and manufacturing goods.

[28]The countries in the data base are the US, Japan, China, Taiwan, Korea, Singapore, Malaysia, Thailand, Indonesia and the Philippines.

The information provided in these tables goes beyond a simple aggregation of country-level input-output tables. Besides the harmonization of product categories, input flows by parent and source countries are estimated by means of additional surveys and thus deviate from the proportionality assumption[29] which, according to Puzello (2012), is rejected in these data. This constitutes an important advantage of using the IDE-JETRO input-output compared to previous attempts at constructing input-output tables based on the proportionality assumption (as in Johnson and Noguera 2012, for example).

5.2 Measuring the position along the chain: indexes N, N^*, D & D^*

To better understand the degree of fragmentation in vertical production chains we adopt four indexes that generalize the two indexes proposed in Fally (2012) and applied there to US data. The indexes are designed to describe industries' position in vertical production chains by exploiting information about relationships in the input-output table. The 'D' index measures an industry's weighted average distance to final demand, where distance is a measured by the apparent number of plants visited by the industry's output before reaching consumers.[30] The 'N' index calculates, for each industry, the number of stages that are embodied in each industry's production. These two calculations are distinct for each industry, and in the US data there is only a weak correlation between them.

Distance to final demand or "upstreamness": We turn to a formal representation of the two indices. Consider a variable D_{ik}, which is intended to measure the distance of a product k from final demand. Some part of product k's sales will be intermediate trade purchased by downstream industries, so the industry in question's distance measure will depend upon which industries buy its output, and in turn how far those downstream industries are from final demand. Because an industry's sales go to several industries, which will vary in their respective measures of D, the industry measure must be weighted, and it must also be defined recursively. Let D_{ik} indicate the distance measure in region i for product k. We define D_{ik} as:

$$D_{ik} = 1 + \sum_{jl} \varphi_{ikjl} D_{jl}$$

where φ_{ikjl} denotes the amount of output from product k in country i that is used to produce

[29] The proportionality assumption is made to construct input purchase by source country and parent industry when only partial information is provided. For instance, traditional country-level input-output tables describe how much steel is used by the auto industry in each country. Using trade flow data (which describe how much steel is imported from a particular country), previous international input-output tables have been constructed by allocating the use of input across source countries on a proportional basis.

[30] The measure is equivalent to the 'upstreamness' measure in Antras, et al. (2012).

one dollar's worth of product in sector l in country j. The entire system of equations that includes a D_{ik} for each industry and country can be solved to produce a measure for each sector-country pair.

As shown in Antras, et al. (2012), this index can be interpreted as the average number of stages that goods cross before reaching final consumers. Using the input-output matrix, we can decompose the different trajectories taken by the good across and within industries. Each trajectory is associated with a specific number of transactions across or within industries. Index D would then correspond to the average number of transactions weighted by the fraction of output corresponding to each trajectory.

Notice that D_{ik} does not only rely on inter-industry linkages but also depends on the extent of fragmentation within each industry. If an industry's production is partly used an input by other firms in the industry (e.g. electronic parts used as parts for other electronic parts within the same country), the coefficient φ_{ikik} would be strictly positive and would contribute to a higher index D_{ik} (since it would also correspond to a higher number of transactions).[31]

How to interpret 'D' in the model? First, suppose that products k correspond to stages f. When f is strictly positive, i.e. when it does not refer to the most dowstream stage in the n^{th} country $i(n)$, then all sales are made to the next plant $f - df$ in the chain:

$$D_{i(n),f+df} = df + D_{i(n),f}$$

If $f = 0$ and $i(n)$ is not the most downstream country $i(1)$, then all sales go towards the most upstream firm in the next country in the chain. After integrating, we obtain that the model counterpart of D_{if} corresponds to the total range of firms located downstream:

$$D_{i(n),f} = f + \sum_{n'<n} F_{i(n')}$$

summing across all downstream countries $i(n')$ with $n' < n$.

In terms of the model, we can also interpret $D_{i,k}$, for any country i, as a semi-elasticity of

[31] One may argue that the industry classifications are too aggregated and create biases in computing D_{ik} and N_{ik} compared to what would be obtained with more precise data. Fally (2012) examines the aggregation properties of indexes D and N and shows that aggregating industries does not much affect the average of D and N across industries.

required quantities w.r.t. to transaction costs.[32] Formally, D_{if} corresponds to:

$$D_{i,f} = \sum_j \frac{\partial \log q_{i,f}}{\partial \gamma_j}$$

Embodied stages: The N_{ik} index captures a weighted average of the number of plants involved sequentially in the production of good k in country i. It is defined recursively by:

$$N_{ik} = 1 + \sum_{jl} \mu_{ikjl} N_{jl}$$

where μ_{ikjl} denotes the amount of input l from country j used to produce one dollar of product k in country i. This is a single equation, but, as with the D index, the system of equations can be solved to produce a measure of N for each sector-country pair. As shown in Fally (2012),[33] this index can also be expressed as a weighted average of the number of stages required to produce good k in country i, weighted by how much each stage of production contributes to the final value of that good.

We can also interpret N in light of our theoretical framework. In the model, the amount of input purchased by other firms corresponds to the price of the good minus the labor cost incurred at each stage, i.e.: $\frac{w_i c_i(s_{if}) df}{p_{if}}$, where p_{if} denotes the price of the good in country i at stage f. The model counterpart of index N would thus correspond to the following recursive definition:

$$N_{i,f} = df + \left(1 - \frac{c_i(s_{if})df}{p_{if}}\right) N_{i,f+df}$$

with a similar equation when the chain crosses a border. The solution to this differential equation equals the average of the number of production stages required to produce a good at stage f in country i.

There is a strong connection between the N and D index. Since the number of stages between firm f' in the m^{th} country $i(m)$ and firm f in the n^{th} country $i(n)$ corresponds to $D_{i(m),f'} - D_{i(n),f}$, we obtain formally:

$$N_{i(n),f} = \frac{1}{q_{i(n),f} p_{i(n),f}} \left[\int_{(m,f')>(n,f)} \left(D_{i(m),f'} - D_{i(n),f}\right) q_{i(m),f'} c_{i(m)}(s_{i(m),f'}) \right]$$

where the integral is taken across all upstream firms either in $i(n)$ at a more upstream stage

[32] In a model with a discrete number of firms instead of a continuum of suppliers, transaction costs would correspond to $\delta = e^{\gamma}$ (as in Kikuchi et al 2014) and D would correspond to the elasticity of quantities w.r.t transaction costs instead of the semi-elasticity.

[33] See Proposition 1 in Fally (2012).

$f' > f$ or in more upstream countries $i(m)$ with $m > n$, and where the price p_{if} can be itself re-expressed as the sum of all costs incurred in upstream stages, adjusting for quantities: $q_{i(n),f} p_{i(n),f} = \int_{(m,f')>(n,f)} q_{i(m),f'} c_{i(m)}(s_{i(m),f'})$.

The connection between the two indexes N and D is clearest if we look at the most downstream stage. For the most downstream country $i = 1$ and the most downstream firm $f = 0$ in the country, index N corresponds to a weighted average of D:

$$N_{i(1),f=0} = \frac{1}{q_{1,0} p_{1,0}} \left[\sum_j \int_{f'=0}^{F_j} D_{j,f'} \; q_{if'} c_j(s_{jf'}) \, df' \right]$$

with the price $p_{i=1,f=0} = \sum_j \int_{f'=0}^{F_j} q_{if'} c_j(s_{jf'}) df'$ being the sum of all upstream costs.

As for D, we can also use the model to interpret N_{ik} as a semi-elasticity of w.r.t. to transaction costs, looking at prices instead of quantities. Formally, N_{if} corresponds to:[34]

$$N_{if} = \sum_j \frac{\partial \log p_{if}}{\partial \gamma_j}$$

Borders vs. stages: The N and D calculations are independent of whether or not the shipments in question cross international borders. Since the organization of international production networks is a key question for this literature, and for this study, we propose alternative forms of the indexes in Fally (2012). Using analogous methods to those used in the calculation of D and N we ask how many international borders are crossed for each region-industry pair's outputs and inputs. D* measures the average number of national borders crossed before final consumption. N* measures the weighted average number of production stages that are embodied in the output of an industry.

Formally,

$$D^*_{ik} = \frac{X_{ik}}{Y_{ik}} + \sum_{jl} \varphi_{ikjl} D^*_{jl}$$

and

$$N^*_{ik} = \frac{M_{ik}}{Y_{ik}} + \sum_{jl} \mu_{ikjl} N^*_{jl}$$

with most of the variables defined above. X_{ik} represents exports by sector k in i, M_{ik} represents imports, and Y_{ik} is sector gross output.

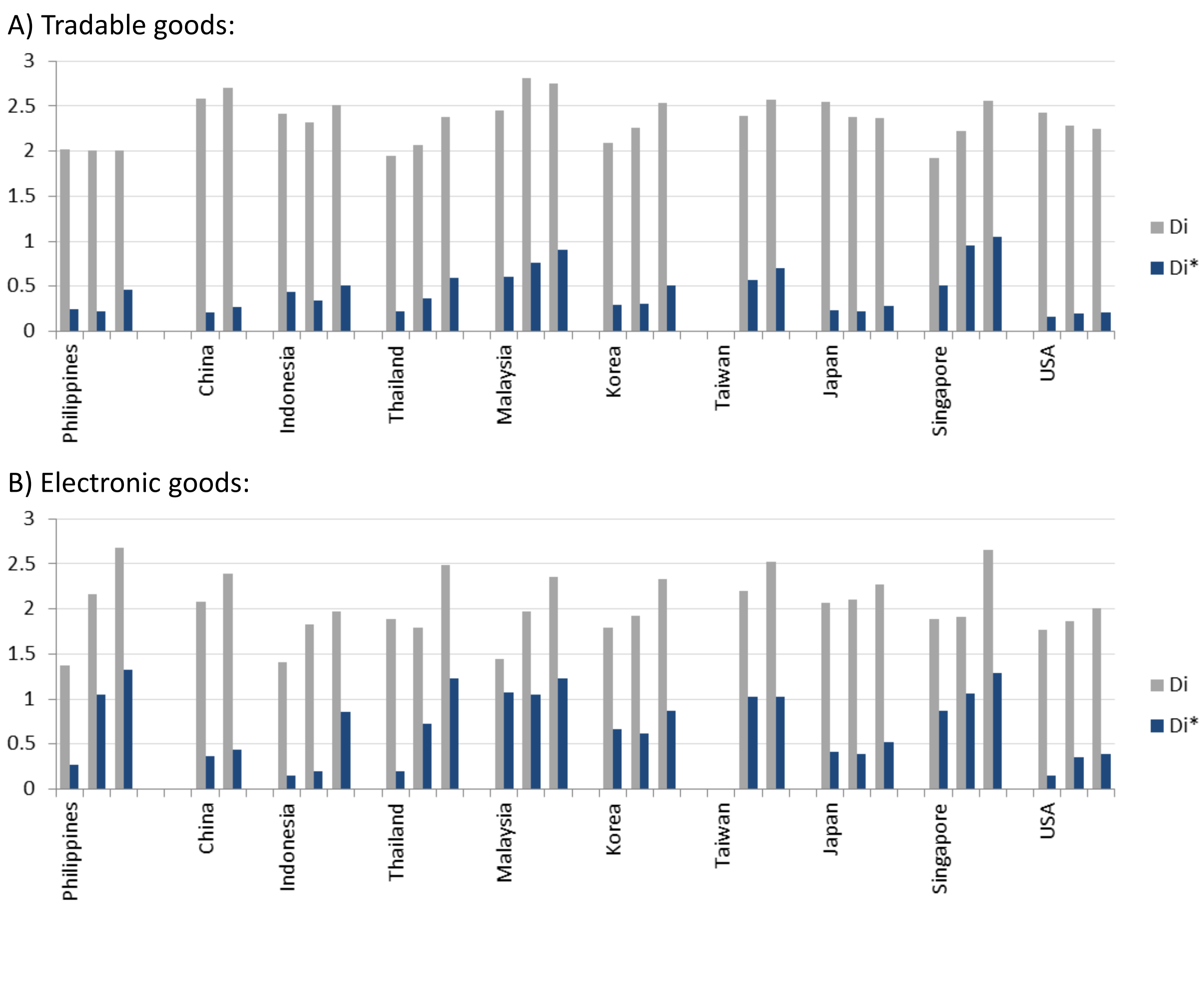
In our model, the equivalent is D^*_{if} reflecting the number of country borders to cross before

[34] Based on the results in Fally (2012), we should also note that N_{if} equals the aggregate gross-output-to-value-added ratio across upstream activities. It can therefore be interpreted as a weighted average of $\frac{\theta_i}{\gamma_i}$ across upstream activities.

reaching final consumers. Index N^*_{if}, on the other hand, reflects the average number of border crossings between the firm f in i and each upstream supplier weighed by value added by each supplier.

In the calibrated model, we calculate these indexes in benchmark and simulated data. The purpose of this exercise is to understand how changes in productivity levels and in trade and transaction costs affect the organization of production. Before we do so, however, it is useful to apply the measures to real data in order to better understand how the measures reflect organization of production in the past.

Figure 3: Indexes D and D^* by country for 1975, 1990 and 2000



Aggregation: Using the IDE-JETRO data, the D, D^*, N and N^* statistics are calculated at the level of country-industry (i, k) pairs. For the calibration exercise that follows a country-level statistic will be useful so as to better describe countries' average position in global supply chains. A weighted average across statistics is most suitable, although there are several options for defining weights, including value added- or export-weighting for D and output-weighting for N. As argued in Fally (2012), a natural weight for the upstreamness index D is value added, and final production by sector-country for index N. We do the same for N^* and D^*.

In order to best capture the relative position of countries on international production chains, our preferred aggregate is an export-weighted average of index D_{ik}. This export-weighted average is the statistic calculated in Antràs et al. (2012) to document countries' comparative advantage along production chains. Formally, we define DX_i by country with the following:

$$DX_i = \frac{\sum_k X_{ik} D_{ik}}{\sum_k X_{ik}} \tag{51}$$

where X_{ik} represents country i's exports of product k.[35]

Descriptive statistics: We calculate these indices using the IDE-JETRO data from 1975, 1990 and 2000. In order to present this information succinctly, we offer two aggregates, D_i and D_i^*, where the aggregates are calculated as averages weighted by sector k value added. We report the aggregate measures below in graphical form in order to show cross-country variation and within country variation over time.

In part A) of Figure 3, we see the results for all tradable goods. There is some variation in the levels and trends of the D across countries. While the cross-country variation is, in some cases, interesting in its own right, the key takeaway is the absence of a consistent story in the simple measures that are neutral with respect to national borders. By contrast, one can see a general pattern of growth in the D^* measures (upstreamness in border crossings), which is partially driven by decreases in trade costs but may also reflect a movement upstream. Both indexes increase in 2000 for most countries (with Japan and the US as notable exceptions), which suggests that the countries in question moved into upstream positions within internationalizing production chains.

The electronics sector is particularly interesting over this period. Complex international production chains are, anecdotally, an important phenomenon in East Asian manufacturing. This is even more notably so within the electronics sector. Moreover, there has been important growth in the region's trade in electronics, which constituted only 8% of Asian exports in 1975, and 34% in 2000. In part B) of Figure 3, we report results of our calculations for the electronics sector only, by country and by year.

[35]While a VA-weighted average of D^* can also capture the relative position on international supply chains, an advantage of the measure DX_i over D^* is that it is less sensitive to the country's level of openness. For instance, we may be concerned that Singapore has high values of D^* mainly because of low trade costs rather than being relatively more upstream. As our sole interest is in countries' relative position along the chain, a measure that conflates position with openness is problematic. The index DX_i puts more emphasis on the composition of exports than openness.

These measures indicate quite a bit more heterogeneity in the levels of both indexes, D and D^*. Production chains appear to be somewhat shorter in the U.S. than in Asia. Moreover there is sharp upward movement for most indexes and for most countries. One explanation would be that this is due to electronics becoming much more important as inputs into production of other sectors (i.e. automobiles), but this upward trend also reflects a specialization of Asian countries into relatively upstream stages within the electronics industry.

Correlation between upstreamness and value-added content: The model predicts that: i) countries with high coordination costs θ should specialize upstream while countries with low θ should specialize downstream; ii) the value-added-to-gross-output ratio decreases with θ since it equals the ratio of θ and transaction costs γ. By combining these two predictions, we should observe a negative correlation between upstreamness and the value-added-to-gross-output ratio.

Figure 4: Value-added-to-gross-output ratio as a function of upstreamness D_{ik}

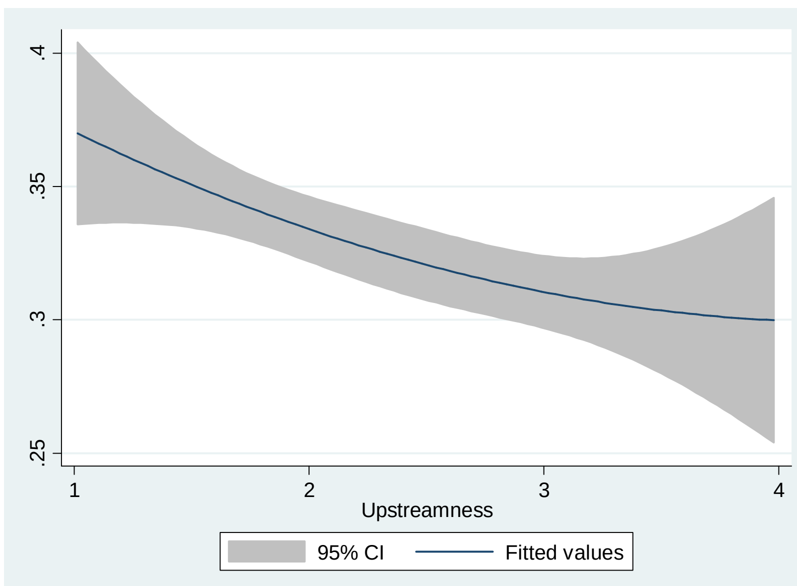


Figure 4 shows that we indeed observe such a negative correlation between upstreamness and value added content.[36] This finding supports the prediction that firm scope tends to be smaller upstream than downstream. In Table 1, we regress the value-added-to-gross-output ratio on upstreamness to find a significant and negative correlation between the two, whether we include no fixed effect (column 1), country fixed effects (column 2), industry fixed effects (column 3) or both (column 4). Consistent with the model, the correlation is the strongest

[36]Figure 4 covers manufacturing industries only, trimming observations with upstreamness above 4.

when most of the variation is driven by cross-country variation, i.e. when we include industry fixed effects but no country fixed effects.

Table 1: Correlation between upstreamness D_{ik} and the value-added-to-gross-output ratio

	(1)	(2)	(3)	(4)
Dependent var.:		VA/GO		
Upstreamness	-0.025	-0.060	-0.019	-0.028
	[0.012]**	[0.008]**	[0.008]*	[0.008]*
Industry FE	No	Yes	No	Yes
Country FE	No	No	Yes	Yes
R2	0.106	0.429	0.152	0.542
N Obs.	478	478	478	478

Notes: Notes: OLS regression with robust s.e.; manufacturing industries, trimming observations with upstreamness above 4; * significant at 5%; ** significant at 1%.

5.3 Calibration

Our calibration exercise focuses on the 10 countries that are covered by the IDE-JETRO input-output tables. The general equilibrium model described above is calibrated so as to reproduce key features of the data. The parsimony of the model allows us to consider only a small number of parameters to calibrate, those listed in the left column of Table 2.

Thanks to Lemma 2 and the analytical results described in section 3.2, we can reduce the optimization problem described in equation (10) to a linear programming problem for each chain. Numerical simulations are performed in Matlab. We approximate a continuum of varieties by assuming 1,000,000 different final goods.

We now describe each calibrated parameter and its targeted moment, as described respectively in the left and right columns of Table 2.

Labor supply: Each country is endowed with an exogenous supply of factors. In the benchmark case, we consider only one factor of production: labor. For each country, we choose the labor force L_i to match aggregate value-added in tradeable goods sectors (i.e. excluding services) divided by the cost of labor (proxied by income per capita in our benchmark simulation).

Labor productivity: There is a tight link between wages and labor productivity in country i. As in Eaton and Kortum (2002), average labor productivity is driven by $\bar{A}_i \equiv T_i^{-\frac{1}{\xi}}$ where T_i is a shift parameter for the Frechet distribution of $A_i(\omega)$ in country i (equation 9). Given the connection between $\bar{A}_i$ and wages, we use data on per capita income as a proxy for labor cost to be matched by the model. In turn, the dispersion parameter ξ is calibrated using the main estimate of Simonovska and Waugh (2014). Data on per capita income is obtained from the Penn World Tables for the year 2000.

Conditional on wages and calibrated parameters, we can compute labor demand for each country. Equality between aggregate labor demand and labor supply is attained by adjusting labor productivity. Of course, productivity is tighly related to wages. As shown in Table 2, there is almost a log-linear (downward-sloping) relationship between wages w_i and $\bar{A}_i$ in our benchmark calibration.

Coordination costs: As shown in Proposition 1, the coordination cost parameter θ_i is a key determinant of the position of a country along the chain, downstream or upstream. A country tends to export final goods when coordination costs are low and export intermediate goods when these costs are higher. Since all countries export a mix of final and intermediate goods, we assume that θ_i is heterogeneous across varieties ω, as discussed in section 4.3. We assume that it is log-normally distributed. In calibration countries are allowed a different shift parameter $\bar{\theta}_i$ and a different standard deviation $\sigma_{i\theta}$. We use DX as the primary moment to calibrate $\bar{\theta}_i$ and $\sigma_{i\theta}$ to fit countries' intermediate share in total exports. While the correlation of the values in Table 2 is weak (0.11), the prediction of the model is confirmed: countries that are more upstream have higher average values of $\bar{\theta}_i$. Empirically we note than most of the countries in the model have values of DX near the center of the distribution. Only Indonesia and China stand out as countries that are respectively, relatively up- and down-stream.

Transaction costs: Another key parameter of the model is γ_i, the cost of transactions between two firms. This cost is assumed to be positive even for transactions that occur within borders. Transaction costs are difficult to estimate in practice, but our model indicates that the gross-output-to-value-added ratio equals the ratio of coordination and transaction costs parameters $\frac{\theta_i}{\gamma_i}$ and thus can be used to retrieve an estimate of γ_i once we know $\bar{\theta}_i$ (on average). Results are provided in Table 2. As a credibility check we compare our results to plausible real-world counterparts of the γ_i parameter using the Doing Business Database (World Bank). Reassuringly, we find expected correlations of our calibrated γ_i variables with the costs of enforcing a contract claim (0.84), the time to enforce contracts (0.29), and the recovery rate in insolvency proceedings (-0.51).[37]

We note that Singapore has unusually low transaction costs in the calibration, which is consistent with its high gross output to value added ratio. Singapore's value of $\bar{\theta}_i$ is also relatively

[37] We could regress γ_i on each type of transaction cost (contract vs. bankruptcy, etc.) but with only 10 countries this regression would be meaningless.

low. Singapore often appears as a special case in the simulations that follow. In addition to its small size, it is likely that the low values of $\bar{\theta}_i$ and γ_i are responsible for the particular behavior of Singapore in our counterfactual analysis. At the other end of the spectrum, Indonesia has the higher transaction costs, which is consistent with the Doing Business indicators and recent literature (e.g. Barron and Olken, 2009).

Trade costs: In addition to transaction costs that are also incurred within borders, cross-border transactions face an additional burden. The nature and size of the extra cost affecting international transactions is the matter of an extensive debate in the trade literature (transport costs, asymmetric information, marketing cost, technical barriers, or cultural differences). There is however a consensus that these costs are large and have large effect on cross-border trade. While distance usually plays a key role in explaining the pattern of international trade (see Disdier and Head, 2008), distance is not as crucial for trade among Asian countries.[38] We therefore assume that cross-border trade costs are uniform across all country pairs in our sample. We fit the trade cost parameter by asking the model to replicate the global ratio of trade to output.

Simulation results on other moments: Before turning to the counterfactual results we briefly describe the benchmark equilibrium. By construction, our model is able to reproduce key indicators of fragmentation such as the gross-output-to-value-added ratio and values of D^*. Alternatively, we can examine how the fitted model fares in terms of other indexes such as indexes D, N and N^* described above.

Table 3 compares indexes from the model vs. data. In broad terms the magnitudes are consistent with the data, even though they are constructed in very different ways. These indexes from the data are computed at the industry level then averaged across industries. Index N in the benchmark calibration is computed for the most downstream firm in the chain while index D is a weighted average across firms weighted by value-added at each stage (same for D^*).

Indexes D and N tend to be too high in our calibrated model (too much fragmentation), but they are strongly correlated with their data counterpart (86% correlation for N and 44% correlation for D). With the exception of Singapore, the richest and poorest countries tend to be more downstream (lower index D) and have a smaller number of embodied stages (index N). With the exception of Singapore, middle-income countries in our sample such as Taiwan and Malaysia end up with the highest indexes D and N both in the calibrated model and the data.

[38] In a standard gravity equation of trade using the IDE-JETRO data, the coefficient for distance is not significant while the estimate border effect is large and significant, economically and statistically. Of course, this is a small sample of only 10 countries.

Table 2: Parameter choice and moments to match

Parameters:			*Moments to match:*		
Average $\bar{A}_i = \tilde{T}^{-\frac{1}{\xi}}$	USA	1.000	GDP per capita	USA	35,080
by country	SGP	4.768	(PWT)	SGP	32,808
(relative to the US)	JPN	1.634		JPN	26,721
	TWN	4.977		TWN	21,891
	KOR	4.521		KOR	17,208
	MYS	14.249		MYS	7,917
	THA	17.885		THA	5,178
	IDN	17.657		IDN	2,549
	CHN	25.049		CHN	2,442
	PHL	41.881		PHL	2,210
Labor supply	USA	53,551	Total value-added	USA	1,878.6
in tradeable goods	JPN	41,665	in tradeable goods	JPN	1,113.3
(x1000 workers)	SGP	735	(in $M)	SGP	24.1
	TWN	3,889		TWN	85.1
	KOR	10,491		KOR	180.5
	MYS	5,637		MYS	44.6
	THA	10,410		THA	53.9
	IDN	36,585		IDN	93.3
	CHN	266,707		CHN	651.3
	PHL	13,618		PHL	30.1
Average coordination costs $\bar{\theta}_i$	USA	1.161	DX Index	USA	2.555
by country	SGP	0.590	(Export weighted)	SGP	2.659
	JPN	1.029		JPN	2.375
	TWN	0.692		TWN	2.613
	KOR	0.825		KOR	2.637
	MYS	0.695		MYS	2.590
	THA	0.742		THA	2.360
	IDN	1.173		IDN	3.002
	CHN	0.809		CHN	1.973
	PHL	0.790		PHL	2.508
Transaction costs γ_i	USA	0.487	aggregate GO / VA ratio	USA	2.666
by country	SGP	0.244		SGP	4.676
	JPN	0.442		JPN	2.773
	TWN	0.285		TWN	3.951
	KOR	0.358		KOR	3.359
	THA	0.325		THA	3.452
	MYS	0.383		MYS	3.101
	IDN	0.740		IDN	2.035
	CHN	0.343		CHN	3.110
	PHL	0.464		PHL	2.629
Simple average border cost	All	50%	Trade/output ratio	All	26%

Table 3: Fragmentation indexes: model vs. data

Index	D		D*		N		N*		M share	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
USA	2.252	2.680	0.208	0.121	2.422	2.804	0.316	0.089	0.657	0.387
SGP	2.558	4.254	1.062	0.234	2.656	4.228	0.535	0.143	0.672	0.231
JPN	2.370	2.683	0.278	0.132	2.364	2.894	0.182	0.133	0.558	0.317
TWN	2.567	3.778	0.699	0.227	2.542	3.832	0.423	0.173	0.644	0.258
KOR	2.537	3.256	0.503	0.240	2.536	3.329	0.358	0.174	0.632	0.310
MYS	2.755	3.471	0.912	0.264	2.515	3.449	0.501	0.155	0.704	0.272
THA	2.373	2.951	0.591	0.252	2.443	3.001	0.400	0.164	0.593	0.265
IDN	2.509	2.893	0.521	0.524	2.109	2.234	0.244	0.104	0.823	0.523
CHN	2.707	2.785	0.278	0.101	2.620	3.160	0.183	0.189	0.472	0.197
PHL	2.006	2.859	0.483	0.329	2.216	2.692	0.307	0.137	0.677	0.315
Correl. with data		0.443		0.321		0.860		0.159		0.726

5.4 Counterfactual simulations

East-Asian economies have been the setting for tremendous changes in recent decades. Arguably the most significant changes are the increased fragmentation of production, China's opening to international trade and its subsequent rapid economic growth. In our theory these phenomena can certainly be related, as China's opening to trade could have facilitated fragmentation along chains in which it is now involved. Rapid economic growth may be associated with trade-related increases in productivity, but multi-factor productivity growth not specifically related to trade might also have been important. With a calibrated model at hand, we can now examine various counterfactual simulations to study how structural changes would affect economic outcomes such as output, trade, welfare and the fragmentation of production.

We see at least four experiments that would provide interesting insight into the reorganization of supply chains in Asia:

- Counterfactual 1): Trade costs have fallen significantly over the past decades and their reduction is cited as the most likely source of the increased fragmentation of production in Asia. Trade costs may decline even further in the near future, as there is still room to improve trade agreements, especially on a multi-lateral basis (Baldwin, 2008). We model this structural change with a 10% reduction in cross-border trade costs.

- Counterfactual 2): Arguably the most dramatic change in the Asian economy over the past two decades has been the very high rates of growth in the Chinese economy, along with its opening to trade. With GDP growth rates reaching 10%, the Chinese economy is inducing very large changes in how production in Asia is organized. To examine the role of China in light of our model, we shock the productivity parameter there. We first simulate a 10% productivity increase in China. Formally, this corresponds to a 10% increase in $T_{CHN}^{\frac{1}{\xi}}$.

- Counterfactual 3): We simulate a reduction in the transaction costs γ_{CHN} for China. This scenario could be used to understand growing transparency in contractual disputes, for example. Reduced transaction costs should encourage relatively more domestic outsourcing in China, and raise the share of domestic value added in production/exports.[39]

- Counterfactual 4): Finally, we consider a bilateral trade cost reduction. This allows us to offer a local estimate of the trade elasticity. In particular we are interested in a quantitative evaluation of the claim in Proposition *4i*, that the elasticity of final goods trade to trade cost changes is larger in the presence of fragmentation. In order to do this we reduce trade costs between China and the US.

5.4.1 Reduction in trade costs

Our first scenario is a 10% reduction in international trade costs τ. These results are reported in Table 4. In addition to the statistics already introduced we report the intermediate share of exports "M share." The distribution of welfare gains across countries (column 2) suggests that countries with larger economies benefit least. This likely reflects the fact that these larger economies are less exposed to changes in international trade costs. Compared to the Arkolakis et al (2012) formula, gains from trade with international production chains tend to be larger. Moreover, our simulations indicate that downstream countries gain relatively more than upstream countries, relative to the ACR formula (we find a -0.54 correlation between index DX and the ratio of gains from trade: simulated vs. ACR).

The general increase in D^* and N^* is consistent with more international fragmentation. On average, the reduction in international trade costs means that production crosses more borders, both upstream of countries' production N^* and downstream of it D^*. Greater international fragmentation can also be seen in the decrease in the VAX ratio (lower value-added content in trade). Interestingly, a falling intermediate share of trade indicates that final goods trade is more sensitive than intermediate goods trade. A consequence is that countries tend to move downstream as trade costs decrease, as reflected in the negative change in export-weighted upstreamness, DX. This overall decrease in upstreamness is consistent with the changes described in Fally (2012, Figure 4).

[39]Kee and Tang (2013) and Li and Liu (2014) provide evidence that the share of domestic value added in Chinese exports has been growing over time.

Table 4: Counterfactual 1): 10% decrease in border trade costs

(10 x change)	Welfare	$\frac{1}{\xi}\log\pi_{ii}^{F}$	DX	D	D*	N	N*	M share	VAX
USA	0.094	0.070	-0.517	-0.083	0.057	-0.051	0.036	-0.121	-0.073
SGP	0.464	0.426	-0.598	-0.335	-0.029	-0.160	-0.005	-0.080	-0.076
JPN	0.139	0.094	-0.546	-0.086	0.052	-0.022	0.048	-0.116	-0.074
TWN	0.381	0.307	-0.624	-0.083	0.001	-0.007	-0.020	-0.092	-0.075
KOR	0.313	0.236	-0.627	-0.027	0.032	-0.014	-0.001	-0.108	-0.072
MYS	0.427	0.373	-0.553	-0.161	-0.013	-0.065	-0.016	-0.092	-0.076
THA	0.411	0.343	-0.508	-0.132	-0.008	-0.065	-0.014	-0.098	-0.078
IDN	0.407	0.298	-0.458	-0.038	0.025	-0.050	0.006	-0.136	-0.056
CHN	0.188	0.110	-0.475	-0.059	0.024	0.041	0.050	-0.087	-0.077
PHL	0.455	0.405	-0.485	-0.204	-0.023	-0.101	-0.003	-0.109	-0.074

5.4.2 Increasing productivity in China

Table 5 reports results from a 10% shock to productivity in China, $T_{CHN}^{\frac{1}{\xi}}$. This is a uniform shock that improves productivity at all points in the chain. Our interest is in seeing how such shocks affect China's relative position in chains, and the degree to which such shocks spill over into other countries.

Table 5: Counterfactual 2): 10% increase in productivity in China

(10 x change)	Welfare	$\frac{1}{\xi}\log\pi_{ii}^{F}$	DX	D	D*	N	N*	M share	VAX
CHN	0.968	-0.047	-0.289	-0.190	-0.052	-0.112	-0.009	-0.047	-0.002
USA	0.014	0.011	0.188	0.052	0.028	0.012	0.002	0.046	0.003
SGP	0.036	0.040	0.191	0.154	0.029	0.079	-0.012	0.026	0.005
JPN	0.015	0.012	0.164	0.066	0.027	0.019	-0.001	0.038	0.004
TWN	0.030	0.029	0.174	0.146	0.029	0.065	-0.012	0.027	0.005
KOR	0.025	0.023	0.173	0.135	0.033	0.049	-0.010	0.031	0.005
MYS	0.035	0.037	0.148	0.143	0.026	0.051	-0.013	0.022	0.005
THA	0.032	0.034	0.140	0.135	0.030	0.041	-0.013	0.026	0.004
IDN	0.040	0.034	0.176	0.188	0.049	0.039	-0.010	0.034	0.005
PHL	0.038	0.039	0.145	0.149	0.031	0.039	-0.012	0.024	0.005

The welfare changes reported in column 2 show that the vast majority of the welfare gains accrue to China, which sees a nearly 10% increase in welfare from a 10% productivity shock. The gains elsewhere are limited, and reasonably similar across countries. The most notable changes have to do with changes in relative position in supply chains. Changes in China's DX, D and D^* indices indicate that the technology shock moves Chinese production significantly closer to final demand, while the other countries move upstream. China's move downstream can also be seen in columns 8 (M share), which shows a reduction in the intermediate share of China's exports.

5.4.3 Reduced transaction costs in China

The welfare effects of a 10% reduction in transaction costs in China are similar to those that were calculated for the Chinese productivity shock. China's welfare rises by almost 10%. The welfare gains of the countries outside of China are, however, somewhat larger than in the case of the Chinese productivity shock. In contrast with the previous counterfactual, the shock to internal transaction costs moves China's production upstream, as indicated by the movements in D^*, D and N^*. Similarly, the increase in N reflects greater fragmentation of production within China. Other countries also move upstream because falling Chinese transaction costs lead to longer chains. In broad terms, a move upstream by China is consistent with the evidence presented in Kee and Tang (2013), who find that Chinese exporters have been shifting their purchases of inputs from foreign to domestic sources.

Table 6: Counterfactual 3): 10% decrease in Chinese transaction costs γ_i

(10 x change)	Welfare	DX	D	D*	N	N*	M share	VAX
CHN	0.953	1.609	3.983	0.033	3.490	-0.027	0.119	0.055
USA	0.026	0.341	0.098	0.025	0.143	0.016	0.030	0.008
SGP	0.050	0.187	0.166	0.016	0.116	-0.003	0.013	0.002
JPN	0.027	0.256	0.096	0.022	0.164	0.015	0.025	0.005
TWN	0.042	0.195	0.159	0.019	0.153	0.003	0.016	0.002
KOR	0.038	0.210	0.161	0.020	0.163	0.007	0.015	0.002
MYS	0.047	0.192	0.179	0.017	0.125	-0.001	0.013	0.002
THA	0.045	0.182	0.169	0.019	0.133	0.002	0.013	0.001
IDN	0.048	0.372	0.363	0.033	0.107	0.001	0.017	0.003
PHL	0.050	0.186	0.194	0.011	0.099	0.001	0.003	-0.001

5.4.4 Reduced trade costs between China and the US

As noted above and in Yi (2010), international fragmentation raises the elasticity of trade to trade costs. In order to explore the quantitative magnitude of this effect we shock trade costs for a single country pair (US and China) and measure trade responses. We compare results in our model to those in a version of the same model, except that no trade in intermediates is allowed in the comparison model. Both models are calibrated in the same manner. We conduct one set of exercises using our baseline calibration, and one that does the same for a lower level

Table 7: Counterfactual 4): 10% bilateral decrease in US-China trade costs

Counterfactual 4A: Baseline calibration with $\tau = 1.5$

	Importer-exporter pair	With cross-border fragmetation		No fragmentation
		All trade	*Final goods*	*Final goods*
$\Delta \log \pi_{ni}$	USA-CHN	0.138	0.151	0.119
	CHN-USA	0.087	0.120	0.090
Trade cost elasticities	USA-CHN	4.399	4.828	3.893
	CHN-USA	3.609	4.071	3.962

Counterfactual 4B: Starting from twice lower trade costs, $\tau = 1.25$

	Importer-exporter pair	With cross-border fragmetation		No fragmentation
		All trade	*Final goods*	*Final goods*
$\Delta \log \pi_{ni}$	USA-CHN	0.095	0.101	0.066
	CHN-USA	0.053	0.066	0.040
Trade cost elasticities	USA-CHN	5.273	5.601	3.794
	CHN-USA	4.265	3.997	3.895

of trade costs. In each exercise we track two outcomes in our model (total trade and trade in final goods alone), and track final goods trade in the model with no trade in intermediates. We report the percentage change in trade flows in each direction, and a trade elasticity measured as $\frac{\Delta \log \frac{\pi_{ni}}{\pi_{nn}}}{\Delta \log \tau_{ni}}$. We hold wages fixed in these simulations because this partial elasticity is conceptually closer to empirical estimates of the trade elasticity than its general equilibrium counterpart.

In the baseline calibration we find somewhat larger trade responses in final goods when fragmentation is allowed. This is consistent qualitatively with the Proposition 4, which demonstrated magnification of the final goods trade elasticity in a two-country version of the model. When we look at trade responses using trade that includes intermediates, the change in the calculated trade elasticity is ambiguous. These results show that the qualitative predictions of magnification in the theory have relatively little quantitative importance at existing levels of implied trade costs.

We impose lower levels of trade costs ($\tau = 1.25$), recalibrate as before and again simulate reductions in τ. In this case, magnification through fragmentation is quantitatively more important. In both directions the elasticity of total trade and trade in final goods to changing trade costs are larger in the fragmentation model than in the model without fragmentation.

This is also consistent with Proposition 4, which showed larger trade responses at lower levels of trade costs. These exercises are similar to those conducted in Johnson and Moxnes (2013), who also found relatively small magnification effects at current levels of trade but larger effects when trade costs were lower.

5.5 Calibrated model with assembly

As an extension in section 4.4 we added a capstone assembly sector to the model. Our quantitative investigation of this extension focuses on the implied changes in the parameterization of the model that occur in this new structure. We treat assembly as a discrete additional stage and rework our indices (D, DX, N, etc.) to incorporate it. As before, we calibrate coordination costs, transaction costs, productivity and trade costs ($\tau = 1.36$) so that the model replicates the data for upstreamness (variable DX_i), gross-output-to-value-added ratios, wages and the overall trade-to-gross-output ratio. In addition, we assume that each final good has two components with equal cost shares in assembly ($\eta_{ik} = 0.5$) and we assume that labor costs account for 10 percent of assembly costs ($\beta = 0.1$). Conditional on our estimate of trade costs, this value is chosen to be sufficiently low so that no country imports both components used to assemble a variety and then re-exports the final good.

Table 8: Calibration with Assembly

	Baseline calibration			Assembly calibration			
Country	$\bar{A}_i$	$\bar{\theta}_i$	γ_i	$\bar{A}_i$	$\bar{\theta}_i$	γ_i	$\frac{X_i^{asm}}{\sum_i X_i^{asm}}$
US	1.000	1.172	0.490	1.000	0.893	0.511	0.000
Singapore	4.783	0.619	0.262	5.122	0.436	0.202	0.000
Japan	1.627	1.044	0.446	1.708	0.731	0.430	0.000
Taiwan	4.933	0.703	0.289	5.142	0.533	0.254	0.000
Korea	4.442	0.812	0.359	4.535	0.631	0.345	0.009
Malaysia	14.585	0.731	0.333	15.018	0.572	0.318	0.001
Thailand	17.993	0.761	0.393	16.563	0.659	0.461	0.019
Indonesia	17.850	1.159	0.726	3.519	2.236	2.671	0.050
China	25.432	0.793	0.339	14.228	1.121	0.691	0.884
Philippines	41.306	0.807	0.480	14.095	1.402	1.551	0.036

The structural parameters from the calibrated assembly model are reported in Table 8 along with each country's exports of assembled goods (X_i^{asm}) as a share of the world total. Cross-country homogeneity of labor productivity in the assembly activity means that the assembly model naturally locates assembly activities in low-wage countries. Since assembly is the most downstream sector, fitting the DX_i in the new structure means that $\bar{\theta}_i$'s must fall in rich countries and rise in poor countries to offset the location of an additional downstream sector in the poor countries. Thus, rich countries are more downstream within production chains while poor countries are more upstream than in the baseline calibration. Calibrated transaction costs remain broadly similar for the rich countries, but γ_i rises substantially in the poor countries in the new calibration in order to offset the increase in $\bar{\theta}_i$. In this calibration both transaction costs and coordination costs are higher in poor countries than in rich countries. One implication is that there is less need for the productivity distribution to generate wage differences. The calibrated values of poor countries' $\bar{A}_i$'s fall as a result. The final column shows that only the poor countries export assembled goods in this calibration, with China playing a dominant role.

6 Concluding remarks

Recent empirical work has documented sizable differences in supply chain length. In this paper we attempt to explain such variation in an integrated framework that links the internalization decisions of firms within a supply chain to the organization of the chain across countries. We develop a continuous firm representation of the optimal organization of a multi-country supply chain, with an endogenous allocation of tasks across firms and countries. We derive formal and intuitive representations of the gains from fragmentation within a chain and relate these to the implicit price of tasks and the price of the final good.

In this Coasian setting, we show that the same parameters that shape the boundaries of firms also determine comparative advantage within international supply chains. Conditional on participation in a supply chain, the lower a country's coordination costs the more downstream it will be. Low within-firm coordination costs also imply an ability to host larger firms. By contrast, countries with high transaction costs will tend to participate downstream because their disadvantages can only be offset in chains for which they have low within-firm-coordination costs and larger firms.

In order to link the model to the prominent literature on the welfare gains of trade we use a conventional Ricardian framework to produce a general equilibrium model with multiple chains, and with exogenous productivity shocks *across* chains. We derive implications for trade elasticities and welfare, relative to standard theoretical benchmarks (Arkolakis et al 2012). Among a number of theoretical results we show that the elasticity of final goods trade to trade costs is larger in the presence of fragmentation. Relative to the Arkolakis et al (2012) formula without fragmentation, we show that welfare effects are smaller for upstream countries and larger for downstream countries in the presence of fragmentation.

To illustrate the quantitative implications of the model we conduct a calibration exercise with counterfactual simulations. In our model, the Coasian structural parameters determine the gross-output-to-value added ratio, a fact that facilitates calibration. We shock international trade costs and find numerical evidence that is consistent with our theoretical results. We find that shocks to Chinese productivity and to the coordination cost parameter in China to highlight different implications for welfare, spillover to other countries and specialization along production chains. In order to better tie the model to available data we develop an extension of the calibrated model with an explicit representation of assembly activities to better formalize the participation of China in international production chains.

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Mathematical Appendix

Proofs for Section 3.1: within-country fragmentation

FOCs: The first-order conditions of this planning program correspond to equations (13) and (14):

$$\begin{aligned} \text{For } s_{fi}: &\quad e^{\gamma_i f} w_i c_i'(s_{if}) \;=\; \lambda_i \\ \text{For } F_i: &\quad e^{\gamma_i F_i} w_i c_i(s_{i,F_i}) \;+\; e^{\gamma_i F_i} P_i^M \gamma_i \;=\; s_{i,F_i}\,\lambda_i \end{aligned}$$

Using our parameterization of the cost function, the first-order condition for s_{if} can be rewritten:

$$e^{\gamma_i f} a_i w_i s_{if}^{\theta_i} = \lambda_i$$

which yields:

$$s_{if} = \left(\frac{\lambda_i}{a_i w_i}\right)^{\frac{1}{\theta_i}} e^{-\frac{\gamma_i f}{\theta_i}}$$

By combining the first-order condition in F_i and the first-order condition in s_{fi}, we obtain:

$$e^{\gamma_i F_i} a_i w_i \frac{s_{i,F_i}^{\theta_i+1}}{\theta_i+1} \;+\; e^{\gamma_i F_i} P_i^M \gamma_i \;=\; s_{i,F_i} \,.\, e^{\gamma_i F_i} a_i w_i s_{i,F_i}^{\theta_i}$$

which can be simplified into:

$$\tfrac{w_i\, a_i\, \theta_i}{\theta_i+1} s_{i,F_i}^{\theta_i+1} = \gamma_i P_i^M$$

and thus:

$$s_{i,F_i} = \left[\frac{(\theta_i+1)\gamma_i}{\theta_i} \frac{P_i^M}{a_i w_i}\right]^{\frac{1}{\theta_i+1}}$$

Lagrangian multiplier: It is the solution of:

$$\int_0^{F_i} s_{if} df = S_i$$

where s_{if} and F_i functions of the Lagrangian multiplier as shown above. The left-hand side can be rewritten:

$$\begin{aligned} \int_0^{F_i} s_{if} df &= \left(\tfrac{\lambda_i}{w_i a_i}\right)^{\frac{1}{\theta_i}} \int_0^{F_i} e^{-\frac{\gamma_i f}{\theta_i}} df \\ &= \frac{\theta_i}{\gamma_i}\left(\tfrac{\lambda_i}{w_i a_i}\right)^{\frac{1}{\theta_i}} \left[1 - e^{-\frac{\gamma_i F_i}{\theta_i}}\right] \\ &= \frac{\theta_i}{\gamma_i}\left(\tfrac{\lambda_i}{w_i a_i}\right)^{\frac{1}{\theta_i}} \left[1 - \left(\tfrac{\lambda_i}{w_i a_i}\right)^{-\frac{1}{\theta_i}} s_{i,F_i}\right] \\ &= \frac{\theta_i}{\gamma_i}\left(\tfrac{\lambda_i}{w_i a_i}\right)^{\frac{1}{\theta_i}} \;-\; \frac{\theta_i s_{i,F_i}}{\gamma_i} \end{aligned}$$

We obtain the following solution in λ_i such that the expression above equals S_i:

$$\lambda_i = w_i a_i \left[\frac{\gamma_i S_i}{\theta_i} + \left(\frac{(\theta_i+1)\gamma_i}{\theta_i} \frac{P_i^M}{a_i w_i} \right)^{\frac{1}{\theta_i+1}} \right]^{\theta_i}$$

Final price: Expression for P_i:

$$\begin{aligned}
P_i &= \int_{f=0}^{F_i} e^{\gamma_i f} w_i c_i(s_{if}) df \ + \ e^{\gamma_i F_i} P_i^M \\
&= \int_{f=0}^{F_i} e^{\gamma_i f} \frac{w_i a_i s_{if}^{\theta_i+1}}{\theta_i+1} df \ + \ e^{\gamma_i F_i} P_i^M \\
&= \frac{w_i a_i}{\theta_i+1} \left(\tfrac{\lambda_i}{w_i a_i}\right)^{\frac{\theta_i+1}{\theta_i}} \int_{f=0}^{F_i} e^{-\frac{\gamma_i f}{\theta_i}} df \ + \ e^{\gamma_i F_i} P_i^M \\
&= \frac{w_i a_i}{\gamma_i} \tfrac{\theta_i}{\theta_i+1} \left(\tfrac{\lambda_i}{w_i a_i}\right)^{\frac{\theta_i+1}{\theta_i}} \left[1 - e^{-\frac{\gamma_i F_i}{\theta_i}}\right] \ + \ e^{\gamma_i F_i} P_i^M \\
&= \frac{w_i a_i}{\gamma_i} \tfrac{\theta_i}{\theta_i+1} \left(\tfrac{\lambda_i}{w_i a_i}\right)^{\frac{\theta_i+1}{\theta_i}} - \frac{w_i a_i}{\gamma_i} \tfrac{\theta_i}{\theta_i+1} \left(\tfrac{\lambda_i}{w_i a_i}\right)^{\frac{\theta_i+1}{\theta_i}} e^{-\frac{\gamma_i F_i}{\theta_i}} \ + \ \frac{w_i a_i}{\gamma_i} \left(\frac{\gamma_i P_i^M}{w_i a_i}\right) e^{\gamma_i F_i} \\
&= \frac{w_i a_i}{\gamma_i} \tfrac{\theta_i}{\theta_i+1} \left(\tfrac{\lambda_i}{w_i a_i}\right)^{\frac{\theta_i+1}{\theta_i}} - \frac{1}{\gamma_i} \tfrac{\theta_i}{\theta_i+1} \lambda_i s_{i,F_i} \ + \ \frac{wa}{\gamma} \tfrac{\theta}{\theta+1} s_{i,F_i}^{\theta_i+1} e^{\gamma_i F_i} \\
&= \frac{w_i a_i}{\gamma_i} \tfrac{\theta_i}{\theta_i+1} \left(\tfrac{\lambda_i}{w_i a_i}\right)^{\frac{\theta_i+1}{\theta_i}} - \frac{1}{\gamma_i} \tfrac{\theta_i}{\theta_i+1} \left[\lambda_i s_{i,F_i} - w_i a_i s_{i,F_i}^{\theta_i+1} e^{\gamma_i F_i}\right] \\
&= \frac{w_i a_i}{\gamma_i} \tfrac{\theta_i}{\theta_i+1} \left(\tfrac{\lambda_i}{w_i a_i}\right)^{\frac{\theta_i+1}{\theta_i}} \ + \ 0
\end{aligned}$$

Using the expression above for λ_i, we obtain equations (20) and (21) in the text:

$$P_i = \left[\frac{S_i}{\theta_i+1} (A_i w_i)^{\frac{1}{\theta_i+1}} + \left(P_i^M\right)^{\frac{1}{\theta_i+1}} \right]^{\theta_i+1}$$

with

$$A_i = a_i \left(\gamma_i \tfrac{\theta_i+1}{\theta_i}\right)^{\theta_i}$$

It is also useful to note that:

$$\lambda_i = (w_i A_i)^{\frac{1}{\theta_i+1}} (P_i)^{\frac{\theta_i}{\theta_i+1}} \tag{52}$$

Labor demand: Each unit of last-stage good produced in country i generates the following demand for labor in i:

$$w_i L_i^D = \frac{S_i}{\theta_i+1} (A_i w_i)^{\frac{1}{\theta_i+1}} \left[\frac{S_i}{\theta_i+1} (A_i w_i)^{\frac{1}{\theta_i+1}} + \left(P_i^M\right)^{\frac{1}{\theta_i+1}} \right]^{\theta_i} \tag{53}$$

Prices along the chain: To obtain a simple expression for the value-added-to-gross-output ratio $\frac{c_i(s_{if})}{p_{if}}$, the first step is to compute p_{if} is the price along the chain.

$$
\begin{aligned}
p_{if} &= \int_{f'=f}^{F_i} e^{\gamma_i(f'-f)} c(s_{if'})df' \;+\; e^{\gamma_i(F_i-f)} P_i^M \\
&= \frac{w_i a_i}{\theta_i+1} \int_{f'=f}^{F_i} e^{\gamma_i(f'-f)}\, s_{if'}^{\theta_i+1} df' \;+\; e^{\gamma_i(F_i-f)} P_i^M \\
&= \frac{w_i a_i}{\theta_i+1} e^{\gamma_i(F_i-f)} \int_{f'=f}^{F_i} e^{-\gamma_i(F_i-f')}\, s_{if'}^{\theta_i+1} df' \;+\; e^{\gamma_i(F_i-f)} P_i^M \\
&= \frac{w_i a_i}{\theta_i+1} e^{\gamma_i(F_i-f)} s_{i,F_i}^{\theta_i+1} \int_{f'=f}^{F_i} e^{-\gamma_i(F_i-f')} e^{\gamma_i\left(\frac{\theta_i+1}{\theta_i}\right)(F_i-f')} df' \;+\; e^{\gamma_i(F_i-f)} P_i^M \\
&= \frac{w_i a_i}{\theta_i+1} e^{\gamma_i(F_i-f)} s_{i,F_i}^{\theta_i+1} \int_{f'=f}^{F_i} e^{\frac{\gamma_i(F-f')}{\theta_i}} df' \;+\; e^{\gamma_i(F_i-f)} P_i^M \\
&= \frac{w_i a_i \theta_i}{(\theta_i+1)\gamma_i} e^{\gamma_i(F_i-f)} s_{i,F_i}^{\theta+1} \left[e^{\frac{\gamma_i(F_i-f)}{\theta}} - 1 \right] \;+\; e^{\gamma_i(F_i-f)} P_i^M \\
&= \frac{w_i a_i \theta_i}{(\theta_i+1)\gamma_i} s_{if}^{\theta_i+1} \;-\; \frac{e^{\gamma_i(F_i-f)}}{\gamma_i} \left[\frac{w_i a_i \theta_i}{\theta_i+1} s_{i,F_i}^{\theta_i+1} \;-\; \gamma_i P_i^M \right] \\
&= \frac{w_i a_i \theta_i}{(\theta_i+1)\gamma_i} s_{if}^{\theta_i+1} \;-\; 0 \\
&= \frac{\theta_i}{\gamma_i}\, w_i c_i(s_{if})
\end{aligned}
$$

Hence the gross-output-to-value-added ratio at the firm level is:

$$
\frac{p_{if}}{c_i(s_{if})} = \frac{\theta_i}{\gamma_i}
$$

We also obtain the same expression for the aggregate gross-output-to-value-added ratio. If we define gross output as the total value of all transactions:

$$
GO_i = \int_0^{F_i} q_{if} p_{if} df
$$

we obtain:

$$
\frac{GO_i}{VA_i} = \frac{\int_0^{F_i} q_{if} p_{if} df}{\int_0^{F_i} q_{if} w_i c_i(s_{if}) df} = \frac{\int_0^{F_i} \frac{\theta_i}{\gamma_i} q_{if} w_i c_i(s_{if}) df}{\int_0^{F_i} q_{if} w_i c_i(s_{if}) df} = \frac{\theta_i}{\gamma_i}
$$

Proof of Proposition 1

To simplify the exposition, we index countries by n, with $n = 1$ referring to the most downstream country and $n = N$ the most upstream country. The goal is to minimize:

$$
\min P_1 \tag{54}
$$

under the constraints:

$$P_{n+1} = \widetilde{P}_n(S_n, \tau P_{n+1}) \qquad \text{and} \qquad \sum_{n=n}^{N} S_n = 1$$

with:

$$\widetilde{P}_n(S, P^M) = \left[\frac{S}{\theta_n+1}(A_n w_n)^{\frac{1}{\theta_n+1}} + \left(P^M\right)^{\frac{1}{\theta_n+1}}\right]^{\theta_n+1}$$

Under which condition can country n be downstream from country $n+1$? Let us take as given the price in country $n+2$ and consider the following function:

$$m(x)^{\theta_n+1} = \widetilde{P}_n(S_n - x, \tau \widetilde{P}_n(S_{n+1}+x, \tau P_{n+2}))$$

This function $m(x)$ indicates by how much the price of output in n will increase if we shift a measure x of tasks from country n to country $n+1$.

$$m(x) = \frac{(S_n - x)}{\theta_n+1}(A_n w_n)^{\frac{1}{\theta_n+1}} + \left[\frac{(S_{n+1}+x)}{\theta_{n+1}+1}(A_{n+1}w_{n+1})^{\frac{1}{\theta_{n+1}+1}} + (\tau P_{n+2})^{\frac{1}{\theta_{n+1}+1}}\right]^{\frac{\theta_{n+1}+1}{\theta_n+1}}$$

If we are at equilibrium, the function $m(x)$ must be at its minimum at $x=0$. The first-order condition imply that $m'(x)=0$. We obtain that:

$$m'(x) = -\frac{(A_n w_n)^{\frac{1}{\theta_n+1}}}{\theta_n+1} + \frac{(A_{n+1}w_{n+1})^{\frac{1}{\theta_{n+1}+1}}}{\theta_n+1}\left[\frac{(S_{n+1}+x)}{\theta_{n+1}+1}(A_{n+1}w_{n+1})^{\frac{1}{\theta_{n+1}+1}} + (\tau P_{n+2})^{\frac{1}{\theta_{n+1}+1}}\right]^{\frac{\theta_{n+1}-\theta_n}{\theta_n+1}} \tag{55}$$

must equal zero at $x=0$.

More importantly, to prove Proposition 1, one needs to examine the second order condition, which imposes $m''(x) > 0$. If $m''(x)$ were negative, $x=0$ would not be a local minimum and it would be more efficient to shift some tasks to either country n or $n+1$.

As one can see in equation (55), the right-hand-side term is increasing in x (i.e. $m''(x) > 0$) only if the exponent $\frac{\theta_{n+1}-\theta_n}{\theta_n+1}$ is positive. This proves that we must have $\theta_{n+1} > \theta_n$ at equilibrium.

Finally, it is not difficult to verify that two consecutive countries cannot have the same $\theta_n = \theta_{n+1}$ as long as we have non-zero trade costs $\tau - 1 > 0$.

Other proofs for Section 3.2

Prices along the chain: Using again equation (55), the first-order condition $m'(0) = 0$ implies:

$$\begin{aligned}
\frac{(A_n w_n)^{\frac{1}{\theta_n+1}}}{\theta_n+1} &= \frac{(A_{n+1}w_{n+1})^{\frac{1}{\theta_{n+1}+1}}}{\theta_n+1}\left[\frac{S_{n+1}}{\theta_{n+1}+1}(A_{n+1}w_{n+1})^{\frac{1}{\theta_{n+1}+1}} + (\tau P_{n+2})^{\frac{1}{\theta_{n+1}+1}}\right]^{\frac{\theta_{n+1}-\theta_n}{\theta_n+1}} \\
&= \frac{(A_{n+1}w_{n+1})^{\frac{1}{\theta_{n+1}+1}}}{\theta_n+1}\left(P_{n+1}\right)^{\frac{1}{\theta_n+1}\cdot\frac{\theta_{n+1}-\theta_n}{\theta_{n+1}+1}}
\end{aligned}$$

This yields expression (32) for the price of goods sold by country $n+1$:

$$\tau P_{n+1} = (A_n w_n)^{\frac{\theta_{n+1}+1}{\theta_{n+1}-\theta_n}} (\tau A_{n+1} w_{n+1})^{-\frac{\theta_n+1}{\theta_{n+1}-\theta_n}}$$

For country i, this gives:

$$P_n = (A_{n-1} w_{n-1}/\tau)^{\frac{\theta_n+1}{\theta_n-\theta_{n-1}}} (A_n w_n)^{-\frac{\theta_{n-1}+1}{\theta_n-\theta_{n-1}}}$$

Allocation of tasks across countries: The range of tasks performed by country i can then be obtained as:

$$\begin{aligned}
\frac{S_n}{\theta_n+1} &= (A_n w_n)^{-\frac{1}{\theta_n+1}} \left[P_n^{\frac{1}{\theta_n+1}} - (\tau P_{n+1})^{\frac{1}{\theta_n+1}} \right] \\
&= (A_n w_n)^{-\frac{1}{\theta_n+1}} \left[(A_{n-1} w_{n-1}/\tau)^{\frac{1}{\theta_n-\theta_{n-1}}} (A_n w_n)^{-\frac{\theta_{n-1}+1}{(\theta_n+1)(\theta_n-\theta_{n-1})}} - (A_n w_n)^{\frac{\theta_{n+1}+1}{(\theta_n+1)(\theta_{n+1}-\theta_n)}} (\tau A_{n+1} w_{n+1})^{-\frac{1}{\theta_{n+1}-\theta_n}} \right]
\end{aligned}$$

which can be simplified into expression (32) given in the text:

$$\frac{S_n}{\theta_n+1} = \left(\frac{A_{n-1} w_{n-1}}{\tau A_n w_n} \right)^{\frac{1}{\theta_n-\theta_{n-1}}} - \left(\frac{A_n w_n}{\tau A_{n+1} w_{n+1}} \right)^{\frac{1}{\theta_{n+1}-\theta_n}}$$

For the last country N in the chain, we obtain:

$$\frac{S_N}{\theta_N+1} = \left(\frac{A_{N-1} w_{N-1}}{\tau A_N w_N} \right)^{\frac{1}{\theta_N-\theta_{N-1}}}$$

Finally, the range of tasks performed by the last country in the chain is:

$$\begin{aligned}
S_1 =& 1 - \sum_{n=2}^{N-1} S_n \\
=& 1 - (\theta_N + 1) \left(\frac{A_{N-1} w_{N-1}}{\tau A_N w_N} \right)^{\frac{1}{\theta_N-\theta_{N-1}}} - \sum_{n=2}^{N-1} (\theta_n + 1) \left[\left(\frac{A_{n-1} w_{n-1}}{\tau A_n w_n} \right)^{\frac{1}{\theta_n-\theta_{n-1}}} - \left(\frac{A_n w_n}{\tau A_{n+1} w_{n+1}} \right)^{\frac{1}{\theta_{n+1}-\theta_n}} \right] \\
=& 1 - \sum_{n=2}^{N} (\theta_n + 1) \left(\frac{A_{n-1} w_{n-1}}{\tau A_n w_n} \right)^{\frac{1}{\theta_n-\theta_{n-1}}} + \sum_{n=2}^{N-1} (\theta_n + 1) \left(\frac{A_n w_n}{\tau A_{n+1} w_{n+1}} \right)^{\frac{1}{\theta_{n+1}-\theta_n}} \\
=& 1 - \sum_{n=1}^{N-1} (\theta_{n+1} + 1) \left(\frac{A_n w_n}{\tau A_{n+1} w_{n+1}} \right)^{\frac{1}{\theta_{n+1}-\theta_n}} + \sum_{n=2}^{N-1} (\theta_n + 1) \left(\frac{A_n w_n}{\tau A_{n+1} w_{n+1}} \right)^{\frac{1}{\theta_{n+1}-\theta_n}} \\
=& 1 - (\theta_1 + 1) \left(\frac{A_1 w_1}{\tau A_2 w_2} \right)^{\frac{1}{\theta_2-\theta_1}} - \sum_{n=1}^{N-1} (\theta_{n+1} - \theta_n) \left(\frac{A_n w_n}{\tau A_{n+1} w_{n+1}} \right)^{\frac{1}{\theta_{n+1}-\theta_n}}
\end{aligned}$$

Final good price: Using the above expressions for S_1 and P_2, we obtain the price of the final good:

$$
\begin{aligned}
P_1 &= \left[\frac{S_1}{\theta_1+1}(A_1w_1)^{\frac{1}{\theta_1+1}} + (\tau P_2)^{\frac{1}{\theta_1+1}}\right]^{\theta_1+1} \\
&= \left[\frac{S_1}{\theta_1+1}(A_1w_1)^{\frac{1}{\theta_1+1}} + (A_1w_1)^{\frac{\theta_2+1}{(\theta_1+1)(\theta_2-\theta_1)}}(\tau A_2w_2)^{-\frac{1}{\theta_2-\theta_1}}\right]^{\theta_1+1} \\
&= \frac{A_1w_1}{(\theta_1+1)^{\theta_1+1}}\left[S_1 + (\theta_1+1)\left(\frac{A_1w_1}{\tau A_2w_2}\right)^{\frac{1}{\theta_2-\theta_1}}\right]^{\theta_1+1} \\
&= \frac{A_1w_1}{(\theta_1+1)^{\theta_1+1}}\left[1 - \sum_{n=1}^{N-1}(\theta_{n+1}-\theta_n)\left(\frac{A_nw_n}{\tau A_{n+1}w_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}}\right]^{\theta_1+1} \\
&= \frac{A_1w_1}{(\theta_1+1)^{\theta_1+1}}\Theta(\mathbf{wA},\tau)
\end{aligned}
$$

This corresponds to expression (34) in the text with the term in Θ reflecting gains from fragmentation:

$$
\Theta(\mathbf{wA},\tau) = \left[1 - \sum_{n=1}^{N-1}(\theta_{n+1}-\theta_n)\left(\frac{w_nA_n}{\tau w_{n+1}A_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}}\right]^{\theta_1+1}
$$

Demand for labor: By the envelope theorem, demand for labor in upstream countries can be obtained by:

$$
\frac{l_nw_n}{P_1} = \frac{d\log P_1}{d\log w_n} = \frac{d\log \Theta}{d\log w_n}
$$

This gives expression (35) in the text:

$$
\frac{l_nw_n}{P_1} = \frac{\left(\frac{w_{n-1}A_{n-1}}{\tau w_nA_n}\right)^{\frac{1}{\theta_n-\theta_{n-1}}} - \left(\frac{w_nA_n}{\tau w_{n+1}A_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}}}{\left(\frac{P_1}{A_1w_1}\right)^{\frac{1}{\theta_1+1}}}
$$

Lagrangian multiplier: The Lagrangian multiplier λ_G is equal to the Lagrangian multiplier λ_1 in the most downstream country (since $q_1 = 1$). Using (52), we obtain:

$$
\lambda_G = (A_1w_1)^{\frac{1}{\theta_1+1}}P_1^{\frac{\theta_1}{\theta_1+1}} = \frac{A_1w_1}{(\theta_1+1)^{\theta_1}}\Theta(\mathbf{wA},\tau)^{\frac{\theta_1}{\theta_1+1}}
$$

Firm scope: For the most downstream firm in the most downstream country, equation (30) becomes:

$$
w_1c_1'(s_{1,f=0}) = \lambda_G
$$

This gives:

$$
w_1a_1s_{1,f=0}^{\theta_1} = \lambda_G
$$

Using the expression above for λ_G, we obtain:

$$\begin{aligned} s_{1,f=0} &= \left(\frac{\lambda_G}{w_1 a_1}\right)^{\frac{1}{\theta_1}} \\ &= \frac{\gamma_1(\theta_1+1)}{\theta_1}\left(\frac{\lambda_G}{w_1 A_1}\right)^{\frac{1}{\theta_1}} \\ &= \frac{\gamma_1}{\theta_1}\Theta(\mathbf{wA},\tau)^{\frac{1}{\theta_1+1}} \end{aligned}$$

To obtain downstream firm scope for other countries (expression 38), we use the first-order condition for firm scope for $f = 0$:

$$w_n c_n'(s_{n,f=0}) = \lambda_n$$

which gives:

$$w_n a_n s_{n,f=0}^{\theta_n} = \lambda_n$$

Using $\lambda_n = (w_n A_n)^{\frac{1}{\theta_n+1}}(P_n)^{\frac{\theta_n}{\theta_n+1}}$ (expression 52) together with the expression for P_n, we obtain:

$$\begin{aligned} s_{n,f=0} &= \left(\frac{\lambda_n}{w_n a_n}\right)^{\frac{1}{\theta_n}} \\ &= \frac{\gamma_n(\theta_n+1)}{\theta_n}\left(\frac{\lambda_n}{w_n A_n}\right)^{\frac{1}{\theta_n}} \\ &= \frac{\gamma_n(\theta_n+1)}{\theta_n}\left(\frac{P_n}{A_n w_n}\right)^{\frac{1}{\theta_n+1}} \\ &= \frac{\gamma_n(\theta_n+1)}{\theta_n}(A_n w_n)^{-\frac{1}{\theta_n+1}}\,(A_{n-1}w_{n-1}/\tau)^{\frac{1}{\theta_n-\theta_{n-1}}}\,(A_n w_n)^{-\frac{\theta_{n-1}+1}{(\theta_n+1)(\theta_n-\theta_{n-1})}} \\ &= \frac{\gamma_n(\theta_n+1)}{\theta_n}\left(\frac{A_{n-1}w_{n-1}}{\tau A_n w_n}\right)^{\frac{1}{\theta_n-\theta_{n-1}}} \end{aligned}$$

We follow similar steps to find the scope of the most upstream firm in each country, s_{n,F_n}:

$$s_{n,F_n} = \frac{(\theta_n+1)\gamma_n}{\theta_n}\left(\frac{A_n w_n}{\tau A_{n+1} w_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}}$$

Proof of Proposition 2

Proposition 2: Results in Proposition 2 are obtained simply by taking the derivative of the expressions above w.r.t trade costs τ. In particular, we find:

$$\frac{\partial\Theta(\mathbf{wA},\tau)}{\partial\tau} > 0$$

which implies that: i) the price in the final good and iv) the shadow cost λ_G decrease when trade costs decrease. Given equation (30)

$$q_n e^{\gamma_n f} w_n c_n'(s_{nf}) = \lambda_G,$$

we obtain that a decrease in λ_G affects firm scope everywhere along the chain and leads to a decrease in s_{nf}, conditional on the position on the chain, wages and the set of countries involved in the chain (point v). As trade costs decrease, however, countries tend to move downstream. Since firms scope is larger downstream, moving up the chain implies larger average firm scope for each country $i > 1$ (except the most downstream one). This can be seen in expressions (38): firm scope at both end of the chain in country i is a decreasing function of τ (point vi in Proposition 2). Finally, we can also see above that S_1 is an increasing function of trade costs (conditional on wages), which proves point iii).

Proof of Proposition 3: Free Trade Equilibrium

To prove part 1, we can use our previous result that, for a given chain ω, the sorting of countries is given by the ranking of θ_i. A key implication is that each chain is divided into a finite number of segments. We structure the proof by contradiction to show that no two countries with different values of θ will produce the same tasks, not even on different chains.

Suppose there exist two varieties over which two countries are involved in parallel on the same range of tasks, i.e. country i products one variety while country j produces another variety on a common segment of tasks $[\bar{S}, \bar{S} + \Delta]$ with $S > 0$.

A key implication of free trade is that prices must be identical across varieties at any point on the production chain, where location on the chain represents the the cumulative range of tasks that have been performed. Denote by P_M the price after completing a range $\bar{S}$ of tasks. For any $s < \Delta$, free trade implies that:

$$\left[\frac{s}{\theta_i+1}(A_i w_i)^{\frac{1}{\theta_i+1}} + \left(P^M\right)^{\frac{1}{\theta_i+1}}\right]^{\theta_i+1} = \left[\frac{s}{\theta_j+1}(A_j w_j)^{\frac{1}{\theta_j+1}} + \left(P^M\right)^{\frac{1}{\theta_j+1}}\right]^{\theta_j+1}$$

(this comes from equation 20).

For this equality to hold for all values of $s < \Delta$, all derivatives must be equal. Taking ratios of the second and first derivatives evaluated at $s = 0$, it is easy to show that the two parameters θ_i and θ_j that govern the convexity of prices (with respect to additional tasks) must be equal. Hence these two countries cannot be distinct.

This proves that there is a partition of the range of tasks that is common across all varieties. Each country corresponds to a specific segment of the range of tasks $[0, 1]$ in which it specializes for all varieties.

To prove part (ii) we characterize aggregate labor demand and equilibrium in the labor market. Denoting labor supply by L_n, we can use equations (34) and (35) from Lemma 2, to obtain a simple expression for aggregate labor demand as a function of the vector of wages $\mathbf{w}$ for all countries $n > 1$:

$$L_n^D(\mathbf{w}) \;=\; \frac{(\theta_1\!+\!1)\left(\sum_i L_i w_i\right)}{w_n} \;\;.\;\; \frac{\left(\frac{w_{n-1}A_{n-1}}{w_nA_n}\right)^{\frac{1}{\theta_n-\theta_{n-1}}} \;-\; \left(\frac{w_nA_n}{w_{n+1}A_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}}}{1 \;-\sum_{n=1}^{N-1}(\theta_{n+1}\!-\!\theta_n)\left(\frac{w_nA_n}{w_{n+1}A_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}}} \tag{56}$$

For the most upstream country N, the right term involving w_{N+1} is dropped. For the most downstream country $n = 1$, we have:

$$L_1^D(\mathbf{w}) \;=\; \frac{\left(\sum_i L_i w_i\right)}{w_1} \;\;.\;\; \frac{1 \;-\sum_{n=1}^{N-1}(\theta_{n+1}\!+\!1)\left[\left(\frac{w_nA_n}{w_{n+1}A_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}} \;-\; \left(\frac{w_nA_n}{w_{n+1}A_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}}\right]}{1 \;-\sum_{n=1}^{N-1}(\theta_{n+1}\!-\!\theta_n)\left(\frac{w_nA_n}{w_{n+1}A_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}}} \tag{57}$$

We can verify that Walras' Law is satisfied: $\sum_n w_n L_n^D(\mathbf{w}) = \sum_n L_n w_n$.

Equilibrium on the labor market requires:

$$L_n^D(\mathbf{w}) \;=\; L_n$$

for each country n.

To prove existence and uniqueness of equilibrium, one would be tempted to define the excess demand function $Z_n(\mathbf{w}) = L_n^D(\mathbf{w}) \;-\; L_n$ as in Alvarez and Lucas (2007). While this excess demand function would satisfy all sufficient conditions to prove existence using standard theorems, it does not satisfy the gross substitute property that is commonly used to prove uniqueness.

Instead, we can prove uniqueness (and existence) with a more simple recursive approach using our analytical results. We normalize wages such that:

$$\frac{(\theta_1\!+\!1)\left(\sum_n w_n L_n\right)}{1 \;-\sum_{n=1}^{N-1}(\theta_{n+1}\!-\!\theta_n)\left(\frac{w_nA_n}{w_{n+1}A_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}}} = 1$$

in order to simply the labor market conditions in each country.

In a first step, using this normalization, we define a function $\widetilde{w}_{N-1}(w_N)$ implicitly such that:

$$L_N^D(\mathbf{w}) = L_N$$

With the normalization above, this is equivalent to imposing:

$$\left(\frac{\widetilde{w}_{N-1}A_{N-1}}{w_NA_N}\right)^{\frac{1}{\theta_N-\theta_{N-1}}} \;=\; L_N w_N$$

which has a unique solution $\widetilde{w}_{N-1}$ as a function of w_N. We can see that this function $\widetilde{w}_{N-1}(w_N)$ is continuous, monotonic and strictly increasing in w_N. Moreover, both the ratio $\frac{\widetilde{w}_{N-1}(w_N)}{w_N}$ and $\widetilde{w}_{N-1}(w_N)$ go from 0 to $+\infty$ as w_N goes from 0 to $+\infty$. We also obtain that the ratio $\frac{\widetilde{w}_{N-1}(w_N)}{w_N}$ is increasing in w_N, i.e. $\widetilde{w}_{N-1}(w_N)$ grows faster than w_N.

Recursively, for any other country n, we can define a continuous function $\widetilde{w}_n(w_N)$ such that:

$$\left(\frac{\widetilde{w}_{n-1}(w_N)A_{n-1}}{\widetilde{w}_n(w_N)A_n}\right)^{\frac{1}{\theta_n-\theta_{n-1}}} - \left(\frac{\widetilde{w}_n(w_N)A_n}{\widetilde{w}_{n+1}(w_N)A_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}} = L_n\widetilde{w}_n(w_N)$$

If $\widetilde{w}_n(w_N)$ and $\frac{\widetilde{w}_n(w_N)}{\widetilde{w}_{n+1}(w_N)}$ both increase with w_N, we find that $\widetilde{w}_{n-1}(w_N)$ and $\frac{\widetilde{w}_{n-1}(w_N)}{\widetilde{w}_n(w_N)}$ also increase with w_N.

If both $\widetilde{w}_n(w_N)$ and the ratio $\frac{\widetilde{w}_n(w_N)}{\widetilde{w}_{n+1}(w_N)}$ go from 0 to $+\infty$ (as w_N goes from 0 to $+\infty$), then we also find that $\widetilde{w}_{n-1}(w_N)$ and the ratio $\frac{\widetilde{w}_{n-1}(w_N)}{\widetilde{w}_n(w_N)}$ go from 0 to $+\infty$ as w_N goes from 0 to $+\infty$.

By recurrence, these monotonicity and limit properties apply to $\widetilde{w}_{n-1}(w_N)$ and $\frac{\widetilde{w}_{n-1}(w_N)}{\widetilde{w}_n(w_N)}$ for all $n \in [1, N-1]$.

Finally, we are left to show that we can find w_N (and thus all other wages) such that the normalization is satisfied:

$$\frac{(\theta_1+1)\left(\sum_n \widetilde{w}_n(w_N)L_n\right)}{1 - \sum_{n=1}^{N-1}(\theta_{n+1}-\theta_n)\left(\frac{\widetilde{w}_n(w_N)A_n}{\widetilde{w}_{n+1}(w_N)A_{n+1}}\right)^{\frac{1}{\theta_{n+1}-\theta_n}}} = 1$$

(where $\widetilde{w}_N(w_N) \equiv w_N$). One can see that the left hand side is strictly increasing in w_N given the monotonicity properties shown above. When w_N approaches to zero, the left hand side has a limit zero. As w_N goes to infinity, the left hand side also goes to infinity. This proves existence and uniqueness of a solution w_N^* such that the above normalization equation holds. Consequently, this also proves existence and uniqueness of wages $w_n = \widetilde{w}_n(w_N^*)$ such that labor markets clear.

Proofs for Section 4.2: Two-country case

Proof of Proposition 4

Final good prices and consumption choices: Using the results from above, final goods prices are:

$$P_D(\omega) = \frac{1}{(\theta_D+1)^{\theta_D+1}}\left[1 - (\theta_U-\theta_D)\,(\tau w_U A(\omega))^{-\frac{1}{\theta_U-\theta_D}}\right]^{\theta_D+1}$$

$$P_U(\omega) = \frac{w_U A(\omega)}{(\theta_U+1)^{\theta_U+1}}$$

respectively when country D or U produces the final stages. This leads to the following thresholds defined in the text:

$$\begin{aligned} P_D(\omega_D^*) &= \tau P_U(\omega_D^*) \\ \tau P_D(\omega_U^*) &= P_U(\omega_U^*) \end{aligned}$$

There is no analytical solution for ω_U^* but it is easy to check the following solution in ω_D^*:

$$\omega_D^* = \frac{\tilde{T}_U\tau^{-\xi}w_U^{-\xi}(\theta_D+1)^{-(\theta_U-\theta_D)\xi}}{\tilde{T}_D + \tilde{T}_U\tau^{-\xi}w_U^{-\xi}(\theta_D+1)^{-(\theta_U-\theta_D)\xi}}$$

We can also verify that the foreign content for the marginal variety ω_D^* is equal to one:

$$\frac{w_U\, l_U(\omega_D^*)}{P_D(\omega_D^*)} = 1$$

where $\frac{w_U\, l_U(\omega)}{P_D(\omega)}$ is the share of value-added from country U in final goods sold by D:

$$\frac{w_U\, l_U(\omega)}{P_D(\omega)} = \frac{(\theta_D+1)\ (\tau w_U A(\omega))^{-\frac{1}{\theta_U-\theta_D}}}{1-(\theta_U-\theta_D)\ (\tau w_U A(\omega))^{-\frac{1}{\theta_U-\theta_D}}} \tag{58}$$

General equilibrium: We prove here that τw_U decreases as trade costs τ decrease.

Wages at equilibrium are determined by a labor market clearing condition or, equivalently, a trade balance condition. Here, trade balance imposes:

$$w_U L_U(1-\omega_U^*) \ = \ L_D\omega_D^* \ + \ L_D\int_{\omega=\omega_D^*}^{1}\frac{w_U\, l_U(\omega)}{P_D(\omega)}d\omega \ + \ w_U L_U\int_{\omega=\omega_U^*}^{1}\frac{w_U\, l_U(\omega)}{P_D(\omega)}d\omega \tag{59}$$

where the left-hand side correspond to exports of final goods by D and the right-hand side corresponds to exports of final and intermediate goods by U.

It is equivalent to a trade balance in value-added content:

$$w_U L_U\int_{\omega=\omega_U^*}^{1}\left(1-\frac{w_U\, l_U(\omega)}{P_D(\omega)}\right)d\omega \ = \ L_D\int_{\omega=0}^{1}\min\left\{\frac{w_U\, l_U(\omega)}{P_D(\omega)}\,,\,1\right\}d\omega \tag{60}$$

where $\frac{w_U\, l_U(\omega)}{P_D(\omega)}$ denotes the foreign value-added content of final goods sold by D (equation 58).

We arrive at a contradiction if we assume that τw_U increases while τ decreases. The right-hand-side term of expression (60) would decrease since it is a strictly decreasing function of τw_U: country D sources less from U if trade-cost-adjusted wages increase in U. On the other hand the term on the left would increase because of higher income $L_U w_U$, a lower import threshold ω_U^* (since goods from D would become relatively cheaper) and higher foreign value-added content $1-\frac{w_U\, l_U(\omega)}{P_D(\omega)}$. Hence it must be that τw_U decreases when τ decreases.

Trade elasticity: For country D, it is easy to check that the elasticity is the same as in Eaton and Kortum (2002):

$$\varepsilon_D^F \ \equiv \ \frac{d\log\left(\frac{\omega_D^*}{1-\omega_D^*}\right)}{d\log\tau} \ = \ -\xi$$

For country U, we take the derivative of $\tau P_D(\omega_U^*) = P_U(\omega_U^*)$ with respect to $\log\tau$, which gives:

$$1+\frac{\partial\log P_D}{\partial\log\tau}\,.\,\left[1+\frac{d\log A(\omega_U^*)}{d\log\tau}\right] = \frac{\partial\log P_U}{\partial\log\tau}\ \frac{d\log A(\omega_U^*)}{d\log\tau}$$

In the expression above, the partial derivative $\frac{\partial\log P_U}{\partial\log\tau}$ is equal to unity. The partial derivative $\frac{\partial\log P_D}{\partial\log\tau}$ is lower than one and equals the share of value coming from U for the threshold variety

ω_U^*:

$$\frac{d\log P_D}{d\log\tau} = \frac{w_U\, l_U(\omega_U^*)}{P_D(\omega_U^*)}$$

After solving for $\frac{d\log A(\omega_U^*)}{d\log\tau}$, we find:

$$\frac{d\log A(\omega_U^*)}{d\log\tau} = \frac{1+\frac{w_U\, l_U(\omega_U^*)}{P_D(\omega_U^*)}}{1-\frac{w_U\, l_U(\omega_U^*)}{P_D(\omega_U^*)}}$$

The trade elasticity in final goods for country U is then:

$$\varepsilon_U^F = \frac{d\log\left(\frac{1-\omega_U^*}{\omega_U^*}\right)}{d\log\tau} = \frac{1}{\xi}\,\frac{d\log A(\omega_U^*)}{d\log\tau} = \frac{1}{\xi}\,\frac{1+\frac{w_U\, l_U(\omega_U^*)}{P_D(\omega_U^*)}}{1-\frac{w_U\, l_U(\omega_U^*)}{P_D(\omega_U^*)}}$$

VAX ratios: For country D, the value-added content of exports to country U is:

$$\begin{aligned} VAX_D &= \frac{1}{1-\omega_U^*}\int_{\omega_U^*}^1 \left(1-\frac{w_U\, l_U(\omega)}{P_D(\omega)}\right) d\omega \\ &= \frac{1}{1-\omega_U^*}\int_{\omega_U^*}^1 \left(1-\frac{(\theta_D+1)\,(\tau w_U A(\omega))^{-\frac{1}{\theta_U-\theta_D}}}{1-(\theta_U-\theta_D)\,(\tau w_U A(\omega))^{-\frac{1}{\theta_U-\theta_D}}}\right) d\omega \end{aligned}$$

For a given ω, the term in the integral sum increases with trade-cost-adjusted wages τw_U, which itself increases with τ (larger domestic value added share as trade costs increase). This is a direct effect. There is also a composition effect: ω_U^* increases with τ, so that country D only exports high-value-added goods (varieties ω closer to one) when trade costs are higher. This second effect also leads to an increase in VAX_D when trade costs increase.

A similar intuition holds for the VAX ratio for country U, as described in the text. The VAX ratio for country U equal the one for country D in this two-country example because we have a trade balance in gross flows as well as in the value-added content of trade.

Gains from trade

Gains from trade for country D**:** For country D, the wage and the price index under autarky are normalized to zero (in log). Hence the log of the price index with trade also reflects the gains from trade. Gains from trade are given by:

$$\Delta\log\left(\frac{1}{P_D}\right) = \int_0^{\omega_D^*}\log\left(\tau w_U A(\omega)\right) d\omega + \int_{\omega_D^*}^1 \log\Theta\left(\tau w_U A(\omega)\right) d\omega$$

with:

$$\Theta(\tau w_U A(\omega)) = \left[1-(\theta_U-\theta_D)\,(\tau w_U A(\omega))^{-\frac{1}{\theta_U-\theta_D}}\right]^{\theta_D+1}$$

and:

$$\Theta(\tau w_U A(\omega_D^*)) = \tau w_U A(\omega_D^*)$$

at the threshold ω_D^*.

The expression for the gains from trade can be rewritten:

$$\begin{aligned}\Delta \log\left(\frac{1}{P_D}\right) &= -\int_0^{\omega_D^*} \log\left(\tau w_U A(\omega)\right) d\omega - \int_{\omega_D^*}^1 \log\Theta\left(\tau w_U A(\omega)\right) d\omega \\ &= -\int_0^{\omega_D^*} \log\left(\frac{A(\omega)}{A(\omega_D^*)}\right) d\omega - \int_{\omega_D^*}^1 \log\Theta\left(\tau w_U A(\omega)\right) d\omega \\ &\quad -\omega_D^* \log\Theta\left(\tau w_U A(\omega_D^*)\right)\end{aligned}$$

There are three terms in the above formula. The first term corresponds to Arkolakis et al (2012) formula. After integrating by part, we can see that it equals the ratio of the change in domestic consumption share and the trade elasticity ξ for country D:

$$\begin{aligned}-\int_0^{\omega_D^*} \log\left(\frac{A(\omega)}{A(\omega_D^*)}\right) d\omega &= \int_0^{\omega_D^*} \frac{\partial \log A(\omega)}{\partial \log \omega} d\omega \\ &= \frac{1}{\xi}\int_0^{\omega_D^*} \frac{d\omega}{1-\omega} \\ &= -\frac{1}{\xi}\log\left(1-\omega_D^*\right)\end{aligned}$$

The second and third terms reflect additional gain from fragmentation:

$$\begin{aligned}-\int_{\omega_D^*}^1 \log\Theta\left(\tau w_U A(\omega)\right) d\omega \;-\; \omega_D^* \log\Theta\left(\tau w_U A(\omega_D^*)\right) &= -\int_{\omega_D^*}^1 \frac{\partial \log\Theta}{\partial \log A}\frac{\partial \log A}{\partial \log \omega} d\omega \\ = \int_{\omega_D^*}^1 \frac{\partial \log\Theta}{\partial \log A}\frac{1}{\xi}\frac{1}{1-\omega} d\omega &= \frac{1}{\xi}\int_{\omega_D^*}^1 \frac{w_U\, l_U(\omega)}{P_D(\omega)}\frac{d\omega}{1-\omega} \;>\; 0\end{aligned}$$

using the equality between $\frac{\partial \log\Theta}{\partial \log A}$ and the foreign labor content $\frac{w_U\, l_U(\omega)}{P_D(\omega)}$.

Gains from trade for country U: For country U, the price index under autarky is:

$$\log P_U^{aut} = \int_0^1 \log\left(w_U^{aut} A(\omega)\right) d\omega$$

where w_U^{aut} denotes the wage in autarky. With trade, the price index is:

$$\log P_U = \int_0^{\omega_U^*} \log\left(w_U A(\omega)\right) d\omega \;+\; \int_{\omega_U^*}^1 \log \tau\Theta\left(\tau w_U A(\omega)\right) d\omega$$

where $\Theta\left(\tau w_U A(\omega)\right)$ is defined like above:

$$\Theta(\tau w_U A(\omega)) = \left[1-\left(\theta_U - \theta_D\right)\left(\tau w_U A(\omega)\right)^{-\frac{1}{\theta_U-\theta_D}}\right]^{\theta_D+1}$$

with the following equality at the threshold ω_U^*:

$$\tau\Theta\left(\tau w_U A(\omega_U^*)\right) = w_U A(\omega_U^*)$$

Gains from trade can then be expressed as:

$$\begin{aligned}\Delta \log\left(\frac{w_U}{P_U}\right) &= \int_{\omega_U^*}^1 \log\left(w_U A(\omega)\right) d\omega - \int_{\omega_U^*}^1 \log \tau\Theta\left(\tau w_U A(\omega)\right) d\omega \\ &= \int_{\omega_U^*}^1 \log\left(\frac{A(\omega)}{A(\omega_U^*)}\right) d\omega - \int_{\omega_U^*}^1 \log\left(\frac{\Theta\left(\tau w_U A(\omega)\right)}{\Theta\left(\tau w_U A(\omega_U^*)\right)}\right) d\omega\end{aligned}$$

Like above, the first term corresponds to Arkolakis et al (2012) formula:

$$\int_{\omega_U^*}^1 \log\left(\frac{A(\omega)}{A(\omega_U^*)}\right) d\omega = -\frac{1}{\xi}\log \omega_U^*$$

The second term yields:

$$\begin{aligned}-\int_{\omega_U^*}^1 \log\left(\frac{\Theta\left(\tau w_U A(\omega)\right)}{\Theta\left(\tau w_U A(\omega_U^*)\right)}\right) d\omega &= -\int_{\omega_U^*}^1 \frac{\partial \log\Theta}{\partial \log A}\frac{\partial \log A}{\partial \omega}(1-\omega)\, d\omega \\ &= -\int_{\omega_U^*}^1 \frac{\partial \log\Theta}{\partial \log A}\frac{1}{\xi\omega}\, d\omega \\ &= -\frac{1}{\xi}\int_{\omega_U^*}^1 \frac{w_U\, l_U(\omega)}{P_D(\omega)}\frac{d\omega}{\omega} \;<\; 0\end{aligned}$$

This term is negative, which means that country U gains less than predicted by the Arkolakis et al (2012) benchmark.

Proof of Proposition 5

Fragmentation and firm scope along the chain: The proof of Proposition 5 relies on the general equilibrium result proven above: τw_U decreases when τ decreases. This leads to unambiguous implications of trade costs for fragmentation.

In particular, the shadow cost of fragmentation:

$$\lambda_G(\omega) = A_D(\omega) w_D\left[\frac{1}{\theta_D+1} - \frac{\theta_U-\theta_D}{\theta_D+1}\left(\tau w_U A(\omega)\right)^{-\frac{1}{\theta_U-\theta_D}}\right]^{\theta_D}$$

decreases when trade costs decrease since τw_U decreases. As described in the text, $s_{D,f}(\omega)$ and $s_{U,f}(\omega)$ can also be expressed as simple functions of τw_U: as τw_U decreases, firm scope decreases. Similar results can be found for $S_D(\omega)$, firm scope of the most upstream firm in country D and firm scope in the most downstream firm in country U as described in Proposition 5.