General Equilibrium Techniques for Policy Modeling

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Detailed and rigorous accounting practices always have been at the foundation of sound and sustainable economic policy. A consistent set of real data on the economy is likewise a prerequisite to serious empirical work with economic simulation model. For this reason, a complete general equilibrium modeling facility stands on two legs: a consistent economywide database and modeling methodology. This chapter gives an overview of the accounting conventions used in applied general equilibrium modeling.

The three governing criteria for development and maintenance of good economywide data are detail, consistency, and currency. Detail in the context of CGE models refers to industrial and domestic institutional (e.g. household) classification, and to capture this, the database should incorporate input-output accounts and other transactions tables. Economywide consistency is achieved primarily by reconciling the input-output accounting information with the standard National Income and Product Accounts (NIPA) such as those published for the United States.[1] This reconciliation is accomplished and maintained with a social accounting matrix (SAM), which details economywide transactions between firms, households, government, and other domestic and foreign institutions at a flexible level of disaggregation. This SAM and other components of the database are estimated to a uniform standard which is consistent with observable information in a single base year.

The discussion below gives general indications about the many sources of data, their unification in the SAM and subsidiary accounts, and the numerical and statistical reconciliation procedures which are used. A typical database development project relies on an extensive applied and theoretical literature, and no attempt is made here to give an exhaustive survey.[2]

[1] See e.g. Young and Tice (1985) for a more detailed description of NIPA accounting.
[2] For more general information, see Pyatt and Round (1985), Stone (1981), and UNSO (1968).

The type of accounting used here is based on a fundamental principle of economics: for every income or receipt there is a corresponding expenditure or outlay.[3] This principle underlies the double-entry accounting procedures that make up the NIPA accounts. A SAM is a form of *single*-entry accounting. SAMs also embody the fundamental principle, but they record transactions *between* accounts in a tableau or matrix format.[4] The number of transactors or accounts constitutes the dimension of the square matrix. By convention, incomes or receipts are shown in the rows of the SAM while expenditures or outlays are shown in the columns. The special merit of SAMs is that they can provide a *comprehensive* and *consistent* record of the inter-relationships of an economy at the level of individual production sectors, factors, and general public and foreign institutions. They can be used to disaggregate NIPA accounts, and they can reconcile these with the economy's input-output accounts.

Traditionally, the database for models with sectoral detail was the input-output accounting tableau, which captures industry linkages through flows of intermediate and factor input. Although it provides sectoral disaggregation, an input-output model does not include enough institutional detail to provide a framework for considering the full impact of policy on an economy. The input-output accounts can be extended to capture income and expenditure flows between other institutions, such as households, government, and the rest of the world in a SAM. Indeed, the development of SAMs was motivated in part by the desire for a unified framework that reconciled input-output accounts with NIPA accounts. The SAM thus provides detail and an economywide policy perspective in a consistent accounting framework.

The first SAM was constructed in the 1960s as a part of the Cambridge Growth Project by Sir Richard Stone, Alan Brown, and their associates. The accounts were for the United

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[3] Pyatt (1988), p. 329.

[4] UNSO (1968) shows that national accounts can be presented in four ways: standard double-entry accounts, balance statements, matrices, and equations.

Kingdom in 1960, and they provided the data base for the Cambridge Growth Model. Since that time, SAMs have been constructed for at least 40 countries and have supported work in input-output analysis, tax-incidence studies, income distribution analysis, sectoral manpower planning, material-balance analysis, and computable, general-equilibrium (CGE) modeling for trade policy analysis. This chapter will introduce to the reader the concept of a SAM with a few examples. It will then present a macro SAM for the United States. Next, it will consider how this macro SAM can be disaggregated to provide a data facility for policy analysis.

Examples of SAMs

This section will present a series of SAMs, varying from abstract to actual numerical estimates. It begins with a general, algebraic representation of a SAM and then considers a set of schematic SAMs.

Algebraically, a SAM may be represented as a square matrix:

$$\mathbf{T} = \{t_{ij}\} \qquad (1)$$

where t_{ij} is the value of the transaction with income accruing to account i from expenditure by account j.

Nominal flows cross the SAM from columns to rows. For transactions involving goods and services, there are corresponding real flows crossing the SAM from rows to columns. For financial transactions, there are corresponding flows of assets from rows to columns. For pure transfers, there are only the nominal flows from column accounts to row accounts.

The fundamental law of economics ensures that the corresponding row and column totals of a SAM, the income and expenditure for each account, must be equal. That is:

$$\sum_j t_{jk} = \sum_j t_{kj} \text{ for all k} \qquad (2)$$

Consider a closed economy where economic activity is divided into three main types: production, consumption, and accumulation.[5] The representative accounts for this economy are presented in Table 1. Production receives its revenue from selling consumption goods in transaction t_{12} and investment goods in transaction t_{13}. The revenue from these sales passes to the consumption account as income paid to the factors of production in transaction t_{21}. The consumption-account income is spent in two ways. Part of it goes to purchase consumption goods in transaction t_{12}, and part is saved in transaction t_{32}. Savings is channeled to investment goods demand in transaction t_{13}, closing this macroeconomic system of income-expenditure flows.

[5] The accumulation account is also known as the capital account. It can be thought of as a loanable funds market. This simple economy is addressed in Stone (1981, Chapter 1).

Table 1: A Closed-Economy SAM

Receipts	Expenditures 1	2	3	Totals
1. Production	-	C	I	Demand
2. Consumption	Y	-	-	Income
3. Accumulation	-	S	-	Savings
Totals	Supply	Expenditure	Investment	

Variables:

t_{12} = C = consumption
t_{13} = I = investment
t_{21} = Y = income
t_{32} = S = savings

Accounting Identities:
1. Y = C + I (GNP)
2. C + S = Y (Domestic Income)
3. I = S (Saving-Investment)

Table 2: An Open-Economy SAM with a Government Sector

Receipts	Expenditures 1	2	3	4	5	Total
1. Suppliers	-	C	G	I	E	Demand
2. Households	Y	-	-	-	-	Income
3. Government	-	T	-	-	-	Receipts
4. Capital Acct.	-	S_h	S_g	-	S_f	Savings
5. Rest of World	M	-	-	-	-	Imports
Total	Supply	Expen-diture	Expen-diture	Invest-ment	Foreign Exchange	

Additional Variables:

$t_{42} = S_h$ = private savings
$t_{32} = T$ = tax payments
$t_{43} = S_g$ = government savings
$t_{15} = E$ = exports
$t_{45} = S_f$ = foreign savings
$t_{51} = M$ = imports
$t_{13} = G$ = government spending

Accounting Identities:

1. $Y + M = C + G + I + E$(GNP)
2. $C + T + S_h = Y$ (Income)
3. $G + S_g = T$ (Govt. Budget)
4. $I = S_h + S_g + S_f$ (Saving-Investment)
5. $E + S_f = M$ (Trade Balance)

The accounts of Table 1 reflect a functional classification. The next set of accounts introduces an institutional classification. First, production and consumption accounts are redefined as the institutions "suppliers" and "households" respectively. Second, the government sector is included as an institution. Third, the economy is opened to the rest of the world. The resulting new accounts are set forth in Table 2.[6] Suppliers receive revenue by selling final consumption goods to households (transaction t_{12}) and government (transaction t_{13}), investment

[6] This five-account economy is addressed in Robinson and Roland-Holst (1988).

goods to the capital account (transaction t_{14}), and export goods to the rest of the world (transaction t_{15}). Revenue from production is spent on value added (transaction t_{21}) and imports from the rest of the world (transaction t_{51}). Household outlays take the form of consumption expenditures (transaction t_{12}), tax payments (transaction t_{32}), and private domestic savings (transaction t_{42}). Government outlays take the form of consumption goods (transaction t_{13}) and government savings (transaction t_{43}). Inflows from the rest of the world take the form of export demand (transaction t_{15}) and foreign savings (transaction t_{45}). Foreign savings is the negative of the U.S. trade balance.

Applying equation (2) to the accounts of Table 2 yields the familiar open-economy identities:

$$\begin{aligned} &Y + M = C + G + I + E \\ &C + T + S_h = Y \\ &G + S_g = T \qquad (3) \\ &I = S_h + S_g + S_f \\ &E + S_f = M \end{aligned}$$

The next step is to provide a numerical example of a SAM by associating values for the United States economy with the transactions indicated in Table 2. This is done with data for the year 1988 in Table 3. The additional transactions t_{52} and t_{53} will be discussed below.

Total receipts of suppliers were \$5,501,963 million. This is the sum of \$4,204,041 million of receipts on sales of consumption goods (t_{12} + t_{13}), \$750,257 million on sales of investment goods (t_{14}), and \$547,665 million in export sales (t_{15}). These figures were obtained from Table 1.1 of the National Income and Product Accounts (NIPA).[7] Total expenditures of suppliers must equal total receipts. There are expenditures of \$4,880,632 million on value

[7] Table 1.1 is entitled "Gross National Product" in Young and Tice (1985).

added (t_{21}). This is the familiar "gross national product", assumed to be equal to national income in the table. There also are expenditures on "imports" of $621,331 million ($t_{51}$).

The expenditures of suppliers on value added correspond in this example to the receipts of the household account. Of this $4,880,632 million, $3,235,095 million is spent on consumption goods (t_{12}) and $728,953 million is saved (t_{42}). The latter figure is calculated from Table 5.1 of NIPA as the sum of "gross private saving" and "statistical discrepancy".[8] Domestic tax payments equal $914,722 million ($t_{32}$), which is calculated as a remainder. There are also $1,862 million in expenditures classified as transfers to foreigners (t_{52}), defined as "transfer payments from persons (net)" from Table 4.1 of the U.S. NIPA accounts.[9]

Table 3: An Aggregate SAM for the United States, 1988
(millions of dollars)

	Expenditures					
Receipts	1	2	3	4	5	Total
1. Suppliers	-	3,235,095	968,942	750,257	547,665	5,501,963
2. Households	4,880,623[a]	-	-	-	-	4,880,632
3. Government	-	914,722	-	-	-	914,722
4. Capital	-	728,953	-96,146	-	117,450	750,257
5. Rest of World	621,331	1,862	41,922	-	-	665,115
Total	5,501,963	4,880,632	914,722	750,257	665,115	

[a]In this highly-aggregated table, it is assumed that gross national product equals national income or that there are no indirect business taxes.

[8] Table 5.1 is entitled "Gross Saving and Investment."
[9] Table 4.1 is entitled "Foreign Transactions in the National Income and Product Accounts."

As mentioned above, the government receives $914,722 million in tax revenues. Government outlays go to three accounts: $968,946 million spent on goods from the suppliers account t_{13}, -$96,146 million saved ($96,146 million dissaved) on the capital account (t_{43}), and $41,922 million transferred to the rest of the world (t_{53}). This last figure is the sum of "transfer payments from government (net)" and "interest paid by government to foreigners" from U.S. NIPA Table 4.1.

The $750,257 million receipts of suppliers on sales of investment goods are the expenditures from the capital account. The receipts of the capital account have three sources. The first of these is the $728,953 million in domestic personal savings just mentioned (t_{42}). The second is -$96,146 million of government saving (t_{43}) from U.S. NIPA Table 1.[10] The third is $117,450 million of foreign savings (t_{45}), the negative of "net foreign investments" recorded in Table 4.1 of the U.S. NIPA accounts. Each of the transactions in the rest of the world account has been described above. The import and export figures appear in Table 4.1 of the U.S. NIPA accounts as well as Table 1.1.

Actual SAMs typically include more detail than Table 3. This comes in part from a more detailed specification of the production side of the economy. The "suppliers" account of Table 3 is usually replaced with four accounts: activities, commodities, factors, and enterprises. The activities accounts buy intermediate inputs and hire factor services to produce commodities, generating value added in the process.[11] The goods sold by activities should be valued at producer prices in the SAM. The commodities accounts combine domestic supply with

[10] Table 1 is entitled "Government Receipts and Expenditures".

[11] The United Nations' System of National Accounts (SNA) defines activity accounts as follows: "Production accounts of industries, producers of government services, producers of private non-profit services to households, and the domestic service of households, in respect of their gross output of goods and services and their intermediate consumption, primary inputs and indirect taxes *less* subsidies" (UNSO, 1968, p. 230).

imports.[12] Commodities should be valued at purchaser prices in the SAM. Factors are a set of accounts for the expenditures and receipts of the factors of production: labor, land, and capital. Enterprises collect gross profits and government transfers and distribute them to other accounts.[13]

In the terminology of the input-output accounts, t_{12} is the "make table."[14] Commodities' receipts fall under five accounts. The first of these, transaction t_{21}, is from activities where commodities receive payments in purchaser prices for the sales of intermediate goods.[15] Input-output accounting refers to t_{21} as the "use table."[16] Transactions t_{25}, t_{26}, t_{27}, and t_{28} are commodity receipts from sales (again at purchaser prices) of consumption goods to households and the government, of investment goods to the capital account, and of exports to the rest of the world. Factor receipts (transaction t_{31}) record the value-added payments from the activities accounts and factor-service exports (transaction t_{38}) from the rest of the world.[17]

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[12] The SNA defines commodity accounts as follows: "Accounts relating to the supply of commodities from domestic production and imports and their disposition to intermediate and final uses" (UNSO, 1968, p. 231). Hanson and Robinson (1988) describe the commodity account as "a giant department store" which "buys goods from domestic producers and foreigners (imports) down the column and sells them to demanders (including exports) along the row" (p. 8).

[13] Hanson and Robinson (1988) describe the difference between activities and enterprises as follows: "(A)ctivities are aggregations of establishments within a sector. They purchase inputs on factor and product markets and sell output on product markets. They are different from enterprises which collect gross capital income and distribute it to other institutions. The distinction provides a framework for capturing an establishment-firm dichotomy, which exists in both data and theory" (pp. 21-22).

[14] "The make table shows the value of each commodity produced by each industry.... The value of the primary product is shown in the diagonal cell.... The secondary products of the industry (products primary to other industries) are shown in the other cells along the row" (United States Department of Commerce, May 1984, pp. 49-50). In terms of the vocabulary of this paper, I can replace the word "industry" in this quote with "activity."

[15] "Purchasers' values are equal to producers' values plus the trade and transport margins appropriate to the purchaser in question..." (UNSO, 1968, p. 54).

[16] "The use table shows the value of each commodity used by each industry" (United States Department of Commerce, May 1984, p. 48). The use table in the U.S. input-output accounts is in producer rather than purchaser prices. However, Hanson and Robinson (1988) report that "the underlying data tapes provide data on trade and transportation margins and allow the construction of a 'use' table to generate commodities in 'purchaser' prices" (p. 14n).

[17] Factor-service exports consist of a flow of profits into the United States from U.S. foreign investments.

Table 4: A More Detailed SAM

Receipts	Expenditures 1 Activities	2 Commodities	3 Factors	4 Enterprises	5 Households	6 Government	7 Capital Acct.	8 Rest of World	9 Total
1. Activities		gross outputs (make table)							total sales
2. Commodities	intermediate demand (use table)				household consumption	government consumption	investment	exports	aggregate demand
3. Factors	value added (net of taxes on activities)							factor service exports	factor income
4. Enterprises			gross profits			transfers			enterprise income
5. Households			wages	distributed profits		transfers		foreign remittances	household income
6. Government	indirect taxes	tariffs	factor taxes	enterprise taxes	direct taxes				government income
7. Capital acct.				retained earnings	household savings	government savings		capital transfers from abroad[a]	total savings
8. Rest of World		imports	factor service imports		transfers abroad	transfers abroad	capital transfers abroad		foreign exchange payments
9. Total	total costs	aggregate supply	factor expenditure	enterprise expenditure	household expenditure	government expenditure	total investment	foreign exchange receipts	

a) Includes increase in reserves.

Now consider the receipts of institutions. Enterprises receive payments from two sources. The first is gross profits from the factors account, transaction t_{43}, and the second is transfers from the government account (t_{46}). Households receive payments from four sources, the first being wages from the factors account (t_{53}). The second and third are from other institutional accounts: distributed profits from enterprises (t_{54}) and transfers from the government (t_{56}). The fourth source is foreign remittances (t_{58}). The government receives payments from the first five accounts: indirect taxes from activities (t_{61}), tariffs from commodities (t_{62}), factor taxes (t_{63}), enterprise taxes (t_{64}), and direct taxes from households (t_{65}).

The capital account receives payments in the form of domestic and foreign savings. Transaction t_{74} comprises the retained earnings of enterprises, while t_{75} and t_{76} represent the savings of households and the government, respectively. Capital transfers from abroad, including any increase in reserves, are received from the rest of the world in transaction t_{78}.

Lastly, there are the receipts of the rest of the world. The first of these is import payments from the domestic commodity account, transaction t_{82}. The second is factor-service imports (t_{83}) of the United States.[18] Lastly, the rest-of-the-world account receives three types of transfers: transfers abroad from persons (t_{86}) and government (t_{85}) and capital transfers abroad (t_{87}).

A Macro SAM for the United States

The final exercise in this series is to construct a macro SAM for the United States based on the 1988 NIPA accounts (Table 5). The macro SAM provides the control totals for all subsequent disaggregated accounts. The macro SAM has twelve accounting categories. Accounts 1 and 2 are the activity and commodity accounts, respectively. There are two factor accounts: labor (account 3) and property (account 4). Gross national product or value added is allocated between accounts 2 and 3 in accordance with the conventions adopted by the Department of Commerce in their input-output accounts (U.S. Department of

Commerce 1984). That is, charges against GNP are broken up into three types: 1) compensation of employees which is received by labor; 2) profit-type income, net interest, and capital consumption allowances, which are received by property;[19] and 3) indirect business taxes, which are received by government.

Account 5 is the enterprise account. Accounts 6 and 7 are the household and government accounts, respectively. Account 8 is the capital account which closes the system of income-expenditure flows. Account 9 is the rest of the world account (ROW) which records international transactions. Account 10 collects tariffs and distributes them to the government. Accounts 11 and 12 are the errors account and the total account, respectively.

To construct the macro SAM requires a mapping between the NIPA account items and the twelve SAM accounts. The mapping used in this project is detailed in Appendix A, and its implementation for the year 1988 is presented in Table 5. The mapping is designed so that factor-service imports (transaction t_{94}) and factor-service exports (transaction t_{49}) are broken out of net output. I assume that all factor-service payments are for capital. In contrast to typical practice, I define gross domestic product (GDP) to be net of imports valued at market prices rather than border prices. Therefore, the $4,830,868 in transaction t_{12} represents the typically-defined GDP *less* customs duties. I do this because it is government, not activities, that engages in tariff collection.

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[18] Imports of factor services consist of a flow of profits from the United States to foreign investors.

[19] Profit-type income consists of proprietors' income, rental income of persons, corporate profits, and business transfer payments, less (subsidies less current surplus of government enterprises).

Table 5: A Macro SAM for the United States, 1988
(millions of dollars)

Receipts	Expenditures 1 Activ.	2 Commod.	3 Labor	4 Prop.	5 Enter.	6 Hsehld.	7 Govt.	8 Capital	9 ROW	10 Tariffs	11 Error	12 Total
1. Activities		4,830,868										4,830,868
2. Commodities						3,235,095	968,946	750,257	430,918			5,385,216
3. Labor	2,907,647											2,907,647
4. Property	1,555,756								116,747			1,672,503
5. Enterprises				1,589,072		96,146	92,292					1,777,510
6. Households			2,463,048		1,045,732		555,683					4,064,463
7. Government	377,065		444,599		137,936	586,649		96,146		16,448		1,658,843
8. Capital acct.					593,842	144,711			117,450		-9,600	846,403
9. Rest of World		537,900		83,431		1,862	41,922					665,115
10. Tariffs		16,448										
11. Errors and Omissions	-9,600											-9,600
12. Total	4,830,868	5,385,216	2,907,647	1,672,503	1,777,510	4,064,463	1,658,843	846,403	665,115	16,448	-9,600	

Property income is passed on to enterprises (t_{54}) and to the rest of the world in the form of factor-service imports (t_{94}), whereas some of labor income is passed to the government in the form of social insurance contributions (t_{73}). Enterprise income is distributed between households (t_{65}), government (t_{75}), and the capital account (t_{85}). Household income is distributed between commodities (t_{26}), enterprises (t_{56}), government (t_{76}), the capital account (t_{86}), and the rest of the world (personal transfer payments in t_{96}). Government receipts are spent on commodities (t_{27}), transfers to enterprises (t_{57}), to households (t_{67}), to the rest of the world (interest and transfer payments to foreigners in t_{97}). Capital account expenditures are divided between commodities (t_{28}) and the government deficit. This last item, \$96,146 million in transaction t_{78}, represents a net deficit for federal, state, and local governments combined. The rest of the world makes payments to the commodities account for exports of goods and non-factor services (t_{29}), to the property account for factor-service exports (t_{49}), and to the capital account in the form of net foreign investment (t_{89}). The tariff account makes payments to the government ($t_{7,10}$).

Construction of the Disaggregated Accounts

The construction of a disaggregated SAM requires a set of detailed input-output accounts. For the present example, I rely on IMPLAN tables from the U.S. Forest Service. Detailed use, make, final demand, and value added tables are available in the U.S. for 1982. In all cases, the input-output data are aggregated slightly up to the 487 sectors. This section describes how these input-output data were updated using supplementary data and balanced to the control totals of the macro SAM.[20]

The first step in constructing the disaggregated accounts is to estimate gross activity outputs. Activity output data for 1988 are available from the U.S. Department of Labor at the level of 226 sectors. These output data are further disaggregated to the 487 sectors based on 1982 gross output shares from the IMPLAN data. Next, I estimate commodity outputs. To do this, I row normalize the 1982 IMPLAN make matrix and premultiply it by a row vector of the activity outputs. The 1982 IMPLAN make matrix is then updated to 1988 using the RAS procedure with the activity and commodity output vectors as control totals.[21]

We next consider the value added sub-matrix of the disaggregated SAM. Control totals for 1988 value added are taken from transactions t_{31}, t_{41}, and t_{71} of Table 5. Two sectors require special attention with respect to value added. These are sectors 486 (government industry) and 487 (household industry). Neither of these use intermediate inputs, and it is therefore necessary to set their value added equal to 1988 gross output. The most recent sectoral breakdown of value added is for 1987 from Tables 6.1 and 6.2 of the U.S. NIPA accounts. These data cover approximately 60 aggregate sectors. Total 1988 value added less the value added for sectors 486 and 487 is allocated among these NIPA sectors based on the 1987 shares. Total value added for the NIPA sectors is then further allocated to the 485 remaining U.S. SAM sectors based on 1988 activity output shares. The use of the NIPA value added data is designed to preserve the broad sectoral structure of value added. The sectoral totals are allocated among labor income, property income, and indirect business taxes based on shares from the 1982 IMPLAN data. Finally, the value added sub-matrix is balanced to the macro-SAM control totals using the RAS procedure discussed in Chapter 4.

We next consider the errors entry in transaction $t_{11,1}$ of Table 5. Errors for sectors 486 and 487 are set equal to zero so that value added and gross output match exactly. Then the error value of -9,600 is allocated among the remaining 485 activity sectors in proportion to the total value added of each sector.

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[20] See Reinert and Roland-Holst(1993) for more details on construction of this SAM.

[21] On the RAS procedure, see Stone and Brown (1965), Bacharach (1970, Chapter 3), and Schneider and Zenios (1989). This procedure is described in Section 6 below.

The next sub-matrix is the import sub-matrix. Control totals for imports are taken from transactions t_{92} and $t_{10,2}$ of Table 5. Transaction t_{92} gives a total value of imports of goods and non-factor services of $537,900 million. Of this, $449,048 million is for imports of goods (merchandise) and $88,852 million is for imports of non-factor services. Import data for 1988 by 7-digit TSUSA line are extracted from U.S. Bureau of the Census data tapes and are matched to SAM sectors 1 to 411.[22] This results in total merchandise imports of $428,785 million. The difference between this total and the merchandise control total, $20,264 million, is allocated among the 411 merchandise sectors in proportion to their shares in the $428,785 million.

The control total for non-factor service imports is allocated among the relevant service sectors of the 1985 BEA input-output table based on shares from this table. Imports for these sectors are then further allocated to the service sectors of the U.S. SAM (sector 412-487) based on shares from the IMPLAN data. This two-step procedure is designed to preserve the sectoral structure of non-factor service imports, giving greater weight to the more recent data.[23]

Transaction $t_{10,2}$ gives a total value for duties collected of $16,448 million. Calculated duties collected data for 1988 by 7-digit TSU.S.A line are extracted from U.S. Bureau of the Census data tapes and are concorded to SAM sectors 1 through 487. This results in a total duties collected of $14,970 million. The difference between this total and the control total, $1,478 million, is allocated among the 487 sectors in proportion to their shares in the $14,9670 million.

The final demand sub-matrix is updated in a two-iteration process. Control totals are taken from transactions t_{26}, t_{27}, t_{28}, and t_{29} of Table 5. In the first iteration, special attention must be given to those

[22] TSU.S.A denotes tariff schedule of the United States.
[23] The U.S. SAM does not separate out noncompeting imports. This was done for two reasons. First, the BEA allocation of TSU.S.A lines between competing and noncompeting imports is inaccessible and dated. A new allocation is needed to properly determine noncompeting imports, and this is a daunting task. Second, removing noncompeting imports from the

commodities without intermediate deliveries. For these commodities, total final demand is set equal to estimated commodity supply (commodity output plus commodity imports). For the remaining sectors, I rely on the sectoral breakdown available from the Bureau of Economic Analysis (BEA) for 1985. These data cover approximately 80 sectors. Total 1988 final demand less the final demand for the above "special" sectors is allocated among the BEA sectors based on the 1985 shares. It is then further allocated to the 487 sectors based on shares of the estimated 1988 commodity supply. I rely on the 1985 data to ensure I have captured the broad sectoral structure of final demand for the most recent year available. The 487 sectoral totals are allocated among the final demand types (household, government, capital, and ROW or exports) based on shares from the 1982 IMPLAN data. Finally, the final demand sub-matrix is balanced to the macro-SAM control totals using the RAS procedure.

The second iteration of the final demand sub-matrix begins by computing the 1988 row control totals for the use matrix. The row control totals are computed as commodity supply (outputs plus imports) less total, iteration-one final demand and equal total intermediate demands. I then calculate the implied intermediate demand-total supply ratios. Since the row control totals or total intermediate demand are calculated as residuals, I want to make sure that this procedure has not "shut down" intermediate deliveries in any sectors. Therefore, I identify those sectors where the intermediate demand-total supply ratios fall by more than 50 percent from 1982 (based on IMPLAN data) to 1988 (based on our calculations).[24] Were this was the case, I project intermediate demand based on the growth rate of commodity supply between 1982 and 1988. I then calculate the new, implied final demands for these sectors. These second-iteration final demands are lower than the first-iteration final demands, and the final demands of other sectors must be raised to compensate for them. This

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imports of each commodity category would result in a SAM incompatible with estimated behavioral parameters and would defeat the purpose of using the SAM for general equilibrium modeling.

compensation is spread out over 35 sectors with the largest absolute increase in intermediate demand between 1982 and 1988. The second-iteration, sectoral totals are allocated among the final demand types based on shares from the 1982 IMPLAN data, and the sub-matrix is again balanced to the macro-SAM control totals using the RAS procedure.

The next step is to update the 1982 IMPLAN use matrix. For the row control vector, I take the estimated vector of commodity outputs, add to them the import vectors, and subtract the second-iteration final demand vectors. For the column control vector, I take the activity output vector and subtract the value added and error vectors. With these control vectors, I update the use matrix using the RAS procedure described in the next chapter.

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[24] This was the case for 20 of the 487 sectors.

3. Numerical and Statistical Methods for Reconciling Economic Accounts

In this section, we survey a number of basic methods for construction and management of large disaggregated economic databases like SAMs. In the course of constructing accounts, it is often necessary to reconcile diverse and partially conflicting information sources, and a number of numerical techniques have been devised for this purpose. To initiate practitioners, the most general of these will by discussed below. For those who wish to explore the subject further, references are also provided. Also covered here are aggregation methods which are often needed in the course of practical modeling work to bring detailed databases down to a manageable and more focused scale. We close with a number of practical examples of direct and weighted aggregation schemes.

SAM Balancing

One of the objectives of the Cambridge Growth Project was to estimate a detailed SAM of the United Kingdom for the year 1960. A transactions matrix was only available for 1954, so Stone (1962) suggested a procedure to update the matrix to 1960. This "RAS" method takes its name from the notation used in Stone's original equations. The RAS method estimated a transactions matrix for the year 1960 by starting with the 1954 transactions matrix, expressing it in 1960 prices, and adjusting rows and columns iteratively so that they add up to the 1960 totals.[25]

Let $\mathbf{R}_0$ be a known, initial matrix of transactions and let $\mathbf{R}$ be the unobservable transaction matrix for the year I desire to estimate. Let $\mathbf{p}$ be a vector whose elements are the ratios of desired period

[25] See Bacharach (1970, Chapter 3).

prices to initial period prices. Let $<\mathbf{z}>$ denote the diagonal matrix having vector **z** on its main diagonal. The **R** matrix in desired period prices then takes the form:[26]

$$\mathbf{R} = <\mathbf{p}>\mathbf{R}_0<\mathbf{p}>^{-1} \qquad (5)$$

The next step is to calculate a column vector of intermediate outputs for the desired year as the difference between gross outputs and final demands. Stone and Brown (1965) denote this vector **u**. The row vector **v** of intermediate inputs for the desired year is the difference between gross outputs and value added.

The following constraints must be satisfied:

$$\mathbf{Ri} = \mathbf{u} \qquad (6)$$

$$\mathbf{i'R} = \mathbf{v} \qquad (7)$$

where **i** is the conformable unit column vector. Equation 6 states that the rows of the new transaction matrix must sum to the observed row totals. Equation 7 states that the columns must sum to the observed column totals.

The problem is then to adjust **R** to obtain an estimate of **R**. The RAS algorithm proceeds as follows:[27]

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[26] The reader might multiply out a 2x2 example to elucidate this adjustment.

[27] See Stone and Brown (1965) and Schneider and Zenios (1989).

Step 0 (Initialization): Set k = 0 and $\mathbf{R}^k = \mathbf{R}$.

Step 1 (Row Scaling):

Define $\mathbf{r}^k = <\mathbf{u}>(\mathbf{R}^k\mathbf{i})^{-1}$

and update $\mathbf{R}^k$ as $\mathbf{R^*} \leftarrow <\mathbf{r}^k>\mathbf{R}^k$

Step 2 (Column Scaling):

Define $\boldsymbol{\sigma}^k = (\mathbf{i}'\mathbf{R^*})^{-1}<\mathbf{v}>$

and define $\mathbf{R}^{k+1}$ by $\mathbf{R}^{k+1} = \mathbf{R^*}<\boldsymbol{\sigma}^k>$

Step 3 : Replace k ← k + 1 and return to Step 1.

The algebraic RAS has a number of limitations. First, it cannot handle negative matrix elements. While this is not a problem for balancing the transactions matrix, it could be a problem for balancing other components of a SAM. Second, it is necessary to rescale the problem if any negative row or column totals appear. This rarely arises in practical work, however. Finally, the method assumes that the elements of the matrix are identically uniformly distributed random variables. This may not always be the case if one is less certain about some elements of the matrix than others. It is this consideration that has lead to research in new matrix balancing techniques.

Byron (1978) proposed the estimation of **R** by the minimizing of a constrained quadratic loss function. Let **r** denote the column vector created from the row vectorization of the nonzero elements of **R** . Similarly, the column vector created from the row vectorization of estimates of the nonzero elements of **R** is denoted by **r**. Now re-express (6) and (7) as:

$$\mathbf{Gr} - \mathbf{h} = \mathbf{0} \tag{7}$$

The objective will be for the estimates **r** to be as close to **r** as possible in a quadratic loss sense subject to the constraints in (7). This can be accomplished using the following constrained quadratic loss function:

$$Z = 1/2(\mathbf{r} - \mathbf{r})'\mathbf{V}^{-1}(\mathbf{r} - \mathbf{r}) + \boldsymbol{\lambda}'(\mathbf{Gr} - \mathbf{h})$$

(8)

The term $\boldsymbol{\lambda}$ is a vector of Lagrange multipliers.[28] The diagonal matrix **V** consists of weights that indicate the degree of certainty (variance) in the original **r**. The less the certainty, the less important are the differences between the estimated element and the original element. Byron proposes a conjugate gradient algorithm for minimizing (8).

Statistical MicroSAM Balancing - Maximum Entropy Tabular Reconciliation (METR)

While the linear reconciliation approach to SAM balancing is intuitive and easy to implement, it lacks any inferential basis, including uncertainty measurements or the capacity to take account of prior information. For this reason, IPALP relies on a more advanced method, termed Maximum Entropy Tabular Reconciliation (METR). This approach originates from the entropy control estimation techniques of information theory (see e.g. Kapur and Kesavan 1992, and Golan et al. 1996) and has been applied to social accounting matrix estimation in e.g. Robinson et al. (1998) and Robinson and El-Said (2000). This section provides a general overview of this reconciliation strategy, but interested readers should consult the literature on this topic before attempting application to large accounting systems.

The entropy technique is a method of solving underdetermined estimation problems. The problem is underdetermined because, for an n x n matrix, we are seeking to identify n^2 unknown, non-negative parameters, i.e. the cells of the SAM. However, there are only $2n-1$ independent row and column adding-up restrictions. In other words, restrictions must be imposed on the estimation problem so that we have enough information to obtain a unique solution and to provide enough degrees of freedom. The underlying philosophy of entropy estimation is to use *all* and *only* the information available for the problem at hand:

[28] See Chiang (1974, Chapter 12) for an explanation of the Lagrange-multiplier method of constrained optimization.

the estimation procedure should not ignore any available information nor should it add any false information. [29]

In the case of SAM estimation, 'information' may be the knowledge that there is measurement error concerning the variables, and that some parts of the SAM are known with more certainty than others. There may be a prior in the form a SAM from a previous year, whereby the entropy problem is to estimate a new set of coefficients 'close' to the prior using new information to update it. Furthermore, 'information' could consist of moment constraints on e.g. row and column sums, e.g. the average of the column sums. In addition to the row and column sums, 'information' may also consist of certain economic aggregates such as total value-added, aggregate consumption, investment, government consumption, exports and imports. Such information may be incorporated as linear adding-up restrictions on the relevant elements of the SAM. In addition to equality constraints such as these, information may also be incorporated in the form of inequality constraints placing bounds the mentioned macro aggregates. Finally, one may want to restrict cells that are zero in the prior to remain so also after the entropy balancing procedure.

Following Robinson et al. (2000) and Robinson and El-Said (2000), let the SAM be defined as a matrix T with elements $T_{i,j}$ representing a payment from the column account j to the row account i. As mentioned above, social accounting matrices are consistent accounting frameworks that do not allow leakages. In other words, every row sum in the SAM must equal the corresponding column sum:

(i) $$y_i = \sum_j T_{i,j} = \sum_j T_{j,i}$$

Dividing each cell entry in the matrix by its respective column total generates a matrix of column coefficients A:

$$A_{i,j} = \frac{T_{i,j}}{y_j}$$

It is assumed that the entropy problem starts with a prior, $\bar{A}$, which perhaps is a SAM from a previous year, or as in this case, a raw and unbalanced assembly of the SAM accounting components described in the previous section. $\bar{A}$ represents the starting point from which the cross-entropy balancing procedure departs in deriving the new matrix of coefficients A^*. The entropy problem is to

[29] See Shannon (1948) and Theil (1967), who motivate these statistical ideas from their roots in information theory.

find a new set of A coefficients which minimize the so-called Kullback-Leibler (1951) measure of the 'cross entropy' (CE) distance between the prior $\overline{A}$ and the new estimated coefficient matrix A*.

$$\min_{\{A\}} I = \left[\sum_i \sum_j A_{i,j} \ln \frac{A_{i,j}}{\overline{A}} \right] = \left[\sum_i \sum_j A_{i,j} \ln A_{i,j} - \sum_i \sum_j A_{i,j} \ln \overline{A} \right]$$

subject to

$$\sum_j A_{i,j} y_i^*$$

$$\sum_j A_{j,i} = 1 \; and \; 0 \le A_{j,i} \le 1$$

Analogous to Walras' Law in general equilibrium theory, note that one equation can be dropped in the second set of constraints: If all but one column and row sum are equal, the last one must also be equal. The solution of the above problem is solved by setting up the Lagrangian. The *k* macro aggregates can be added to the set of constraints on the problem above as follows:

$$\sum_i \sum_j G_{i,j}^{(k)} T_{i,j} = \gamma^{(k)}$$

where *G* is an *n* x *n* aggregator matrix with ones for cells that represent the macro constraints and zeros otherwise, and γ is the value of the aggregate constraint.

As mentioned above, in the real world one faces economic data measured with error. The cross entropy problem can also be formulated as an 'error-in-variables' system where the independent variables are measured with noise. If, for example, we assume the known column sums are measured with error, the row/column consistency constraint can be written as:

$$y = \bar{x} + e$$

where y is the vector of row sums and $\bar{x}$, the known vector of column sums, is measured with error e. The prior estimate of the column sums could be the initial column sums, the average of the initial column and row sums, or e.g. the row sums.

Following Golan et al. (1996) the errors are written as weighted averages of known constants *v*:

$$e_i = \sum_w w_i \bar{v}_{i,w}$$

where *w* is a set of weights that fulfill the following constraints:

$$\sum_w w_{i,w} = 1 \text{ and } 0 \le w_{i,w} \le 1$$

In the estimation problem the weights are treated as probabilities to be estimated, and the prior for the error distribution in this case is chosen to be a symmetric distribution around zero with predefined lower and upper bounds, and using either three or five weights. Naturally, not only the column and row sums can be measured with error. The macro aggregates by which we constrain our estimation problem may also be measured with error and so we can operate with two sets of errors with separate weights *w1*'s on the column sum errors, and weights *w2*'s on the macro aggregate errors. The optimization problem in the 'errors-in-variables' formulation is now the problem of finding *A*'s, *w1*'s and *w2*'s that minimize the cross entropy measure including a terms for the error weights:

$$\min_{\{A,w1,w2\}} I = \left[\sum_i \sum_j A_{i,j} \ln A_{i,j} - \sum_i \sum_j A_{i,j} \ln \bar{A}\right] + \left[\sum_i \sum_w w1_{i,w} \ln w1_{i,w} - \sum_i \sum_w w1_{i,w} \ln \overline{w}1_{i,j}\right] + \left[\sum_i \sum_w w2_{i,w} \ln w2_{i,w} - \sum_i \sum_w w2_{i,w} \ln \overline{w}2_{i,j}\right]$$

The cross-entropy measures reflect how much the information we have introduced has moved our solution estimates away from the inconsistent prior, whilst also accounting for the imprecision of the moments assumed to be measured with error. Hence if the information constraints are binding, the distance from the prior will increase. If they are not binding, the cross entropy distance will be zero.

The IPALP application of the cross entropy estimation to a raw and unbalanced MicroSAM uses the 'error-in-specification' formulation described above, and the standard errors for both the column sum and macro aggregate constraints have been set to 1%. The prior for the column sums equal to the average of the initial column and row sums since that there is no *a priori* belief that the one should be more accurate than the other. In addition to the column constraints, a number of macro aggregates have been introduced as constraints on the estimation process. The total value of factor payments is fixed to the aggregate value as specified in the MacroSAM. In other words total GDP at factor costs is constrained to its original value. Furthermore, the foreign trade entries are constrained to their macro aggregates, as are the entries for total private consumption, total government consumption and total investments. Hence also total GDP at market prices and measured from the expenditure side is also bound to the macro figures, taking into account the margin allowed for measurement errors.

For the IPALP approach, we have also developed computer software to implement METR. While RAS methods can be carried out in ordinary spreadsheet applications, METR requires dedicated higher level programming to implement its optimization features.

Global Aggregation Schemes for CGE Modeling

Economists are called upon to assess the impacts of commercial policies at many different levels of aggregation, from a tariff on a particular 4-digit SIC item to broad, sectoral average tariffs. In addition, they are often asked to assess the inter-sectoral effects of commercial policies. For example, what are the effects of the steel quotas on downstream users (e.g. autos) or upstream suppliers (e.g. coal)? Moreover, how can one trace the detailed income-expenditure links of these production effects to factors, consumption units, and capital accounts? These considerations suggest that a disaggregated SAM with interindustry detail would facilitate a more complete analysis of trade policy. However, it would facilitate analysis and interpretation to aggregate those parts of the SAM that are not of detailed relevance to the issues under consideration.

The primary objective, then, has been to develop a consistent, economy-wide data base containing a significant amount of inter-industry detail along with exogenous parameter estimates. This data base will be accompanied by a flexible aggregation facility which will allow each user to focus on particular sectors of the economy, aggregating the remainder of the economy into a few, more generic sector, consumption, and rest-of-the-world accounts.

The role of the SAM and the aggregation facility in supporting CGE modeling of trade policy is represented in Figure 4.1. The fully-disaggregated SAM is denoted as "SAM I" in this figure, and the initial exogenous parameter estimates are denoted EPE I.[30] The aggregation procedure takes the information in this disaggregated data base and creates a second SAM and corresponding parameter set at a level of aggregation specified by the user. I call these "SAM II" and "EPE II" in the Figure. SAM II composes the benchmark equilibrium data set for the CGE model.[31] The set EPE II of aggregate parameters represents only a subset of the structural parameters of the model. The rest of these (share parameters, etc.) are obtained by calibrating the model to the benchmark data.[32] The analyst introduces an exogenous, counterfactual policy change, such as a tariff cut, and the behavioral model simulates the response of the economy to such a policy change. This results in a counterfactual equilibrium which can be expressed as a new SAM, denoted "SAM III". In this way, *the modeling exercise begins and ends*

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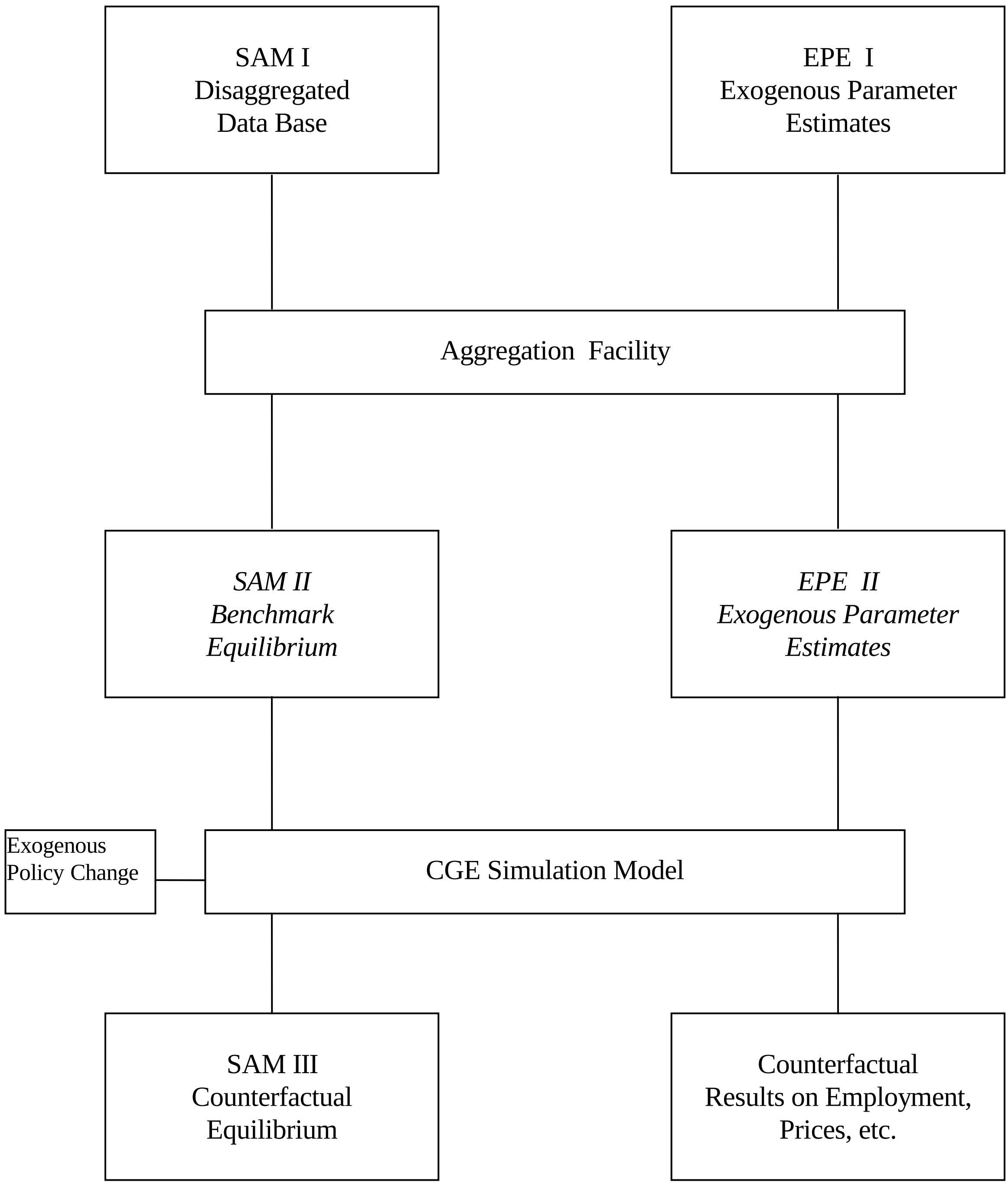
[30] For a description of the construction of the exogenous parameter estimate database, see Reinert and Roland-Holst (August 1990).

[31] See Shoven and Whalley (1984) for a description of the role of benchmark equilibrium data sets in CGE analysis.

[32] Calibration is the process of calculating a set of share parameters based on the benchmark equilibrium data set given exogenous elasticity parameters. This is done so that the model will reproduce the initial data set (SAM II) as an equilibrium given the initial levels of exogenous policy variables and elasticity parameters. See Shoven and Whalley (1984).

with a SAM. At the third stage, the model also produces a large volume of subsidiary counterfactual results on changes in employment, prices, etc..

Figure 4.1: Schematic of SAM, CGE Model and Flexible Aggregation Facility



Consolidation of commodity accounts

Some applications of SAMs require that the make and use matrices be consolidated into a transactions matrix. There are two possible approaches: to eliminate the commodity accounts or to eliminate the activity accounts. I present here consolidation via elimination of activity accounts and explain how the resulting SAM is more suitable for commercial-policy analysis than the SAM resulting from the elimination of the commodity accounts.

Consider a simple case with two activity accounts (A1,A2), two commodity accounts (C1,C2), a demand account (D), and a rest-of-the-world account (R). I partition the SAM with the accounts to be retained in the upper left-hand corner:

Table 4.1: Schematic Activity-Commodity Breakdown

	C1	C2	D	R	A1	A2
C1			**F1**	**E1**	**U11**	**U12**
C2			**F2**	**E2**	**U21**	**U22**
D					**V1**	**V2**
R	**I1**	**I2**				
A1	**M11**	**M12**				
A2	**M21**	**M22**				

The use matrix has coefficients U_{ij}, and the make matrix has coefficients M_{ji}. Final demands are F_i, and the value added values are V_j. Imports and exports are I_i and E_i, respectively.

As Pyatt (1985) demonstrates, the consolidated accounts may be expressed in matrix form as :

$$T = \begin{bmatrix} 0 & 0 & F1 & E1 \\ 0 & 0 & F2 & E2 \\ 0 & 0 & 0 & 0 \\ I1 & I2 & 0 & 0 \end{bmatrix} + \begin{bmatrix} U11 & U12 \\ U21 & U22 \\ V1 & V2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} M11 & M12 & 0 & 0 \\ M21 & M22 & 0 & 0 \end{bmatrix} 0$$

where the underbars denote column-sum normalized coefficients.

As can be seen from this expression, the trade accounts are preserved in their original form by commodity. This is *not* the case for the elimination of the commodity accounts under which the imports (including tariffs) and exports are apportioned. Therefore, relying on commodity technology is useful for commercial-policy analsis.

Practical Examples of Aggregation

The analysis of trade policy at the industry level requires a high level of detail to capture specific protection mechanisms and market interactions. At the same time, detailed results on all sectors of the economy are of little direct interest to a given case study and could make CGE analysis prohibitively expensive. For this reason, aggregation will play a prominent role in the ITC modeling facility. A typical application would consist of about ten to twenty sectors, five to ten (including the target sector) chosen at the 528 sector level of detail or a little more aggregated, and five to ten others representing the remaining sectors of the economy aggregated into more generic sectors. This approach is very appealing on efficiency grounds, and there is a growing body of evidence supporting its empirical validity.[33]

The aggregation problem takes two forms in our analysis, exact and weighted aggregation. Exact aggregation is the usual adding up of sectoral flow or stock data, while weighted aggregation will be used for structural parameters which apply to the aggregated flows. The former method is a special case of the latter where the weights are unitary. The general problem of symmetric aggregation from n to m sectors (or other accounting categories) can be stated as:

$$\mathbf{B} = \mathbf{W}'\mathbf{A}\mathbf{W} \tag{9}$$

where **A** is an n x n disaggregated matrix of data, **B** is the m x m aggregated matrix, and **W** is an n x m matrix of aggregation weights. A typical column **W**j represents an n vector of weights, zero for those of the n original sectors not to be included in the new jth category of **B**, nonzero otherwise. In the case of flow data, the nonzero values can be unity (exact aggregation), where they must be mutually exclusive between the **W**j, or they can be weights which sum to unity across j, apportioning the ith original (**A**) flow between new (**B**) categories. With flow data, there is no necessary relationship between the elements of **W**j. When performing weighted aggregation of parameters, however, the columns of **W** should each sum to unity and be mutually exclusive in nonzero values.

As an elementary example of exact weighting, consider an aggregation of the 3 x 3 SAM of Table 3.1 into a 2 x 2 SAM that separates production from consumption and accumulation. Account 1 will consist of production alone, so $\mathbf{W}_1$ = [1,0,0]'. Account 2 will consist of consumption and accumulation, so $\mathbf{W}_2$ = [0,1,1]'. The equivalent of equation (9) is:

[33] Results of Morkre (1989), de Melo and Roland-Holst (1989), and Reinert and Roland-Holst (1990) indicate that the aggregation chosen for other sectors has no substantive effect on estimated interactions between the

$$\begin{bmatrix} 0 & C+I \\ Y & S \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & C & I \\ Y & 0 & 0 \\ 0 & S & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}^{0}$$

Next, consider the aggregation of consumption flows and income elasticities in the Linear Expenditure System. This example generalizes the above discussion to the aggregation of nonsquare tables of flows and elasticities. The information on the LES is given in Table 4.1. There are four types of goods (food, durables, nondurables, and services) and three consumption groups (rural, urban union, and urban nonunion). Our objective is to aggregate this information to a two-by-two case of food and non-food goods and rural and urban consumers.

We first aggregate the expenditure matrix. As in the case of the aggregation of a square matrix, I pre- and post-multiply the matrix by weighting matrices of units and zeros. In the nonsquare case, however, these are not the simple transpose of each other. The premultiplication matrix aggregates the rows of the original matrix. Since I want to aggregate the second through fourth rows, the first row of the premultiplication matrix is [1, 0, 0, 0], while the second row is [0, 1, 1, 1]. The postmultiplication matrix aggregates the columns of the original matrix. Since I want to aggregate the second and third column, the first column of the postmultiplication matrix is [1, 0, 0]' while the second column is [0, 1, 1]'. Putting together these elements, the expenditure aggregation is:

detailed sectors.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 50 & 40 & 75 \\ 10 & 15 & 25 \\ 30 & 30 & 50 \\ 10 & 15 & 50 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 50 & 115 \\ 50 & 185 \end{bmatrix}$$

We next aggregate the income elasticity matrix. Due to the fact that different weights must be applied to each column, I transform the original matrix into a block diagonal matrix with the income elasticity blocks for each consumer group on the diagonal. A premultiplication matrix aggregates the rows of this matrix. The aggregation of rows must maintain the Engel aggregation condition that the consumption-weighted sum of the income elasticities must equal unity.[34] This condition can be maintained by weighting each of the income elasticities of the goods to be aggregated together by the shares of the expenditure on these goods in the total expenditure on the group to be aggregated. The second row of the premultiplication matrix aggregates the income elasticities of the nonfood items. The aggregation of goods is as follows:

Table 4.2: Expenditure and Income Elasticities

	Consumption Flows			Income Elasticities		
Goods	Rural	Urban Union	Urban Nonunion	Rural	Urban Union	Urban Nonunion
Food	50	40	75	0.90	0.70	0.70
Durables	10	15	25	1.05	1.10	1.10
Nondurables	30	30	50	1.05	1.10	1.10
Services	10	15	50	1.30	1.50	1.30
Totals	100	100	200			

[34] See Henderson and Quandt (1980, Chapter 2) for a discussion of the Engel aggregation condition.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0.2 & 0.6 & 0.2 & 0 & 0.25 & 0.5 & 0.25 & 0 & 0.2 & 0.4 & 0.4 \end{bmatrix} \begin{bmatrix} 0.90 & 0 & 0 \\ 1.05 & 0 & 0 \\ 1.05 & 0 & 0 \\ 1.30 & 0 & 0 \\ 0 & .70 & 0 \\ 0 & 1.10 & 0 \\ 0 & 1.10 & 0 \\ 0 & 1.50 & 0 \\ 0 & 0 & .70 \\ 0 & 0 & 1.10 \\ 0 & 0 & 1.10 \\ 0 & 0 & 1.30 \end{bmatrix} = \begin{bmatrix} 0.90 & 0.70 & 0.70 \\ 1.10 & 1.20 & 1.18 \end{bmatrix}$$

To aggregate the column of the resulting matrix, I use a postmultiplication matrix. The aggregation of columns must be done so that the Engel aggregation condition holds in the resulting aggregate column. This is so if I weight each of the income elasticities of the consumers to be aggregated by their shares in the total expenditure on the good in question by these consumers. The aggregation of columns is given by:

$$\begin{bmatrix} 0.90 & 0.70 & 0.70 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.10 & 1.20 & 1.18 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0.348 \\ 0 & 0.652 \\ 1 & 0 \\ 0 & 0.324 \\ 0 & 0.676 \end{bmatrix} = \begin{bmatrix} 0.90 & 0.70 \\ 1.10 & 1.19 \end{bmatrix}$$

Through weighting schemes similar to this, I will aggregate the various elasticity values of the parameter database EPE I of Figure 4.1.

A Flexible Aggregation Facility

Despite the importance of detailed base accounting information, implementing a very detailed SAM in a CGE model can be quite unwieldy, impractical, and would generate vast amounts of extraneous information for most policy studies. For this reason, a flexible aggregation facility can be useful to consolidate the database around the sectors of principal interest in a given policy exercise. This consolidation is done with respect to a reference aggregation of nine sectors which are consistent with production accounts published by the Department of Commerce in their annual NIPA accounts (see Table 6 series, U.S. Department of Commerce, 1988). These nine reference sectors are defined in Table 4.2, and the reference SAM is given in Table 4.3.

The nine reference sectors capture generic interactions among productive sectors in the economy, but they are too aggregate to clearly identify the role of specific sectoral policies. When a given sector is chosen for study, it is instead drawn out from the 487 sector database and carried individually through the nine sector aggregation, yielding a total of ten sectors for analysis. If more detailed indirect effects are of interest, then a number of related sectors can also be segregated in the aggregate database. For example, if I were interested in the agricultural sector sugar crops (13) and the manufacturing sector sugar (74), the resulting SAM would be that given in Table 4.4.

Table 4.3: Reference U.S. Industrial Classifications.

Sector	Label	Title
1	AgForFsh	Agriculture, Forestry, and Fishing
2	Mining	Mining and Mineral Resources
3	Construct	Construction
4	NDurMfg	Nondurable Manufacturing
5	DurMfg	Durable Manufacturing
6	TrComUt	Transportation, Communication, Utilities
7	Trade	Wholesale and Retail Trade
8	FinInsRE	Finance, Insurance, and Real Estate
9	Services	Personal, Business, and Public Services

Table 4.4: A Reference SAM

		AgForFsh	*Mining*	*Construct*	*NDurMfg*	*DurMfg*	*TrComm*	*Trade*	*FinInsRE*	*Services*
		1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
1	***AgForFsh***	42174	7	2408	98260	7783	74	2683	7610	6565
2	***Mining***	68	9626	2244	81959	8091	34823	1	25	25
3	***Construct***	1806	11668	625	6767	8747	20927	5527	35989	17626
4	***NDurMfg***	29973	1015	34995	370421	83276	37101	24004	14290	149157
5	***DurMfg***	4073	2594	174911	54660	479542	18816	7494	4395	80976
6	***TrComm***	4511	1240	16564	66440	64757	78291	45619	30976	83618
7	***Trade***	8202	753	72451	57265	72983	10949	13764	7129	49736
8	***FinInsRE***	10083	2667	9646	17949	25210	14466	51925	193663	79024
9	***Services***	4989	1410	52562	68116	74358	30890	123868	93098	213502
1	***Labor***	32505	18242	197013	218389	429879	211905	384751	217417	1197545
11	***Property***	60036	55682	31662	141784	68905	207225	146709	511312	332442
12	***Enterprise***	0	0	0	0	0	0	0	0	0
13	***Household***	0	0	0	0	0	0	0	0	0
14	***Government***	7755	11736	7014	27723	18290	35207	126693	113027	29621
15	***CapAcct***	0	0	0	0	0	0	0	0	0
16	***ROW***	8167	31302	0	114621	294959	74768	0	11769	2315
17	***ROWTaxes***	176	192	0	8341	7739	0	0	0	0
18	***Error***	-222	-189	-521	-858	-1144	-1005	-1456	-1862	-2344
	Total	214296	147945	601574	1331837	1643375	774437	931582	1238838	2239808

		Labor	*Property*	*Enterprise*	*Household*	*Governmen*	*CapAcct*	*ROW*	*ROWTaxes*	*Error*
		10	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*
1	***AgForFsh***	0	0	0	17573	6940	659	21562	0	0
2	***Mining***	0	0	0	877	473	1600	8132	0	0
3	***Construct***	0	0	0	0	133789	357941	160	0	0
4	***NDurMfg***	0	0	0	452646	38311	3511	93137	0	0
5	***DurMfg***	0	0	0	236374	96719	295724	187098	0	0
6	***TrComm***	0	0	0	310041	33654	12788	25938	0	0
7	***Trade***	0	0	0	528885	11051	55747	42668	0	0
8	***FinInsRE***	0	0	0	771344	15741	22287	24832	0	0
9	***Services***	0	0	0	917354	632269	0	27391	0	0
10	***Labor***	0	0	0	0	0	0	0	0	0
11	***Property***	0	0	0	0	0	0	116747	0	0
12	***Enterprise***	0	1589072	0	96146	92292	0	0	0	0
13	***Household***	2463048	0	1045732	0	555683	0	0	0	0
14	***Government***	444599	0	137936	586649	0	96146	0	16448	0
15	***CapAcct***	0	0	593842	144711	0	0	117450	0	-9600
16	***ROW***	0	83431	0	1862	41922	0	0	0	0
17	***ROWTaxes***	0	0	0	0	0	0	0	0	0
18	***Error***	0	0	0	0	0	0	0	0	0
	Total	2907647	1672503	1777510	4064462	1658844	846403	665115	16448	-9600

Table 4.5: A SAM with Disaggregated Sugar Sectors

		SugarCrop	*SugarRef*	*AgForFsh*	*Mining*	*Construct*	*NDurMfg*	*DurMfg*	*TrComm*	*Trade*	*FinInsRE*	*Services*
		1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
1	***SugarCrop***	33	1392	1	0	0	10	0	0	0	0	0
2	***SugarRef***	0	2035	42	0	0	4184	3	0	0	0	80
3	***AgForFsh***	151	9	41989	7	2408	96849	7783	74	2683	7610	6565
4	***Mining***	1	30	67	9626	2244	81929	8091	34823	1	25	25
5	***Construct***	15	32	1792	11668	625	6734	8747	20927	5527	35989	17626
6	***NDurMfg***	187	452	29743	1015	34995	363750	83273	37101	24003	14290	149077
7	***DurMfg***	50	40	4023	2594	174911	54620	479542	18816	7494	4395	80976
8	***TrComm***	17	768	4495	1240	16564	65672	64757	78291	45619	30976	83618
9	***Trade***	68	542	8134	753	72451	56723	72983	10949	13764	7129	49736
10	***FinInsRE***	186	77	9897	2667	9646	17872	25210	14466	51925	193663	79024
11	***Services***	43	201	4945	1410	52562	67915	74358	30890	123868	93098	213502
12	***Labor***	104	963	32402	18242	197013	217426	429879	211905	384751	217417	1197545
13	***Property***	572	652	59464	55682	31662	141132	68905	207225	146709	511312	332442
14	***Enterprise***	0	0	0	0	0	0	0	0	0	0	0
15	***Household***	0	0	0	0	0	0	0	0	0	0	0
16	***Government***	17	53	7739	11736	7014	27669	18290	35207	126693	113027	29621
17	***CapAcct***	0	0	0	0	0	0	0	0	0	0	0
18	***ROW***	0	611	8167	31302	0	114010	294959	74768	0	11769	2315
19	***ROWTaxes***	0	0	176	192	0	8341	7739	0	0	0	0
20	***Error***	-2	-4	-220	-189	-521	-854	-1144	-1005	-1456	-1862	-2344
	Total	1442	7853	212855	147945	601573	1323984	1643376	774437	931582	1238837	2239808

		Labor	*Property*	*Enterprise*	*Household*	*Governmen*	*CapAcct*	*ROW*	*ROWTaxes*	*Error*
		12	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*
1	***SugarCrop***	0	0	0	0	0	0	6	0	0
2	***SugarRef***	0	0	0	1363	25	0	120	0	0
3	***AgForFsh***	0	0	0	17573	6940	659	21556	0	0
4	***Mining***	0	0	0	877	473	1600	8132	0	0
5	***Construct***	0	0	0	0	133789	357941	160	0	0
6	***NDurMfg***	0	0	0	451283	38285	3511	93017	0	0
7	***DurMfg***	0	0	0	236374	96719	295724	187098	0	0
8	***TrComm***	0	0	0	310041	33654	12788	25938	0	0
9	***Trade***	0	0	0	528885	11051	55747	42668	0	0
10	***FinInsRE***	0	0	0	771344	15741	22287	24832	0	0
11	***Services***	0	0	0	917354	632269	0	27391	0	0
12	***Labor***	0	0	0	0	0	0	0	0	0
13	***Property***	0	0	0	0	0	0	116747	0	0
14	***Enterprise***	0	1589072	0	96146	92292	0	0	0	0
15	***Household***	2463048	0	1045732	0	555683	0	0	0	0
16	***Government***	444599	0	137936	586649	0	96146	0	16448	0
17	***CapAcct***	0	0	593842	144711	0	0	117450	0	-9600
18	***ROW***	0	83431	0	1862	41922	0	0	0	0
19	***ROWTaxes***	0	0	0	0	0	0	0	0	0
20	***Error***	0	0	0	0	0	0	0	0	0
	Total	2907647	1672503	1777510	4064463	1658844	846403	665115	16448	-9600

4. Multilateral Trade Flow Estimation

Part II: Theory and Specification of Economic Behavior

5. Household Behavior

The choice of functional form for the consumer demand system depends to a large extent on the availability of data, particularly elasticity estimates. If a demand system has only *n* parameters, then normally it will be calibrated to the *n* budget shares, and no external data is necessary. This is the case with the CES demand system (and its Cobb-Douglas derivative) which has exactly *n* parameters to calibrate. However, the use of a simple demand system such as the CES imposes severe assumptions about income and price elasticities which are unlikely to be even remotely consistent with observed data. In the case of the more often used linear expenditure system (LES), there are 2*n* parameters to calibrate and hence this system requires at least *n* additional parameters to fully calibrate the system. These additional parameters are typically estimates of income elasticities generated by econometric estimation performed on the relevant base data (for example household income surveys), or else by pulling results from other studies. (N.B. The functional forms may impose restrictions on the elasticities in which case they may not all be independent. An example of this is shown in the section on the LES).

With a full scope of elasticities, including price, income and cross-price elasticities, it is possible to calibrate a demand system with a large number of parameters. These types of functions have been known as flexible functional forms. They do have drawbacks including the fact that they tend to be more complex than other demand systems. Another potential drawback is that they are not always integrable, in which case it is not always possible to undertake consistent welfare analysis.

This chapter will start with the easiest demand system derived from a CES utility function. It is easy to implement and calibrate, but it does impose strong assumptions concerning income and price elasticities. Given the rather large literature on income and price elasticities, and the ease of implementation of the LES, its use is strongly recommended over the CES. Nonetheless, it also imposes restrictions on price elasticities which are not always consistent with observed data (for example, it assumes all goods are gross substitutes, and no good is an inferior good). Both the LES and an extension of the LES are presented in sections 2 and 3. Section 4 introduces a flexible functional form known as the Almost Ideal Demand System (AIDS), which though more general than either the CES or the LES, is more difficult to implement, and generally requires more underlying data for calibration purposes. Section 5 provides a comparison of the CES, the LES, and the AIDS demand system using a standard neo-classical GE model. This section does not prove that one system is better than another, but it does show that specification of the consumer demand system does matter.

The CES Utility Function and its Derivatives

The consumer demand system derived from the CES utility function is generated by the following framework:

$$\max U = \left[\sum_{i=1}^{n} a_i C_i^{\rho} \right]^{1/\rho}$$

subject to:

$$\sum_{i=1}^{n} P_i C_i = Y$$

where C is the vector of consumer demand for goods and services, P is the vector of consumer prices, and Y is disposable income. The a parameters are share parameters and will be interpreted below.

Solution of the optimisation leads to the following demand system (see derivation of CES reduced form in Chapter 4):

$$C_i = \alpha_i \left(\frac{P}{P_i} \right)^{\sigma} \frac{Y}{P}$$

$$P = \left[\sum_i \alpha_i P_i^{1-\sigma} \right]^{1/(1-\sigma)}$$

where we have the following relationship between the primal and dual parameters:

$$\alpha_i = a_i^{\sigma} \qquad \text{and} \qquad \sigma = \frac{1}{1-\rho} \geq 0$$

Notice that in the case of unit substitution elasticity, the budget shares (i.e. C_iP_i/Y) are constant, irrespective of income and relative prices. Calibration of the CES is straightforward with the given substitution elasticity and base year consumption and prices. Also note, that all formulas hold for the Cobb-Douglas case, except for the utility primal function, and the definition of the price index. These formulas are:

$$U = \prod_i C_i^{\alpha_i}$$

$$P = \prod_i \left(\frac{P_i}{\alpha_i} \right)^{\alpha_i}$$

where the α coefficients must sum to 1 and are in fact the (constant) budget share parameters, also equal to s_i below. All the formulas below hold for the Cobb-Douglas case (i.e. σ equal to 1), with the appropriate price index formula.

The income elasticity for all commodities is 1, which is seriously contravened by econometric estimates of income elasticities. The own price elasticity is given by:

$$\frac{\partial C_i}{\partial P_i} = -\sigma \frac{C_i}{P_i} + (\sigma - 1)\alpha_i \left(\frac{P}{P_i} \right)^{\sigma} \frac{C_i}{P_i} = -\sigma \frac{C_i}{P_i} + (\sigma - 1) \frac{C_i}{Y} C_i$$

$$\varepsilon_{ii} = \frac{\partial C_i}{\partial P_i} \frac{P_i}{C_i} = -\sigma + (\sigma - 1) s_i = \sigma (s_i - 1) - s_i$$

where s_i is the budget share ($s_i = C_iP_i/Y$). Likewise, the cross-elasticities can be derived to yield:

$$\varepsilon_{ij} = \frac{\partial C_i}{\partial P_j} \frac{P_j}{C_i} = (\sigma - 1) s_j = \sigma s_j - s_j$$

Note that the derivative of the price index with respect to P_i is given by:

$$\frac{\partial P}{\partial P_i} = \alpha_i \left(\frac{P}{P_i} \right)^{\sigma} = s_i \frac{P}{P_i}$$

The indirect utility function, $v(P,Y)$ is derived by inserting the optimal consumption function into the primal utility function. This yields the following indirect utility function:

$$v(P,Y)=\frac{Y}{P}$$

It is easy to verify that the Marshallian demand function can be derived from the indirect utility function:

$$C_i(P,Y)=-\frac{\partial v/\partial P_i}{\partial v/\partial Y}=-\frac{-Y\ P^{-2}(\partial P/\partial P_i)}{1/P}=\alpha_i\left(\frac{P}{P_i}\right)^{\sigma}\frac{Y}{P}$$

The expenditure function is the solution to the following minimisation problem, where *u* and *P* are exogenous:

$$\min\sum_i P_i C_i$$

subject to

$$u=\left[\sum_i a_i C_i^{\rho}\right]^{1/\rho}$$

This yields:

$$E(P,u)=uP$$

The compensated (or Hicksian) demand function are given by:

$$H_i(P,u)=\frac{\partial E}{\partial P_i}=\alpha_i\left(\frac{P}{P_i}\right)^{\sigma}u$$

The compensated own-price elasticity is:

$$\xi_{ii}=\sigma(s_i-1)$$

And the compensated cross-price elasticity is:

$$\xi_{ij}=\sigma\ s_j$$

LES

One of the more frequently used demand systems in applied GE work is the so-called *linear expenditure system* (LES), also known as the Stone-Geary demand system owing to the early development of the system by Richard Stone and XXX Geary.[35] It turns out that the LES demand system is derived from a rather simple modification of the Cobb-Douglas utility function.[36] The modification permits the income elasticity for each demanded commodity to differ from unity, which is a significant improvement. The LES utility function has the following form:

$$U=\prod_{i=1}^{n}(C_i-\theta_i)^{\mu_i}$$

with

$$\sum_{i=1}^{n}\mu_i=1$$

where *U* is utility, *C* is the vector of consumption goods, and μ and θ are consumer demand parameters which are interpreted below. The reason for the normalisation constraint on the μ parameters will also be explained below. There are *n* consumer goods.

[35] Get reference for the Stone-Geary work.

[36] The only modification to the Cobb-Douglas function are the shift parameters represented by θ.

It is easy enough to derive the demand equations, the consumer solves the following problem:

$$\max \prod_{i=1}^{n} (C_i - \theta_i)^{\mu_i}$$

subject to[37]

$$\sum_{i=1}^{n} P_i C_i = Y$$

where P is the vector of consumer prices, and Y is disposable income (after taxes and disposition of household saving). The first order conditions are:

(1) $$\frac{\mu_i}{C_i - \theta_i} u = P_i \lambda \qquad 1 \le i \le n$$

(2) $$\sum_{i=1}^{n} P_i C_i = Y$$

Through substitution we derive the following demand functions:

(3) $$C_i = \theta_i + \frac{\mu_i}{P_i}\left(Y - \sum_{j=1}^{n} P_j \theta_j\right) \qquad 1 \le i \le n$$

The usual interpretation of the demand function is that consumer demand is the sum of two elements. The first part is the so-called *subsistence minima*, θ (also referred to as the *floor* consumption). The second element is a share of disposable income after the purchase of the aggregate subsistence minima (the μ parameter is sometimes called the marginal propensity to consume). The expression

$$Y^* = Y - \sum_{j=1}^{n} P_j \theta_j$$

is sometimes referred to as the *supernumerary income*, it is the value of residual disposable income after purchases of the subsistence minima. From equation (3), it is clear that the normalisation restriction on the μ parameters must obtain, i.e. the marginal budget shares must sum to 1.

From the demand equation we can derive the income and price elasticities:

(4) $$\eta_i = \frac{\mu_i Y}{P_i C_i} = \frac{\mu_i}{s_i}$$

(5) $$\varepsilon_{ii} = \frac{\theta_i (1 - \mu_i)}{C_i} - 1$$

(6) $$\varepsilon_{ij} = -\frac{\mu_i \theta_j P_j}{P_i C_i} = -\frac{\mu_i \theta_j P_j}{s_i Y}$$

where η_i are the income elasticities, ε_{ii} are the own price elasticities, and ε_{ij} are the cross-price elasticities. The income elasticity is the ratio of the marginal propensity to consume out of supernumerary income over the average budget share, s_i.

[37] It is assumed that all disposable income is consumed, i.e. the budget constraint is binding.

The Lagrangian multiplier, λ, is the marginal utility of income, i.e. it represents the increment to utility by relaxing the budget constraint. Inserting equation (3) into (1) yields the following expression for λ:

$$(7) \qquad \lambda = \frac{1}{P} = \prod_{i=1}^{n} \left(\frac{P_i}{\mu_i} \right)^{-\mu_i}$$

where the variable P defines the (dual) price index of the consumer prices.

Welfare

The indirect utility function is immediately derived by inserting equation (3) into the definition of the utility function:

$$v(P,Y) = \prod_{i=1}^{n} (C_i - \theta_i)^{\mu_i}$$

or

$$v(P,Y) = \prod_{i=1}^{n} \left(\frac{\mu_i}{P_i} Y^* \right)^{\mu_i} = Y^* / P$$

where Y^* is defined as supernumerary income. (The simplification is due to the normalisation rule on the μ parameters). The indirect utility function represents the maximum level of utility obtainable given income Y, and the vector of prices P. Using duality theory, it is possible to derive the Marshallian demand function starting from the indirect utility function:

$$C_i(P,Y) = -\frac{\partial v / \partial P_i}{\partial v / \partial Y}$$

where

$$\frac{\partial v}{\partial P_i} = \frac{1}{P}\frac{\partial Y^*}{\partial P_i} - \frac{Y^*}{P^2}\frac{\partial P}{\partial P_i} = -\frac{\theta_i}{P} - \frac{\mu_i}{P_i}\frac{Y^*}{P} \qquad \text{and} \qquad \frac{\partial v}{\partial Y} = \frac{1}{P}\frac{\partial Y^*}{\partial Y} = \frac{1}{P}$$

and

$$\frac{\partial P}{\partial P_i} = \frac{\mu_i}{P_i} P$$

The resulting expression is equivalent to equation (3). The expenditure function is derived by minimising the cost of achieving a given level of utility, u. It is set-up as:

$$\min \sum_{i=1}^{n} P_i C_i$$

subject to

$$\prod_{i=1}^{n} (C_i - \theta_i)^{\mu_i} = u$$

The first order conditions for the expenditure function are:

$$\lambda u = \frac{P_i}{\mu_i}(C_i - \theta_i)$$

$$\prod_{i=1}^{n} (C_i - \theta_i)^{\mu_i} = u$$

Inserting the first equation into the second equation, we derive the following expression:

$$\lambda = P$$

Next, we can re-insert the first equation into the primal function, replacing λ with P to derive:

$$E(P,u) = \sum_{i=1}^{n} P_i C_i = \sum_{i=1}^{n} P_i \theta_i + uP$$

where

$$P = \prod_{i=1}^{n} \left(\frac{P_i}{\mu_i} \right)^{\mu_i}$$

The expenditure function represents the minimum level of expenditure required to achieve the level of utility u with the given vector of prices P. The Hicksian (compensated) demand functions are given by the derivative of the expenditure function with respect to P:

$$H_i(P,u) = \frac{\partial E}{\partial P_i} = \theta_i + \frac{\mu_i}{P_i} uP$$

Hicksian equivalent variation[38] (*EV*), a measure of welfare is given by the following formula:

$$EV = E(P^1, u^1) - E(P^0, u^1)$$

i.e. the value of expenditure necessary to compensate a consumer at base year prices to achieve the new level of utility. If *EV* is positive, there is a net welfare loss.

The compensated own-price elasticities are given by:

$$\xi_{ii} = \frac{H_i - \theta_i}{H_i} (\mu_i - 1) = (\mu_i - 1) \frac{\mu_i Y^*}{s_i Y}$$

and the cross-price elasticities by:

$$\xi_{ij} = \frac{\mu_i}{s_i} s_j - \frac{\mu_i}{s_i} \frac{P_j \theta_j}{Y} = \mu_j \frac{\mu_i Y^*}{s_i Y}$$

The two formulas can be combined to yield:

$$\xi_{ij} = [\mu_j - \delta_{ij}] \frac{\mu_i Y^*}{s_i Y}$$

where δ_{ij} is the Kronecker product, i.e. equal to 1 when $i = j$, otherwise equal to 0. It is easy to verify the Slutsky equation:

$$\xi_{ij} = \varepsilon_{ij} + s_j \eta_i$$

where the uncompensated demand elasticities are summarised by:

$$\varepsilon_{ij} = -\frac{\mu_i}{s_i Y} \left[P_j \theta_j + \delta_{ij} Y^* \right]$$

Finally, the substitution elasticities are given by the following formula:

$$\sigma_{ij} = \frac{\xi_{ij}}{s_j} = \frac{\mu_i Y^*}{s_i s_j Y} \left[\mu_j - \delta_{ij} \right]$$

The matrix of substitution elasticities is clearly symmetric.

Calibration

There are several ways to calibrate the LES system. A straightforward method uses the vector of consumption, consumer prices, and estimates of the income elasticities to derive the parameters for the LES system. The key constraint is that the sum of the marginal propensities to consume

[38] See Varian (1992).

must add up to one. The estimated income elasticities may be inconsistent with the household budget shares, in which case it is necessary to make adjustments to individual income elasticities or scale all of them.

The calibration process is based on four items: a) a consumer price vector (often assumed to be a unit vector); b) a vector of consumer purchases (from an input-output table or a SAM); c) household disposable income (from national accounts); and d) a set of income elasticities (from household surveys and/or literature searches).

Given P, C, Y and η, calculate the marginal propensity to consume (μ), using equation (4) from above:

$$\mu_i = \frac{\eta_i P_i C_i}{Y} = \eta_i s_i$$

where s_i are the consumption shares. However, there is a constraint on the μ parameters since they must sum to 1. There is no guarantee that the initial levels of the income elasticities, combined with the consumption shares will insure the consistency of the μ parameters. There are two solutions. The initial estimates of the μ parameters can be re-scaled to sum to 1 and the resulting income elasticities are then derived from equation (4). For example:

$$\mu_i^* = \frac{\mu_i}{\sum_i \mu_i}$$

$$\eta_i^* = \frac{\mu_i^*}{s_i}$$

where the starred parameters represent the final values after adjustment of the initial marginal budget shares. A second solution is to assume that (n-1) estimates of the income elasticities are correct, resulting in (n-1) μ parameters. The nth μ parameter can be derived from the unit constraint. This implies that the nth income elasticity is also derived from the consistency constraint, as opposed to being econometrically estimated or derived from other sources. Either solution has its advantages and drawbacks. In either case, the final derived income elasticities should be verified for plausibility. For example, in the case of the second option, if the residual sector is services, for example, one would expect the income elasticity to be at least 1, or higher.

There are n parameters left to calibrate, θ, and apparently n equations to invert in terms of θ, the consumer demand functions, (3). The system of equations are linear in the θ parameters, however, they are not all independent (in other words the matrix is not of full rank). This is easy to see since the μ parameters sum to unity. Again, there are two potential solutions. The first is to assign a particular value to one of the θ parameters, for example 0, and then to invert the remaining system of equations of full rank (n-1). How to set this up will be explained in the subsequent section. A second solution is to add an equation. This second solution has often been used in many applied GE models and relies on estimates of a particular elasticity which has gone by the name of the Frisch parameter. The Frisch parameter in the case of the LES[39] is defined by:

$$\text{(8)} \qquad \varphi = -\frac{Y}{Y^*} = -\frac{Y}{Y - \sum P_j \theta_j}$$

In other words, the Frisch parameter is the inverse ratio of the supernumerary income to total income. The Frisch parameter will converge towards -1 as the share of committed expenditures declines towards 0. Or, in another way of looking at it, the greater the share of committed expenditures, the higher is the value of the Frisch parameter (in absolute terms). Hence, estimates of the Frisch parameter tend to show that it is higher (in absolute terms) for poorer countries, where committed expenditures represent a larger portion of total income.[40]

Equation (8) can be used to re-write the price elasticity relations in terms of the Frisch parameter, the budget shares, and the income elasticities.

$$\begin{aligned}
\varepsilon_{ii} &= \frac{\theta_i}{C_i}(1-\mu_i) - 1 \\
&= \frac{P_i\theta_i}{Y}\frac{(1-\mu_i)}{s_i} - 1 \\
&= \left(s_i - \frac{\mu_i Y^*}{Y}\right)\left(\frac{1 - s_i\eta_i}{s_i}\right) - 1 \\
&= \left(s_i + \frac{s_i\eta_i}{\varphi}\right)\left(\frac{1}{s_i} - \eta_i\right) - 1 \\
&= 1 - s_i\eta_i + \frac{s_i\eta_i}{\varphi}\left(\frac{1}{s_i} - \eta_i\right) - 1
\end{aligned}$$

$$\text{(9)} \qquad \varepsilon_{ii} = -s_i\eta_i + \frac{\eta_i}{\varphi}(1 - s_i\eta_i)$$

[39] The Frisch parameter has also been interpreted as the marginal utility of money, or the flexibility of money. If the LES utility function is defined as:

$$\sum \mu_i \ln(C_i - \theta_i)$$

then the marginal utility of money (i.e. the budget shadow price) is:

$$\lambda = 1/Y^*$$

The flexibility of money is the elasticity of the marginal utility of money with respect to income:

$$\frac{\partial\lambda}{\partial Y}\frac{Y}{\lambda} = -\frac{Y}{Y^*}$$

This latest expression is the same as the definition of the Frisch parameter. However, the Frisch parameter can only be equated with the flexibility of money in the case of the additive form of the LES utility function. In the case of the multiplicative form of the LES utility function, the budget shadow price is (1/*P*). For calibration purposes, only estimates of the ratio of total income to supernumerary income is needed (i.e. Y/Y^*), not the flexibility of money.

[40] See de Janvy and Sadoulet (1995) pages 37 and 354, and Deaton and Muellbauer (1980) page 141 for Frisch parameter estimates.

$$\begin{aligned}
\varepsilon_{ij} &= -\frac{\mu_i \theta_j P_j}{s_i Y} \\
&= -\frac{\mu_i \left(P_j C_j - \mu_j Y^*\right)}{s_i Y} \\
&= -\eta_i \left(s_j - s_j \eta_j \frac{Y^*}{Y} \right)
\end{aligned}$$

$$\varepsilon_{ij} = -\eta_i s_j \left(1 + \frac{\eta_j}{\varphi} \right) \tag{10}$$

$$\begin{aligned}
\theta_i &= C_i - \frac{\mu_i}{P_i} Y^* \\
&= C_i - \frac{s_i \eta_i}{P_i} Y^* \\
&= C_i - C_i \frac{\eta_i}{Y} Y^*
\end{aligned}$$

$$\theta_i = C_i \left(1 + \frac{\eta_i}{\varphi} \right) \tag{11}$$

Equation (11) can be used to calibrate the θ parameters based on the consumption vector *C*, the income elasticities η, and the Frisch parameter φ.

ELES

Household savings behaviour has been ignored thus far in the discussion on consumer demand systems. Many models assume separability in household decision making between saving and current consumption. Lluch and Howe[41] introduced a relatively straightforward extension of the LES to include the saving decision simultaneously with the allocation of income on goods and services, this has become known as the extended linear expenditure system or the ELES. The ELES is based on consumers maximising their intertemporal utility between a bundle of current consumption and an expected future consumption bundle represented in the form of savings. The ELES has several attractive features. The utility function of the ELES has the following form:

$$U = \prod_i \left(C_i - \theta_i\right)^{\mu_i} \left(\frac{S}{P^s} \right)^{\mu_s}$$

with

$$\sum_{i=1}^{n} \mu_i + \mu_s = 1$$

where *U* is utility, *C* is the vector of consumption goods, *S* is household saving (in value), P^s is the price of saving, and μ and θ are ELES parameters. The algebraic properties of the ELES are similar to those of the LES, and hence only final functional forms will be presented. There are two key functional differences between the LES and the ELES, otherwise all the formulas can be replicated and saving can be assumed to be the (n^{th}+1) good. First, saving is represented almost always as a value, not as a volume. The price of saving, P^s will be interpreted below. The

[41] See Lluch (19xx) and Howe (19xx).

second difference is that it is explicitly assumed that the floor level of saving is 0, hence the θ parameter is 0.

The demand equations are derived similarly to above, the consumer solves the following problem:

$$\max \prod_i (C_i - \theta_i)^{\mu_i} \left(\frac{S}{P^s}\right)^{\mu_s}$$

subject to

$$\sum_{i=1}^{n} P_i C_i + S = Y$$

where P is the vector of consumer prices, and Y is disposable income. The demand functions are:

(12) $$C_i = \theta_i + \frac{\mu_i}{P_i}\left(Y - \sum_{j=1}^{n} P_j \theta_j\right) \qquad 1 \le i \le n$$

(13) $$S = \mu_s\left(Y - \sum_{j=1}^{n} P_j \theta_j\right) = Y - \sum_{j=1}^{n} P_j C_j$$

From the demand equation we can derive the income and price elasticities:

(14) $$\eta_i = \frac{\mu_i Y}{P_i C_i} = \frac{\mu_i}{s_i} \qquad \eta_s = \frac{\mu_s Y}{S} = \frac{\mu_s}{s}$$

(15) $$\varepsilon_i = \frac{\theta_i (1 - \mu_i)}{C_i} - 1 \qquad \varepsilon_s = -1$$

(16) $$\varepsilon_{ij} = -\frac{\mu_i \theta_j P_j}{P_i C_i} = -\frac{\mu_i \theta_j P_j}{s_i Y} \qquad \varepsilon_{sj} = -\frac{\mu_s \theta_j P_j}{sY} = -\frac{\theta_j P_j}{Y^*}$$

where s is the average propensity to save.

Welfare

With the addition of saving, the indirect utility function is given by:

$$v(P,Y) = \prod_i \left(\frac{\mu_i}{P_i} Y^*\right)^{\mu_i} \left(\frac{\mu_s}{P^s} Y^*\right)^{\mu_s}$$

or

$$v(P,Y) = \frac{Y^*}{P} \quad \text{where} \quad P = \prod_i \left(\frac{P_i}{\mu_i}\right)^{\mu_i} \left(\frac{P^s}{\mu_s}\right)^{\mu_s}$$

The expenditure function is derived by minimising the cost of achieving a given level of utility, u. It is set-up as:

$$\min \sum_{i=1}^{n} P_i C_i + S$$

subject to

$$\prod_i (C_i - \theta_i)^{\mu_i} \left(\frac{S}{P^s}\right)^{\mu_s} = u$$

The final expression for the expenditure function is:

$$E(P,u)=\sum_{i=1}^{n}P_i\theta_i+uP$$

where

$$P=\prod_i\left(\frac{P_i}{\mu_i}\right)^{\mu_i}\left(\frac{P^s}{\mu_s}\right)^{\mu_s}$$

Calibration

In some respects calibration of the ELES is simpler than calibration of the LES. It still uses the budget share information from the base SAM, including the household saving share. Typically, calibration uses income elasticities for all of the *n* commodities represented in the demand system and uses equation (14) to derive the marginal budget shares, μ_i. This procedure again leads to a residual income elasticity, which in this case is the income elasticity of saving. The derived savings income elasticity may be implausible, in which case adjustments need to be made to individual income elasticities for the goods, or adjustments can be made on the group of goods, assuming some target for the savings income elasticity.

The first step is therefore to calculate the marginal budget shares using the average budget shares and the initial income elasticity estimates.

$$\mu_i=\frac{\eta_i P_i C_i}{Y}=\eta_i s_i$$

The savings marginal budget share is derived from the consistency requirement that the marginal budget shares sum to 1:

$$\mu_s=1-\sum_{i=1}^{n}\mu_i$$

Assuming this procedure leads to a plausible estimate for the savings income elasticity[42], the next step is to calibrate the subsistence minima, θ. This can be done by seeing that the demand

---

[42] It would also be possible to calibrate on a given saving income elasticity, and to re-scale uniformly (or individually) the other marginal propensities to consume. For example, if there is a desired value for η_s, then μ_s is derived using the formula above. The other income elasticities can be scaled uniformly using the following formula:

$$\chi=(1-\mu_s)/\sum_i s_i\eta_i$$

where η_i are the initial income elasticity estimates, and the re-scaled income elasticities are given by:

$$\eta_i^*=\chi\eta_i$$

The μ parameters will be scaled by the same factor χ. An alternative is to fix some of the income elasticities and re-scale the others using least squares. The problem is to minimize the following objective function:

$$\sum_{i\in\Omega}\left(\eta_i-\eta_i^0\right)^2$$

subject to

$$\sum_{i\in\Omega}s_i\eta_i=1-\sum_{i\notin\Omega}s_i\eta_i$$

where the set Ω contains all sectors (including possibly savings) where the income elasticity is not fixed, i.e. its complement contains those sectors with fixed income elasticities. The solution is:

equations, (12), are linear in the θ parameters. Note that in the case of the ELES the system of equation are of full rank because the μ parameters do not sum to 1 (over the *n* commodities. They only sum to 1 including the marginal saving share. This may lead to calibration problems if the propensity to save is 0, which may be the case in some SAMs with poor households.) The linear system can be written as:

$$C = I\theta + MY - M\ \Pi\ \theta$$

where *I* is an *n* x *n* identity matrix, *M* is a diagonal matrix with μ_i / P_i on the diagonal, and Π is a matrix, where each row is identical, each row being the transpose of the price vector. The above system of linear equations can be solved via matrix inversion for the parameter θ:

$$\theta = A^{-1}C^*$$

where

$$A = I - M\ \Pi$$

$$C^* = C - MY$$

The matrices *A* and C^* are defined by:

$$A = [a_{ij}] = \begin{cases} 1-\mu_i & \text{if} \quad i=j \\ -\mu_i \dfrac{P_j}{P_i} & \text{if} \quad i \neq j \end{cases}$$

$$C^* = [c_i] = C_i - \frac{\mu_i Y}{P_i}$$

The *A* and C^* matrices are greatly simplified if the price vector is initialised at 1:

$$A = [a_{ij}] = \begin{cases} 1-\mu_i & \text{if} \quad i=j \\ -\mu_i & \text{if} \quad i \neq j \end{cases}$$

$$C^* = [c_i] = C_i - \mu_i Y$$

Box: **Calibration of the ELES in GAMS**

Calibration of the subsistence minima in GAMS is not necessarily straightforward since GAMS does not contain an explicit function for inverting matrices. There are, as usual, two potential solutions. The first is brute force, which in this case uses the Gauss-Seidel algorithm for solving linear equations. Equation (12) can be re-written to isolate θ and be used as an iterating equation for Gauss-Seidel:

$$\theta_{i,it+1} = \frac{P_i(C_i - \mu_i\theta_{i,it}) - \mu_i\left(Y - \sum_j P_j\theta_{j,it}\right)}{P_i(1-\mu_i)}$$

The model variables *P*, *C*, and *Y* are fixed at their base year values, and the μ parameters are calibrated first using equation (14). The index *it* is an iteration counter. The algorithm stops when the distance between θ_{it+1} and θ_{it} is less than a given tolerance level.

$$\eta_i = \eta_i^0 + s_i \frac{1 - \sum_{i \notin \Omega} s_i\eta_i - \sum_{i \in \Omega} s_i\eta_i^0}{\sum_{i \in \Omega} s_i^2} \quad \forall i \in \Omega$$

The second solution uses one of the GAMS solver to solve a trivial optimisation problem. The code below shows both solutions. Note that this procedure is not necessary under the LES since to calibrate the subsistence minima under the LES requires an estimate of the Frisch parameter, and then equation (11) can be used to calculate the θ parameters.[43]

```
* Consumer demand system

* Calibrate the marginal budget shares

mu(i)  = yelas(i)*pa0(i)*cons0(i)/yd0 ;
mus    = 1 - sum(i,mu(i)) ;
yelass = mus*yd0/sh0 ;

* Check for consistency

Abort$(sh0 eq 0) "No household saving in SAM, choose different calibration method"
Display "Saving income elasticity:", yelass ;

* Calibrate the subsistence minima

* 1. First method -- using Gauss-Seidel

* The parameter maxres contains the sum of the differences of the subsistence
*    minima between two iterations of gauss-seidel
* cwork is a working vector which contains an interim copy of the subsistence
*    minima during the iteration procedure
* count is an iteration counter
* reltol is the tolerance level
* iter is the iterating set

* Declare parameters for calibration

parameter cwork(i)   Working vector for holding interim solution ;
scalars
   ystar    Working variable for containing supernumerary income
   maxres   Error term            / 1 /
   count    Iteration counter     / 0 /
   reltol   Tolerance level       / 1.0e-9 / ;

set iter set containing maximum number of iterations / 1*100 / ;

* Initialise interim solution to one-half of consumption

cwork(i) = cons0(i)/2.0 ;

* Loop over the number of maximum iterations

loop(iter $ (maxres gt reltol),
   ystar    = yd0 - sum(j,pa0(j)*cwork(j)) ;
   theta(i) = (pa0(i)*(cons0(i)-mu(i)*cwork(i))-mu(i)*ystar)/(pa0(i)*(1-mu(i))) ;
   maxres   = sum(i,abs(cwork(i) - theta(i))) ;
   count    = count + 1 ;
   cwork(i) = theta(i)
) ;

display count, maxres, reltol ;
Abort $ (maxres gt reltol) "Convergence not achieved" ;

$ontext

* 2. Second method -- using optimisation

* Declare dummy variables for setting up model
```

[43] However, if it were decided to fix one of the subsistence minima, i.e. not use the Frisch parameter, then the matrix inversion needs to be used to calculate the other (n-1) subsistence parameters. In effect, this is what the ELES does since it fixes the subsistence minima for household saving.

```
variables
   thetav(i)      Holding variables for theta parameters
   err            Temporary objective function

* Declare model equations

equations
   thetaeq(i)     Definition of subsistence minimna
   erreq          Minimize sum of errors ;

* Define model equations

thetaeq(i)..
   cons0(i)*pa0(i) =e= thetav(i)*pa0(i) + mu(i)*(yd0-sum(j,pa0(j)*thetav(j))) ;

erreq..
   err =e= sum(i,(cons0(i)*pa0(i)
        - (thetav(i)*pa0(i) + mu(i)*(yd0-sum(j,pa0(j)*thetav(j)))))) ;

* Initialise subsistence minima at one-half base consumption

thetav.l(i) = cons0(i)/2 ;
err.l = sum(i,(cons0(i)*pa0(i)
      - (thetav.l(i)*pa0(i) + mu(i)*(yd0-sum(j,pa0(j)*thetav.l(j)))))) ;

model thetam / thetaeq, erreq / ;

solve thetam using LP minimizing err ;

theta(i) = thetav.l(i) ;

$offtext

frisch   = -yd0/(yd0 - sum(i,pa0(i)*theta(i))) ;
display theta, cons0 ;
display "Frisch parameter:", frisch ;
```

The Almost Ideal Demand System

While the LES (and its ELES derivative) are significant improvements over the Cobb-Douglas utility function, they nonetheless impose some restrictions on the parameters which empirical evidence show to be suspect. Deaton and Muellbauer have proposed an alternative which is in the class of flexible functional forms, in other words, it is able to replicate a wider range of income and price elasticities than the LES demand system. The demand system is known as the Almost Ideal Demand System (AIDS) and is derived from the following expenditure function:

$$E(P,u) = e^{a(P)} e^{ub(p)}$$

where

$$a(P) = \alpha_0 + \sum_{i=1}^{n} \alpha_i \ln(P_i) + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij}^{*} \ln(P_i) \ln(P_j)$$

$$b(P) = \beta_0 \prod_{i=1}^{n} P_i^{\beta_i}$$

The Hicksian demand function can be derived by taking the partial derivative of E with respect to P_i:

$$
\begin{aligned}
H_i(P,u)=\frac{\partial E}{\partial P_i} &= e^{a(P)}e^{ub(P)}\frac{\partial a(p)}{\partial P_i}+e^{a(P)}e^{ub(P)}u\frac{\partial b(p)}{\partial P_i} \\
&= E\left[\frac{\alpha_i}{P_i}+\frac{1}{2P_i}\sum_{j=1}^{n}\gamma_{ij}^{*}\ln(P_j)+\frac{1}{2P_i}\sum_{j=1}^{n}\gamma_{j,i}^{*}\ln(P_j)\right]+E\left[ub(p)\frac{\beta_i}{P_i}\right] \\
&= \frac{E}{P_i}\left[\alpha_i+\sum_{j=1}^{n}\gamma_{ij}\ln(P_j)+u\beta_i b(p)\right]
\end{aligned}
$$

where the γ coefficients are defined by:

$$\gamma_{ij}=\frac{1}{2}\left(\gamma_{ij}^{*}+\gamma_{ji}^{*}\right)=\gamma_{ji}$$

Replacing u by (ln(E)-a)/b, and multiplying both sides by the factor (P_i/E) yields:

$$s_i=\alpha_i+\sum_{j=1}^{n}\gamma_{ij}\ln(P_j)+\beta_i(\ln(E)-a(P))$$

where s_i is the budget share allocated to commodity i. At the optimum, E is identically equal to the budget Y, and a price index P can be defined by:

$$\text{(17)}\qquad \ln(P)=\alpha_0+\sum_{j=1}^{n}\alpha_j\ln(P_j)+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\gamma_{ij}\ln(P_i)\ln(P_j)$$

Then the budget share equation has the following reduced form:

$$\text{(18)}\qquad s_i=\alpha_i+\sum_{j=1}^{n}\gamma_{ij}\ln(P_j)+\beta_i\ln(Y/P)$$

This equation is almost linear in the (logs) of price and real income which proves to be particularly convenient for estimation purposes. The price index P can be replaced by an alternative price index which can be determined independently of the estimated coefficients, yielding a tractable linear equation for estimation purposes. The elasticities are derived by taking the appropriate partial derivatives. Taking the partial derivative of C_i with respect to P_i yields the following:

$$
\begin{aligned}
\varepsilon_{ii}=\frac{\partial C_i}{\partial P_i}\frac{P_i}{C_i}=-1+\frac{P_i}{s_i}\frac{\partial s_i}{\partial P_i} &= -1+\frac{P_i}{s_i}\left[\frac{\gamma_{ii}}{P_i}-\frac{\beta_i}{P_i}\frac{\partial P}{\partial P_i}\right] \\
&= -1+\frac{P_i}{s_i}\left[\frac{\gamma_{ii}}{P_i}-\beta_i P\left(\frac{\alpha_i}{P_i}+\frac{1}{P_i}\sum_{k=1}^{n}\gamma_{ik}\ln(P_k)\right)\right]
\end{aligned}
$$

$$\text{(19)}\qquad \varepsilon_{ii}=-1+\frac{\gamma_{ii}}{s_i}-\frac{\beta_i\alpha_i}{s_i}-\frac{\beta_i}{s_i}\sum_{k=1}^{n}\gamma_{ik}\ln(P_k)$$

The cross-price elasticities are:

$$
\begin{aligned}
\varepsilon_{ij}=\frac{\partial C_i}{\partial P_j}\frac{P_j}{C_i}=\frac{\partial s_i}{\partial P_j}\frac{P_j}{s_i} &= \left[\frac{\gamma_{ij}}{P_j}-\frac{\beta_i}{P}\frac{\partial P}{\partial P_j}\right]\frac{P_j}{s_i} \\
&= \left[\frac{\gamma_{ij}}{P_j}-\frac{\beta_i}{P}P\left(\frac{\alpha_j}{P_j}+\frac{1}{P_j}\sum_{k=1}^{n}\gamma_{jk}\ln(P_k)\right)\right]\frac{P_j}{s_i}
\end{aligned}
$$

$$\text{(20)}\qquad \varepsilon_{ij}=\frac{\gamma_{ij}}{s_i}-\frac{\beta_i\alpha_j}{s_i}-\frac{\beta_i}{s_i}\sum_{k=1}^{n}\gamma_{jk}\ln(P_k)$$

Equations (19) and (20) can be combined to yield:

(20') $$\varepsilon_{ij} = -\delta_{ij} + \frac{\gamma_{ij}}{s_i} - \frac{\beta_i \alpha_j}{s_i} - \frac{\beta_i}{s_i}\sum_{k=1}^{n}\gamma_{jk}\ln(P_k)$$

where δ_{ij} is known as the Kronecker delta and equals 1 if *i*=*j*, and 0 otherwise. The income elasticities are:

$$\eta_i = \frac{\partial C_i}{\partial Y}\frac{Y}{C_i} = \frac{Y}{s_i}\frac{\partial s_i}{\partial Y} + 1 \quad = \quad \frac{Y}{s_i}\frac{\beta_i}{Y} + 1$$

(21) $$\eta_i = 1 + \frac{\beta_i}{s_i}$$

There is a restriction on the weighted sum of the income elasticities, using the budget shares as weights, and that is they must sum to 1. It is clear from the definition of the income elasticities that this places a restriction on the β parameters:

(22) $$\sum_{i=1}^{n}\beta_i = 0$$

Other restrictions apply as well to ensure the normal properties of adding up, homogeneity and symmetry. Adding up requires the following additional conditions:

(23) $$\sum_{i=1}^{n}\alpha_j = 1$$

(24) $$\sum_{i=1}^{n}\gamma_{ij} = 0 \quad \forall j$$

Homogeneity requires:

(25) $$\sum_{j=1}^{n}\gamma_{ij} = 0 \quad \forall i$$

Symmetry is satisfied provided:

$$\gamma_{ij} = \gamma_{ji}$$

The compensated price elasticities can be derived from the Slutsky equation:

$$\xi_{ij} = \varepsilon_{ij} + s_j \eta_i = \varepsilon_{ij} + s_j + \frac{s_j}{s_i}\beta_i$$

The matrix of substitution elasticities is derived from the following formula:

$$\sigma_{ij} = \frac{\xi_{ij}}{s_j}$$

$$\sigma_{ij} = \frac{1}{s_j}\left[-\delta_{ij} + \frac{\gamma_{ij}}{s_i} - \frac{\beta_i \alpha_j}{s_i} - \frac{\beta_i}{s_i}\sum_{k=1}^{n}\gamma_{jk}\ln(P_k) + s_j + \frac{s_j}{s_i}\beta_i\right]$$

Using the following identity, from equation (18):

$$s_j - \alpha_j - \sum_{k=1}^{n}\gamma_{jk}\ln(P_k) = \beta_j \ln(Y/P)$$

we can derive:

$$
\begin{aligned}
\sigma_{ij} &= \frac{1}{s_j}\left[-\delta_{ij}+\frac{\gamma_{ij}}{s_i}-\frac{\beta_i\alpha_j}{s_i}-\frac{\beta_i}{s_i}\sum_{k=1}^{n}\gamma_{jk}\ln(P_k)+s_j+\frac{s_j}{s_i}\beta_i\right] \\
&= \frac{1}{s_j}\left[-\delta_{ij}+\frac{\gamma_{ij}}{s_i}+\frac{\beta_i}{s_i}\left[s_j-\alpha_j-\sum_{k=1}^{n}\gamma_{jk}\ln(P_k)\right]+s_j\right] \\
&= \frac{1}{s_j}\left[-\delta_{ij}+\frac{\gamma_{ij}}{s_i}+\frac{\beta_i}{s_i}\beta_j\ln(Y/P)+s_j\right] \\
&= 1-\frac{\delta_{ij}}{s_j}+\frac{\gamma_{ij}}{s_is_j}+\frac{\beta_i\beta_j}{s_is_j}\ln(Y/P)
\end{aligned}
$$

It is readily seen that the matrix of substitution elasticities is symmetric.

Calibration

Calibration of the AIDS demand system is significantly more difficult than calibration of the three previous demand systems. There are 1+2*n*+*n* x *n* parameters to calibrate. The definition of the income elasticities can be used to derive the β coefficients. However, before calculating these coefficients, the income elasticities may need some adjustments in order to guarantee that the adding up condition holds. Either one of the income elasticities can be determined residually, or else they can all be adjusted by the same adjustment factor:

$$\eta_i = \frac{\eta_i^0}{\sum_i \eta_i s_i}$$

where the η^0 are the initial estimates of the income elasticities, and *s* represents the budget shares. (An alternative formula is to minimise the sum of squared errors subject to the adding up constraint, i.e.:

$$\max \sum_i \left(\eta_i - \eta_i^0\right)^2$$

subject to:

$$\sum_i s_i\eta_i = 1$$

The solution of this optimisation leads to the following formula:

$$\eta_i = \eta_i^0 - s_i\frac{1-\sum_i s_i\eta_i^0}{\sum_i s_i^2}$$

The adjustments to the coefficients will be weighted by their respective shares.) The β coefficients can then be calculated using the following formula:

$$\beta_i = s_i(\eta_i - 1)$$

This leaves *n*(*n*+1)+1 parameters to calibrate: α_i, α_0, and γ_{ij}. Equations (17) and (18) can be used to calibrate the α_i and α_0 parameters, however, they depend on the γ parameters. If all prices are initialised at unit value, then the γ parameters drop from these equations, and the α parameters can be calibrated directly. In this case it is also true that the adding up constraint on the α parameters, equation (23), holds, since by construction, equation (22) obtains.

Assuming for the moment that prices are initialised at 1, this leaves only the γ parameters, of which there are *n*x*n*, but due to the symmetry requirements, there are only *n*+*n*(*n*-1)/2 degrees of freedom—*n* for the diagonal elements, plus *n*(*n*-1)/2 for the off-diagonal elements. With the availability of the matrix of own- and cross-price elasticities, it is possible to use equation (20')

to calibrate the remaining γ coefficients. It is also necessary to invoke either equation (24) or (25). (Both are not necessary because the symmetry requirement implies that they are equivalent.) This results in n+(nxn) equations for only $n(n+1)/2$ variables, i.e. the system is over-determined. There is no guarantee that the matrix of elasticities is consistent with the AIDS functional form. The elasticities may derive from different sources. One solution to the problem is to adjust the price elasticities under the constraints imposed by the AIDS function. A simple way is to allow the price elasticities to be free variables and minimise the sum of squared deviations from their initial estimates, subject to equations (20') and (24). This method can be generalised to include calibration of the α parameters.
An alternative is to use the matrix of substitution elasticities to calibrate the AIDS model. Starting with an initial guess of the substitution elasticities, the optimisation program can be set up as:

$$\min \sum_{i=1}^{n} \sum_{j \neq i}^{n} \left(\sigma_{ij} - \sigma_{ij}^{0}\right)^2$$

subject to:

$$(26) \qquad \ln(P) = \alpha_0 + \sum_{j=1}^{n} \alpha_j \ln(P_j) + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} \ln(P_i) \ln(P_j)$$

$$(27) \qquad s_i = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln(P_j) + \beta_i \ln(Y/P)$$

$$(28) \qquad \sum_{i=1}^{n} \alpha_i = 1$$

$$(29) \qquad \sigma_{ij} = 1 + \frac{\gamma_{ij}}{s_i s_j} + \frac{\beta_i \beta_j}{s_i s_j} \ln(Y/P) \quad \text{for } i < j$$

$$(30) \qquad \sum_{j=1}^{n} \gamma_{ji} = 0$$

$$(31) \qquad \gamma_{ij} = \gamma_{ji} \quad \text{for } i < j$$

The system is linear in all of the parameters, therefore this is a quadratic programming problem with linear constraints. This should ensure relatively well-behaved convergence properties. However, it is good practice to compare the resulting elasticity estimates with the original ones. They should be checked for deviations from their initial values, as well as their overall consistency with economic theory. For example, it is unusual (though not impossible) to see positive own-price elasticities. This system is underdetermined, but since it is an optimisation programme, the extra degrees of freedom should lead to a valid solution. The following table shows the list of free variables and equation restrictions:

Variables	***Dimensions***	***Restrictions***	***Dimensions***
α_0	1	(26)	1
α_i	n	(27), (28)	n+1
γ_{ij}	n x n	(30), (31)	$n + n(n-1)/2$
σ_{ij} for $i < j$	$n(n-1)/2$	(29)	$n(n-1)/2$
Total	$1+n(3n+1)/2$		$2+n(n+1)$

AIDS and the Armington Assumption

The most ubiquitous form of implementing the Armington assumption in the description of import demand is to use a CES specification (including eventually nested CES functions to capture more interesting substitution possibilities). In order to introduce some notation, let *XA* represent aggregate Armington demand (in region *r* for sector *i*), let *XD* represent domestic demand for domestic goods, and *XM* represent domestic demand for aggregate imports. The respective prices of the three goods are *PA*, *PD*, and *PMT*. The solution to the first nest of the CES-specified Armington specification results in the following three equations:

$$(32) \qquad XD_{r,i} = \alpha^d_{r,i}\left(\frac{PA_{r,i}}{PD_{r,i}}\right)^{\sigma_{r,i}} XA_{r,i}$$

$$(33) \qquad XM_{r,i} = \alpha^m_{r,i}\left(\frac{PA_{r,i}}{PMT_{r,i}}\right)^{\sigma_{r,i}} XA_{r,i}$$

$$(34) \qquad PA_{r,i} = \left[\alpha^d_{r,i} PD^{1-\sigma_{r,i}}_{r,i} + \alpha^m_{r,i} PM^{1-\sigma_{r,i}}_{r,i}\right]^{1/\left(1-\sigma_{r,i}\right)}$$

The first two equations determine the respective domestic and import shares, while the third equation defines the Armington aggregate price. The substitution of elasticity is given by the σ parameter. The second nest decomposes the aggregate demand for imports, *XM*, into demand for imports from the various trading partners. (Note that additional nests could be used to add more distinct substitution possibilities across trading partners.) Let *WTF* represent import demand into region *r* originating in region *r*'. We use the convention that imports in the trade flow matrix are read down a column, hence cell (*r*',*r*) represents imports from region *r*' into region *r*. Analogous to equations (32)-(34) the equations for the world trade flow matrix are given by:

$$(35) \qquad WTF_{r',r,i} = \mathrm{A}_{r',r}\left(\frac{PMT_{r,i}}{PM_{r',r,i}}\right)^{\omega_{r,i}} XM_{r,i}$$

$$(36) \qquad PMT_{r,i} = \left[\sum_{r'=1}^{R} PM^{1-\omega_{r,i}}_{r',r,i}\right]^{1/\left(1-\omega_{r,i}\right)}$$

The variable *PM* represents the bilateral price of imports into region *r* from region *r*', inclusive of trade and transport margins and bilateral tariffs. The substitution elasticity is given by ω. In models with aggregate regions, the diagonal of *WTF* may not necessarily be zero, i.e. this specification is also valid for integrating intra-regional (or self) imports.

An alternative to the CES version of the Armington specification is to implement a version of the AIDS function. There are two advantages to the AIDS specification. First, the AIDS specification captures in a more transparent fashion different cross-price substitution possibilities across trading partners. Second, the AIDS specification can also capture income effects. By definition the CES specification has unitary income elasticities. A relatively easy way to implement AIDS while maintaining tractability and intra-regional imports is to use a nested specification. The top nest allocates import demand across regions of origin. The diagonal component consists of an aggregate bundle of the pure domestic component, *XD* using the notation above, and intra-regional imports, i.e. $WTF_{r,r}$. The second nest decomposes this diagonal bundle into its two separate components. Using similar notation to above, the implementation of AIDS in the Armington specification leads to the following top-level equations:

(37) $$s_{r',r,i} = \alpha_{r',r,i} + \sum_{r''=1}^{R} \gamma_{r',r'',r,i} \ln\left(PM^a_{r'',r,i}\right) + \beta_{r',r,i} \ln\left(XA_{r,i}\right)$$

It is easier to read this equation by dropping the (*r*,*i*) indices. It can be interpreted as follows. For each importing region *r* and for each sector *i*, the share originating in region *r*' is equal to the sum of three components. The first component is a shift parameter reflecting the basic (or initial) import penetration. The second component reflects the changes to the import share emanating from changes in the partner-specific import prices. The g coefficients reflect the impacts of cross-price effects. The third component measures the impact on import shares coming from changes in overall aggregate demand. The income impact vary by region of origin. The following equation defines the AIDS price index:

(38) $$\ln\left(PA_{r,i}\right) = \alpha_{0,r,i} + \sum_{r'=1}^{R} \alpha_{r',r,i} \ln\left(PM^a_{r',r,i}\right) + \frac{1}{2} \sum_{r'=1}^{R} \sum_{r''=1}^{R} \gamma_{r',r'',r,i} \ln\left(PM^a_{r',r,i}\right) \ln\left(PM^a_{r'',r,i}\right)$$

The price vector in the top-level AIDS, PM^a, is not exactly the same as the import price vector by region of origin. For the off-diagonal elements, the price is the same. For the diagonal element, i.e. $PM^a_{r,r}$, the price represents the aggregate price of *XD*, that is *PD*, and the price of intra-regional imports, $PM_{r,r}$. The off-diagonal elements of the world trade flow matrix are defined by the following equation:

(39) $$WTF_{r',r,i} = s_{r',r,i} \frac{PA_{r',r,i} XA_{r',r,i}}{PM^a_{r',r,i}} \quad \text{for} \quad r' \neq r$$

(40) $$PM^a_{r',r,i} = PM_{r',r,i} \quad \text{for} \quad r' \neq r$$

Let *XDM* represent the diagonal element of the AIDS function. It is decomposed into *XD* and $WTF_{r,r}$ using a CES specification, and the price of the *XDM* bundle is given by *PDM*. The following equations finish the specification:

(41) $$XDM_{r,i} = s_{r,r,i} \frac{PA_{r,i} XA_{r,i}}{PA^a_{r,r,i}}$$

(42) $$XD_{r,i} = \alpha_d \left(\frac{PDM_{r,i}}{PD_{r,i}}\right)^{\sigma} XDM_{r,i}$$

(43) $$WTF_{r,r,i} = \alpha_m \left(\frac{PDM_{r,i}}{PM_{r,r,i}}\right)^{\sigma} XDM_{r,i}$$

(44) $$PDM_{r,i} = \left[\alpha_d PD^{1-\sigma} + \alpha_m PM^{1-\sigma}_{r,r,i}\right]^{1/(1-\sigma)}$$

(45) $$PM^a_{r,r,i} = PDM_{r,i}$$

Equation (41) equates the *XDM* bundle with the diagonal share parameter determined by the AIDS specification. Equations (42) and (43) represent the CES disaggregation of the *XDM* bundle, respectively into *XD* and $WTF_{r,r}$. Equation (44) determines the CES dual price of the *XDM* bundle, *PDM*. Finally, equation (45) sets the diagonal AIDS price component to be identically equal to *PDM*.

6. Labor Market Structure and Conduct

Introduction

In an era of globalization, linkages between international trade and labor markets are receiving intensified scrutiny. Many OECD countries are preoccupied with the implications of expanded trade for employment growth, employment diversion (referred to in Europe as *delocalisation*), and real wages. At the same time, more and more developing countries are concerned about how best to facilitate human resource development for trade-driven economic expansion. With increasing capital mobility and technology diffusion, the quantity and quality of domestic labor is an ever more important determinant of comparative advantage. Structure and conduct in domestic labor markets can be just as important as labor endowments, however. As expanding trade has imbued commodity markets with greater competitiveness and flexibility, trade-induced domestic growth is placing new adaptive pressures on labor markets. Increasingly, labor market rigidities are being viewed as impediments to more effective participation in the global economy, as well as to more sustainable growth of output, employment, and average living standards.[44]

At least as important as the level and composition of employment are real wage trends and policies that influence these directly and indirectly. While government and labor groups are understandably reluctant to abandon the social priorities which underlie many labor market interventions, the efficiency costs these confer upon their economies are often significant and usually not well understood. Despite a vast body of labor market research emerging in the last two decades, only a small part focuses on trade or empirical estimates of efficiency effects. The main objective of this paper is to review and synthesize the new labor market theories, embedding them in an empirical general equilibrium framework so they can be used to answer policy concerns about employment and wage effects which arise from both external and domestic influences.

Rather than exhaustively testing competing labor market specifications and evaluating real cases, our present purpose is expository. In the following sections, we provide a rational menu of generic labor market specifications, with relatively parsimonious numerical examples of how each can be implemented in a single prototype calibrated general equilibrium (CGE) model. The CGE model is a real one, based on a complete dataset for Mexico, but its application in this paper is more methodological than empirical. From the basic tool kit presented here, it is hoped that other practitioners will join with us to enlarge the very incomplete basis of empirical evidence on how international trade and domestic labor markets interact.

Each section covers different genera of labor market theory with the same three-part structure: conceptual motivation, literature survey, and numerical example. No attempt has been made to cover every contending theory, contributor, or alternative specification. The sample here is intended to represent the main streams of this rapidly growing research area, cite their leading contributors, and offer simple entry points for more detailed empirical research.

[44] The recent *OECD Jobs Study* (1994) provides a comprehensive historical overview of such trends. See also the OECD's annual *Employment Outlook*.

Wage Rigidities

Wage rigidities are one of the most pervasive distortions in labor markets. These arise from essentially two sources: 1) government interventions which seek to secure basic living standards or, in rarer cases, to limit wage growth; 2) distortions against competitive wage adjustment which arise from market power held by workers, employers, or both. In this section, we consider general examples of both cases, where wage rigidities are exogenous or endogenous to the labor market.

Exogenous Wage Rigidity

A broad spectrum of government policies exist in different countries to legislate minimum wage levels directly or support reservation wages via social insurance programs. Although these policies use economic instruments and have pervasive economic effects, they are rarely implemented with economic efficiency criteria in mind. In this section, we conduct a variety of simulation experiments to see how minimum wage policies can affect the adjustment process ensuing from trade liberalization.

The first major contribution to the analysis of minimum wage is presented in Stigler (1946). He demonstrates that the imposition of a minimum wage above the equilibrium wage reduces employment. An alternative version recognizes that minimum wage regulations may apply only to a covered sector, with an uncovered sector in which workers displaced by the higher minimum wage could find jobs. This approach can be extended further to allow for job queuing at the minimum wage, either by those earning the lower wage in the uncovered sector, or by those dropping out of the labor force[45]. Holzer, Katz and Krueger (1991) demonstrate that jobs paying around the minimum wage have a greater number of applicants than other jobs, suggesting the presence of significant rents.[46] Edwards and Edwards (1990) provide an excellent analytical survey of a number of international trade models with wage rigidities. For further discussions on the theory underlying the economic impact of minimum wage policy, see Riveros (1990) and Fiszbein (1992). Econometric evidence on minimum wage policies includes Brown, Curtis, Kohen (1982), Riveros and Paredes (1988), and Lopez and Riveros (1988). Using a time-series approach, Santiago (1989) estimates labor market effects of higher effective minimum wage levels.

In this set of simulations, we shall examine four alternative types of minimum wage policy. Each represents different target groups or different social insurance objectives, and together they cover the main policy alternatives and generic types of distortionary effects. The prototype general equilibrium model is described, and its structural equations set forth, in the appendix. All notation used in the following discussions is based on the conventions of the prototype.

Minimum Wage by Occupation

In this case, the government attempts to guarantee a nominal hourly minimum to one or more specific labor categories. We assume fixed labor supplies throughout, and in the event that the minimum wage is binding, unemployment will be created in the target occupational

[45] See Mincer (1976) for a standard model of queuing.
[46] See also Hamermesh (1993)(p.182-191) for a stylized version of labor market effects of minimum wages and a brief survey of relevant empirical work.

groups. We assume that these workers respond by entering the informal labor market and finding jobs there, putting downward pressure on the informal wage. The wage equation for a given target occupational group (l) is modified from the prototype to take the form

$$w_l \geq \overline{w}_l \qquad (1)$$

where w_l and $\overline{w}_l$ represent, respectively, the average and minimum wage to the target occupational group.

Some observations about this specification are in order. Firstly, note that we assume the minimum applies to occupational average wages rather than to individual wages of workers. Distributional effects within occupations are ignored. Secondly, inter-sectoral wage differentials are also ignored, so the incidence of the minimum wage policy will be distorted, i.e. sectors with low wage premia may still pay below the target minimum on average. Third, note that the inequality above makes the prototype model under-determined. The eliminate the extra degree of freedom, we add an orthogonality condition

$$(w_l - \overline{w}_l)(L_l^S - L_l^D) = 0 \qquad (2)$$

where L_l^S and L_l^D represent, respectively, the labor supply and demand of the given target occupational group.

Finally, we modify the labor supply equation for the informal occupational group (N) to allow for spillover of unemployed workers in the minimum wage target group, i.e.

$$L_N^S = \overline{L}_N^S + (L_l^S - L_l^D) \quad (3)$$

Minimum Real Wage by Occupation

Although most minimum wage policies are enunciated in terms of nominal hourly rates, some have escalation clauses to reflect the social objectives of real purchasing power maintenance. In the case of an occupational target group, such a policy can be simply specified as

$$w_l \geq \overline{w}_l P_l \qquad (4)$$

where P_l represents an endogenous price index. This might be an aggregate GDP deflator or an index more focused on the needs of a target group, such as a consumption-weighted purchaser price index. In any case this simple modification may increase or decrease the distortionary effects of the wage minimum, depending upon whether deflationary or inflationary pressures dominate a given adjustment process.

Minimum Wage by Sector

In some instances, minimum wage policies are targeted at workers in specific occupations and sectors. This more focused approach may be designed to correct severe inter-sectoral differentials or could be the result of sector-specific political forces. In this case, the wage determination equation for a given target occupational (l) and sectoral (i) group takes the form

$$\omega_{li} w_l \geq \overline{w}_{li} \qquad (5)$$

where the average occupational wage, w_l, is tied to the sectoral wage premium, ω_{li}, and where $\overline{w}_{li}$ represents the target occupational and sectoral minimum wage. The other modifications above are unchanged.

Minimum Real Wage by Sector

A final variation concerns real wage maintenance in a specific sector. This kind of policy is especially common in public sector employment, where wages are normally legislated in any case and often indexed. Here the wage constraint takes the form

$$\omega_{li} w_l \geq \overline{w}_l P_l \quad (6)$$

Simulation Experiments

We now compare the results of the reference simulation with those obtained under a variety of minimum wage specifications. The first experiment is the reference case used throughout this exercise, a trade liberalization scenario entailing abolition of Mexican tariffs and NTBs on all imports.[47] Five alternative experiments then follow, including a minimum fixed at the observed wage for unskilled workers, a real (GDP deflated) minimum for the same group, sectoral minimum wages for export-intensive (Energy) and import-intensive (Durables) sectors, respectively, and a minimum real wage for service sector workers.

The reference experiment is typical of CGE trade liberalization scenarios, with modest aggregate GDP growth arising from sectoral productivity gains in this fixed employment setting (Table 1). Removing import protection, other things equal, will induce real exchange rate and domestic price depreciation, exerting downward pressure on real wages in most occupational groups. When labor markets are competitive, as in Experiment 1, unskilled workers take most of the brunt of this.

Assuming instead that unskilled wages are protected by official minimum wage policy, nominally in Experiment 2 and in real terms in Experiment 3, changes the results significantly. The results in the two differ only in the magnitude of the adjustment necessary to offset unskilled wage rigidity, but are otherwise identical in qualitative terms. Because of the factor market rigidity, the real exchange rate must depreciate even further to align domestic and international resource costs.[48] Consumer prices also fall further, this time because of the significant wage repression in the residual, informal labor market which receives a significant influx of newly unemployed unskilled formal workers. This result clearly illustrates the regressive nature of minimum wage policies which has been emphasized by many authors.[49] In per capita terms, however, the fixed nominal wage policy is less wage repressive than the reference, while the fixed real wage policy is more so.

Finally, one might at first be startled by the increase in aggregate efficiency under distortionary policies. Recall, however, that this is a second-best situation, where we have assumed inter-sectoral labor productivity differences and calibrated these into a fixed wage distribution.[50] This means that reallocating labor can raise aggregate productivity per unit of resource cost, and especially so if the labor is induced to migrate from higher to lower wage categories. Under the assumption that sectoral wage differences correspond to labor

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[47] See Reinert, Roland-Holst, and Shiells (1995), for a more detailed discussion of such liberalization experiments.
[48] This point is omitted by Edwards and Edwards (1990) in their otherwise detailed treatment of this subject.
[49] Compare, e.g. Devarajan ,Ghanem, and Thierfelder (1994).
[50] This kind of labor market distortion, where sector-specific wage differences correspond to productivity differences, has been observed by a number of authors. See e.g. Krueger and Summers (1987, 1988).

productivity differences, re-allocating workers from low to high wage (productivity) sectors increases real GDP. This effect is amplified when workers also cross over to informal employment. Like economies of scale, then, labor market distortions appear to have the potential to amplify efficiency gains, but of course subject to other economic and social costs which may not be incorporated in this model.

Sectoral fixed wages have smaller absolute and distributional effects, except for the large and relatively low wage service sector. Efficiency effects vary with the skill and productivity composition of the target sectors. Real exchange rate depreciation is smaller when the distortion is on the income (export) side (Energy) of the trade balance than on the expenditure (import) side (Durables), but highest when the distortion is in the large, relatively nontradeable service sector (reverse Dutch Disease).

Table 6
Minimum Wage Scenarios
(percentage changes)

Selected Aggregates	*Experiment* 1	2	3	4	5	6
Real GDP	.8	3.0	2.3	.9	1.2	3.7
Real Exchange Rate	-5.7	-7.8	-7.2	-5.9	-6.8	-7.6
Consumer Price Index	-9.3	-9.8	-9.9	-9.4	-9.8	-8.5
Real Wages						
Unskilled	-10.2	9.8	.0	-9.8	-11.0	-16.6
Skilled	-3.6	-10.1	-9.2	-3.2	-1.2	-11.6
Informal	-.5	-38.9	-33.5	-.1	3.4	.7
Val. Added Wgt. Ave.	-5.5	-8.3	-10.6	-5.1	-4.0	-11.0
Employment Wgt. Ave.	-7.0	-4.9	-9.8	-6.7	-6.3	-11.5
Premia for Sectoral Real Wage Maintenance				*Energy*	*Durables*	*Services*
Unskilled	.0	.0	.0	23.7	26.3	23.2
Skilled	.0	.0	.0	14.4	12.4	15.6

Experiment 1: Mexican tariff and NTB abolition with competitive labor markets.
Experiment 2: Experiment 1 with a nominal minimum wage for unskilled labor.
Experiment 3: Experiment 1 with a real minimum wage for unskilled labor.
Experiment 4: Experiment 1 with minimum nominal wages for formal workers in Energy.
Experiment 5: Experiment 1 with minimum nominal wages for formal workers in Durables.
Experiment 6: Experiment 1 with minimum real wages for formal workers in Services.

Table 2 gives an overview of sectoral results associated with the reference and minimum wage experiments. As is typical, sectoral adjustment to liberalization and with respect to different labor market policies are more dramatic than aggregate results. In all cases, however, they follow intuitively from the economic structure, pattern of prior protection, and occupational composition of sectoral employment (see the summary table in the appendix).

Table 7
Sectoral Changes Resulting from Trade Liberalization
(percentages)

Output		*Output Exp 1*	*Exp 2*	*Exp 3*	*Exp 4*	*Exp 5*	*Exp 6*
1	Agriculture	-9	-12	-11	-8	-7	-7
2	Energy	9	5	6	3	11	11
3	NonDurables	-2	-4	-4	-2	-1	-1
4	Durables	6	9	7	6	0	8
5	Services	2	4	3	2	2	1
	Weighted Ave.	1	1	1	1	0	1
Exports							
1	Agriculture	50	30	35	51	57	56
2	Energy	18	11	13	9	21	21
3	NonDurables	48	38	41	49	54	52
4	Durables	50	57	53	51	38	55
5	Services	32	44	43	32	35	22
	Weighted Ave.	40	44	43	40	39	39
Demand for Dom. Goods							
1	Agriculture	-11	-14	-13	-11	-10	-10
2	Energy	-4	-5	-4	-5	-5	-3
3	NonDurables	-6	-7	-7	-6	-5	-5
4	Durables	-10	-8	-9	-10	-13	-9
5	Services	0	1	1	0	0	-1
	Weighted Ave.	-4	-3	-3	-4	-4	-3
Imports							
1	Agriculture	138	156	151	137	132	134
2	Energy	217	232	228	239	205	215
3	NonDurables	52	58	56	52	48	51
4	Durables	27	28	29	27	28	26
5	Services	-24	-29	-29	-24	-26	-19
	Weighted Ave.	38	42	41	38	37	37

The uniformity of weighted average adjustments across experiments is striking but logical, being the result of the macroeconomic components of the model such as fixed aggregate factor supplies and constant external policy. Individual sectoral differences are significant across experiments, however, indicating that important differences in relative competitiveness can emerge under different labor market specifications.

Endogenous Wage Rigidity

Simple Rent Sharing

By definition, wage rigidities arise when wages do not move fast enough to reflect the changing value of labor productivity. One of the simplest cases of this arises when firm-level excess profits exist and labor takes a share of these in addition to its competitive wages. This rent sharing partially de-couples wages from the first order relationship characteristic of neoclassical labor markets. Before looking at more complex bargaining models, we extend the prototype model with a simple rent sharing rule to see how it may compromise economic efficiency.

The idea that the behavior of labor markets could be represented satisfactorily by standard competitive models was first criticized by Schlichter (1950). He argued that competitive models failed to account for the empirically tested significant wage differentials among observationally homogeneous types of workers. Recent empirical work supports these results (see Dickens and Katz 1987, Krueger and Summers 1987 and 1988, Katz and Summers 1989, Christofides and Oswald 1989 and 1992, and Abowd, Kramarz and Margolis 1994). Several authors advanced the hypothesis that rent-sharing behavior can significantly affect the wage determination process[51]. For a discussion on the implications of industry rents, refer to the excellent work by Katz and Summers (1989) who present an insightful literature review and relevant empirical evidence on the subject.[52] While Blanchflower and Oswald (1989 and 1992) present empirical evidence on the negative relationship between workers' earnings and local unemployment level, Blanchflower, Oswald, Sanfey (1992) find that the real wage is an increasing function of employers' past profitability[53]. Christofides and Oswald (1989 and 1992) support both of these results, which are consistent with rent-sharing theory[54].

Assume that, in a given sector, a given occupational group has bargaining power for rent sharing which can be represented by a simple index β_{li} whose value lies between zero and unity. In this case, a premium ω_{li} above the competitive wage w_l will accrue to these workers, given by rent sharing rule

$$\omega_{li} = 1 + \frac{\beta_{li}}{1-\beta_{li}} \frac{r}{w_l L_{li}^D} \qquad (7)$$

where r represents firm operating rents.[55] As a practical matter in this implementation, we calibrated the parameter β_{li} and rents r equal the total wage premium and labor value added in the sector under consideration.

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[51] The observed wedge between the marginal productivities of factors in different uses is a type of market imperfection which is likely to cause certain factors to earn rents. In the present context of rent-sharing, the idea is that workers are able to capture a large part of the rents earned by firms.

[52] Their empirical results find that a large portion of monopoly rents earned by product markets may be captured by workers rather than shareholders.

[53] The argument is that workers benefit of higher wages when the firm or industry is booming. Local unemployment, however, tends to weaken workers' bargaining power, producing a negative relationship between wages and unemployment.

[54] A standard competitive framework would expect factor prices to be equalized across sectors and firms to hire factors of production up to the point where their marginal productivity equals their cost. Consequently, wages should be affected by labor supply forces rather than by unemployment and the profitability of a firm or industry should not prevent employers from paying exactly the "competitive" wage.

[55] Compare to Blanchflower, Oswald, and Sanfey (1992) for details.

Wage Bargaining

A more elaborate view of endogenous wage determination recognizes the existence of labor unions as explicit bargaining agents. When labor is organized to negotiate the terms of employment, wages may be above and employment below their competitive levels. In this and the next subsection, two cases are considered. Here, we look at the case where unions bargain over wages only and firms choose the level of employment to maximize profits. Next, we shall examine joint wage-employment contracts.

Unions can be viewed as instruments used by employees to extract rents from firms. There exist two broad categories of wage bargaining models, namely the *monopoly union model* and the *efficient bargaining model*[56]. Essentially, there is a tradeoff between wages and employment. The monopoly union model is a special case where the firm has no bargaining power in wage setting and the union has no power in employment. The wage is set unilaterally by the union. However, bargaining over the wage alone will generally not permit an efficient outcome[57]. For a simplified presentation of standard wage bargaining models, see Blanchard and Fischer (1989)[58]. Extensive surveys of work on the economic theory of union behavior are found in Oswald (1985) and Farber (1986). Penclavel (1985) reviews microeconomic research on union models and extends them to the macroeconomic level[59]. Excellent empirical work for Britain is presented in Layard and Nickell (1986). Blanchflower, Oswald and Garrett (1990) estimate the relative importance of inside power enjoyed by unionized workers in the wage determination process[60].

Extending the prototype model to incorporate labor negotiation requires a specification of the union's objective function. Assume that union members are homogeneous with individual utility represented by $U(\omega_{li}w_l)$ and that their group utility can be represented by

$$V(\omega_{li}w_l, L_{li}^D) = \left(\frac{L_{li}^D}{L_{li}^0}\right)U(\omega_{li}w_l) + \left(1 - \frac{L_{li}^D}{L_{li}^0}\right)U(w_l) \qquad (8)$$

where we assume that employment in the base situation, L_{li}^0, represents maximum union membership.[61] Thus the welfare of the union is a convex combination of utilities for those who remain in the sector, earning the negotiated wage, and those who find employment elsewhere, assumed to earn the average wage. Note now that

$$V(\omega_{li}w_l, L_{li}^D) - V(w_l, L_{li}^D) = \frac{L_{li}^D}{L_{li}^0}\left[U(\omega_{li}w_l) - U(w_l)\right] \quad (9)$$

so that, in a wage-only contract, the net gain for the members who remain employed is independent of the utility of unemployed members. The bargaining problem facing the union is then given by the Lagrangian

$$Max_{\omega} L_{li}^D[U(\omega w_l) - U(w_l)] + \lambda[L_{li}^o - L_{li}^D] \qquad (10)$$

whose interior (i.e. $\lambda = 0$) solution is obtained by solving the following expression

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56 This latter category of models is also referred to as the *right-to-manage* model.
57 See Farber (1986) for a formal discussion. Generally, most of the existing applied work assumes that unions bargain over wages and employers select the employment level.
58 See Blanchard and Fischer, Chapter 9, p. 438-546.
59 See also Calmfors (1985) for discussion on trade union behavior and its macroeconomic implication.
60 The "inside power" hypothesis is also disussed in Solow (1985) and Lindbeck and Snower (1986,1987) in the context of efficiency wages.
61 A number of authors (e.g. de Melo and Tarr (1990)) use a single utility function for the union, but this is more difficult to motivate from principles of demand theory. See Oswald (1987) for more on this point.

$$\frac{U_{\omega}(\omega w_l)}{\sigma} = \frac{U(\omega w_l) - U(w_l)}{\omega} \quad (11)$$

where σ denotes endogenous wage elasticity of labor demand in the prototype CES specification of production. Intuitively, this expression represents an equivalence of ratios for marginal (subjective and technical) substitution rates and values. Using the Extended Linear Expenditure System in the prototype model, this specification can be implemented without difficulty.

Efficient Contracts

Most anecdotal evidence indicates that unions bargain over wages and firms generally have discretion about employment levels.[62] Despite this, however, wage-only bargaining can produce outcomes which are not on the firm-union, wage-employment contract curve and are therefore inefficient. To remedy this, we extend the prototype below to incorporate simultaneous bargaining over both wages and employment levels.

Under efficient bargaining models, firms and unions share equal bargaining powers in wage end employment setting. In their seminal paper, McDonald and Solow (1981) argue that a contract is efficient, when it lies at a point of tangency between an indifference curve and an isoprofit locus, that is at a point on the contract curve. Which point is chosen on the contract curve will depend on the relative bargaining power of the firm and of the union. If the union is relatively weak, the outcome may be close to the competitive equilibrium; if the union is relatively powerful, it may be close to the firm's zero profit point[63]. In terms of efficient contracts, the bargaining outcomes are most likely going to lie off the demand curve. This occurs because at the bargained wage level, employers would prefer to cheat by reducing the level of employment. Abowd and Lemieux (1993) estimate a simple model of efficient wage-setting. Espinosa and Rhee (1989) extend standard bargaining models to allow for repeated bargaining.[64] Empirical evidence supporting efficient bargaining models include MaCurdy and Penclavel (1986), Brown and Medoff (1986), and Brown and Ashenfelter (1986)[65].

The basic implementation for wage-employment bargaining relies on a Nash solution to the following joint optimization problem:

$$Max_{\omega,L}\, L\left[U(\omega w_l) - U(w_l)\right]\left[F_i(L;\ldots) - \omega w_l L - C_i(\ldots)\right] + \lambda\left[L_{li}^0 - L\right] \quad (12)$$

where F_i and C_i denote the production and (nonlabor) cost functions in sector i, respectively. Omitting second-order cost effects, the solutions to this problem can be approximated with the following two expressions

$$\omega = \frac{\alpha_E}{2w_l}\left[\frac{F}{L} + F_L\right] \quad (13)$$

$$F_L L_{\omega} - \omega w_l = -\beta_E\left(\frac{U(\omega w_l) - U(w_l)}{U_{\omega}(\omega w_l)}\right) \quad (14)$$

[62] See Oswald (1987) for discussion.

[63] See also Penclavel (1985), Oswald (1985) and Farber (1986) for further discussion.

[64] The authors show that when choosing the level of employment, firms may often give up short-term profits (i.e. cheating on the level of employment) for better contracts in the future.

[65] For different point of views, see Layard and Nickell (1990) who show that employment may not be always higher under efficient bargaining than under monopoly union models, and Alogoskoufis and Manning (1991) who reject both the monopoly union model and the efficient bargaining model in favor of a generalized model of inefficient bargaining for wages and employment.

where α_E and β_E are calibrated parameters. These two equations are easily interpreted. The first represents a rent sharing rule like that in equation (7) above. It states that the wage premium equals an arithmetic mean of the average and marginal products of labor.[66] The second expression is the equation representing the locus of efficient wage-employment bargains, the firm-union contract curve. The right-hand side represents the firm's iso-profit loci, the left-hand side the union's indifference curve.[67]

The results of three endogenous wage experiments are presented in Table 3 below, accompanied by the reference simulation. Since each of these experiments is confined to a single occupational group (skilled labor) and sector (durables), aggregate differences are negligible.

[66] More on rent sharing can be found in Abowd and Lemieux (1993).
[67] For more details, see MacDonald and Solow (1984) and Oswald (1987).

Table 8
Endogenous Wage Rigidity Experiments
(percentage changes)

Selected Aggregates	*Experiment* *1*	*7*	*8*	*9*
Real GDP	.8	.7	.8	.8
Real Exchange Rate	-5.7	-5.5	-5.8	-5.5
Consumer Price Index	-9.3	-9.1	-9.3	-9.2
Real Wages				
Unskilled	-10.2	-11.2	-9.9	-11.5
Skilled	-3.6	-1.6	-4.1	-2.9
Informal	-.5	-.7	-.4	-1.0
Val. Added Wgt. Ave.	-5.5	-5.1	-5.6	-5.8
Employment Wgt. Ave.	-7.0	-7.6	-6.8	-8.0
Sectoral Wage Premium and Employment				
WP.Skilled.Durables	.0	-10.9	3.6	1.6
LD.Unskilled.Durables	29.5	18.1	32.7	19.8
LD.Skilled.Durables	-16.1	18.2	-25.3	1.7
LD.Informal.Durables	-30.8	-40.3	-28.0	-39.1
LD.Durables	7.7	9.2	7.3	9.0
Output.Durables	5.8	6.9	5.5	6.8

Experiment 7: Experiment 1 with rent sharing by skilled labor in Durables.
Experiment 8: Experiment 1 with wage bargaining by skilled labor in Durables.
Experiment 9: Experiment 1 with wage and employment bargaining in Durables.

Wage and employment results are affected in significant and revealing ways, however. In the rent sharing scenario (Experiment 7), skilled labor takes a significant, -10.9 - 1.6 = 12.5 per cent wage cut, thereby reversing a 16.1 per cent employment loss to an 18.2 per cent gain. This permits output and total employment expansion in the durables sector, but still comes at the expense of unskilled and informal workers. The latter suffer less than under minimum wage policies, however, in part because we assume no crossover from skilled to informal labor markets.

Then the same group bargains over wages only, their sectoral gain in wage premium (3.6 per percent) is only just offset by a 4.1 per cent decline for skilled workers across the economy, implying they achieve significant own-wage protection. This comes at a price in terms of job security, however, when 25.3 per cent of skilled workers in this sector are laid off.[68] As has been observed in some long term union bargaining situations, labor shedding induced by wage escalation contributes to the economywide wage losses, ultimately undermining the original group's bargaining power. Despite this mixed result, however, skilled workers better their lot vis-à-vis the reference case in terms of the target variable, wages.

[68] The laid off workers join the rest of the skilled labor pool and, on average, experience greater wage losses than their former co-workers. This, and the minimum wage effect on informal workers, illustrates two important effects of wage distortions, own-regressive (within occupational group) and cross-regressive (spilt over to another occupational group) wage linkages. These are among the most complex and interesting aspects of incidence which can be analyzed with labor-oriented CGE models, but detailed analysis extends beyond the scope of the present exposition.

When both wages and employment are negotiated, skilled workers gain job increases of 1.7 per cent and wage premia in durables rise slightly. As a group, skilled workers in Durables still see slight (1.6 - 2.9 = -1.3 per cent) wage depreciation, resulting mainly from firm substitution with unskilled workers. All in all, however, it appears that combined wage and employment bargaining yields significant improvement in the latter (1.7 against -16.1 per cent) without too much sacrifice in the former (-1.3 against -3.6 per cent), particularly with respect to the reference case.

Efficiency Wage Models

Incentive Wages and Fair Wages

Traditional neoclassical production theory views wages as determined by prices and labor productivity, which in turn is determined by exogenously given technologies and economic conditions outside the worker-employer contract. In reality, compensation has complex incentive properties, and there are causal links running not just from productivity to wages, but from wages to productivity. In modern labor market theory, such issues come under the rubrics of efficiency wages and fair wages. These theories recognize that a worker's productivity depends not only on human endowments, but on the perceived reward for effort. This section derives a basic specification where worker effort depends upon wages, and we give indications about how such behavior might qualify the conclusions drawn from the prototype model.

At first, the efficiency wage hypothesis was formulated by Leibenstein (1957) to highlight linkages among wages, nutrition, and health in less-developed countries. Then, Solow (1979) transferred the efficiency wage concept to developed economies with a model in which increased wages improve morale and thus directly affect productivity through an increase in worker effort. Akerlof (1984) develops a "gift exchange" model in which firms can raise effort by offering a "gift" of higher wages in return for higher individual effort. Another school of thought emphasizes sociological evidence supporting the view that workers' effort level may significantly depend on the perceived fairness of their wage[69]. Excellent surveys of works on efficiency wage theories are presented in Katz (1986) and Blanchard and Fischer (1989)[70]. Efficiency wage models have been advanced as providing a coherent explanation for empirically observed "noncompetitive" wage differentials across firms and workers with similar productive characteristics[71]. Bulow and Summers (1986) introduce a model of dual labor markets based on

[69] See Akerlof and Yellen (1990) who introduce the "fair-wage-effort" hypothesis and explore its implication. For an alternative specification of the effort function, see Wadhavani and Wall (1991).

[70] See Yellen (1984), and Murphy and Topel (1990) for additional survey on the theory and evidence of efficiency wages.

[71] Recent empirical studies indicate that large and substantial wage differentials remain even after controlling for observed worker and job characteristics. See, for example, Dickens and Katz (1986), Krueger and Summers (1988), Katz and Summers (1989), Blanchflower and Oswald (1992), and Abowd, Kramarz and Margolis (1994). The theory of equalizing differences in the labor market reflects an alternative explanation for the existence of true wage differentials across industries. For a comprehensive review of the theory of compensating differentials, see Rosen (1986).

employers' need to motivate workers. Gibbons and Katz (1992) present evidence that wage differentials reflect unmeasured differences in workers' productive abilities[72].

Assume that worker effort can be represented by a twice continuously differentiable increasing function of the wage premium, denoted by $e(\omega)$ and satisfying $0 \leq e(\omega) \leq 1$. This function will then enter the firm production function multiplicatively, e.g. *F(L)* is replaced by $F(e(\omega)L)$ to represent effective labor input. For firms facing a market wage then, the optimal employment level is that where the marginal product of an additional worker equals the wage, taking account of effort as determined exogenously by wage levels.

To implement this specification, we choose a general functional form

$$e(\omega_{li} w_l) = \frac{\omega_{li} w_l}{\omega_{li} w_l + \alpha_w e^{-\beta_w \omega_{li} w_l}} \qquad (15)$$

where the parameters α_w and β_w are calibrated to exogenously specified base effort levels and wage elasticity of effort, σ_{ew}, satisfying

$$\sigma_{ew} = \frac{\partial e}{\partial w}\frac{w}{e} = \alpha_W e^{-\beta_W w} \frac{1+\beta_W w}{w + \alpha_W e^{-\beta_W w}} \qquad (16)$$

where, for the sake of brevity, $w = \omega_{li} w_l$.

Principal-Agent Relations (Shirking and Monitoring)

A significant component of labor productivity is thought to be governed by pecuniary incentives and worker supervisory mechanisms. Wage premia might be offered to bias recruitment in favor of higher productivity workers and motivate workers already on the job. Monitoring may be a complement to or substitute for this, a means of overcoming moral hazard and seeing to it that workers perform as expected. Both these approaches entail costs which exceed those which would be incurred by a firm with perfect information which could perfectly discriminate in the labor market, but the degree to which these second-best approaches compromise efficiency is an empirical question.

When shirking detection is uncertain, the firm attempts to pay wages in excess of market clearing to induce workers not to shirk[73]. Then, if a worker is caught shirking and is fired, he will pay a penalty. Considering the threat of firing a worker as a method of discipline is not novel. The works of Calvo (1979) and Shapiro and Stiglitz (1984) have highlighted the moral hazard problem underlying the employer and wage-earner relationship[74]. However, the equilibrium unemployment rate must be sufficiently large that it pays workers to work rather than to take the risk of being caught shirking. Shapiro and Stiglitz (1984) develop a model introducing a "non-shirking constraint".[75] For a formal discussion on the reasons why firms

[72] A number of empirical studies suggest the existence of wage differentials, focusing on specific aspects. See Bishop (1987) for employee's performance, Brown and Medoff (1989) for plant size, and Groshen (1991) for establishment type.

[73] Models of this type have been analyzed by Bulow and Summers (1986), Calvo (1985), Eaton and White (1983) and Shapiro and Stiglitz (1984).

[74] In this type of models, unemployment is involuntary, in the sense that workers without jobs would be happy to work at the market-clearing wage, but cannot credibly signal not to shirk at this wage. For further discussion on this issue, see also Nalebuff, Rodriguez and Stiglitz (1993), and Akerlof and Katz (1987, 1989).

[75] It has been argued that upfront performance bonds could provide incentives for adequate employee productivity. Bulow and Summers (1986), Dickens, Katz, Lang and Summers (1987), and Shapiro and Stiglitz (1984) provide

monitor their workers, see Dickens, Katz, Lang and Summers (1990). For empirical evidence on the substantial resources devoted to monitoring workers, see Dickens, Katz and Lang (1986). Empirical evidence that efficiency wages are paid to elicit effort includes Raff and Summer's (1987) examination of Henry Ford's five dollar day, Bullow and Summer's (1986) analysis of the impact of sectoral wage declines on employment and Cappelli and Chauvin's (1991) finding of a negative relationship between wage premia and dismissal rates.

In this section, the prototype model is extended to incorporate a simple shirking and monitoring specification, giving an indication of how principal-agent relations might affect empirical conclusions from general equilibrium models. Consider a given sector (i) and labor occupational category (l), and assume that workers in this sector have an exogenously defined quite rate (q) and, if they shirk, a probability (f) of being fired. In a steady state, it can be shown that the wage premium necessary to make workers just indifferent between shirking and not doing so is given by

$$\omega_{li} = 1 + \alpha_S \frac{L_l^S}{L_l^S - L_l^D} \frac{q}{f} \qquad (17)$$

where L_l^S and L_l^D denote total labor supply and labor demand for occupational group l, respectively.[76] The parameter α_S is calibrated from base data on sectoral wage differentials and f may be exogenous or endogenous, depending upon whether the firm uses monitoring in an effort to influence worker productivity. In a relatively simple case, such a firm would choose monitoring resources M to impose firing risk $f(M)$ on shirking workers. Assume, as is common in this literature, that $f(M)$ is twice continuously differentiable and $f_M > 0$ and $f_{MM} < 0$ in the relevant range. Then the firm will use monitoring inputs just until their marginal cost, c_M, equals the marginal benefit they occasion in terms of reduced wage premia, i.e.

$$c_M = -\frac{\partial \omega_{li} w_l L_l^D}{\partial M} = \frac{f_M}{f}(\omega_{li} - 1) w_l L_l^D \qquad (18)$$

In other words, the marginal cost of the last unit of monitoring inputs should equal the per cent change in monitoring effectiveness, times the premium component of the wage bill.

To implement this specification, we assume that workers in another occupational category (k) are monitors, and unit monitoring costs equal their wage (i.e. $c_M = w_k$). We then choose a generalized logistic function to represent how the monotone and bounded ($0 < f < 1$) risk of firing depends upon the level of monitoring. Thus $f(M)$ takes the general form

$$f(M) = \frac{M}{M + \alpha_M e^{-\beta_M M}} \qquad (19)$$

where the parameters α_M and β_M are calibrated from an exogenously specified number of supervisory workers M and elasticity of firing risk with respect to monitoring inputs, $\sigma_{f,M}$.

Table 4 presents the results of four experiments, which are compared to the reference case as usual. Again, activity is largely confined to sector and occupational groups and aggregate effects are relatively small.

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detailed discussions of why firms may be limited in requiring workers to exhibit performance bonds, pay fines or charge entrance fees.

[76] See e.g. Bulow and Summers (1986) for a discussion of no shirking constraints.

Table 9
Incentive Wage and Monitoring Experiments
(percentage changes)

	Experiment				
Selected Aggregates	*1*	*10*	*11*	*12*	*13*
Real GDP	.8	.3	.6	1.1	.8
Real Exchange Rate	-5.7	-6.7	-5.8	-6.5	-5.6
Consumer Price Index	-9.3	-10.1	-9.4	-9.7	-9.2
Real Wages					
Unskilled	-10.2	-10.2	-9.6	-12.0	-10.0
Skilled	-3.6	10.5	-1.1	1.2	-4.0
Informal	-.5	10.5	1.5	3.1	-.7
Val. Added Wgt. Ave.	-5.5	2.5	-3.8	-3.4	-5.6
Employment Wgt. Ave.	-7.0	-3.0	-5.9	-6.8	-7.0
Sectoral Wage Premium and Employment					
WP.Unskilled.Durables	.0	.0	.0	27.6	-2.2
LD.Unskilled.Durables	29.5	-53.2	19.2	-17.8	34.0
LD.Skilled.Durables	-16.1	105.0	4.8	33.4	-20.4
LD.Informal.Durables	-30.8	104.2	-10.1	19.5	-34.8
LD.Durables	7.7	-7.2	4.7	1.1	8.4
Output.Durables	5.8	-5.4	3.7	1.0	6.3
Effort.Unskilled.Durables	.0	-45.7	-9.0	.0	.0
Firing Risk	.0	.0	.0	.0	3.4
Monitors	.0	.0	.0	.0	9.1

Experiment 10: Experiment 1 with basic effort function, elasticity = 2.0.
Experiment 11: Experiment 1 with basic effort function, elasticity = 0.5.
Experiment 12: Experiment 1 with constant effort, endogenous wage premium.
Experiment 13: Experiment 1 with monitoring.

Experiments 10 and 11 use two simple specifications of the effort function to evaluate efficiency or incentive wage effects for unskilled workers in durables, one with a wage elasticity of effort of 2.0 and the other with $\sigma_{ew} = 0.5$. In these simulations, declining incentive wages generally lead to falling effort (depending in magnitude on the relevant elasticity), falling efficiency, and a competitive disadvantage for the sector of employment. Where effort falls faster than wages (Experiment 10), durables employers substitute away from unskilled labor. If the wage elasticity of effort is less than unity, an employment shift in favor of this group occurs.

Experiment 12 poses the question: What wage premium in durables would be necessary to maintain constant effort in the face of declining economywide unskilled wages, and what would be its ultimate effect on the rest of the adjustment process? The answer in this case is 27.6 per cent, driving many unskilled workers (17.8 per cent) out of durables employment, but keeping sectoral output relatively constant. Thus a significant own-regressive effect emerges, where firms are induced by the incentive problem to choose a new occupational mix, including fewer unskilled worker who receive higher wages to maintain their effort levels, but shedding a significant number of them to face unemployment or sharply lower wages in new jobs. Vis-à-vis the reference case, unskilled employment in durables reverses a 29.5 per cent gain to a -17.8 per cent lay off, while skilled workers switch from -16.1 per cent laid off to 33.4 per cent more employed.

A final simulation implements our simple monitoring specification, with the result that both durables employment and output can exceed reference levels by employing more monitors. Under trade liberalization, the opportunity cost of supervisory (skilled) workers or monitors falls, making it economic to have (9.1 per cent) more of them, thereby raising the firing risk for unskilled shirkers 3.4 per cent (from 80 per cent in the base) and lowering the sector's unskilled, constant effort, wage premium by 2.2 per cent.

Transaction Costs

The prototype model assumes that the process of job creation and destruction is castles for workers and firms, but in general both parties may incur significant expenses from labor market participation. Workers may engage in costly search activities and purchase goods and services designed to increase their search effectiveness. For firms, labor market transactions costs fall into four broad categories: 1) recruitment; 2) training; 3) severance; 4) costs arising from labor relations. Although some of these costs might affect a worker's ultimate productivity, they must be factored into firm profits in addition to basic wage compensation.[77] For this reason, transactions costs drive a wedge between labor productivity in the firm's production function and the hiring/firing decision, with a commensurably detrimental effect on efficiency.

The role of labor turnover costs in the efficiency wage mechanism is analyzed in Salop (1979) and Stiglitz (1985). Turnover costs is costly to firms in terms of search for new workers, lost production during vacancies, and a loss of specific training. If firms must bear part of the costs of turnover and if quit rates are a decreasing function of wages paid, firms will attempts to pay above market clearing wages in order to reduce costly labor turnover costs[78]. However, the same wage may not clear simultaneously the market for new hires and the market for trained workers[79]. There is almost no available data on the size or breakdown of labor market transaction costs. While few surveys have attempted to analyze the costs of firing and hiring, even fewer studies have tried to infer the accounting costs of turnover within particular firms. Taken together, the diversity of the reported estimates illustrate the difficulty of clearly identifying and measuring these costs.[80] Given these constraints, turnover models predict high wages where hiring and training costs are substantial. Empirical studies indicate that industry wage premiums reduce voluntary turnover (Brown and Medoff 1978, Dickens and Katz 1987, Krueger and Summers 1986 and 1988).[81] These results provide additional evidence that wage premiums may not reflect compensating differences.[82]

[77] Training costs can in some cases be amortized into the wage.

[78] In most types of efficiency wage models, firms' willingness to pay higher relative wages lead to involuntary unemployment equilibrium, mainly because the wage is unable to clear the labor market when it must simultaneously allocate labor and provide adequate incentives. See Krueger and Summers (1988) for discussion.

[79] The dual role of wages causes a type of market failure which induces a non-unique market-clearing wage equilibrium for workers with different quit functions (Salop 1979). Following this line of thought, Stiglitz (1985) provides a rationale for wage distributions within an industry for similar workers.

[80] Penclavel (1972) presents a general discussion on training and labor turnover in US manufacturing industries and Hamermesh (1993) reports available data on this issue.

[81] Krueger and Summers (1988) find a positive and statistically significant effect of industry wage premiums on job tenure, and a negative but statistically insignificant effect on quit rates. Moreover, Brown and Medoff (1978)

Because of their symmetry and complexity, transactions costs can lead to a broad array of distortions on both sides of labor markets, including underemployment or over-employment, wage premia or wage discounts, excessive worker retention and employment stability or excessive layoffs and employment volatility. Higher costs and more limited information both confer strategic disadvantage on those who possess them. Ultimately, qualitative results will depend upon relative recruitment/severance cost and information quality for firms and workers, while magnitudes can only be assessed empirically.

To illustrate the role of labor market transactions costs, we extend the prototype model with a simple specification for both workers and firms. For workers, it is assumed that employment is associated with a cost equal to a fixed proportion of their entry wage representing turnover costs.[83] For firms, we assume that both recruitment and severance are associated with a cost in fixed proportion to wages. In a competitive labor market, one might expect these costs to be passed through equilibrium wages, while in a bargaining or rent-sharing environment they might be shifted from strategically stronger to weaker agents.

Transaction costs can be incorporated into all the endogenous wage determination models discussed in the previous section, but for illustrative purposes we only evaluate them in the competitive labor market setting. To do this, the labor demand and supply equations for the prototype must be amended to include the parameters δ_h and δ_f, denoting coefficients for transaction costs for employment (from the worker perspective), hiring, and firing (both from the firm perspective), represented as unit costs discounted over the expected term of employment.[84] It is also a simple matter to incorporate search costs from the worker perspective, but this omitted in the interest of brevity. The results of these experiments are given in Table 5 below and discussed in that section.

Selection Models

A large component of modern labor market theory focuses attention upon the process of employee selection by firms. In a simplified neoclassical setting, firms and workers are each homogeneous populations with perfect information, making costless contracts in a frictionless labor market. In reality, of course, both employers and candidates are very diverse and considerable uncertainty governs their interactions. These practical limitations will undermine the efficiency of the labor market and can lead to behavior which has complex incentive properties. In this section, we consider a representative example which indicates how the standard neoclassical model and information set must be expanded to account for these phenomena.

Imperfect information by firms about the quality of workers provides a selection rationale for efficiency wage payments. If workers are heterogeneous in ability and if ability and reservation wages are positively correlated, firms that offer higher wages will attract higher-quality job applicants. The simplest reason for the dependence of productivity on wages is

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estimate a mean elasticity of quits with respect to the wage premium of about -0.3. Dickens and Katz (1987) find qualitatively the same results for nonunion workers. See also Freeman (1980) and Leonard and Jacobson (1990).

[82] See also Gavin (1986) and Lazear (1990) for a discussion and econometric results concerning severance pay.

[83] For convenience only, we assume the payment is made to the government. In general, this turnover cost would appear as worker demand for goods and services associated with employment. We assume there is no direct worker cost associated with a lay-off.

[84] This discounting is necessary in a comparative static framework, where there is only on wage bill during the term of labor market clearing.

adverse selection (Stiglitz 1987, Weiss 1980 and Greenwald 1986). With a continuum of worker types, steepening the wage profile will be a profitable strategy for selecting a subset of types[85]. Some rents will exist because it is not worthwhile to achieve perfect sorting. Nalebuff, Rodriguez and Stiglitz (1993) present a model with asymmetric information in which wages serve as an effective screening device. For an excellent overview of the theory of contracts, see Hart and Holmström (1987). See also Nalebuff and Stiglitz (1983) for a presentation on the role of compensation in economies with imperfect information[86]. Weiss (1980) and Malcomson (1981) apply the efficiency wage concept in the context of a pool of heterogeneous workers, where firms can only roughly estimate the quality of each applicant.

To illustrate how different assumptions about the underlying labor market selection process can affect empirical simulation results, consider two alternative explanations of inter-sectoral wage differentials. In both cases, we assume that the wage differences reflect equilibrium differences in sectoral labor productivity. The first scenario is used in the prototype and is standard in most CGE models. Here one assumes that productivity differences are specific to the firm, and workers who enter a sector "inherit" that sector's productivity and wage premium. Thus workers moving from high low productivity sectors experience a corresponding drop in their individual productivity. At the other extreme, we assume that labor productivity is specific to workers, and the existing wage distribution reflects equilibrium differences in recruitment which place more productive workers in higher wage sectors. In this case, workers take their productivity levels with them when they change jobs. As usual, the truth probably lies somewhere between these two extremes, but their implications for the adjustment process are very different.

To implement the second scenario in the prototype model is a simple matter. We need only to convert the base sectoral employment levels from worker units to efficiency units. This is accomplished by simply rescaling employment in each sector and occupation by the observed wage differential, then setting the latter to unity. The results are presented and discussed in the next section.

Labor Market Search and Matching

The prototype neoclassical model represents an extreme simplification of the process by which workers seek employment and firms seek recruits. The true underlying dynamics of labor market search and matching are of course very complex, and an extensive theoretical and econometric literature has developed to elucidate it. Most of this work simplifies this task considerably, representing the underlying process by a functional form which, while parsimonious in most cases, has enough structure to capture the essential behavioral features of search and matching.[87] We incorporate one such functional form in the prototype to give an indication about how its general properties are affected and as an example of how more empirical work might be done in this area.

[85] See Stiglitz (1985) for the implications of imperfect information on the equilibrium wage distribution.
[86] The implications of imperfect information in competitive markets is discussed in the seminal paper of Rotschild and Stiglitz (1976).
[87] At the mircoeconomic level, the work of Pissarides (1981, 1985b, 1986, 1987) is representative, while the work of Blanchard and Diamond (1990) on the Beverege Curve shows how search and matching is approached from a macroeconomic perspective.

It is the large literature on the Unemployment-Vacancy (*UV)* curve which has fueled new interest in the analysis of structural change in the labor market.[88] Search theory has emerged from the idea that trade in the labor market is an economic activity which yields crucial implications for unemployment[89]. Mortensen (1986) presents a useful survey of the literature on job search[90]. Stochastic job matching functions were first developed by Jovanovic (1979)[91]. Standard references in the matching literature include Diamond (1981, 1982a, 1982b), Mortensen (1982b), and Pissarides (1985b, 1987, 1990). For an insightful discussion on the methodology and empirical evidence of search and matching models, see Eckstein and Wolpin (1990) and Stern (1990).[92] In these models, the labor market is characterized by unemployed workers searching for jobs and firms recruiting workers to fill their vacancies. The potential trading partners are brought together pairwise by a given stochastic matching technology and the probability of matching a worker-firm pair depends on the number of active searching workers and recruiting firms. A number of authors have examined the (in)efficiency of search equilibria[93]. Pissarides (1984) presents a model with endogenous demand for labor, later extended to include a dynamic dimension (1985a, 1987, 1990). Mortensen (1982a) and Howitt and McAfee (1987) introduce models with variable search intensities.[94] Jackman, Layard and Pissarides (1989) present empirical evidence on variable intensities and Pissarides (1986) provides a search model with interesting econometric results for Britain.

Assume as in the prototype that notional labor demand is given by the number of vacant jobs v, number employed is given by L, and number of employable workers equals T. In a neoclassical labor market, efficiency would prevail and these notional levels would be realized at some equilibrium wage rate. Assume instead that labor market pairing of prospective workers ($u = T\text{-}L$) with vacant jobs (v) is inefficient and can be modeled by a generalized function or matching technology of the form

$$m(v,u,w) = v\left(1+\alpha_m e^{-(\beta_u v+\beta_v u+\beta_w w)}\right)^{-1} \quad \textbf{(20)}$$

where the $\beta_i > 0$ are elasticities of effective job creation with respect to each explanatory variable and α is a calibrated scale parameter. This multi-nomial logistic function is a generalized version of a variety of specifications discussed and estimated in the literature on this subject.[95]

Since the matching function is asymptotic to the number of vacancies, the labor market will never clear completely, and thus underemployment plus a wage premium are likely to

[88] Early studies on vacancy-unemployment interactions were motivated by the desire to find a way of measuring the excess labor demand discussed in Phillips curve studies. For recent empirical work on the *UV* curve, see Jackman, Layard and Pissarides (1989) for Britain, and Blanchard and Diamond (1989) for the United States.

[89] Lucas and Prescott (1974) present a theoretical paper in which the theory of job search is used to develop an equilibrium theory of employment.

[90] See also Layard, Layard and Pissarides (1991) for various theoretical extensions and empirical evidence on job search theory.

[91] In the context of matching-bargaining models, Howitt's (1985) model of transaction should also be noted. Empirical studies on the probability of leaving include Lancaster (1979), Nickell (1979), Yoon (1981), Flinn and Heckman (1982), Narendreanathan and Nickell (1985) and McKenna (1987).

[92] See also Pissarides and Wadsworth's evidence (1994) of on-the-job search for Britain.

[93] Mortensen (1982b) argues that agents' search and recruitment expenditures are generally inefficient because no agent internalizes the value of his increased search activity to other searchers. See also Diamond (1982a) and Pissarides (1984, 1985b and 1987).

[94] Extending Hosios' work (1990), Pissarides (1990) considers variable intensities as input-augmenting technical progress.

[95] See e.g. Hosios (1990) for more discussion.

emerge among the efficiency costs of imperfect matching. The matching function is calibrated to an assumed 10 per cent and two hypothetical different elasticity regimes.

Table 5 presents the results of illustrative experiments with transactions costs, labor market selection, and a search/matching specification. The direct adjustments ensuing under transactions costs are completely intuitive, with hiring costs (Experiment 14) increasing unskilled unemployment and reducing wages and firing costs reducing lay offs and wage declines. A 10 per cent hiring premium depresses new employment by almost an equal amount (29.5 - 19.0 = 10.5 per cent), but firing costs cannot be compared directly since this requires a reference case with lay offs.

Also significant, and much less obvious, are the spillover effects on other occupational groups. Even though the latter labor markets have been assumed to be competitive, they move with the unskilled group in ways which would be difficult to predict from simple rules of thumb. Of particular interest is Experiment 15, where, despite that fact that firing costs are not incurred directly, their presence induces a distortion which reduces wage and employment losses for the other groups.

Experiment 16 represents the simple but illuminating labor market selection experiment. Assuming that labor productivity is embodied in those workers employed in the base equilibrium, removing trade distortions confers no efficiency gains in the presence of resource constraints. This is because worker reallocation cannot raise average efficiency levels. The assumption that base wages and employment reflect worker-specific differences in productivity has very different implications for structural adjustments within the economy, however. The reference simulation indicates that the 1990 Mexican system of prior import protection may have been relatively “worker friendly” in the sense that all three occupation groups’ real wages decline as a result of liberalization. When productivity is embodied in those workers, however, they benefit from removing distortions, since they can allocate their skills more “efficiently” (in terms of factor rewards) when distortions are removed. Since we now assume that any sector can pay premium wages to premium workers, and durables had relatively superior average wages in the base case, they expand less than other sectors which are, for example, more export competitive and can bid away high quality workers.

The final two simulations indicate how more general labor market inefficiencies, captured by a generic matching function, can effect adjustment to trade liberalization. Among a three-dimensional continuum of cases, we chose only five regimes for the three elasticity values in expression (20) above. The first two correspond to uniformly flexible and inflexible cases, i.e. all three β‘s = 5.0 and 0.2, respectively. In between these hypothetical extremes, we consider three cases, one where each β equals 5.0 while the other two equal 0.2, thereby imputing most of the new matching to each of the three constituient influences, vacancies, unemployment, and wages. While the results do differ at the sectoral and occupational level, it is difficult to generalize from these experiments. Apparently, greater sensitivity of the matching function to vacancies (Experiments 17 and 19) leads to more job creation for unskilled workers, in part because the declining wage permits firms to recruit more. This does not imply, however, that wage sensitivity (Experiment 21) leads to the smallest unskilled wage decline. While the qualitative results are comparable in all cases, and the three intermediate elasticity specfications yield intermediate outcomes, more intensive investigation of this specification is obviously needed. In particular, some detailed econometric work could do much to narrow the acceptable range of functional forms and parameter values.

Table 10
Transactions Cost, Selection, and Search/Matching Experiments
(percentage changes)

Selected Aggregates	*Experiment* 1	14	15	16	17	18	19	20	21
Real GDP	.8	.6	.8	.0	1.0	.7	1.0	.7	.7
Real Exchange Rate	-5.7	-5.9	-5.6	-5.6	-5.6	-5.7	-5.7	-5.7	-5.6
Consumer Price Index	-9.3	-9.5	-9.2	-9.3	-9.2	-9.3	-9.2	-9.3	-9.3
Real Wages									
Unskilled	-10.2	-11.7	-9.9	4.3	-14.3	-9.8	-13.0	-9.9	-11.5
Skilled	-3.6	-4.5	-3.0	5.3	-6.0	-3.4	-5.0	-3.4	-4.5
Informal	-.5	-1.4	-.2	6.2	-2.5	-.4	-1.6	-.4	-1.4
Val. Added Wgt. Ave.	-5.5	-6.6	-5.1	5.1	-8.5	-5.3	-7.4	-5.3	-6.6
Employment Wgt. Ave.	-7.0	-8.3	-6.7	4.9	-10.5	-6.8	-9.3	-6.8	-8.2
Sectoral Wage Premium and Employment									
LD.Unskilled.Durables	29.5	19.0	30.0	7.0	35.8	28.1	34.4	28.2	29.9
LD.Skilled.Durables	-16.1	-18.4	-10.3	1.4	-20.9	-14.1	-20.1	-14.3	-15.2
LD.Informal.Durables	-30.8	-32.9	-30.6	-3.6	-36.4	-28.7	-35.5	-28.8	-29.9
LD.Durables	7.7	8.2	7.6	3.9	8.5	7.5	8.3	7.5	7.7
Output.Durables	5.8	6.1	5.8	3.8	6.1	5.7	6.1	5.7	5.7

Experiment 14: Experiment 1 with unskilled *ad valorem* Hiring cost of 10 per cent.
Experiment 15: Experiment 1 with unskilled *ad valorem* Firing cost of 10 per cent.
Experiment 16: Experiment 1 with selection via labor-embodied productivity.
Experiment 17: Experiment 1 with matching function in unskilled labor, β={5,5,5}.
Experiment 18: Experiment 1 with matching function in unskilled labor, β={.2,.2,.2}.
Experiment 19: Experiment 1 with matching function in unskilled labor, β={5,.2,.2}.
Experiment 20: Experiment 1 with matching function in unskilled labor, β={.2,5,.2}.
Experiment 21: Experiment 1 with matching function in unskilled labor, β={.2,.2,5}.

Conclusions and Extensions

This paper offers a practical taxonomy of more recent labor market theories, combined with a menu of specifications to implement them in empirical simulation modeling. After reviewing an extensive theoretical literature and providing guidelines for using these ideas empirically, the task ahead is very clear. Even the focused and parsimonious examples used above show how challenging it can be to understand trade and employment linkages, particularly when taking account of labor market imperfections. The universe of discourse is an essentially general equilibrium one, where second-best properties are endemic. Thus policy makers cannot reasonably rely only on simple theoretical intuition or rules of thumb.

We have seen how social protection measures, like minimum wages, can be regressive, how the same policy applied to different sectors or occupational groups can have very different direct and indirect effects, how the same distortions can hinder efficiency in one case and promote it in another, and how behavioral information unlikely to be unavailable to the average policy maker can undermine or even reverse intended outcomes. Given these variegated results, in a relatively aggregated a single country application, generalization to more detailed interactions or across countries would be even more tenuous. While theoretical work can and has produced important insights, only detailed, case by case, empirical work will elucidate the workings of real labor market structures, conduct, and policy interventions.

Labor Supply and Migration

Two aspects of labor markets are of special importance to developing countries: aggregate labor supply functions and rural-urban migration. These are relatively parsimonious specifications, intending as guidelines only, but they represent an important direction for extending standard neoclassical CGE models. Both of these phenomena are essential to understanding the Chinese development process, and we describe them briefly below.

Labor Supply

As in most developing countries at the early stages of industrialization, the rate of aggregate Chinese participation in the formal labor force has been steadily rising with real wages. This positive relationship can be expected to continue as real wages rise, but to level off as dependency limits are reached in the population. We capture this phenomenon in the model with a logistic labor supply function. Assume that households elect to supply labor in response to the average real wage in their labor market, discounted by their household consumer price index. Formally, the labor force participation rate of a given household, r_h, is given by the calibrated logistic function

$$r_h = \alpha + \frac{\beta}{1 + \gamma e^{-\delta \bar{w}_h}}$$

where

$$\overline{w}_h = \sum_l w_l \frac{\theta_{lh}}{CPI_h}$$

and θ_{lh} denotes the share of workers of occupation l in household h. Schematically, this function looks like

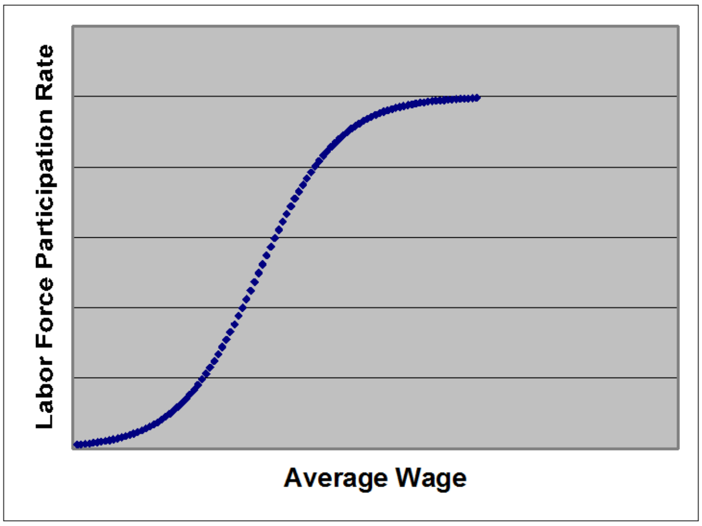

Figure 2.1: Aggregate Labor Supply

The calibrated upper bound for this function is one minus the household dependency ratio, i.e. the total percentage of eligible workers in each household group. The calibrated inflection point is the observed base year participation rate at the observed real wage.

Thus we assume that labor supply elasticity is highest around the observed wage (i.e. a labor surplus economy) and decreases as wages rise, becoming completely inelastic as labor force participation rates exhaust the eligible labor force. The current implementation of the model uses an economywide aggregate labor supply function, i.e. all household are aggregated together.

Harris- Todaro Migration

Migration is modeled with a simple variant of the Harris-Todaro paradigm, where workers move between regions in response to relative wages in those labor markets. For this implementation of the China model, we make five assumptions.

1. Workers migrate, leaving and joining their respective households.
2. Migrants seek new employment in the same labor classification they left.
3. Each occupational group has it's own migration function.
4. There are two regions, Rural and Urban.
5. Workers consider only relative market wages in the migration decision.

This leads to two regions (Rural, Urban), three labor classifications (Manual, Clerical, Professional) and the following constant elasticity calibrated migration function

$$\frac{L_{lr}}{L_{lu}} = \alpha_l \left(\frac{W_{lr_{rk}}}{W_{lu}} \right)^{\mu_l}$$

CET Migration

A formally similar approach to Harris-Todaro is the CET specification. Discussion of this specification of migration is confined rural-urban linkages at the moment, although this can easily be generalized to other regional frameworks if data exist to support it. The basic model uses a CET function to capture imperfect labor mobility between the two (rural and urban) regions. We make the following assumptions for convenience:

- Workers migrate, leaving and joining their respective households.
- Migrants enter the new labor force in the same labor classification they leave.
- Each occupational group has its own migration function.

Formally, migration is characterized as an optimization problem, where the objective is to maximized expected income for the whole occupational group by allocating labor across regions. This takes the form:

$$\begin{aligned} &\max \quad && W_{rk}L_{rk} + W_{uk}L_{uk} \\ &s.t. \quad && FS_k = \alpha_k \left[\beta_k L_{rk}{}^{\lambda} + \gamma(1-\beta_k)L_{uk}{}^{\lambda} \right]^{1/\lambda} \end{aligned}$$

where L_{rk} denotes rural supply of category k (e.g. Unskilled, Skilled, or Professional) labor, L_{uk} it's urban counterpart, and FS_k is economywide supply for occupation k. The variables W_{rk} and W_{uk} are corresponding expected returns to employment in each region. For the moment, we simply model these with market wages, but they might incorporate search cost, unemployment risk, or other migration considerations. The terms α_k and β_k are CET share parameters, and λ is the CET exponent. The CET exponent is related to the CET substitution (or migration) elasticity, μ via the following relation:

$$\lambda = \frac{\mu+1}{\mu} \Leftrightarrow \mu = \frac{1}{\lambda-1}$$

The migration decisions, in reduced form, are given by Equations (1) and (2), where the share parameters are α^t and the CET substitution elasticity is σ^t.[96]

[96] Note the difference between the Armington CES and the CET. First, the relation between the exponent and the substitution elasticity is different. Second, the ratio of the prices and the share parameter in the reduced forms are inverted. This is logical since the goal of the producer is to maximize revenues. For example, an increase in the price of exports, relative to the composite aggregate price, will lead to an increase in export supply.

A Case Study: Labor Markets and Dynamic Comparative Advantage

Although international migration has received more public attention because of its complex political implications, domestic migration is often of greater historical significance, both numerically and economically.[97] While in some countries like the United States, the first type is a necessary condition for the second, the latter still deserves attention in its own right. Patterns of domestic migration have dramatically influenced both internal economic structure and trade orientation in many nations. This has often followed a two-stage process, where migration into the hinterland to develop the primary resource base is succeeded by migration to the cities seeking opportunity in the modern sector. Such demographic trends can influence and be influenced by shifting trade orientation, i.e. changes in the composition of human resources have important implications for dynamic comparative advantage and terms-of-trade changes can exert significant pressure on domestic labor markets.

The magnitudes of this kind of population adjustment can truly be impressive. In the last century, Latin America was about 15% urban, and the figure now is 90%. Indonesia expects its total population to grow modestly, from 180M in 1990 to about 240M by 2040, but at the same time the urban population is projected to grow from 24% to 65%. China's population, while still predominately rural, is experiencing strong migratory pressures. According to official estimates, some 100M Chinese are currently classified as migrants, while officials at the Chinese Academy of Social Sciences estimate (conservatively) a labor surplus in agriculture of about 250M. These figures imply the existence of a volatile labor force exceeding the population of the EEC.[98] As most of these individuals shift their labor from rural to urban areas and from traditional to more modern modes of production, China's production possibilities and trade opportunities will shift accordingly.[99]

The present paper has two objectives, set forth in the following sections. First, a formal approach to incorporating migration in an explicit dynamic economic model is developed. This is followed in section 3 by an empirical application of the approach to the Chinese economy. Section 4 closes with conclusions and some remarks on extensions of this work.

The Basic Model

Imagine an envelope function E(x,y)=0, characterizing maximal output combinations for fixed endowment composition. In response to the classic situation where prices are more favorable to the modern sector with the opening of an economy, one might expect to see resource shifts (e.g. migration) which deform E into E' (see Figure 1). This is the process that underlies regional and occupational migration (i.e. from unskilled to skilled, informal to formal, etc.) in many

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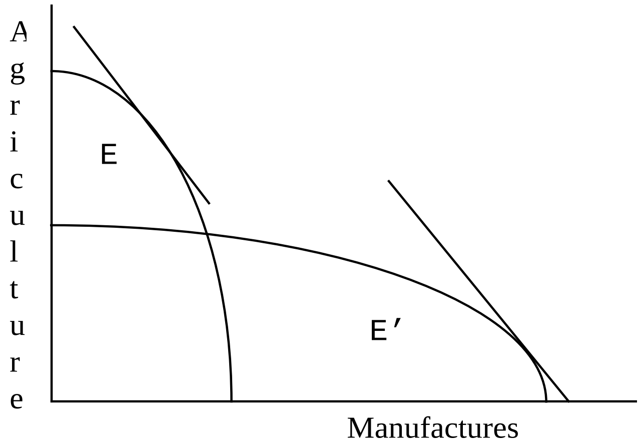
[97] For example, while Burfisher et al (1994) have examined cross-border migration issues related to the North American Free Trade Area (NAFTA), Levy and van Wijnbergen (1994) have argued that the most important migration results of the NAFTA are likely to be realized within Mexico.

[98] Personal communication.

[99] The Chinese government already recognizes what this trend made lead to exploding demand for urban infrastructure in the form of residential housing and a wide array of public goods. If they are to avoid the fate of urban Latin America, the implied fiscal commitments are prodigious.

developing countries, although it is often accompanied by market failures and other institutional problems.



A hybrid overlapping generations model will be used to capture such linkages between migration, economic growth, and trade.[100] Although it can be implemented in more elaborate contexts, for the present discussion consider a two sector model of a small open economy. What emerges from this simple framework is an economy with endowments that are sector-specific in the short run, but fungible in the longer run. As relative prices influence factor prices, labor will migrate, shifting the economy's comparative advantage dynamically in response to externally determined price signals.[101]

Formally, assume that the two goods, called manufactures (x) and agriculture (y), are made with C^2, linearly homogeneous technologies utilizing capital (K) and sector-specific, Urban (U) and Rural (U) labor, which take the forms

$$x = F(U,K) \quad \textbf{(1)}$$

$$y = G(R,K) \quad \textbf{(2)}$$

Assume further that manufactures are the numeraire good, so the economywide relative price

$$p = \frac{p_y}{p_x} \qquad (3)$$

denotes the agricultural terms-of-trade. In any given period t, with endowments K_t, U_t, and R_t, factor rewards are given by

$$w_t = F_U(U_t, K_{xt}) \qquad (4)$$

$$v_t = p_t G(R_t, K_{yt}) - r_t(K_t - K_{xt}) \qquad (5)$$

$$r_t = F_K(U_t, K_{xt}) \qquad (6)$$

$$r_t = p_t G_K(R_t, K_t - K_{xt}) \qquad (7)$$

$$K_t = K_{xt} + K_{yt} \qquad (8)$$

yielding within-period equilibria for all the above, given commodity price p_t.

Capital is mobile between sectors within each period. The labor types are defined in an implicitly regional way (rural and urban), but with a broader definition of migration could be distinguished more functionally (informal-formal, unskilled-skilled, etc.).[102] Within periods,

[100] See e.g. Samuelson (1958).
[101] See Lee and Roland-Holst (1994) for an extensive empirical example of this process.
[102] See Maechler and Roland-Holst (1996) for a broader discussion of these specifications.

the two types of labor are fixed in total supply and specific to the two production sectors of the present discussion. Between periods, labor mobility between the two groups is described by a C^2 concave transformation surface G(U,R)=0. This function implicitly maximizes employment of each type, given relative wages and other end of period equilibrium conditions.[103]

To represent inter-temporal economic linkages within this framework, the basic structure outlined in equations is expanded to include a variant of the standard overlapping generations model. The representative agent lives for two periods, working, consuming, and saving in the first, and consuming from savings in the second. Assume that total population remains constant throughout the analysis and that an individual works in one location (rural or urban) in the first period of life. However, prior to entering the work force, progeny are allowed to take a decision to migrate, after which they work at the destination during the first period of their life.

Assuming that the numeraire good is also the capital good, consumption is given by

$$c_t = \overline{w}_t - s_t = \overline{w}_t - k_{t+1} \qquad (9)$$

$$c_{t+1} = (1 + r_{t+1})k_{t+1} \qquad (10)$$

where

$$\overline{w}_t = w_t U_t + v_t R_t \qquad (11)$$

denotes the average wage and

$$s_t = s(w_t, v_t, r_{t+1}, p_t, p_{t+1}) \qquad (12)$$

is the representative savings function. On a period-by-period basis, this model satisfies the factor market conditions

$$K_{t+1} = (U_t + R_t)k_{t+1} = (U_t + R_t)s(w_t, v_t, r_{t+1}, p_t, p_{t+1}) \qquad (13)$$

and

$$U_{t+1} = M(z_t) \quad (14)$$

where $z_t = w_t/v_t$ and M() denotes the reduced-form, end of period migration function.

From this information, within-period variational equations for factor prices can be derived, given exogenous and between period changes in prices (p), capital stock (K_t), and (e.g.) the urban labor force (U_t), i.e.

$$\hat{w}_t = \left[-\alpha_{Kx}\beta_y\gamma_y\hat{p}_t + \alpha_{Kx}\alpha_R\left(\hat{K}_t - \beta_x\hat{U}_t\right)\right]\Delta \qquad (15)$$

$$\hat{v}_t = \left[\left(\beta_x\gamma_x + \alpha_U\beta_y\gamma_y\right)\hat{p}_t + \alpha_U\alpha_{Ky}\left(\hat{K}_t + \beta_x\hat{U}_t\right)\right]\Delta \qquad (16)$$

$$\hat{r}_t = \left[\alpha_U\beta_y\gamma_y\hat{p}_t - \alpha_U\alpha_R\left(\hat{K}_t - \beta_x\hat{U}_t\right)\right]\Delta \qquad (17)$$

where

$$\alpha_U = F_U\frac{U_t}{x_t}$$

$$\alpha_{Kx} = F_K\frac{K_{xt}}{x_t} = 1 - \alpha_U$$

$$\alpha_R = G_R\frac{R_t}{y_t}$$

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103 Here we are placing a hierarchy on market adjustments, assuming migration takes longer than commodity market clearing, for example. This assumption is much like gestation of capital in standard discrete time growth models, but it has nontrivial implications for the way equilibria are determined. For more discussion of hierarchical market specifications, see Roland-Holst (1995).

$$\alpha_{Ky} = G_K \frac{K_{yt}}{y_t} = 1 - \alpha_R$$

$$\beta_i = \frac{K_{it}}{K_t}$$

$$\gamma_1 = \frac{F_U F_K}{F_{UK}}$$

$$\gamma_2 = \frac{G_U G_K}{G_{UK}}$$

$$\Delta = \left(\alpha_R \beta_1 \gamma_1 + \alpha_U \beta_2 \gamma_2\right)^{-1}$$

Expressions (15) - (17) reflect the expected links between factor markets and commodity prices. An exogenous rise in the agricultural terms of trade (p) leads to a rise in rural wages and rental rates and a fall in urban wages. Rental rates vary inversely with the capital stock and wages of both groups vary in different positive proportions. A re-allocation of labor from rural to urban areas raises rural and lowers urban wages.

To embed this in the dynamic framework, consider now the following steady state conditions

$$\overline{K} + (U + R)s(\overline{K}, \overline{p}) = (U + R)s(w(\overline{K}, \overline{p}), v(\overline{K}, \overline{p}), r(\overline{K}, \overline{p}), \overline{p}, \overline{p}) \qquad (18)$$

$$\overline{z} = \frac{w(\overline{K}, \overline{p})}{v(\overline{K}, \overline{p})} \qquad (19)$$

Assuming sufficient conditions for existence of the steady state, we then have additional variational equations of the form[104]

$$\hat{K} = (\hat{U} + \hat{R})s + \sigma_w \hat{w} + \sigma_v \hat{v} + \sigma_r \hat{r} + \sigma_p \hat{p} \qquad (20)$$

$$\hat{z} = \hat{w} - \hat{v} \qquad (21)$$

$$\hat{U} = \mu_z \hat{z} \qquad (22)$$

where $\sigma_a = \frac{s_a a}{s}$ and $\mu_a = \frac{M_a a}{U}$ are the appropriate savings and migration elasticities, respectively. For a complete set of comparative static results, the six expressions (14)-(17) and (20)-(22) can be solved for changes in the terms-of-trade and other exogenous factors. Unfortunately, however, these solutions have little to offer in terms of general qualitative results. Consider for example the two reduced-form expressions for relative wage and capital stock changes

$$\hat{z} = -\left[1 - \Delta\left(\alpha_U - \alpha_R\right)\beta_1 \mu_z\right]^{-1} \Delta\left[\left(\beta_1 \gamma_1 + \beta_2 \gamma_2\right)\hat{p}_t - \left(\alpha_U - \alpha_R\right)\hat{K}_t\right] \qquad (23)$$

$$\hat{K} = (1 - \Delta E_3)^{-1} \Delta\left[(\beta_1 \gamma_1 \sigma_v + \beta_2 \gamma_2 E_1 + E_2)\hat{p}_t - \beta_1 E_3 \mu_z \hat{z}\right] \qquad (24)$$

where

$$E_1 = \alpha_U \sigma_r + \alpha_U \sigma_v - \alpha_{K1} \sigma_w$$

$$E_2 = \Delta^{-1} \sigma_p$$

$$E_3 = \alpha_{K1} \alpha_R \sigma_w + \alpha_U \alpha_{K2} \sigma_v - \alpha_U \alpha_R \sigma_r$$

It is evident from these expressions that, while some direct effects might be interpreted in this two-sector framework, factor market links here and in higher dimensional cases can better be elucidated by empirical means. For this reason, we devote the next section to an empirical application of the above framework.

An Application to China

This section presents simulation results obtained with a calibrated general equilibrium (CGE) model of China, specified with endogenous rural-urban migration of the type set forth above. This particular CGE model has been extensively documented elsewhere and will not be discussed in detail here.[105] Suffice to say for the present that the model is based on a detailed, 1987 social accounting matrix for China and is calibrated dynamically over the intervals 1987, 1990, 1995, 2000, 2005, and 2010. Although the full form of the model details 64 sectors and 10 different household types, an aggregate, 4 sector, 2 household version is used for this exercise.

To simulate the intertemporal process of migration discussed in section 2, a simple CET function is used to specify a "transformation" of rural into urban labor (or vice-versa). As can be seen in the following reduced-form, this function relies on two parameters, an elasticity and a calibrated intercept relating the base employment and wage ratios.

$$\frac{U}{R} = \alpha\left(\frac{w}{v}\right)^{\gamma} \quad (25)$$

The elasticity represents sensitivity of the rural and urban labor forces to changes in the wage ratio, and adjustments represent migration. The intercept, on the other hand, represents a reference level of "tolerance" for rural-urban wage differentials. In the case of China, nominal average wages are conservatively estimated at over four to one in favor of urban workers.

By analogy to the two parameters of this simple migration function, the experiments reported here take two approaches to the forces inducing internal migration. In the first instance, we examine cases where nominal wages change in favor or urban workers in response to an exogenous shift in terms-of-trade. Secondly, we examine a change in the tolerance of rural workers for existing wage differentials. Put another way, migrants generally respond not to the ratio of market wages, but to the ratio of risk-adjusted (or search-cost adjusted) expected income. In this context, rising risk in the agricultural sector (e.g. drought, rising labor productivity and concomitantly rising labor surpluses) or falling risk of unemployment in the urban sector makes migration more attractive even at constant market wage ratios. Our results indicate that these two types of migration, the one induced by rising market wage ratios, the other by rising expectations with respect to existing differentials, have very different economic consequences.

A set of seven simulations is reported below. The first two represent external shocks of about the same magnitude, one expanding demand for Chinese manufacturing exports tenfold against the baseline trend over the period 1987-2010. The second case specifies a thirty percent

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[104] See Eaton (1987) for discussion of these conditions.
[105] See Dessus, Roland-Holst and van der Mensbrugghe (1998) for complete model documentation.

appreciation of manufacturing export prices over the same period. In the case of no rural-urban migration, economywide real GDP is unchanged and urban households benefit while rural ones lose as a result of other resource (mainly capital) diversion to manufacturing. When migration is permitted, using a long run elasticity over the five-year intervals period of 10, international adjustments are comparable but domestic ones are quite different. Migration does occur, with about three percent more urban workers in the terminal period under both trade scenarios. Although positive, the migration is still smaller than might be expected, but this is due in part to the effectiveness with which it stifles (by over 75%) the rise in urban wages and partially offsets the fall in rural wages. The offset in EV incomes is even greater, and is only slightly larger than the gain in real GDP. This latter effect arises from reallocating labor from lower to higher wage (read productivity) activities.[106]

It is perhaps surprising that these strong trade effects do not induce more migration and output expansion, but the current specification of the model biases such results downwards. In particular, the magnitude of population and output shifts is probably underestimated by the use of a full employment base case in agriculture. As mentioned in the introduction, official sources estimate that China has a few hundred million workers in agriculture who could take up other activities and even residences without significant reductions in rural output. We are also not modeling differential productivity for new urban labor force entrants, a well-established trend in expanding Asian economies. These factors all imply higher levels of migration, manufacturing growth, and smaller contractionary effects on agriculture.

Experiment 5 takes a different approach to the migration question, examining the effect of lowering the "tolerance" wage differential or, from a different perspective, a change in the ratio of expected or risk-adjusted wages. In this experiment, the migration function is re-calibrated to impose a unitary rural-urban wage differential. This extreme but illustrative case occasions a 74.9 per cent drop in urban wages and a 17.4 per cent rise in their rural counterpart. To achieve this, only 5.4 per cent of the rural population is needed, while the urban workforce swells 22.9 per cent. This type of migration, basically impelled from the supply side without a corresponding expansion of labor requirements, has an effect more analogous to the African or Latin American urban experiences than to Asian ones.[107] Real GDP rises significantly, again because of a Katz-Summers labor re-allocation effect, but urban real incomes are severely hit and incomes of the remaining rural population actually rise. This illustrates an important difference between expectations and changes in economic fundamentals.

In experiments 6 and 7, expectations-driven migration is coupled with each of the external trade shocks. Although the resulting impacts are generally better than additive, benefits of improved trading opportunities cannot significantly offset the negative effects of the larger migratory shifts.

Table 1: Selected Aggregate Results

	Experiment						
	1	**2**	**3**	**4**	**5**	**6**	**7**
Real GDP	.0	.0	.7	.7	5.3	5.9	5.9
Urban EV Income	9.6	9.9	1.1	1.2	-45.4	-43.4	-43.3

[106] This essentially reiterates the logic of Katz and Summers (1989) concerning the productivity implications of sectoral wage differentials.
[107] Compare, e.g. results in Collado, Roland-Holst, and van der Mensbrugghe (1995).

Rural EV Income	-2.5	-2.6	-1.2	-1.3	10.4	9.5	9.5
Urban Wage	23.5	24.2	5.0	5.2	-74.9	-73.2	-73.2
Rural Wage	4.7	4.9	4.6	4.7	17.4	24.8	25.0
Urban Employment	.0	.0	3.0	3.1	22.9	25.6	25.7
Rural Employment	.0	.0	-.7	-.7	-5.4	-6.1	-6.1
Ag. Terms-of-trade	-6.7	-6.9	-4.9	-5.1	19.1	16.4	16.3
Intl. Terms-of-trade	20.4	21.1	20.4	21.0	-.2	20.2	20.9
Total Exports	14.1	14.5	14.5	14.9	2.4	16.6	17.0
Total Imports	36.1	37.3	36.5	37.7	2.2	38.7	39.9

All results expressed as percentage change with respect to trend values in the terminal year.
Experiment 1: Tenfold expansion of Manufacturing export demand. No migration.
Experiment 2: Thirty per cent appreciation of Manufacturing export prices. No migration.
Experiment 3: Experiment 1 with migration against existing wage differential.
Experiment 4: Experiment 2 with migration against existing wage differential.
Experiment 5: Migration in response to changing relative wage expectations.
Experiment 6: Experiment 5 with export demand growth as in Experiment 1.
Experiment 7: Experiment 5 with export demand growth as in Experiment 2.

Table 2 presents more detailed sectoral results of the seven experiments. Most of these adjustments are intuitive, particularly those in agriculture. The difference between demand and supply driven migration is even more striking on a sectoral basis, however. Because of the greater urban labor supply shifts in experiments 5-7, all urban sectors expand employment, even when their output may be shrinking. This is particularly evident in the low wage service sector. Despite this dramatic job growth, however, urban price-adjusted incomes have plummeted.

Table 2: Sectoral Adjustments

	Experiment						
Exports	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Agriculture	-20.4	-20.9	-22.9	-23.5	-37.2	-52.3	-52.7
Energy	-41.5	-42.3	-43.2	-44.1	-5.9	-45.6	-46.4
Manufacturing	34.2	35.1	34.6	35.5	2.3	37.5	38.4
Services	-35.5	-36.2	-29.3	-30.0	64.6	12.2	11.0
Output							
Agriculture	-.2	-.2	-1.1	-1.1	-5.6	-6.5	-6.5
Energy	-7.8	-8.0	-8.3	-8.4	-2.2	-9.7	-9.8
Manufacturing	.1	.1	.5	.4	1.9	2.1	2.0
Services	1.1	1.2	3.2	3.3	9.4	11.5	11.6
Labor Demand							

Agriculture	.0	.0	-.7	-.7	-5.4	-6.1	-6.1
Energy	-8.2	-8.4	-6.7	-6.9	15.9	8.4	8.3
Manufacturing	-.4	-.4	2.0	2.0	20.0	21.8	21.9
Services	.8	.8	4.4	4.5	25.4	29.4	29.5

Conclusions and Extensions

In many economies, domestic migration has been one of the primary forces animating economic modernization. In other contexts, it has intensified both urban and rural poverty and led to chronic social and economic problems. Whether it leads to long term benefits or hardship, however, it has been and will remain a powerful force to be reckoned with by policy makers. This is particularly true in populous Asia, where the main adjustments in the rural-urban balance have yet to run their course. A better understanding of the preconditions for beneficial migration and the warning signs of detrimental trends may help avert unpleasant experiences that have already occurred in some parts of Africa and Latin America.

This paper sets out a theoretical framework for analyzing migration in the context of a dynamic trade model. Using a hierarchical approach to market adjustment, a between-period migration function was embedded between the within-period equilibria in an overlapping generations model of a small open economy. This specification is simple enough to elucidate the main forces at work, but it also reveals that this process is in significant respects still too complex to admit general interpretation. It was apparent from the analytics that, even in a simple two-sector, three factor model, important inferences about linkages and policy effects could be made only by recourse to empirical analysis.

This conclusion led to the second part of the paper, applying the migration specification in a dynamic CGE model of China. While this exercise was intended only to give general indications about how to implement such a model, two important insights arose from the empirical results. Firstly, the impetus for migration can arise from the demand or the supply side of labor markets, and this can have dramatically different implications for its economic consequences. As one might reasonably expect, demand-driven migration is more likely to be beneficial, particularly to those at the destination (including the migrants).

The second conclusion from this simple example regards the importance of embedding migration in a more complete specification of labor markets generally. While it is instructive to incorporate a transfer process of the type presented here, its practical implications cannot be clearly understood without consideration of structural features in labor markets at both the origin and destination. The most important of these include labor surplus conditions, which would influence both the output effects of departures and the employment prospects of arrivals. Historically, market imperfections at both ends have undermined the potential of an economy to shift comparative advantage by reallocating its labor force geographically and functionally. Indeed, market and institutional failures, notoriously difficult for economists to model, are probably the main reason the economic promise of migration has so often gone unfulfilled.

7. Enterprise Behavior

Production Functions

Because of their evolutionary role as institutions that assemble production teams, the fundamental behavioral model for enterprises is the production function. This mathematical specification of how factor services combine to transform resources and components into goods and services lies at the heart of the neoclassical paradigm. In this sub-section, we review a variety of widely accepted specifications for production functions and discuss how they can be implemented empirically.

Generalised CES and its derivatives

The constant elasticity of substitution (CES) function is the most ubiquitous functional form used in standard GE applications. It is often used to describe production technologies, as well as being used in consumer demand and trade specification. The first part of this chapter will focus on a theoretical description of the generalised CES technology. Subsequent sections will show practical applications of its usage.

From a producers point of view, under perfectly competitive markets for inputs (i.e. with prices unaffected by producers' demand), the choice of the optimal combination of inputs is modelled by assuming the producer's objective function is to minimise costs, subject to a given production technology. If the technology is given by the CES primal function, the decision can be described by the following:

$$\min \sum_{i=1}^{n} P_i X_i$$

subject to

$$V = A\left[\sum_{i=1}^{n} a_i \left(\lambda_i X_i\right)^{\rho}\right]^{1/\rho}$$

where X_i are the inputs in production, and P_i their respective prices. V is output which is given by the CES production function. The CES exponent, ρ, is related to the substitution elasticity, which will be described below. The CES share parameters are given by the a_i coefficients. The parameter represented by A is a uniform shift parameter which can be applied to all inputs, and the λ_i coefficients are shift parameters which apply to each individual input. For example, in the two factor production function — labour and capital — neutral productivity growth would be applied by shifting the A parameter. Hicks neutral productivity growth would only impact the λ coefficient on labour. (Note, that this function is over specified, both for calibration purposes, as well as for simulation purposes. Neutral technological growth can be obtained by shifting all of the λ parameters by the same uniform amount. Calibration will be discussed below. But in all cases, it is clear the A parameter can be integrated into the λ parameters without any loss in degrees of freedom. It may be preserved simply for ease of use and exposition. It will also be necessary for the special form of the CES function when the substitution elasticity is 1).

To solve the producer's optimisation problem, start with the Lagrangian:

$$L = \sum_i P_i X_i + P\left(V - A\left[\sum_i a_i\left(\lambda_i X_i\right)^{\rho}\right]^{1/\rho}\right)$$

and take the partial derivative with respect to X_i and P and set them equal to 0. (P is the shadow price of the production constraint and will be given an explicit interpretation and expression below). The first order conditions are:

$$P_i = P\left[\sum_i c_i X_i^{\rho}\right]^{\frac{1-\rho}{\rho}} c_i X_i^{\rho-1} = P\,V^{1-\rho} c_i X_i^{\rho-1}$$

$$V = \left[\sum_i c_i X_i^{\rho}\right]^{\frac{1}{\rho}}$$

where the shift and share parameters have been merged so that:

$$c_i = a_i\left(A\lambda_i\right)^{\rho}$$

Using the first expression, it is possible to express all X_i terms in terms of P, V, and P_i:

$$X_i = \left[\frac{c_i P}{P_i}\right]^{\frac{1}{1-\rho}} V$$

which can be substituted into the second term to yield:

$$V = \left[\sum_i c_i \left[\frac{c_i P}{P_i}\right]^{\frac{\rho}{1-\rho}} V^{\rho}\right]^{1/\rho}$$

Some manipulation yields:

$$\text{(1)} \qquad P = \left[\sum_i c_i^{\sigma} P_i^{1-\sigma}\right]^{1/(1-\sigma)} = \left[\sum_i a_i^{\sigma}\left(\frac{P_i}{A\lambda_i}\right)^{1-\sigma}\right]^{1/(1-\sigma)} = \frac{1}{A}\left[\sum_i a_i^{\sigma}\left(\frac{P_i}{\lambda_i}\right)^{1-\sigma}\right]^{1/(1-\sigma)}$$

where we have the following relationship between the substitution elasticity and the CES exponent:

$$\sigma = \frac{1}{1-\rho} \Leftrightarrow \rho = \frac{\sigma - 1}{\sigma}$$

The shadow price turns is the dual of the CES primal function. Re-inserting this latter expression into the first order condition yields the following demand function for inputs:

$$\text{(2)} \qquad X_i = \left(A\lambda_i\right)^{\sigma-1} a_i^{\sigma}\left(\frac{P}{P_i}\right)^{\sigma} V$$

Equations (1) and (2) represent the reduced forms from the generalised CES, i.e. given the vector of prices P_i and the aggregate level of production V, then the unit cost of production is P and is determined by equation (1), and the individual demands are given by equation (2). In most practical applications, A is almost always assumed to be equal to 1, and the exponent on the share parameters is merged into the primal share parameter to yield the following equations:

$$(1') \qquad P = \left[\sum_i \alpha_i \left(\frac{P_i}{\lambda_i} \right)^{1-\sigma} \right]^{1/(1-\sigma)}$$

$$(2') \qquad X_i = \alpha_i \lambda_i^{\sigma-1} \left(\frac{P}{P_i} \right)^{\sigma} V$$

where

$$\alpha_i = a_i^{\sigma} \Leftrightarrow a_i = \alpha_i^{1/\sigma}$$

Equation (1') defines the CES dual price which is an average of the input prices, where the CES dual price function is the aggregator, with the share and productivity parameters providing the appropriate weights. Equation (2') represents the optimal demand for each input. Individual demand equals a constant share of the level of output, V, adjusted by a term in the relative price of the input (compared to the aggregate cost of inputs). Hence, if an input's price increases (relative to overall costs), then demand for that factor will decrease. The percentage decrease will depend on the elasticity of substitution. At the lower limit, when the substitution elasticity is 0, input demand is a constant share of output and relative prices are irrelevant. This latter case is the so-called Leontief technology, or fixed coefficients technology. It will be explored in greater detail below.

Taking the ratio of the demand of any two inputs, say i and j, yields the following expression:

$$\frac{X_i}{X_j} = \frac{\alpha_i \lambda_i^{\sigma-1}}{\alpha_j \lambda_j^{\sigma-1}} \left(\frac{P_j}{P_i} \right)^{\sigma}$$

Taking the partial derivative of this expression with respect to the ratio P_i/P_j, and multiplying the resulting expression by $(P_i/P_j)/(X_i/X_j)$, yields the so-called elasticity of substitution, i.e. the percentage change in the ratio of two inputs, with respect to a percentage change in their relative prices:

$$\frac{\partial \left(\frac{X_i}{X_j} \right)}{\partial \left(\frac{P_i}{P_j} \right)} \frac{\frac{P_i}{P_j}}{\frac{X_i}{X_j}} = -\sigma$$

The resulting expression explains the designation of this functional form as the constant elasticity of substitution. For example, in a 2-factor CES, say labour and capital, if the wage rate were to increase by 10 percent with respect to the rate of return on capital, then the labour-capital ratio would decline by approximately σ times 10 percent.

Calibration

Calibration of the CES function is relatively straightforward. First, it is assumed that all prices and volumes have been assigned initial values. It is further assumed that the productivity shifters are given an initial value of 1, though the calibration process really only requires that they be given some initial value. With a specified substitution elasticity, equation (2') can be inverted to yield values for the share parameters:

$$(3) \qquad \alpha_i = \left(\frac{P_{i,0}}{P_0}\right)^{\sigma} \frac{X_{i,0}}{V_0}$$

In many cases, prices will be initialised at unit value, in which case, the share parameters are simply the value shares of the relevant inputs. Note that the calibration formula is valid for all legitimate values of σ. If applications require the more general form of the CES function, i.e. equation (2), there are two possible calibration procedures. The first is to simply assign the value 1 to the uniform shift parameter *A*, and then to use equation (3) to calibrate the share parameters. A second procedure is to use equation (3) to calculate initial share parameters, say $\alpha_{i,0}$. The latter can then be scaled to sum to 1 (which is necessary for example in the case of the Cobb-Douglas function, see below). The uniform shift parameter can then be calculated to be consistent with the base data and the re-scaled share parameters. (Note, for an *n*-factor input function, there are only *n* degrees of freedom for calibrating (*n*+1) parameters. The first solution fixes *A*, and then calibrates the *n* share parameters. The second solution adds an additional equation, thereby increasing the degrees of freedom by 1). The calibration equations under the second procedure become:

$$\alpha_{i,0} = \left(\frac{P_{i,0}}{P_0}\right)^{\sigma} \frac{X_{i,0}}{V_0}$$

$$\alpha_i = \frac{\alpha_{i,0}}{\sum_i \alpha_{i,0}}$$

$$A = \left[\sum_i \alpha_{i,0}\right]^{1/(\sigma-1)}$$

In most applications, neither the original primal share parameters (a_i) nor the primal exponent (ρ) are ever needed in the model implementation. They can be derived from the relevant formulas above relating the a_i parameters to the α_i parameters, and ρ to σ.

The Cobb-Douglas Production Function

The Cobb-Douglas production function has been used in numerous applications and is a special case of the CES function. It is clear from equation (1') that this formula fails for the special case when σ equals 1 (though equation (2') is still valid). The Cobb-Douglas function is the limiting case of the primal function as ρ approaches 0. The primal function then takes the following form:

$$V = A\prod_i \left(\lambda_i X_i\right)^{\alpha_i}$$

where there is a restriction on the share parameters (α_i) to insure constant-returns-to-scale technology. The restriction is that the sum of the share parameters must be identically equal to 1. Without re-deriving the first order conditions, the reduced forms for the case of the Cobb-Douglas function are:

(1")
$$P = \frac{1}{A}\prod_i \left(\frac{P_i}{\alpha_i \lambda_i}\right)^{\alpha_i}$$

(2")
$$X_i = \alpha_i \frac{P}{P_i} V$$

The demand function implies that input value shares ($P_i X_i / PV$) are constant. Also, note that the demand function is not a direct function of productivity. Equation (2") can easily be used to calibrate the share parameters since they are simply equal to the initial value shares. Either the primal or dual functions can be used to calibrate the uniform shift parameter A. Since the functional form of the CES dual price function for the Cobb-Douglas case is different from the general CES dual price function, there are three practical options for implementing a Cobb-Douglas function. Equation (2) can be specified for all legitimate values of the substitution elasticity, including the case of σ equal to 1. (In the latter case, the productivity term simply drops out). The problem occurs for the price equation. The dual price equation (1) is only valid for σ different from 1. The first solution is to implement (1") for the Cobb-Douglas case. (To keep the computer implementation as general as possible, some sort of programming flag would be necessary to differentiate the different cases). A second strategy would be to approximate the Cobb-Douglas by setting the substitution elasticity to some value close to 1, for example either 0.99 or 1.01. The numerical results from implementing this strategy would not be significantly different from the first strategy. The third strategy would be to calculate the aggregate price using the accounting identity equating aggregate cost of inputs with the unit cost of production:

$$P\,V = \sum_i P_i V_i$$

This identity is always true and can replace the dual price formula. However, experience has shown that this equation tends to generate convergence problems, i.e. the dual price formulas are significantly more robust. See Box 4.1 for a GAMS implementation of the CES function and its derivatives.

The Leontief Specification

The other special case of the CES function is when the substitution elasticity takes the value 0. In this special case, the r has the value of -∞. The primal function has the following form:

$$V = \min\left(\frac{\lambda_i X_i}{\alpha_i}\right)$$

Both formulas (1') and (2') still obtain in this case, though they take the following particular form:

(1''') $$P = \sum_i \alpha_i \left(\frac{P_i}{\lambda_i} \right)$$

(2''') $$X_i = \frac{\alpha_i}{\lambda_i} V$$

The unit cost is a simple weighted average of the input prices where the weights are given by the share parameters adjusted for changes in productivity. The demand function (2''') shows that input demand is invariant to changes in input prices. Moreover, the input-output ratio will only change with changes in relative productivity.

Box 4.1: **GAMS Implementation of the CES function**

The code below shows how the CES function can be calibrated and implemented in the GAMS framework. There are three strategies available to deal with the Cobb-Douglas functional form. The simplest strategy is strategy 2, i.e. change the substitution elasticity to something close to 1, e.g. 1.01, and use the general form of the CES function for both calibration and model implementation.

```
* Declare and initialise variables
   .
   .
* Strategy 1 – flag use of Cobb-Douglas function
* Calibration

alpha(i) = (x0(i)/v0)*(lambda0(i)**(1-sigma))*(px0(i)/pv0)**sigma ;
a$(sigma eq 1) = v0/prod(i,(lambda0(i)*x0(i))**alpha(i)) ;
a$(sigma ne 1) = (sum(i,alpha(i)))**(1/(sigma-1)) ;
alpha(i)$(sigma ne 1) = alpha(i)/sum(j,alpha(j)) ;

* Equation declaration

xeq(i)   Input demand
peq1     Definition of unit cost (for Cobb-Douglas)
peq2     Definition of unit cost (for sigma not equal to 1)

*. Definition of equations
xeq(i)..            x(i)*px(i)**sigma =e= alpha(i)*v*((a*lambda(i))**(sigma-
1))*pv**sigma ;
peq1$(sigma eq 1).. a*pv =e= prod(i,(px(i)/(alpha(i)*lambda(i)))**alpha(i)) ;
peq2$(sigma ne 1).. (a*pv)**(1-sigma) =e= sum(i,alpha(i)*(px(i)/lambda(i))**(1-
sigma)) ;
* Strategy 2 – Change substitution elasticity to 1.01 from 1
* Calibration

sigma$(sigma eq 1) = 1.01 ;
alpha(i) = (x0(i)/v0)*(lambda0(i)**(1-sigma))*(px0(i)/pv0)**sigma ;   .
* Equation declaration

xeq(i)   Input demand
peq      Definition of unit cost

*. Definition of equations
xeq(i)..  x(i)*px(i)**sigma =e= alpha(i)*v*(lambda(i)**(sigma-1))*pv**sigma ;
peq..     pv**(1-sigma) =e= sum(i,alpha(i)*(px(i)/lambda(i))**(1-sigma)) ;
* Strategy 3 – Use accounting identity to calculate unit cost
* Calibration

alpha(i) = (x0(i)/v0)*(lambda0(i)**(1-sigma))*(px0(i)/pv0)**sigma ;
a$(sigma eq 1) = v0/prod(i,(lambda0(i)*x0(i))**alpha(i)) ;
a$(sigma ne 1) = (sum(i,alpha(i)))**(1/(sigma-1)) ;
alpha(i)$(sigma ne 1) = alpha(i)/sum(j,alpha(j)) ;

* Equation declaration

xeq(i)   Input demand
peq      Definition of unit cost

*. Definition of equations
xeq(i)..   x(i)*px(i)**sigma =e= alpha(i)*v*((a*lambda(i))**(sigma-1))*pv**sigma ;
peq..      pv*v =e= sum(i,px(i)*x(i)) ;
```

Productivity

Equations (1') and (2') incorporate productivity shifters, which can be either uniform across all inputs, or input specific. This section will describe the mechanics of productivity changes using a small numerical example. Let's assume a level of production of 100, produced only with two inputs, labour and capital. Let labour represent 65 percent of value added, and capital

35 percent, and further assume that all prices are equal to 1. The share parameters are therefore equal to 0.65 and 0.35 respectively for labour and capital. Table 4.1 shows the impact of three partial equilibrium experiments. The first increases only labour productivity by 4 percent. The second increases only capital productivity by 7.4 percent. The third is a uniform increase across all inputs by 2.6 percent. In all experiments, the level of production and factor prices are held constant. Each experiment is conducted for different values of the substitution elasticity, starting from a value of 0 (i.e. the Leontief technology), to a value of 1 (the Cobb-Douglas function). The aggregate results are virtually identical in all of the experiments: the unit cost of production drops by 2.5 percent. (The experiments were designed to produce this result. Given the base share parameters, a 4 percent increase in labour productivity, is approximately equivalent to an increase in TFP of 2.6 percent (0.65*0.04), and an increase in capital productivity alone of 7.4 percent (0.026/0.35). The results from the first experiment show that the impact on labour demand decreases as the substitution elasticity increases (at constant factor prices). As the effective price of labour decreases with respect to the effective price of capital, the impact on labour demand decreases with rising substitution elasticities. Analogously, the impact on capital demand goes in the opposite direction. The conclusion is that the greater the substitution elasticity the greater labour and capital share the burden of rising labour productivity. Experiment 2 is perfectly symmetric with experiment 1, though it is the capital productivity factor which is being varied. In the case of uniform shifts in the productivity factor, there is absolutely no change in relative factor proportions. This is also the case with the Cobb-Douglas function, i.e. with a substitution elasticity of 1, factor proportions are invariant to changes in productivity (be it neutral or not).

Table 4.1: **The Impact of Increase in Factor Productivity in a 2-Factor CES Function**

	Labour Productivity increase of 4%			Capital Productivity increase of 7.4%			Uniform Productivity increase of 2.6%		
Substitution Elasticity	Price	Labour	Capital	Price	Labour	Capital	Price	Labour	Capital
Change in Level									
0.0	0.975	62.5	35.0	0.976	65.0	32.6	0.975	63.4	34.1
0.2	0.975	62.7	34.8	0.976	64.7	32.9	0.975	63.4	34.1
0.4	0.975	62.8	34.6	0.976	64.4	33.2	0.975	63.4	34.1
0.6	0.975	63.0	34.5	0.975	64.0	33.5	0.975	63.4	34.1
0.8	0.975	63.2	34.3	0.975	63.7	33.8	0.975	63.4	34.1
1.0	0.975	63.4	34.1	0.975	63.4	34.1	0.975	63.4	34.1
Change in Percentage									
0.0	-2.5	-3.8	0.0	-2.4	0.0	-6.9	-2.5	-2.5	-2.5
0.2	-2.5	-3.6	-0.5	-2.4	-0.5	-6.0	-2.5	-2.5	-2.5
0.4	-2.5	-3.3	-1.0	-2.4	-1.0	-5.2	-2.5	-2.5	-2.5
0.6	-2.5	-3.0	-1.5	-2.5	-1.5	-4.3	-2.5	-2.5	-2.5
0.8	-2.5	-2.8	-2.0	-2.5	-2.0	-3.4	-2.5	-2.5	-2.5
1.0	-2.5	-2.5	-2.5	-2.5	-2.5	-2.5	-2.5	-2.5	-2.5

Note: Output and factor prices remain constant in all experiments.

The Constant-Elasticity-of-Transformation Function

The constant-elasticity-of-transformation (CET) function is in many ways identical to the CES function, so this section will not develop it as fully. While the CES function is typically used to choose an optimal combination of demands subject to either a CES production technology or a CES utility function, the CET function is used to optimally allocate supplies across markets subject to a CET production technology. The formulation is the following:

$$\max \sum_{i=1}^{n} P_i X_i$$

subject to

$$V = \left[\sum_{i=1}^{n} g_i X_i^{\nu} \right]^{1/\nu}$$

i.e. the supplier desires to maximise revenues across all markets, subject to the transformation frontier, where X_i represents supply to market i at price P_i, and V is aggregate supply. Without re-deriving the reduced forms, they are given by:

$$P = \left[\sum_{i} \gamma_i P_i^{1+\omega} \right]^{1/(1+\omega)} \tag{4}$$

$$X_i = \gamma_i \left(\frac{P_i}{P} \right)^{\omega} V \tag{5}$$

where we have the following relation between the CET transformation elasticity and the primal and dual share parameters:

$$\omega = \frac{1}{\nu - 1} \Leftrightarrow \nu = \frac{\omega + 1}{\omega} \qquad \text{and} \qquad \omega > 0$$

$$\gamma_i = g_i^{-\omega} \Leftrightarrow g_i = \gamma_i^{-1/\omega}$$

The parameter ω is the transformation elasticity which is either estimated or designated by the user. The other parameters are calibrated using the base year values for the variables and the transformation elasticity. Typically in the model implementation there is no need for the CET primal exponent ν, nor the primal share parameters, the g_i. The γ_i are readily calibrated by inverting equation (5):

$$\gamma_i = \left(\frac{X_{i,0}}{V_0} \right) \left(\frac{P_0}{P_{i,0}} \right)^{\omega}$$

The resemblance with the CES reduced forms, equations (1') and (2') are clear. The obvious differences are in the definition of the CET dual price, where the sign on the elasticity is positive, and the exponent is therefore always positive. In the supply equation, one will also notice that the component price, P_i, is now in the numerator. This is clearly the desired expression, since a rise in the price on one of the markets being supplied (compared to the average market price), would naturally incite suppliers to increase supply to that market.

The only interesting special case for the CET is when the transformation elasticity is infinite, in which case the CET primal exponent is 1. In this case, the primal function reduces to simply the arithmetic sum of supply to all markets. The implication of an infinite transformation elasticity is the law of one price, i.e. the price on all markets is uniform. In this case, equations (4) and (5) reduce to the following:

(4’) $$X = \sum_i X_i$$

(5’) $$P_i = P$$

Nested CES

CRESH/CRETH

8. Industry Structure and Conduct

The links between trade and domestic markets have received intensive scrutiny in recent years.[108] During the last decade, ideas borrowed from industrial organization have helped trade theorists effect an extensive reappraisal of long-held ideas about the resource basis for trade orientation and specialization according to Heckscher-Ohlin concepts of comparative advantage.[109] This large and provocative body of literature has established the importance of domestic industry structure and conduct as determinants of international competitiveness.

Over roughly the same period, developing countries have been grappling with the consequences of dramatic changes in their trading opportunities and domestic economic structure. In transit from the growth years of the sixties and seventies, they passed through the downturn and international market dislocations of the early eighties and into the more recent era of relative stability and liberalized international trade. The adjustments of the eighties were especially difficult for many countries, who were forced to shift from inward-looking emphasis on domestic sources of growth to export orientation, intensified competition from more outward-oriented economies in the Asian Pacific, and the efficiency disciplines of more flexible international capital markets.

It is ironic that the new school of trade theorists has largely confined its attention to developed countries, since nowhere has the link between trade and industry structure and conduct been more apparent in recent times than in developing countries. During the last decade, developing countries have passed through stabilization and adjustment experiences which rival those of the OECD countries at any time since the last war. With only a few exceptions, the vivid and diverse lessons of these countries still await closer examination by mainstream students of international trade.[110]

In this paper, we examine a variety of alternative specifications for domestic industry structure and conduct. Using a single prototype CGE model of Indonesia as an illustrative vehicle, we examine the roles played by returns to scale and competitive and oligopolistic firm interactions, in the general equilibrium responses of the economy to trade reform.

While the simulations reported here are pedagogical and should not be interpreted as policy perscriptions for Indonesia, this economy's adjustment experience is in some ways

[108] This chapter is based on joint work with Joseph Francois.
[109] See e.g. Krugman(1986).
[110] See Roberts and Tybout (1990) and Devarajan and Rodrik (1989ab), Rodrik (1988), and de Melo (1988) for more on the perspective from development studies.

typical of many rapidly emerging developing countries. A deepening inward orientation in the sixties and early seventies was sharply reversed into export orientation in the seventies, only to crash on the global recession of 1982. After extensive structural adjustments during the remainder of that decade, Indonesia emerged with one of the more reformist trade orientations in the developing world. Despite this, however, its domestic markets were still characterized by extensive public ownership, high concentration levels in leading sectors, and firm conduct which could limit competition and the efficiency that might otherwise ensue from external reforms.

The next section gives a brief overview of conventions for specifying of firm structure, with particular emphasis on the technology of production. This is followed by the main empirical section of the paper, which sets out a menu of specifications for market structure and conduct, assessing these with a variety of prototype simulations. Section 4 is devoted to conclusions and extensions, followed by a bibliography. The sixth and final section consists of an appendix summarizing the structure of the basic prototype model.

Firm Structure

The production unit in CGE models is most commonly a neoclassical firm, representing an SIC based production activity for which accounting data on intermediate use, factor demand, taxes, margins, and the composition of supply are available. As such, it is viewed conceptually as a transformation function which has minimal structural features and behavioral properties. Structural features must respect theoretical coherence, offering reasonable flexibility, parsimony, and some substitution possibilities. Behavioral features should at least accommodate input and output price sensitivity and strategic considerations related to these. We review the structural components in this section, while behavioral properties are discussed at the industry level in the next section.

Production Structure

Intermediate Use

The fixed-coefficient technology for use of intermediate inputs, due originally to Leontief (1939), still enjoys nearly universal acceptance in multisectoral economic modeling. While a broad spectrum of alternatives has been applied and compared, this one remains the most serviceable because of its parsimony, tractability, and the lack of significant theoretical or empirical interest in the behavior or market forces governing intermediate products. Most attention in practical work goes to final goods and factors. In the present model, we follow these conventions with a standard Leontief intermediate technology of the form

$$N_i^D = \sum_{j=1}^{n} \alpha_{ji} X_i \qquad (2.1)$$

Lack of intermediate substitutability is certainly an unattractive restriction, since real firms engage in this activity continuously and often seem quite responsive to individual input price shocks. Generalizing this anecdotal evidence is another matter, however, and little definitive progress beyond the Leontief paradigm has been made to date. Work in this direction has taken two general forms. Neoclassical modelers have attempted to use continuous aggregation functions for all firm inputs, intermediates as well as factors. Applied econometricians and input-output analysts have sometimes specified price-sensitive demand equations for individual inputs. Neither approach has yet yielded results which appear to justify the effort, however, in terms of improving our understanding or prescience about what is going on at the firm or sectoral level.

When one or more inputs appear to be of special importance (e.g. energy, substitutable raw materials, etc.), these are sometimes partitioned with factors into a nested aggregation function while the rest of the intermediates remain in the Leontief scheme. This approach gives greater play to leading own-price input effects without undue complexity.[111]

Value Added

Value added at the firm level consists of factor services, operating margins, and any policy instruments incident upon these. Factor services are incorporated in the essential production theory behind these models and will be discussed in the next section. The other components generally enter as *ad valorem* distortions on the price of value added. An example of the latter is given in the following equation, decomposing producer prices into their intermediate and value added constituents

$$\begin{aligned} PX_i &= PN_i + PVA_i \\ &= \sum_{j=1}^{n} \alpha_{ji} PQ_i + (1+t_i^{VA})(1+\mu_i)(w_i\lambda_i + r_i\kappa_i) \end{aligned} \qquad (2.2)$$

where t^{VA} and μ denote value added taxes and distribution margins, respectively.

Production Functions

While intermediate technology has remained relatively simple, theoretical and applied microeconomics have lavished attention on modeling the process of creating value added. The result of this effort it a vast literature on alternative specifications of neoclassical production functions.[112] While all of these can in principle be implemented empirically, only a few of the many alternatives are represented in most CGE work. The most attractive candidates share two virtues, parsimony and flexibility, these are desirable in light of data constraints and the need for broad-spectrum application across diverse sectors.

The detailed properties of these alternative functional forms have been intensively investigated elsewhere and need not be discussed here. Suffice to say that the majority of CGE models have thus far relied on quite elementary specifications, generally containing only one or two more parameters than the number of factors. The clear favorite has been the CES family of aggregation functions, which allow for continuous factor substitution at a constant rate. This group is about the simplest which is analytically well-behaved and accords with basic economic

[111] An example energy in this way can be found in Beghin et al (1994).
[112] See e.g. Chambers (1992) or Blackorby

intuition about adjustments to changes in relative factor prices. They require only one exogenous parameter per aggregation level, the rest being calibrated from a single observation on output and factor use. In the present exposition, we rely on a canonical two-factor CES specification of the form

$$VA = \alpha\left[\beta L^{(\sigma-1)/\sigma} + (1-\beta)K^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)} \qquad (2.3)$$

where the sectoral subscript is suppressed, the intercept (α) and share (β) parameters are calibrated from output and factor use data, and the elasticity of labor-capital substitution (σ) is specified exogenously.

A second value added specification which is beginning to attract attention is the so-called Constant Different in Elasticity (CDE) functional form.[113] This specification is nearly as parsimonious as CES, but it allows for the important possibility of bilateral complementarity between factors of production.

For those who want to examine a richer universe of factor price interactions, a large family of so-called flexible functional forms, including translog production functions, have been developed and estimated for use in production and utility analysis. These allow for more extensive second-order interactions, with concomitant requirements for initial data and ex post interpretation.

Multi-output Specifications

Some authors have argued that the standard multi-input, single output specification of the productive unit is inappropriate. This criticism has been leveled most persuasively at CGE models of agriculture, where the production unit is often highly diversified in outputs as well as inputs.[114] While this is an important issue, evaluating its effect on competitive behavior would take the present exposition outside reasonable limits.

Firm-level Costs

The cost structure of the firm of course follows from the choice of technique and observed data to which it is calibrated. Since it is so important to both firm conduct and the ensuing results in CGE models, however, the specification of firm-level cost deserves separate discussion. One aspect which has received intensive scrutiny in the CGE literature in recent years is returns to scale. Beginning with a study by Harris (1984), a large literature on empirical modeling arose to evaluate trade liberalization under various specifications of returns to scale.[115] This new research initiative was abetted by an intense interest among trade theorists in applying concepts from industrial organization to trade theory.[116] Both these strains of work indicated that conclusions from empirical and theoretical work grounded in classical trade theory could be contradicted, both in magnitude and direction, when scale economies or diseconomies played a significant role in the adjustment process.

To illustrate the logic behind this, consider a general example of an economy with one representative consumer and m sectors. Domestic demand is characterized by g(p) for m domestic prices p_i Each sector is represented by a number n_i of identical firms with production

[113] The definitive statement of this specificaiton is Hanoch (1967). See Roland-Holst and van der Mensbrogghe (1995) for a recent application.
[114] See DeJanvry and Sadoulet (1990) for discussion.
[115] See de Melo and Tarr (1992) for methodological discussion, Reinert, Roland-Holst, and Shiells (1994) and Francois (1995) for more recent applications.
[116] See e.g. Krugman (1985) for examples from this literature.

and cost functions x_i and c_i, respectively. The aggregate social welfare function for this economy consists of the three components

$$W(p) = \int_0^{x(p)} \left\{ D^{-1}(x) - p \right\} dx + n' \left[px(p) - c(x) \right] + t'M$$

representing, respectively, classical consumer and producer surplus, in addition to any net tariff revenues captured from foreign sources.

The relevant element of this expression for the present discussion is producer surplus, which can be shown by total differentiation

$$n' \left[p - c_x \right] \qquad (2.4)$$

to depend upon the direction and degree to which market prices (p) diverge from marginal costs (c_x). Under constant returns, scale economies are nonexistent and therefore only demand distortions and import price changes affect welfare. When increasing returns prevail, rising per firm output will confer efficiency gains upon the entire economy, while falling per firm output will reduce economywide efficiency.[117] The latter would only aggravate the distortionary costs of tariff protection (the first component of W(p) above), while the former might reverse this classical welfare loss.[118] The case of decreasing returns is exactly analogous, with efficiency costs rising instead of falling with firm output.

Constant Returns to Scale

Constant returns to scale (CRTS) is a attractive property in terms of flexibility and parsimony, but its empirical veracity is open to question. In the real world, factors are so heterogeneous in quality and mobility that even a constant average product technology usually operates under increasing or decreasing marginal costs. There may be uncertainty about their precise magnitude, but scale economies are a fact of life and appear to be pervasive even in mature industries with diverse firm populations.

Despite these facts, CRTS is a property of the most popular empirical production functions and thus is incorporated in most of CGE models. While this facilitates practical data gathering, calibration, and interpretation of results, a growing body of work with non-CRTS specifications suggests that a re-appraisal of CRTS-based results is probably justified.

Increasing Returns to Scale

The most common extension of CRTS incorporates unrealized economies of scale into production. This increasing returns to scale (IRTS) specification usually takes the form of a monotonically decreasingly average cost function, calibrated to some simple notion of a fixed cost intercept. In other words, one assumes that marginal costs are governed by the preferred CRTS production function (usually CES), but that some subset of inputs are committed *a priori* to production and their costs must be covered regardless of the output level. Thus average costs are given by a reciprocal function of the form

[117] Note that per firm output also depends upon market entry and exit. The case of endogenous n will be discussed below.
[118] The effect of the third component of W(p) depends upon terms-of-trade adjustments and is of no direct relevance to the present discussion.

$$AC = \frac{FC}{X} + MC \qquad (2.5)$$

where marginal cost comes directly from a CRTS technology. It would also be a simple matter for scale-dependent costs to enter multiplicatively, but this alternative has been little explored. To calibrate the above equation, one needs only an engineering estimate of the distance between average and marginal cost, along with some idea about how to impute fixed cost to initial factor and/or intermediate use. In practice, it has become customary to appeal to the concept of cost disadvantage ratio. This measure of unrealized scale economies is generally defined as

$$CDR = \frac{AC - MC}{AC}$$

Decreasing Returns to Scale

In a variety of practical market settings, capacity constraints prevent firms of expanding at decreasing or even constant marginal cost. Constraints like this can arise as a result of limitations on the quantity of quality of primary resources used in production. Generally speaking, these are outside the scope of the limited set of productive factors considered in simulation models, and thus they undermine the CRTS assumption when other factor use rises proportionately. To capture these effects in a parsimonious way, it suffices to specify a component of the firm cost function which is nonlinear in output such as the following

$$TC = \alpha\left[\log(X) - \log(X_o)\right] + wL + rK \qquad (2.6)$$

where X_o denotes base output. We choose the above specification for illustrative purposes because, locally, it exhibits constant elasticity of marginal cost with respect to output $\varepsilon = \alpha/TC_o$.

Factor Demands

Labor

In the vast majority of general equilibrium models, labor demand is determined by neoclassical first-order conditions, subject to the workings of perfectly competitive labor markets. While the choice of closure rule for labor markets can influence simulation results, the universe of alternatives between fixed wages and fixed employment is now better understood. In a dual representation used here for the prototype model, labor demand takes the form

$$L^D = \left[\frac{\beta P^{VC}}{w}\right]^{\frac{\sigma-1}{\sigma}} \left[\frac{\alpha}{X}\right]^{\sigma} \qquad (2.7)$$

where P^{VC} denotes the price of variable cost or a composite value added price index.

There is ample scope for extending both the supply and demand sides of labor market specifications. The former are discussed at length elsewhere in this book, and while the latter is of interest, we do not expand upon it here. It is worth noting, however, that some degree of labor market power in the hands of either firms or workers could change their cost/productivity situation in significant ways. In a world of skill-intensive employment, labor market competition is also an increasingly important component of inter-firm competition.

Capital

The prototype model treats labor and capital demand in completely analogous way. Indeed, we discriminate between them in the present exposition only because it is relevant to the composition of income. Thus the reduced-form capital demand for a representative firm in sector i is given by

$$K^D = \left[\frac{(1-\beta)P^{VC}}{r}\right]^{\frac{\sigma-1}{\sigma}} \left[\frac{\alpha}{X}\right]^{\sigma} \quad (2.8)$$

Several aspects of capital are relevant to industry structure and conduct and should be mentioned as areas for future research. Varietal and mobility issues tend to be important with this factor. Substitution between capital types and with respect to other factors is a complex question, and depreciation and gestation lags are important considerations in dynamic specifications. Vintages of capital and innovation have been incorporated into a few CGE models.[119]

Capital can also have empirically important limitations in its mobility, both spatially and temporally. Capital markets are at once more and less flexible than labor markets, depending upon whether one considers only financial capital or physical factors like land and machines. Most of these considerations are potentially important determinants of firm and industry interactions, but they are outside the scope of this exposition. Generally speaking, much work still needs to be done on the empirical (partial and general equilibrium) treatment of capital.

Market Structure and Conduct

The Perfectly Competitive Representative Firm

The standard paradigm for individual industry structure and conduct in CGE models, and the reference point for the discussion that follows, is a single neoclassical representative firm facing perfectly competitive factor markets and behaving competitively in its output market. Such a firm can be characterized by the following set of equations, drawn from the prototype model, where the sectoral subscript has been suppressed

$$P = AC \quad (3.1)$$

$$V = a_j S \quad (3.2)$$

$$L^D = \left[\frac{\beta P^{VC}}{w}\right]^{\frac{\sigma-1}{\sigma}} \left[\frac{\alpha}{X}\right]^{\sigma} \quad (3.3)$$

$$K^V = \left[\frac{(1-\beta)P^{VC}}{r}\right]^{\frac{\sigma-1}{\sigma}} \left[\frac{\alpha}{X}\right]^{\sigma} \quad (3.4)$$

$$K^D = K^F + K^V \quad (3.5)$$

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[119] See e.g. van der Mensbrugghe (1994).

$$S_i = \overline{A}_{S_i}\left[\sum_k \delta_i^k \left(S_i^k\right)^{(\tau_i+1)/\tau_i}\right]^{\tau_i/(\tau_i+1)} \qquad (3.6)$$

$$S_i^f / S_i^d = g_S\left(P_{Si}^f / P_{Si}^d ; \tau_i\right) \qquad (3.7)$$

$$D_i^d = S_i^d \qquad (3.8)$$

Note that the first equation, which specifies average cost pricing, is equivalent to the supply function for the representative firm. Given market price determination, subject to (3.8), this expression determines output while intermediate and factor demands follow from the production function.

Monopoly

Albeit of limited empirical interest, monopoly can be instructive as a reference case. The monopoly specification is a straightforward extension of the perfect competition. Of course there is still a representative firm in the sector under consideration, the difference lies its pricing behavior. In particular, the monopolist chooses the price level instead of output, the latter being determined by demand. In the structural equations of the model, we thus replace equation (3.1) above with a pricing rule of the form

$$\frac{P - MC}{P} = \frac{1}{\varepsilon} \qquad (3.9)$$

where the market elasticity of demand is given by

$$\varepsilon = -\frac{\partial Q}{\partial P}\frac{P}{Q} \qquad (3.10)$$

Oligopolies with Homogeneous Products

Between the perfectly competitive and monopoly paradigms lies a continuum of possible firm distributions. When the number of firms is small enough for them to influence one another, complex strategies can arise. The scope of this paper is too limited to cover the full spectrum of oligopoly theory, but we give some representative specifications which indicate the decisive role that firm interactions can play in determining prices, quantities, efficiency and welfare.

The first vehicle used to explore oligopoly interactions is the so-called Cournot conjectural variations model. In particular, we assume that each firm produces a homogeneous product, faces downward sloping demand and adjusts output to maximize profits, with a common market price as the equilibrating variable. We further assume, following Frisch (1933), that firms anticipate or conjecture the output responses of their competitors. Assume further that the industry is populated by n identical firms producing collective output $Q = nQ_i$. When the i^{th} firm changes its output, its conjecture with respect to the change in industry output is represented by

$$\Omega_i = \frac{dQ}{dQ_i} \qquad (3.11)$$

which equals a common value Ω under the assumption of identical firms. Given a representative profit function

$$\Pi_i = PQ_i - TC_i \qquad (3.12)$$

which yields the first-order condition

$$\frac{d\Pi_i}{dQ_i} = P + Q_i \frac{dP}{dQ}\frac{dQ}{dQ_i} - \frac{dTC_i}{dQ_i} = P^D - \frac{Q}{n\varepsilon}\frac{P}{Q}\Omega - MC = 0 \qquad (3.13)$$

and finally the oligopoly pricing rule

$$\frac{P - MC}{P} = \frac{\Omega}{n\varepsilon} \qquad (3.14)$$

The above expression encompasses a variety of relevant cases. The classic Cournot specification corresponds to Ω=1, where each firm believes that the others will not change their output, and industry output changes coincide with their own. Price-cost margins vary inversely with the number of firms and the market elasticity of demand, as logic would dictate. In the extreme cases, a value of Ω=0 corresponds to perfectly competitive, average cost pricing, while Ω=n is equivalent to a perfectly collusive or monopolistic market.

Market Entry and Exit

The previous section defined Cournot interactions with respect to a fixed number of incumbent firms. If one allows for the possibility of market entry and exit, then N becomes endogenous and the competitive climate in the industry under consideration varies accordingly. Note first of all that price-cost margins in the last expression vary accordingly and intuitively, i.e. margins shrink with increasing number of firms.[120] Beyond this, the major effect of entry and exit is on firm level scale economies. Entry and exit can alter the average scale of firm operations, other things equal, and in the increasing and decreasing returns cases this can alter aggregate efficiency effects.

The ultimate scope of entry, exit, or realization of scale economies is an empirical question which can be illustrated here, but not explored in detail, with our prototype model. Entry and exit is equivalent to a model closure problem, generally taking the form of limiting rules for incumbent profits, prices, or some other indicator of the return on existing operations. We consider two illustrative cases. The first is equivalent to simple prohibition of entry or exit, arising from a contestable market assumption of fixed profit rates for incumbents. In this case, the scale of individual (representative) firm operations varies proportionately with industry output, and changes in scale economies are easy to predict. Secondly, one might allow firm numbers to be endogenous and specify a secondary rule on incumbent pricing, for example so-called endogenous Cournot conjectures

$$\Omega = \frac{\Omega_o n_o}{n} \qquad (3.15)$$

which implies that firms see their markets as becoming more competitive with increasing numbers of entrants.

Dynamic Interactions

The conjectural variation approach to Cournot competition has been criticized by a number of authors as an unrealistic approach to dynamic market interactions.[121] In the last decade, significant advances have been made in the theory of repeated games, and these appear to hold more promise for simulating market dynamics. The repeated game approach is appealing not only because it explicitly considers the sequential and historical aspects of competition, but because it opens up a richer universe of strategic opportunities and solution concepts.[122]

[120] That is, margins narrow with increasing numbers of incumbent firms.
[121] See e.g. Shapiro (1989).
[122] See e.g. Fudenberg and Tirole (1986) or textbook surveys such as Osborne and Rubinstein (1990).

For present expository purposes, we choose a standard case of dynamic games with complete information. Consider the same n firms and other attributes above, this time imagining that firm interactions occur or anticipate an infinite sequence of discrete, simultaneous quantity strategies, each maximizing the present value of future profits. Assume that all firms have the same discount rate δ and therefore value a stream of constant future profits π as

$$\pi + \delta\pi + \delta^2\pi + \delta^3\pi + \ldots = \frac{\pi}{1-\delta} \qquad (3.16)$$

It is well known that the simple Cournot strategy is a Nash equilibrium for such a repeated game, and indeed for any subsequence of repeated plays (i.e. it is subgame-perfect).[123] The Cournot strategy does not maximize profits for the industry or the individual firms, however, which could all make more money if they colluded and acted like a monopolist. The question then arises: Can a competitive strategy be sustained which is individually superior to Cournot? The answer in general is yes, but depends upon the number of incumbents and the discount rate.

Here we consider only the case of no entry or exit, postulating the following strategic rules for an infinitely repeated game:

> Produce the sustainable individual optimum quantity, q*, in the first period. In the t^{th} period, produce q* if all other firms have done so in each of the t-1 preceding periods. Otherwise, revert to the Cournot quantity q_C.

In any given period, a representative oligopolist can make π* by producing q* and will only deviate from this if its discounted present value is less than that of deviating (i.e. choosing q_d and earning π_d) once and playing Cournot (q_C and π_C) thereafter. This means at the margin that

$$\frac{\pi^*}{1-\delta} = \pi_d + \frac{\delta}{1-\delta}\pi_C \quad or \quad \pi^* = (1-\delta)\pi_d + \delta\pi_C \quad (3.17)$$

The deviation quantity is that which would give the one-time profit maximum against the other firms play of q* each, i.e. q_d solves

$$Max_{q_d}\ p(q_d + (n-1)q^*)q_d - TC(q_d) \qquad (3.18)$$

which takes the form

$$q_d = \frac{(n-1)\rho\varepsilon}{(1-\rho\varepsilon)}q^* = \alpha q^* \qquad (3.19)$$

where ρ denotes the price-cost margin in expression (3.14) above. This quantity equation and the dynamic profit equilibrium condition (3.17) together yield solutions for q_d and q*. The latter is the quantity with the maximal dynamically sustainable profit level for all incumbents. It varies with n and inversely with δ, approaching the collusive or monopoly quantity q_M as δ approaches a trigger level (dependent upon n) and approaching q_C as δ approaches unity. Rising n lowers the δ trigger level for collusion but also lowers the Cournot output and profit levels.

Next, we compare the Cournot quantity setting framework with its Bertrand, price setting counterpart. It will be recalled that, in a world of homogeneous goods, the static Cournot Nash equilibrium is competitive, marginal cost pricing. This is a rather sterile outcome for oligopoly interactions, but a repeated game framework yields a richer set of Bertrand solution strategies. Consider the following example: What is the discount rate which would sustain collusive monopoly pricing as a subgame-perfect Nash equilibrium?

[123] Freidman (1971) and Fudenberg and Tirole (1986).

Oligopoly with Heterogeneous Goods

The industrial organization literature has long since elaborated on the overly simplistic idea of competition among domestic producers of perfect substitutes, but this is a less developed aspect of trade theory and simulation work.[124] While it was essential almost from the beginning to differentiate goods by country of origin, domestic product differentiation is a more recent innovation in simulation modeling.[125]

In this section, we use a relatively parsimonious specification of domestic product differentiation to evaluate the dynamic Cournot and Bertrand oligopoly specifications discussed above. We evaluate them both in the context of complete and incomplete information to give an indication about how uncertainty can be incorporated in these models and how it might affect the results.[126]

Conclusions and Extensions

[124] Krugman (1979, 1980) and others have extended the theoretical literature, while recent examples in more applied work are Francois et al (1995) and Brown (1992).

[125] The former consists mainly of CES import and CET export differentiation by country of origin and destination (respectively) to avoid over-specialization in trade adjustments.

[126] See Rotemberg and Saloner (1986) for more on this.

9. A Canonical GE Model in Excel

In this chapter we describe a canonical, two-sector single country CGE model and its implementation in Excel. We implement this example by calibrating the model to a SAM from China derived from Version 5.0 of the GTAP data set. The first section describes the specification for the single country model. It contains many of the structural features of the GTAP data set, though with a simplified trade structure.

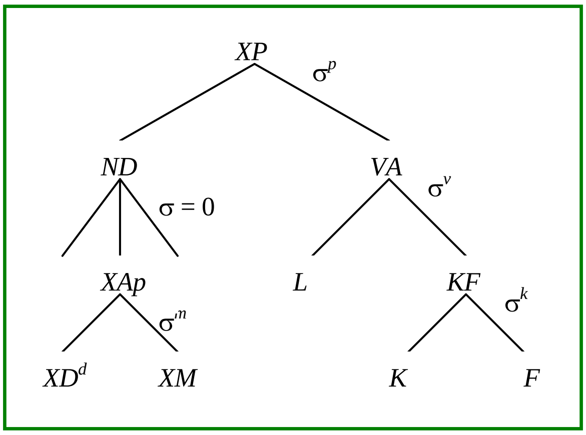
The second section describes the implementation this model in an Excel worksheet. The Excel worksheet contains data for the 57-sector version of the GTAP SAM for Vietnam. An aggregation feature is contained in the worksheet which enables the user to aggregate to any 2-sector SAM.

Model Specification[127]

Production

Production, *XP*, is modeled as a series of nested CES functions which determines the substitution and complementarity relations across the different inputs into production (figure 1).

Figure 1: **CES Production Nest**



XP: Output
ND: Aggregate intermediate demand
VA: Value added bundle
L: Demand for labor
KF: Capital/sector specific capital bundle
K: Demand for capital
F: Demand for sector specific factor
XAp: Input/output matrix (at the Armington level)
XD^d: Domestic demand for domestic goods
XM: Demand for imports
σ^p: Top level substitution elasticity (*ND* and *VA*)
σ^v: Substitution elasticity between *L* and *KF*
σ^k: Substitution elasticity between *K* and *F*
σ^m: Armington elasticity

[127] The model specification is written in its most general form. The indices *i* and *j* refer to sectors. Summation signs only refer to the sectoral index and it is implicitly assumed that the summation is from 1 to *N*, where *N* is the number of sectors. A subscript of 0 refers to an initial value.

The top nest determines demand for an aggregate bundle of intermediate goods, *ND*, and the value added bundle, *VA*. The relevant prices of these two bundles are *PND* and *PVA*, respectively. At this level the CES cost function will determine the final unit cost of production, *PX*. Equations (1) and (2) reflect the reduced form CES demand functions, and equation (3) determines the unit cost of production.

$$(1) \qquad ND_i = \alpha_i^{nd}\left(\frac{PX_i}{PND_i}\right)^{\sigma_i^p} XP_i$$

$$(2) \qquad VA_i = \alpha_i^{va}\left(\frac{PX_i}{PVA_i}\right)^{\sigma_i^p} XP_i$$

$$(3) \qquad PX_i = \left[\alpha_i^{nd} PND_i^{1-\sigma_i^p} + \alpha_i^{va} PVA_i^{1-\sigma_i^p}\right]^{1/\left(1-\sigma_i^p\right)}$$

where the substitution elasticity (the *ND*-*VA* substitution) is given by σ^p.
Assuming perfectly competitive markets, the output price, *PP*, is equal to the unit cost of production multiplied by an *ad valorem* production tax.

$$(4) \qquad PP_i = \left(1+\tau_i^p\right)PVA_i$$

The value added bundle is composed of three factors: labor (*L*), capital (*K*), and a sector-specific factor (*F*). It is also decomposed using nested CES functions. At the top level, labor is combined with a capital-fixed factor bundle (*KF*). And at the next level, the *KF* bundle is decomposed into capital on the one hand and the fixed factor on the other hand. Equations (5) and (6) determine demand for labor and the *KF* bundle, respectively, where the relevant prices are *W*, the wage rate, and *PKF*, the price of the *KF* bundle. The substitution elasticity is given by σ^v.
Equation (7) determines the price of the value added bundle using the CES cost function. Note that the labor demand function is expressed in efficiency units, where the parameter λ^l incorporates (potentially sector-specific) changes to labor productivity. Note that the wage rate is not sector specific. The model explicitly assumes that labor is fully mobile across sectors and hence there is a uniform economy-wide wage rate.

$$(5) \qquad L_i^d = \alpha_i^l\left(\lambda_i^l\right)^{\sigma_i^v-1}\left(\frac{PVA_i}{W}\right)^{\sigma_i^v} VA_i$$

$$(6) \qquad KF_i = \alpha_i^{kf}\left(\frac{PX_i}{PKF_i}\right)^{\sigma_i^v} VA_i$$

$$(7) \qquad PVA_i = \left[\alpha_i^l\left(\frac{W}{\lambda_i^l}\right)^{1-\sigma_i^v} + \alpha_i^{kf} PKF_i^{1-\sigma_i^v}\right]^{1/\left(1-\sigma_i^v\right)}$$

Equations (8) and (9) express the decomposition of the *KF* bundle, where the parameters λ^k and λ^f incorporate productivity changes for capital and the sector specific factor respectively, and the substitution elasticity is given by σ^k. Equation (10) determines the price of the *KF* bundle.

(8) $$K_i^d = \alpha_i^k \left(\lambda_i^k\right)^{\sigma_i^k - 1} \left(\frac{PKF_i}{R_i}\right)^{\sigma_i^k} KF_i$$

(9) $$F_i^d = \alpha_i^f \left(\lambda_i^f\right)^{\sigma_i^k - 1} \left(\frac{PKF_i}{PF_i}\right)^{\sigma_i^k} KF_i$$

(10) $$PKF_i = \left[\alpha_i^k \left(\frac{R_i}{\lambda_i^k}\right)^{1-\sigma_i^k} + \alpha_i^f \left(\frac{PF_i}{\lambda_i^f}\right)^{1-\sigma_i^k}\right]^{1/\left(1-\sigma_i^k\right)}$$

The left-hand most branch of the nest, aggregate intermediate demand (*ND*), is decomposed into the input-output matrix of the production side. A simple Leontief structure is assumed, therefore there is no substitution across intermediate inputs. Equation (11) determines intermediate demand for goods and services, *XAp*.[128] Finally, equation (12) determines the price of aggregate intermediate demand. Given the assumption of the Leontief technology, it is equal to the weighted sum of the tax inclusive Armington prices, where the weights are given by the Leontief share coefficients.

(11) $$XAp_{ij} = a_{ij} ND_j$$

(12) $$PND_j = \sum_i a_{ij} \left(1 + \tau_{ij}^{itp}\right) PA_i$$

Household Income and Final Demand

Consumption

Household income, *YH*, is derived from factor income augmented by transfers from the government.[129] Disposable income, *YD*, is equal to after-tax household income adjusted by depreciation.

(13) $$YH = \sum_i \left[WL_i^d + R_i K_i^d + PF_i F_i^d\right] + P.TR_h^g$$

(14) $$YD = \left(1 - \kappa\right) YH - DeprY$$

Consumer demand is modeled using the extended linear expenditure system (ELES) which is similar to the (LES), but incorporates household saving into the consumer's objective function. Equation (15) specifies consumer demand for the Armington good, *XAc*. It is the sum of two components. The first component, θ, is the so-called *subsistence minima*. The second is a share, μ, of *supernumerary income*, Y^*, which is residual income after aggregate expenditures on the subsistence minima. Supernumerary income is defined in equation (16). Equation (17) defines household saving by residual. Equation (18) defines the depreciation allowance.

(15) $$XAc_i = \theta_i + \frac{\mu_i Y^*}{\left(1 + \tau_i^{itc}\right) PA_i}$$

[128] All domestic demand components are expressed as Armington goods. The suffix *p* is used to refer to production demand, *c* for private consumption, *g* for public consumption, and *i* for investment demand.
[129] Exogenous transfers are multiplied by a price index to insure model price homogeneity. The variable *P* is the GDP price deflator (at factor cost).

(16) $$Y^{*} = YD - \sum_{j}\left(1+\tau_{j}^{itc}\right)PA_{j}\theta_{j}$$

(17) $$S^{h} = YD - \sum_{i}\left(1+\tau_{i}^{itc}\right)PA_{i}XAc_{i}$$

(18) $$DeprY = P.DeprY_{0}$$

Government

The volume of aggregate government expenditures, *XG*, is fixed. The government is assumed to have a CES expenditure function (potentially with a zero elasticity). Equation (19) determines the volume of aggregate government expenditures. Equation (20) specifies sectoral government demand, *XAg*. And equation (21) determines the government expenditure price, *PG*.

(19) $$XG = XG_{0}$$

(20) $$XAg_{i} = \alpha_{i}^{g}\left(\frac{PG}{\left(1+\tau_{i}^{itg}\right)PA_{i}}\right)^{\sigma^{g}} XG$$

(21) $$PG = \left[\sum_{i}\alpha_{i}^{g}\left[\left(1+\tau_{i}^{itg}\right)PA_{i}\right]^{1-\sigma^{g}}\right]^{1/\left(1-\sigma^{g}\right)}$$

Investment

Investment is savings determined. The value of domestic investment is identically equal to the value of domestic savings augmented (or diminished) by the level of foreign savings. The volume of aggregate investment is given by *XI*, and the investment price deflator is given by *PI*. Equation (22) represents the investment-savings closure rule, with public savings given by S^g, foreign savings given by S^f, and the *ER* representing the exchange rate which will be discussed below. Similar to government expenditures, a CES expenditure function is assumed to allocate aggregate investment into sectoral demand, *XAi*.

(22) $$PI.XI = S^{h} + S^{g} + ER.S^{f} + DeprY$$

(23) $$XAi_{i} = \alpha_{i}^{i}\left(\frac{PI}{\left(1+\tau_{i}^{iti}\right)PA_{i}}\right)^{\sigma^{i}} XI$$

(24) $$PI = \left[\sum_{i}\alpha_{i}^{i}\left[\left(1+\tau_{i}^{iti}\right)PA_{i}\right]^{1-\sigma^{i}}\right]^{1/\left(1-\sigma^{i}\right)}$$

Trade Volumes

Through now, domestic demand has been determined at the so-called Armington level. This model assumes that there is a single Armington agent who allocates aggregate demand into two components: demand for goods produced domestically and imports.[130] Given the uniformity in preference, Armington demand is aggregated across all domestic agents into a single variable, *XA*, which is allocated to domestic goods, XD^d, and to imports, *XM*. Equation (25) determines

[130] The GTAP data set allows implementation of agent-specific Armington preferences. This specification, though richer, would significantly increase the dimensions of the model.

aggregate Armington demand. Equation (26) is the reduced form demand for domestic goods using a CES preference function with a substitution elasticity of σ^m. Equation (26) determines the demand for imports. Finally, equation (27) expresses the aggregate Armington price, *PA*, which is the CES aggregation of the domestic price, *PD*, and the import price, *PM* (which is tariff inclusive).

$$(25) \qquad XA_i = \sum_j XAp_{ij} + XAc_i + XAg_i + XAi_i$$

$$(26) \qquad XD_i^d = \alpha_i^d \left(\frac{PA_i}{PD_i} \right)^{\sigma_i^m} XA_i$$

$$(27) \qquad XM_i = \alpha_i^m \left(\frac{PA_i}{PM_i} \right)^{\sigma_i^m} XA_i$$

$$(28) \qquad PA_i = \left[\alpha_i^d PD_i^{1-\sigma_i^m} + \alpha_i^m PM_i^{1-\sigma_i^m} \right]^{1/\left(1-\sigma_i^m\right)}$$

It is typical to treat the export supply decision in a symmetric fashion using a CET transformation function. Thus a producer has the capacity to supply domestic and export markets, but the supply decision is constrained by a transformation frontier, where the transformation elasticity determines the degree to which suppliers can switch from one market to the other as a function of relative prices. At one extreme, the transformation elasticity is zero and the markets will be supplied in constant proportions of output. At the other extreme, the transformation elasticity is infinite, and suppliers can seamlessly switch from one market to the other. In the case of the latter, goods to each market are uniform and the law of one price holds. The equations below are formulated for all possible cases.

With *XP* representing aggregate output, the component supplied to the domestic market is XD^s, and the component allocated to foreign markets is *ES*. Equations (29) and (30) specify the allocation decision. When the transformation elasticity is finite, equations (29) and (30) reflect the reduced form CET supply functions, where the transformation elasticity is given by σ^x. If the transformation elasticity is infinite, the supply functions are replaced with the law-of-one-price conditions. Equation (31) in fact represents an equilibrium condition in both cases. In the first case, with a finite transformation elasticity, aggregate supply equals the aggregation of supply across both markets, using the CET aggregation function. Since it is equivalent to the CET revenue function (the CET dual), equation (31) uses the dual formulation (which tends to have better numerical properties).[131] With an infinite elasticity, aggregate supply is identically equal to the sum of supply to the individual markets.

[131] The CET primal expression is given by the following formula:

$$XP_i = \left[g_i^d \left(XD_i^s \right)^{\nu_i} + g_i^e \left(ES_i \right)^{\nu_i} \right]^{1/\nu_i}$$

where the following relations hold:

$$\nu_i = \left(\sigma_i^x + 1 \right) / \sigma_i^x \qquad \gamma_i^d = \left(g_i^d \right)^{-\sigma_i^x} \qquad \gamma_i^e = \left(g_i^e \right)^{-\sigma_i^x}$$

$$(29)\quad \begin{cases} XD_i^s = \gamma_i^d \left(\dfrac{PD_i}{PP_i} \right)^{\sigma_i^x} XP_i & \text{if} \quad \sigma_i^x \neq \infty \\ PD_i = PP_i & \text{if} \quad \sigma_i^x = \infty \end{cases}$$

$$(30)\quad \begin{cases} ES_i = \gamma_i^e \left(\dfrac{PE_i}{PP_i} \right)^{\sigma_i^x} XP_i & \text{if} \quad \sigma_i^x \neq \infty \\ PE_i = PP_i & \text{if} \quad \sigma_i^x = \infty \end{cases}$$

$$(31)\quad \begin{cases} PP_i = \left[\gamma_i^d PD_i^{1+\sigma_i^x} + \gamma_i^e PE_i^{1+\sigma_i^x} \right]^{1/\left(1+\sigma_i^x\right)} & \text{if} \quad \sigma_i^x \neq \infty \\ XP_i = XD_i^s + ES_i & \text{if} \quad \sigma_i^x = \infty \end{cases}$$

Export demand is allowed to respond to price signals, i.e. the small country assumption does not necessarily hold for export markets. Equation (32) determines export demand, *ED*, using a constant elasticity demand function (with a demand elasticity of ε), where WPE^* represents a world price index which is exogenous, and *WPE* is the world export price of domestic exports, i.e. the FOB price (in international currency units). If the small country assumption holds, the world export price is constant, and the world is assumed to be able to absorb any quantity of exports at the given price.

$$(32)\quad \begin{cases} ED_i = \alpha_i^e \left(\dfrac{WPE_i^*}{WPE_i} \right)^{\varepsilon_i} & \text{if} \quad \varepsilon_i \neq \infty \\ WPE_i = WPE_i^* & \text{if} \quad \varepsilon_i = \infty \end{cases}$$

Trade Prices

World import prices, *WPM*, are given and are converted to domestic import prices, *PM*, using the exchange rate, *ER*, and applying the tariff rates, τ^m. World export prices will be determined by an equilibrium equation (see below) in the case of a finite export demand elasticity, or are given otherwise. They are converted to domestic export prices using the exchange rate and adjusted for export taxes/subsidies, τ^e (which are applied to the producer price, not the world price).

$$(33)\qquad PM_i = ER\left(1+\tau_i^m\right)WPM_i$$

$$(34)\qquad PE_i = ER.WPE_i / \left(1+\tau_i^e\right)$$

Goods equilibrium

In this model, there are two goods market[132], the domestic market for domestic production, and the export market. The domestic price of domestic goods, *PD*, is determined by the equilibrium expressed in equation (35).[133] The world price of domestic exports is determined by the equilibrium expressed in equation (36).[134]

(35) $$XD_i^d = XD_i^s$$

(36) $$ED_i = ES_i$$

Factor market equilibrium

There are three factor markets which will be dealt with separately. The labor market is assumed to clear at the national level, with labor perfectly mobile across sectors. There is a uniform wage rate which equilibrates supply and demand. Supply is allowed to be a function of the real wage. The labor supply function is given in equation (37), with a supply elasticity of ω^l. Equation (38) determines the equilibrating wage rate.

(37) $$L^s = \chi^l \left(\frac{W}{P}\right)^{\omega^l}$$

(38) $$L^s = \sum_i L_i^d$$

The capital market is modeled using a CET supply allocation function. Aggregate capital is allocated across sectors according to sector-specific rates of return. Unless the transformation elasticity is infinite, the allocation is imperfect and sectoral rates of return will not be uniform. In the extreme, with a zero transformation elasticity, capital would be completely sector specific. Equation (39) sets the aggregate capital stock, TK^s. Equation (40) determines its sectoral allocation, K^s, using the reduced form CET supply functions. (With an infinite elasticity, the law of one price holds.) Equation (41) determines the average rate of return, *TR*, using the CET dual price aggregator. (With an infinite elasticity, the aggregate rate of return is determined by an equilibrium condition.) Equation (42) determines the equilibrium rate of return specific to each sector. The transformation elasticity is given by ω^k.

(39) $$TK^s = TK_0^s$$

(40) $$\begin{cases} K_i^s = \chi_i^k \left(\dfrac{R_i}{TR}\right)^{\omega^k} TK^s & \text{if} \quad \omega^k \neq \infty \\ R_i = TR & \text{if} \quad \omega^k = \infty \end{cases}$$

---

132 The small country assumption is assumed for imports, and thus there is no equilibrating price mechanism on this market.

133 If the transformation elasticity is infinite, these equations trivially set supply equal to demand, and the price is determined via the law-of-one-price. If further, the small country assumption holds, all producer prices will equal the prevailing world export price (adjusted by the export tax/subsidy).

134 In the case of an infinite export demand elasticity, equation (36) trivially sets world export demand to domestic export supply.

(41) $$\begin{cases} TR = \left[\sum_i \chi_i^k \left(R_i\right)^{1+\omega^k} \right]^{1/\left(1+\omega^k\right)} & \text{if} \quad \omega^k \neq \infty \\ TK^s = \sum_i K_i^d & \text{if} \quad \omega^k = \infty \end{cases}$$

(42) $$K_i^d = K_i^s$$

The sector specific factor is modeled using a constant elasticity supply function (for each sector). Equation (43) reflects sectoral supply, F^s. Equation (44) is the equilibrium condition determining the equilibrating factor price, *PF*.

(43) $$F_i^s = \chi_i^f \left(\frac{PF_i}{P} \right)^{\omega_i^f}$$

(44) $$F_i^d = F_i^s$$

Closure

Investment-savings closure has been discussed above. Fiscal closure usually has a fixed government fiscal balance with the direct tax rate, κ, adjusting to achieve the fiscal target. The components of government revenue are broken out into three equations. The first, (45), determines revenues generated by indirect taxes. The second, (46), determines revenues generated by the trade distortions. The third determines aggregate government revenues. Equation (47) is the fiscal balance equation determining the value of government savings. Equation (48) determines real government savings. Finally, equation (49) reflects the fiscal closure rule.

(45) $$ITaxY = \sum_i PA_i \left(\sum_j \tau_{ij}^{itp} XAp_{ij} + \tau_i^{itc} XAc_i + \tau_i^{itg} XAg_i + \tau_i^{iti} XAi_i \right)$$

(46) $$TradeY = ER \sum_i \tau_i^m WPM_i XM_i + \sum_i \tau_i^e PE_i ES_i$$

(47) $$GRev = ITaxY + TradeY + \sum_i \tau_i^p PX_i XP_i + \kappa.YH$$

(48) $$S^g = GRev - PG.XG - P.TR_g^h$$

(49) $$RS^g = S^g / P$$

(50) $$RS^g = RS_0^g$$

External closure assumes that the trade balance (equal to the current account balance in this model) is fixed. Or equivalently, that foreign capital flows are exogenous. The real exchange rate is the equilibrating mechanism. For example, removal of tariffs, which would tend to increase import demand, needs to be met by increasing exports. A real exchange rate depreciation (as measured by the GDP deflator for example), would tend to increase exports. Equation (51) reflects the external closure rule (note that it is expressed in terms of the foreign currency).

(51) $$\sum_i WPE_i E_i^d + S^f = \sum_i WPM_i XM_i$$

Any price can be chosen as the model numéraire. It is perhaps natural to use the exchange rate, *ER*. The exchange rate is not the normal nominal exchange rate of macroeconomic models. Instead, it reflects a price index which evaluates a foreign bundle, for example foreign savings or imports, in terms of a domestic bundle.

The remaining equations are used to calculate the domestic price index. Equation (52) defines nominal GDP at factor cost, *GDPFC*. Equation (53) defines real GDP at factor cost, *RGDP*. Finally, equation (54) determines the domestic GDP price deflator.

(52) $$GDPFC = \sum_i WL_i^d + R_i K_i^d + PF_i F_i^d$$

(53) $$RGDP = \sum_i W_0 \lambda_i^l L_i^d + R_{i,0} \lambda_i^k K_i^d + PF_{i,0} \lambda_i^f F_i^d$$

(54) $$P = GDPFC / RGDP$$

The model specified in this section has $N(30+N)+23$ equations and $N(30+N)+22$ endogenous variables. Invoking Walras' Law, one equation is redundant. It can be proven that the balance of payments equation can be derived as a linear combination of other equations in the model, and it will be dropped from the model specification. It can be calculated at each equilibrium to verify that the model is consistent. A two-sector model has 86 equations. The next section describes an implementation of the two-sector version of this model in Excel.

Excel Implementation of the 2-sector Model for China

The single country model described above has been implemented in Excel® using the Chinese SAM available in GTAP (release 5.0). The implemented model has two sectors which are aggregated up from the 57 sector GTAP database. The two-sector version of the SAM is built up from the fully articulated 57-sector SAM available in GTAP. The next sections describe each of the components of the Excel worksheet and how to simulate the single country CGE model. Note that many cells in the Excel worksheet contain a small red triangle in the upper right hand corner. The triangle indicates that the cell contains a comment. Passing the cursor over the cell and holding it there will open a small text box containing the cell comment.

The Base Social Accounting Matrix (SAM)

The base 57-sector SAM is stored in the worksheet BaseSAM. It would normally not be modified unless it was overwritten with a new SAM. In such a case, the new SAM should normally conform to the same structure as the original SAM or some work would be involved to make the rest of the worksheets consistent with the initial conception.

The user defines a two-sector aggregation using the *aggregation* matrix in the worksheet AggMat. An aggregation matrix is a matrix of 0's and 1's. The user should only modify the 0's and 1's in the first two columns of the aggregation matrix and the first 57 rows. This is the part of the aggregation matrix which maps the 57 sectors to the 2 sectors needed for the model. This 57 x 2 matrix must contain only 0's and 1's. Each of the 57 sectors must be assigned to one and only one of the 2 sectors, i.e. the sum across the two columns must be 0, and each column must have at least one cell with a 1 in it, i.e. the sum down the column must be at least 1. The original file comes with a two sector aggregation composed of agriculture and other goods and services. The user should also change the two labels at the top of the aggregation matrix to match the intended 2-sector aggregation.

The worksheet SAM contains two SAMs. The first is the initial base SAM which is the aggregated SAM. It is created by the following formula:

```
=mmult(mmult(transpose(aggmat),BaseSAM),aggmat)
```

which allows for the aggregation of the base 57-sector SAM into the base 2-sector SAM. The second SAM is the one resulting from a model simulation. This will be described further below.

The base SAMs are evaluated in millions of 1997 U.S. dollars. This may be inappropriate from a numerical point of view in terms of model convergence. A scaling factor has been introduced in the *SAM* worksheet which will convert the base units into more appropriate units. A good rule of thumb is that GDP should be of the order of magnitude of 10,000 or less. The scale variable is located in cell `A47`.

The base SAM, combined with some key parameter values, are the basis of calibrating the other parameters of the CGE model, defined below.

The Model

The model is implemented in the *Model* worksheet. The worksheet is divided intro three main blocks, plus some auxiliary blocks. The three main blocks are: a) the block of endogenous variables; b) the equations block; and c) the block of exogenous variables. The two blocks of variables each contain four columns. The first column contains the name of the variables. The second column contains the initial (or base) values for the variables. The third column contains the simulation values of the variables. In the case of the exogenous variables, these are user determined. In the case of the endogenous variables, the Excel *Solver* will solve the model equations and replace the values of the endogenous variables with the model solution. The fourth column determines the percentage change from the base levels of the variables.

The equations block has two columns. The first column contains the relevant equation name. The second column contains a single equation of the model. All equations are written such that when an equilibrium has been solved the value of the equation is 0, i.e. y = f(x) is written as y-f(x) = 0. Any deviations from 0 indicate one of the following conditions: a) the equation has been mis-specified; b) a parameter has been mis-calibrated; c) an initial variable has been miscalculated; or d) the model solution is not an equilibrium solution.

The endogenous variables and equations have been written such that each variable is lined up with its "own" equation. Of course this is a heuristic device since all variables depend on all equations, but it is useful to make sure the model is square, i.e. that the number of endogenous variables matches the number of independent equations.

Another feature of the model is that all variables, parameters, and equations have a name, i.e. formulas never refer to individual cells, but always to named cells.[135] This reduces the number of

[135] There is an exception to this rule. When initializing variables and parameters based on values from the base SAMs, the formulas make specific calls to cell references in the SAM.

errors and makes auditing of formulas more straightforward. It also facilitates reading and comprehension of the model. Each variable name corresponds to a specific mnemonic related to the variable as described in the model specification above. Since the model is multi-sectoral (and Excel has no indexing mechanism), each sectoral variable has a suffix. These suffixes are _a and _o respectively, for *agriculture* and the *other* sector. The input-output matrix (and its related variables) are doubly indexed using _aa, _ao, _oa, and _aa. All variables have two values (and names). The first has a suffix of zero indicating it contains the variable's initial value. The second is without the zero suffix. The initial value will never change unless a new aggregation is specified or the user changes an initial price index (see below). Variable names can be seen by selecting a cell and looking at the name of the cell in the range name window (below the font window).

Initialization of variables

Variables can be divided into three categories. First, there are the price indices. In most CGE models, prices are typically represented as price indices, not as monetary values. They can therefore be initialized to any level, with 1 representing the most natural initialization value.[136] Only base prices are initialized, such as factor prices, prices of composite goods, and producer prices. Other prices are normally equal to base prices adjusted by some price wedge such as a tax/subsidy or a trade margin, or by model specification (for example, domestic export prices are equal world export prices times the exchange rate). In this model the base prices include factor prices, producer prices (aggregate, domestic, and export), production cost prices, the Armington price, government and investment price deflators, the world price of imports, the price of world exports, and the exchange rate. These basic prices are visible in the green-shaded cells and can be modified by the user. The model equations should not be affected by changes in the base price indices. (A good test of the model calibration is to change the base price indices.)

A second set of variables are derived from the base SAM. Since the base SAM is expressed in value terms, and most variables being initialized are volumes, the SAM values will normally be divided by an appropriate price variable. For example, to set the volume of labor, labor remuneration will be divided by the base wage level. The final set of initialized values may be derived from other initial variables using accounting formulas or equations from the model specification. Except for the basic price indices, users should not modify any of the other cells in the columns representing the initial values.

Key parameters and model calibration

The model has four sets of key parameters: production elasticities, final demand elasticities, trade elasticities, and supply elasticities. Key parameters are user-determined. All other model parameters are calibrated such that the model represents an equilibrium when expressed in base levels of the variables. Table 1 lists the key parameters.

The key parameters are defined in the *Parameters* worksheet. They are in the pink-shaded cells and can be modified by the user. (A value of 1 should be avoided for the CES elasticities since the CES dual price functions are not defined for substitution elasticities of 1. To emulate a Cobb-Douglas function, use the value 0.99 or 1.01). The other parameters in the *Parameters* worksheet are calibrated. They are CES share parameters, shift parameters (for the supply and demand constant elasticity functions), and the consumer demand parameters. The calibration formulas use the model

[136] There can be various reasons for initializing prices to actual base year nominal levels. For example, in the case of energy/climate change models, the absolute price of energy can be relevant when introducing new alternative energies (which are typically identified by their cost of introduction). In such cases, it is possible to initialize the price of oil and other sources of conventional energy to their actual nominal values. In this case, volumes will present actual physical units, for example barrels of oil, or tons of coal.

equations to calculate these parameters, using the initial values of the variables and the key elasticities as inputs. For example, the labor demand function is specified as:

$$L^d = \alpha^l \left(\frac{PVA}{W} \right)^\sigma XP$$

The share parameter will be calculated by solving the above equation for the α^l parameter and inserting the base values for the variables:

$$\alpha^l = \left(\frac{L_0^d}{XP_0} \right) \left(\frac{W_0}{PVA_0} \right)^\sigma$$

(Note that in formulating the calibration formulas, it is easy to use the value of the variables rather than their initial values (for example, `Ld_a`, instead of `Ld_a0`). This can lead to problems when solving the model since the calibrated parameter depends on an endogenous variable and thus becomes endogenous.)

Table 1: **Key Parameters**

Production		
σ^p	sigmap	Substitution elasticity between total intermediate demand, *ND*, and value added, *VA*.
σ^v	sigmav	Substitution elasticity between labor, and the capital-sector specific factor bundle, *KF*.
σ^k	sigmak	Substitution elasticity between capital and the sector specific factor.
Final demand		
η	eta	Income elasticity. Note that in using the ELES, the income elasticity is only used in calibration of the other parameters of the model. The income elasticity is in fact an endogenous outcome of a model. Also note that there are consistency requirements on the income elasticities. The way calibration of the consumer demand system is constructed, the income elasticity of savings is determined residually to achieve the consistency requirements. Users should choose income elasticities of the two goods which lead to a plausible value of the savings income elasticity.
σ^g	sigmag	Government expenditure substitution elasticity (across goods).
σ^i	sigmai	Investment expenditure substitution elasticity (across goods).
Trade elasticities		
σ^m	sigmam	Armington elasticity.
σ^x	sigmax	CET transformation elasticity (between domestic and export supply). Note that an infinite elasticity cannot be specified. Use a high value to approximate the law-of-one-price.
ε	epse	Export demand elasticity. Note that an infinite elasticity cannot be specified. Use a high value to approximate the small-country assumption.
Supply elasticities		
ω^l	omegal	(Aggregate) labor supply elasticity.
ω^k	omegak	Capital mobility elasticity. 0 emulates sector-specific capital. An infinite value cannot be specified. Use a high value to approximate perfectly mobile capital.
ω^f	sigmak	Sector-specific supply elasticities.

Equations Block

The equations block is relatively straightforward. Each model equation is formulated with all variables on the left hand side so that at equilibrium the value of the constraint is 0. Equations are written using variable and parameter names, and thus make no references to Excel cells. The region's redundant equation has been chosen to be the balance of payments equation. This expression of Walras' Law is defined at the bottom of the list of endogenous variables. After calibration and a model solution, this formula should be verified to make sure that the model solution is consistent.

Model re-initialization

After switching to a new region or changing a key model parameter the model solution may no longer be in equilibrium. For convenience, a copy of the model initialization formulas have been saved in a separate area in the *Model* worksheet. A copy of the endogenous variables is bounded by the area `O4:O89`. The initial exogenous variables are bounded by the area `P4:P32`. Re-initialization involves copying each one of these areas from this auxiliary storage in the worksheet and pasting into the relevant areas defining the model and its variables. Since this is a repetitive task, a macro has been created which simplifies the initialization procedure. The macro can either be run from the Tools\Macro\Macro.. options of the menu bar. Choose the InitMod macro and click on the Run button. Alternatively, simply click on a floating button in the worksheet labeled Initialize Model. This button is located around cell `H93` (or just below the equations block).

Model simulation

The following describes the various steps to simulate the model.

1. *Region selection*. Select a region from one of the 65 countries/regions in the pull-down menu at the top left of the SAM pivot table in the worksheet *SAM*.

2. *Data consistency check*. Verify that the base SAM is consistent in the *SAM* worksheet.

3. *Parameter choice*. Choose and enter the key parameters in the *Parameters* worksheet.

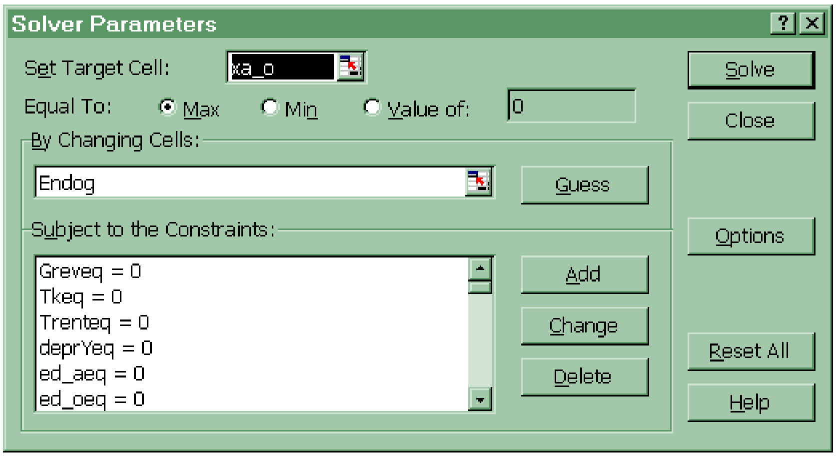
4. *Model initialization*. Re-initialize model variables. Click on the Initialize Model button located near cell `H93` in the *Model* worksheet.

5. *Initialization consistency*. Verify that all model equations are consistent, i.e. that they evaluate to zero, or a small number. Verify that all percentage differences in the variables section (Endogenous and Exogenous) evaluate to 0. Verify that the expression of Walras' Law, at the bottom of the Endogenous variables section evaluates to 0 (or a small number).

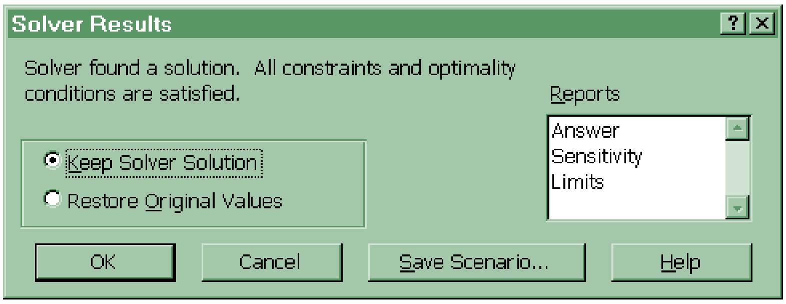
6. *Model consistency*. Solve the model with no shock. This is simply to test that the model is able to reproduce the base solution. A mis-specified equation will normally lead to non-convergence, or an inconsistent solution. Excel contains a powerful equation solving package known as the *Solver*.[137,138] *Solver* is located under the Tools\Solver.. option of the menu bar. The following dialog box should appear:

---

[137] *Solver* is in fact more than just an equation solver, it is a non-linear optimizing algorithm. It will solve for a maximum (or minimum) of a given objective function, subject to non-linear constraints which can be either equalities or inequalities. The regional CGE model is specified as a square systems of equations. Assuming there is a



All the options have already been entered and the *Solver* is ready to start. There are three main options. The first is the objective function. Since it is basically redundant for this model, it is simply a single endogenous variable, by default, Armington absorption of the *other* good (xa_o). It will be maximized. The second option is the list of endogenous variables. A range name has been designated for the endogenous variables and it is called *Endog*. It corresponds to the area `C4:C89` (i.e. it is a column vector with 86 cells). After initializing the model, the range name *Endog*, contains the initialization formulas. After running *Solver* it will contain actual numerical values, i.e. the solution of the model. The third option is the list of constraints. This list contains all 86 equation names which are all constrained to evaluate to zero. The user should not change these options. However, if convergence appears to be a problem, the user may want to modify some of the algorithm's convergence parameters which can be modified in a different dialog box by clicking on the Options button of the *Solver* screen. The *Solver* solution algorithm is invoked by clicking on the Solve button. The status bar at the bottom of the Excel screen displays (minimal) information on each iteration, including iteration count and value of the objective function. If successful, the solver will display the following dialog box:



To have the *Solver* overwrite the values of the endogenous variables, simply click on the OK button. Users can experiment with the other options.

If solution convergence was achieved, the model should have re-produced the base data set (within the limits of the convergence tolerance). All equations should evaluate to 0. The expression of

---

single solution, the choice of objective function should not affect the final solution (though presumably it could impact on the convergence properties).

[138] The *Solver* algorithm is not necessarily loaded with Excel upon installation. If *Solver* is not installed, restart the Excel setup program, and click on the *Solver* option to have it installed.

Walras' Law should evaluate to 0. All deviations from initial values should evaluate to 0. A final test is to check the consistency of the resulting SAM.

The *SAM* spreadsheet, contains the solution SAM. The solution SAM is expressed in terms of the model solution. For example, the labor remuneration cell (in agriculture) contains the formula[139]:

```
=wage*ld_a/scale
```

If the SAM is not consistent, either the solution is inconsistent, the model has been mis-specified, or the formulas in the SAM have been mis-specified.

7. *Homogeneity consistency*. If this is a new model, it is recommended to check model homogeneity. This involves a perturbation of the model numéraire. If the model is homogeneous in prices, perturbation of the model numéraire should leave all volumes constant, and adjust all prices and value variables by the same percentage amount as the percentage change in the numéraire (i.e. all relative prices remain constant). To check homogeneity, multiply the initial value of the numéraire by some constant (e.g. 1.1 would increase the model numéraire by 10 percent). The expression is contained in cell `L23` (also labeled as `ER`). Enter the following formula, for example:

```
=er0*1.1
```

Initially, the only equations which will be affected by this change are the domestic investment equation, the domestic trade prices, and the tariff revenue equation because these are the only equations where the numéraire (the exchange rate) appears. Invoke *Solver* to find a new solution to the model. If the homogeneity test fails (other than due to the lack of convergence), at least one of the equations has been mis-specified, or there could be a built-in nominal rigidity, such as a fixed nominal wage.[140] If both tests succeed, the model should be re-initialized, and the next step is to run one or more shocks to the model.

8. *Simulation of exogenous shocks*. Assessing the impacts of shocks to the exogenous variables is relatively straightforward. Table 2 lists the exogenous variables which can be modified.

Table 2: **List of Exogenous Variables**

Variable Name	*Description*
`itp_aa, itp_ao, itp_oa, itp_oo`	Indirect tax on intermediate demand
`tp_a, tp_o`	Output tax
`lambdal_a, lambdal_o`	Labor productivity
`lambdak_a, lambdak_o`	Capital productivity
`lambdaf_a, lambdaf_o`	Sector specific factor productivity
`itc_a, itc_o`	Indirect tax on private consumption
`itg_a, itg_o`	Indirect tax on public consumption
`iti_a, iti_o`	Indirect tax on investment expenditures
`er`	Exchange rate (model numéraire)
`wpebar_a, wpebar_o`	Price of world exports
`wpm_a, wpm_o`	World price of imports (in foreign currency)
`tm_a, tm_o`	Tariff rates

[139] The formula is adjusted by the scale variable to make the solution SAM comparable with the initial SAM.
[140] Convergence is sometimes achieved, and all consistency tests are valid, and yet the homogeneity test fails. This can sometimes occur because multiple solutions cannot be ruled out. Trying a different percent change to the numéraire may yield the desired result, or else re-scaling the model.

`te_a, te_o`	Export taxes/subsidies
`savf`	Foreign saving (in foreign currency).

Implementing a shock requires modifying the formula in column L.[141] By default, the no-shock value is written as:

```
=Lambdal_a0*1
```

In other words, the default shocked value is expressed as the initial value (with the 0 suffix) times 1. A shock of x% would be written as:

```
=Lambdal_a0*(1+x/100)
```

For example, a 3% increase in labor productivity in the agriculture sector would be specified as:

```
=lambdal_a0*(1.03)
```

After introducing the modifications to the exogenous variables, *Solver* needs to be invoked to find the new equilibrium solution. If *Solver* converges successfully, the new equilibrium will replace the variables in the range named *Endog*. After each successful simulation run, the standard consistency checks should be verified: all equations evaluate to 0, Walras' expression evaluates to 0, and the solution SAM is consistent.

Two macros have been provided to perform trade reforms scenarios. The first sets import tariffs to 0. It can be invoked by clicking on the Zero Tariff button located near cell `G98`. (It is a good idea to re-initialize the model before each separate shock simulation). The second trade reform simulation sets export taxes/subsidies to 0, in addition to nullifying tariff rates. It can be invoked by clicking on the Full Trade Reform button located near cell `G103`.

9. *Post-simulation assessment*. Sample post-simulation results are expressed at the bottom of the *Model* worksheet. Users are of course free to modify and expand these. The *Model* worksheet also expresses deviations from baseline values of all endogenous and exogenous variables.

[141] Other shocks can be implemented by directly modifying a model equation. For example, a labor supply shock can be introduced by adjusting the labor supply shift parameter (als) in the labor supply equation.

10. GE Modeling in the GAMS Programming Language

This chapter provides a brief introduction to a software tool which is often used in CGE modeling. Generalized Algebraic Modeling System (GAMS) is a high-level programming language which allows nonspecialist computer users to specify and implement economywide models calibrated to datasets of the kind described in the preceding chapters.[142] While the GAMS language is relatively easy to learn for a computer-literate individual with some knowledge of linear algebra, it is flexible enough to implement very large models on a PC platform.[143] In the following discussion, the GAMS language will be introduced via a practical example, i.e. the specification and calibration of a simple CGE specification.

One of the first computable general equilibrium (CGE) models was that of Johansen (1960). A Johansen-style CGE model is written as a system of equations linear in proportional changes of the variables. CGE models of this form include Taylor and Black (1974), Dixon et al. (1982), and Deardorff and Stern (1986).

Perhaps the best-known analytical statement of this type of model was given by Jones (1965). In fact, the Jones algebra and the Johansen CGE formulation are completely analogous techniques. Both are designed to solve nonlinear equation systems by using local first-order approximations. GAMS will be applied to an elementary example of the Johansen-Jones approach to general equilibrium modeling. We first set out the Jones algebra and then describe its translation into the GAMS language. An appendix gives the full listing of the GAMS program.

---

[142] GAMS is the property of the GAMS Development Corporation, 1217 Potomac Street NW, Washington DC 20007, who has sole authority over its use and entitlement to fees derived therefrom. See Brooke, Kendrick, and Meeraus (1988) for more details.

[143] See e.g. Devarajan et al (1994), and Lee and Roland-Holst (1994) for models in excess of 10,000 equations.

The Jones Algebra

Consider a two-sector model with the following production structure $Y_j=F_j(L_j,K_j)$, where j=1,2, the first sector produces importable goods (Y_1), and the second produces exportable goods (Y_j).[144] The factors are defined by L_j, labor input into sector-j production, with $L_1 + L_2 = L$, where L is the employment level, and K_j, capital input into sector-j production, and $K_1 + K_2 = K$, where K is the current stock of capital.

In order to formulate a complete model, more notation is needed. Let w denote the wage rate, r the capital rental rate. Now let p_j and p_{wj} denote the domestic and world prices of good j, respectively, while a_{ij} is the input coefficient for input i into the production of good j. Finally, let t_1 denote an import tariff and s_2 is an export subsidy.

This notation and the assumptions of constant returns to scale in production and perfect competition yield the following general equilibrium system:

Fixed-employment conditions:

$$a_{L1}Y_1 + a_{L2}Y_2 = L \qquad (6.1)$$

$$a_{K1}Y_1 + a_{K2}Y_2 = K \qquad (6.2)$$

Average-cost pricing conditions:

$$wa_{L1} + ra_{K1} = p_1 \qquad (6.3)$$

$$wa_{L2} + ra_{K2} = p_2 \qquad (6.4)$$

Conditional input coefficient functions:

$$a_{L1} = a_{L1}(w,r) \qquad (6.5)$$

$$a_{L2} = a_{L2}(w,r) \qquad (6.6)$$

$$a_{K1} = a_{K1}(w,r) \qquad (6.7)$$

$$a_{K2} = a_{K2}(w,r) \qquad (6.8)$$

Domestic price equations:

$$p_1 = (1+t_1)p_{w1} \qquad (6.9)$$

$$p_2 = (1+s_2)p_{w2} \qquad (6.10)$$

In the above ten-equation system, the exogenous variables are L, K, p_{w1}, and p_{w2}, the endogenous variables are Y_1, Y_2, a_{L1}, a_{L2}, a_{K1}, a_{K2}, w, r, p_1. The terms p_2. t_1 and s_2 are parameters. In order to put the equations into proportional change form, we need to introduce some additional notation. The circumflex, "^", denotes percentage change in the indicated variable. The parameter λ_{ij} denotes the proportion of factor i used in sector j, while θ_{ij} denotes the share of factor i in the output of sector j and σ_j denotes the elasticity of substitution between labor and capital in sector j.

With these conventions in mind, one can obtain the following system of equations by total differentiation of equations (6.1)-(6.10):[145]

$$\lambda_{L1}Y_1 + \lambda_{L2}Y_2 = L - \lambda_{L1}\hat{a}_{L1} - \lambda_{L2}\hat{a}_{L2} \qquad (6.11)$$

$$\lambda_{K1}Y_1 + \lambda_{K2}Y_2 = K - \lambda_{K1}\hat{a}_{K1} - \lambda_{K2}\hat{a}_{K2} \qquad (6.12)$$

$$\theta_{L1}\hat{w} + \theta_{K1}\hat{r} = \hat{p}_1 \qquad (6.13)$$

$$\theta_{L2}\hat{w} + \theta_{K2}\hat{r} = \hat{p}_2 \qquad (6.14)$$

[144] It is possible to include both nontraded and intermediate goods into the Jones algebra. For an example of this, see Tobey and Reinert (1991).

$\hat{a}_{L1} = \theta_{K1}\sigma_1(\hat{r} - \hat{w})$ (6.15)

$\hat{a}_{L2} = \theta_{K2}\sigma_2(\hat{r} - \hat{w})$ (6.16)

$\hat{a}_{K1} = \theta_{L1}\sigma_1(\hat{w} - \hat{r})$ (6.17)

$\hat{a}_{K2} = \theta_{L2}\sigma_2(\hat{w} - \hat{r})$ (6.18)

$\hat{p}_1 = \hat{p}_{w1} + dt_1/(1+t_1)$ (6.19)

$\hat{p}_2 = \hat{p}_{w2} + ds_2/(1+s_2)$ (6.20)

GAMS Implementation

The next step is to translate the system 6.11-6.20 into the GAMS programming language. Before doing so, some discussion of solution strategy, arithmetic operators, and relational operators is warrented. Since the above system is linear and square (number of endogenous variables equals number of equations), it can be solved by matrix inversion, which is how Johansen models generally have been solved.[146] The GAMS software, however, was designed to solve more general linear and nonlinear programming problems. We adapt it to exactly determined CGE models by simply specifying the model's equations as the system of constraints and including an arbitrary objective function. The latter is superfluous since a fully specified general equilibrium model should have a unique solution.[147]

Like most programming languages, GAMS has a variety of operators. These are divided into three principal groups, arithmetic, relational, and conditional. The arithmetic operators used in GAMS are of the following:

**	exponentiation

[145] Equations 6.13 and 6.14 require the application of the envelope theorem. See Jones (1965).
[146] See Dervis, de Melo, and Robinson (1982), Appendix B.3.
[147] The optimization features of GAMS have been used by a number of authors to study policy responses to changing eonomic conditions. See e.g. Lee and Roland-Holst (1993).

* /	multiplications and division
+ -	addition and subtraction

These are listed in order of the precedence which would be applied in the absence of parentheses. Exponentiation is performed first, and multiplication and division precede addition and subtraction. Finally, computation proceeds from right to left through an open (i.e. parentheses-free) expression.

Relational operators used in GAMS are as follows:

lt, le, eq, ne, ge, gt	less than, less than or equal, not equal, etc.
not	not
and	and
or xor	or, either or

These again are listed in order of open precedence. Liason between arithmetic and logical operations is provided by the usual zero-false, nonzero-true standard.

GAMS programs consist of a series of statements followed by semicolons:

```
Statement ;
          .
          .
          .
Statement ;
```

GAMS programs are commonly structured as follows:

```
Data:
        SAM
        Parameters and other data

Definitions:
        Sets
        Parameters
        Initial values
        Variables
        Equations

Model:
Solution:
Display:
```

We begin the example general equilibrium program with the data input. This data consists of two components, a social accounting matrix and supporting tables of structural parameters and other data. Generally, these components are loaded into the model from two separate files with the GAMS "include *filename* ;" statement. In the present, simpler Jones model, we omit these two files an proceed directly with definitions. The first type of definition is a set declaration, which generally take the form:

```
sets
        setname name1   text  /elements/
                        .
                        .
                        .
        setname namen text  /elments/
        ;
```

In the example model, indices are required for sectors and factors, which leads to the following sets definition:

```
sets
        i               industries  / 1*2 /
        f               factors / L,K /
;
```

Note that there are two ways to list set elements. They may be listed individually, separated by commas, or they may be listed as a range, indicated by an asterisk. These two options also can be used together. For example, a more complex set of elements might be / e1*e10, e12, e14 /.

There are essentially four ways of introducing data into a GAMS program:

1) A scalars statement;

2) A parameters statement with assigned values;

3) A parameters statement without assigned values, followed by a table statement;

4) A parameters statement without assigned values, followed by assignment statements.

The present CGE example will utilize all four means of entering data, beginning with a scalars statement.

A scalars statement can be used to declare and assign a value to a parameter with zero dimension (i.e. not indexed by a set) and takes the form:

```
scalars
        scalar          name1  text  /value/
```

```
                    .
                    .
                    .
          scalar  namen  text  /value/
;
```

As mentioned above, a dummy objective function is usually used to solve a square CGE model with the GAMS optimization software. It is often convenient to simply assign this function a constant scalar value as follows:

```
scalars
          dummy  named  / 1.0 /
;
```

Now consider the parameters statement, which declares and (optionally) assigns values to the parameters of the model. The parameters of the example model are λ_{ij}, θ_{ij}, σ_j, t_1, s_2, dt_1, and ds_2. We assign values for σ_j directly in the parameters statement. Values for λ_{ij} and θ_{ij} will be assigned in table statements. Values for the remaining parameters will be given in assignment statements. In order to illustrate particular GAMS features, we also introduce three further parameters with the labels "tarhat," "subhat," and "cphat."

The parameters statement has the general form:

```
parameters
          parameter  name1  text  /values/
                    .
                    .
                    .
```

```
            parameter namen  text  /values/
    ;
```

For the Johansen/Jones model, the statement takes the form:

```
parameters
        lambda(f,i)     factor allocation
        theta(f,i)      factor income share
        sigma(i)        elasticity of factor substitution
                        / 1  0.8
                          2  0.9 /
        t(i)                    initial tariff
        s(i)                    initial subsidy
        dt(i)                   change in tariff
        tarhat(i)       proportional change in tariff
        subhat(i)       proportional change in export subsidy
        cphat(i)                proportional change in price due to commercial policy
        ;
```

The assignment statements used to enter parameter values have been designed to feature the GAMS dollarsign control character, which can be used in two ways. A $ on the left-hand-side of an assignment statement is a conditional assignment: "[I]f the logical relationship is true, the assignment is made; if it is not, however, the existing value is retained, zero being used if no previous value has been given".[148] A $ on the right-hand-side of an assignment statement implies an if-then-else sequence and an assignment is always made.[149]

[148] Brooke, Kendrick, and Meeraus (1988), p.72.
[149] The reader might find it useful to think of the $ operator as a "such that" operator.

The assignment statements for the Johansen/Jones model are

```
t('1') = 0.20 ;
t('2') = 0.30 ;
dt('1') = 0.10 ;
dt('2') = 0.15 ;

tarhat(i) $ ( t(i)  gt  0 )  =  dt(i)/(1 + t(i)) ;
subhat(i) $ ( s(i) gt 0 )  =  ds(i)/(1 + s(i)) ;

cphat(i)  =  tarhat(i) $ t(i) + subhat(i) $ s(i) ;
```

The first four statements refer to specific elements of index i, and these elements must be put in single or double quotations. The fifth and sixth statements make assignments to tariff and subsidy proportional change variables, respectively, if the conditions following the dollar operators are true. If the conditions are not true, no assignment is made; the existing value is retained, zero being the default if no previous value was assigned. In the seventh statement, the dollar operators on the right-hand-side of the equation govern which of the two values, tarhat(i) or subhat(i) are assigned to cphat(i). The expressions $ t(i) and $ s(i) are the conditions that t(i) and s(i), respectively, be nonzero.

Next, we will demonstrate how values for λ_{ij} and θ_{ij} can be entered with a table statement. Table statements can come in many different forms, of which only one example is provided here:

```
table  lambda(f,i)

                    1           2
        L           0.50        0.50
        K           0.25        0.75
   ;

table theta(f,i)
                    1           2
        L           0.60        0.40
        K           0.40        0.60
   ;
```

The first line of a table statement begins with the word 'table'. This is followed by the variable name, including set domains. Labels are used to generate a grid, and values are entered into this grid. Any blanks in the grid denote zeros. It is not necessary to list all elements of a set as row or column labels. Where an element is left out, the corresponding row or column will be a vector of zeros. Labels cannot be repeated, however. The table statement ends with a semicolon. In contrast to the scalars and parameters statements, only one parameter can by initialized in a table statement. Therefore, separate table statements are required for each parameter to be initialized.

Next, we will display the parameters using a GAMS display statement. The important thing to remember about a display statement is that, in listing the parameters to be displayed, set domains are not included. For the present example, the parameters are displayed using the following statement:

```
display lambda, theta, sigma, t, dt ;
```

This completes the data component of our GAMS general equilibrium model. This is generally followed by the model component, which begins with a variables declaration statement. The general form of the GAMS variables statement is as follows:

```
variables
        variable  name1  text
                  .
                  .
                  .
        variable  namen  text
        ;
```

In the case of our model, the variables to be declared are the endogenous variables, the exogenous variables, and a dummy variable. These are declared as follows:

```
variables
        yhat(i)         proportional change in production
        ahat(f,i)       proportional change in input
        what            proportional change in wage rate
        rhat            proportional change in capital rental rate
        phat(i)         proportional change in domestic price

        lhat            proportional change in labor endowment
        khat            proportional change in capital endowment
        psthat(i)       proportional change in world price

        omega           dummy variable for objective function
        ;
```

Equation identifiers are declared in a GAMS program using an equations statement. In general, the equations statement appears as:

```
equations
        equation  name1  text
                  .
                  .
                  .
        equation  namen  text
        ;
```

For the present CGE model, the equations statement is as follows:

```
equations
        fxelab          fixed employment of labor
        fxecap          fixed employment of capital
        acp(i)          average cost pricing
        linp(i)         labor input
        kinp(i)         capital input
        domp(i)         domestic prices
        obj             objective
        ;
```

Next, equations must be defined. This is done in a series of statements. For equations which are equalities, the general form is as follows:

equation *name1*.. left-hand side =e= right-hand side ;

.

.

.

equation *namen*.. left-hand side =e= right-hand side ;

Two decimal or period points '..' are required between the equation name and the equation algebra. The '=e=' notation represents the equality sign for equation definitions. It is distinct from the more usual '=' symbol used in parameter assignments. Each equation definition is a GAMS statement and ends in a semicolon. Equation definitions may be indexed in those cases where the variable being determined is defined as a set.

For our model, the definitions are as follows:

```
fxelab..   sum(i, lambda('l',i)*yhat(i)) =e= lhat -  sum(i, lambda('l',i)*ahat('l',i));

fxecap..   sum(i, lambda('k',i)*yhat(i)) =e= khat - sum(i, lambda('k',i)*ahat('k',i));

acp(i)..   theta('l',i)*what + theta('k',i)*rhat =e= phat(i);

linp(i)..  ahat('l',i) =e= theta('k',i)*sigma(i)*(rhat-what);

kinp(i)..  ahat('k',i) =e= theta('l',i)*sigma(i)*(what-rhat);

domp(i)..  phat(i) =e= psthat(i) + cphat(i);

obj..      omega =e= dummy;
```

Note that, when referring to a particular element in an assignment statement or an equation definition statement, the element name is put in quotation marks. The 'sum' function is used to calculate sums over the domain of a set. Its general form is sum(set name, expression). It is used in the first two equation definitions to sum expressions over set i. It is also possible to include a dollar control operator after the set name in a sum function in order to restrict the elements of the set which are included in the summation.

The above set of equations determine the ten endogenous variables and the dummy variable. Still to be specified is the model closure.[150] The closure is given as follows:

```
lhat.fx = 0.00 ;
```

[150] "(P)rescribing closure boils down to stating which variables are endogenous or exogenous in an equation system" (Taylor, 1990, pp. 15-16).

```
khat.fx = 0.00;
psthat.fx('1') = 0.00;
psthat.fx('2') = 0.00;
```

While a GAMS parameter has a single value associated with it, a GAMS variable has four such values. They are

```
.lo    the lower bound
.up    the upper bound
.l     the activity level
.m     the marginal value
```

The lower and upper bounds are the minimum and maximum values, respectively, that a variable can take on during optimization. The activity level is the current value of a variable, and the marginal value is the effect of the variable value after optimization on the objective function. In cases where the lower and upper bound coincide, the variable is fixed, and the suffix 'fx' is used to assign the fixed value. This is what is done in the above model closure. The first two equations address factor market closure, fixing factor supplies, while the second two equations address external sector closure, fixing world prices. The user can introduce exogenous changes in any or all of these four variables.[151]

Finally, we need a model statement, a solve statement, and a final display statement for the activity levels of the variables after solution. These are as follows:

[151] Other types of closures are, of course, possible. For example, Tobey and Reinert (1991) use an export demand function to specify rest-of-the world behavior. This replaces the fixed world export price used here.

```
model simple /all/;

solve simple maximizing omega using nlp;

display yhat.l, ahat.l, what.l, rhat.l, phat.l, lhat.l,
        khat.l, psthat.l;
```

The model statement declares a model named 'simple' which consists of all the declared equations. The model is solved by maximizing omega. Since omega is set equal to the dummy parameter, the outcome of this maximization procedure is simply to solve the ten constraint equations of the maximization problem for the ten endogenous variables. The term 'nlp' refers to non-linear programing. The solve statement invokes a solver called MINOS. Since the system of equations in our model is linear, it solves very quickly.

Why GAMS?

This module presented a linearized, Johansen-Jones approach to general equilibrium modeling. As we mentioned above, it is possible to solve this class of CGE models using matrix inversion. What, then, is the utility of GAMS? The linearization technique is a local approximation, useful for small changes in exogenous variables. A more general approach to CGE modeling is to specify functional forms, constructing a square but *nonlinear* system of equations. Such a system is not solvable by matrix inversion. For these problems, the GAMS package and the MINOS solver are quite useful.

Annex: GAMS Listing for a Small CGE Model

```
$title   A Small Computable General Equilibrium model implemented in GAMS
$offsymlist offsymxref

sets
    i    industries   /1 * 2/
    f    factors      /L, K/
    ;

scalars
    dummy        dummy parameter      /1.00/
    ;

parameters
    lambda(f,i)   factor allocation share
    theta(f,i)    factor income share
    sigma(i)      elasticity of substitution
        /1  0.8
         2  0.9/
    t(i)          initial tariff
    s(i)          initial subsidy
    dt(i)         change in tariff
    ds(i)         change in export subsidy
    tarhat(i)     proportional change in tariff
    subhat(i)     proportional change in export subsidy
    cphat(i)      proportional change in price due to commercial policy
    ;

       variables
    yhat(i)       proportional change in production
    ahat(f,i)     proportional change in input
    what          proportional change in wage rate
    rhat          proportional change in capital rental rate
    phat(i)       proportional change in domestic price

    lhat          proportional change in labor endowment
    khat          proportional change in capital endowment
    psthat(i)     proportional change in world price

    omega         dummy variable
    ;
```

```
equations
     fxelab        fixed employment of labor
     fxecap        fixed employment of capital
     acp(i)        average cost pricing
     linp(i)       labor input equations
     kinp(i)       capital input equations
     domp(i)       domestic prices
     obj           objective
     ;

* calibration
t('1')  = 0.20;
s('2')  = 0.30;
dt('1') = 0.10;
ds('2') = 0.15;

tarhat(i) $ (t(i) gt 0) = dt(i)/(1+t(i));

subhat(i) $ (s(i) gt 0) = ds(i)/(1+s(i));

cphat(i) = tarhat(i) $ t(i) + subhat(i) $ s(i);


table lambda(f,i)

              1              2
       L      0.50           0.50
       K      0.25           0.75
       ;


table theta(f,i)

              1              2
       L      0.60           0.40
       K      0.40           0.60
       ;

display lambda, theta, sigma, t, dt;
```

```
* equation definitions

fxelab..    sum(i, lambda('l',i)*yhat(i)) =e= lhat
                    - sum(i, lambda('l',i)*ahat('l',i));

fxecap..    sum(i, lambda('k',i)*yhat(i)) =e= khat
                    - sum(i, lambda('k',i)*ahat('k',i));

acp(i)..    theta('l',i)*what + theta('k',i)*rhat =e= phat(i);

linp(i)..   ahat('l',i) =e= theta('k',i)*sigma(i)*(rhat-what);

kinp(i)..   ahat('k',i) =e= theta('l',i)*sigma(i)*(what-rhat);

domp(i)..   phat(i) =e= psthat(i) + cphat(i);

obj..       omega =e= dummy;


* model closure (exogenous variables)

lhat.fx = 0.00;
khat.fx = 0.00;
psthat.fx('1') = 0.00;
psthat.fx('2') = 0.00;


* model declaration

options solprint=off;
options iterlim=100,limrow=0,limcol=0,domlim=0;

model simple /all/;


solve simple maximizing omega using nlp;


display yhat.l, ahat.l, what.l, rhat.l, phat.l, lhat.l,
        khat.l, psthat.l;
```

11. A Dynamic Prototype Model for Single Economy Policy Analysis

The Development Research Center of the State Council has asked for World Bank assistance in developing new analytical tools for economic policy analysis. These tools are intended to assist Chinese policy makers in assessing the impacts of various policy options—fiscal, structural, development and financial. The tools need to elucidate the impacts on the composition of output and demand, income distribution, poverty and macroeconomic indices such as GDP growth and trade. In a first phase of the project, a *real* computable general equilibrium model will be developed. The model will be multisectoral (based on the level of detail of the latest input-output table for China), multi-factor (differentiating different labor skills and operating surplus), and multi-household to assist in distribution and poverty analysis. In a second phase, this model will be extended to include the increasingly important financial linkages in the Chinese economy.

This chapter presents a dynamic prototype model for the CGE analytical tool. The prototype has some key features for assessing structural and poverty impacts:

- Labor markets disaggregated by skill level
- Land and capital markets disaggregated by type of capital/land
- A production structure which differentiates the substitutability of unskilled labor on the one hand, and skilled labor and capital on the other hand
- Differentiation of production of like-goods (e.g. small- and large-scale farms, or public versus private production)
- Detailed income distribution
- Intra-household transfers (e.g. urban to rural), transfers from government, and remittances
- Multiple households
- A tiered structure of trade (differentiating across various trading partners)
- Possibility of influencing export prices
- Internal domestic trade and transport margins
- Various potential factor mobility assumptions

The rest of the document proceeds to describe all of the model details using the standard circular flow description of the economy. It starts with production (*P*), income distribution (*Y*), demand (*D*), trade (*T*), domestic trade and transport margins (*M*), goods market equilibrium (*E*), macro closure (*C*), factor market equilibrium (*F*), macroeconomic identities (*I*), and growth (*G*).

Table 1 describes the indices used in the equations. Note that the model differentiates between production activities, denoted by the index *i*, and commodities, denoted by the index *k*. In many models, the two will overlap exactly. However, this differentiation allows for the same commodity to be produced by one or more sectors, and to differentiate these commodities by source of production. For example, it could be used in a model of economies in transition where commodities produced by the public sector have a different cost structure than commodities produced by the private sector, and the commodities themselves could be differentiated by consumers.[152] Another example, could be small- versus large-scale agricultural producers.

Table 1: Indices used in the model

i	Production activities
k	Commodities
l	Labor skills
ul	Unskilled labor
sl	Skilled labor[a]
kt	Capital types
lt	Land types
e	Corporations
h	Households
f	Final demand accounts[b]
m	Trade and transport margin accounts[c]
r	Trading partners

Notes: a. The unskilled and skilled labor indices, *ul* and *sl*, are subsets of *l*, and their union composes the set indexed by *l*.
b. The standard final demand accounts are '*Gov*' for government current expenditures, '*ZIp*' for private investment, '*ZIg*' for public investment, '*TMG*' for international export of trade and transport services, and '*DST*' for changes in stocks.
c. The standard trade and transport margin accounts are '*D*' for domestic goods, '*M*' for imported goods, and '*X*' for exported goods.

12. Model Equations

Production

Production, like in most CGE models, relies on the substitution relations across factors of production and intermediate goods. The simplest production structure has a single constant-elasticity-of-substitution (CES) relation between capital and labor, with intermediate goods being used in fixed proportion to output. In the production structure described below, there are multiple types of capital, land and labor, and they are combined in a nested-CES structure which is intended to represent the various substitution possibilities across these different factors of production. Typically, intermediate goods will enter in fixed proportion to output, though at the aggregate level, the model allows for a degree of substitutability between aggregate intermediate demand and value added.[153] The decomposition of value added has several components (see *figure* 1 for a representation of the multiple nests). First, land is assumed to be a substitute for an aggregate capital labor bundle.[154] The latter is then decomposed into unskilled labor on the one hand, and skilled labor cum capital on the other hand. This conforms to recent observations which suggest that capital and skilled labor are complements which can substitute for unskilled

[152] The model allows for perfect substitution, in which case consumers are indifferent regarding who produces the good. An example might be electricity.

[153] Deviations from this structure might include isolating some key inputs, for example energy, or agricultural chemicals in the case of crops, and feed in the case of livestock.

[154] In some sectors the model also allows for a sector-specific factor of production, for example, coal mining and oil production require reserves which cannot be used for any other activity. In this case, the nesting follows the same general structure as depicted in Figure 1.

labor. The four aggregate factors—unskilled and skilled labor, land and capital, are decomposed by type in a final CES nest.

Top-level nest and producer price

The top-level nest has output, *XP*, produced as a combination of value added, *VA*, and an aggregate demand for goods and non-factor services, *ND*. In most cases, the substitution elasticity will be assumed to be zero, in which case the top-level CES nest is a fixed-coefficient Leontief production function. Equations (P-1) and (P-2) represent the optimal demand conditions for the generic CES production function, where *PND* is the price of the *ND* bundle, *PVA* is the aggregate price of value added, *PX* is the unit cost of production, and σ^p is the substitution elasticity. If the latter is zero, both *ND* and *VA* are used in fixed proportions to output, irrespective of relative prices. Equation (P-3) represents the unit cost function, *PX*. It is derived from the CES dual price formula. The model assumes constant-returns-to-scale and perfect competition in all sectors. Hence, the producer price, *PP*, is equal to the unit cost, adjusted for a producer tax/subsidy, τ^p, equation (P-4).

$$ND_i = \alpha_i^{nd}\left(\frac{PX_i}{PND_i}\right)^{\sigma_i^p} XP_i \qquad \text{(P-1)}$$

$$VA_i = \alpha_i^{va}\left(\frac{PX_i}{PVA_i}\right)^{\sigma_i^p} XP_i \qquad \text{(P-2)}$$

$$PX_i = \left[\alpha_i^{nd} PND_i^{1-\sigma_i^p} + \alpha_i^{va} PVA_i^{1-\sigma_i^p}\right]^{1/(1-\sigma_i^p)} \qquad \text{(P-3)}$$

$$PP_i = \left(1+\tau_i^p\right) PX_i \qquad \text{(P-4)}$$

Second-level production nests

The second-level nest has two branches. The first decomposes aggregate intermediate demand, *ND*, into sectoral demand for goods and services, *XAp*. The model explicitly assumes a Leontief structure. Thus equation (P-5) describes the demand for good *k* by sector *j*, where the coefficient *a* represents the proportion between *XAp* and *ND*. The price of the *ND* bundle, *PND*, is the weighted average of the price of goods and services, *PA*, using the technology coefficients as weights, equation (P-6). The so-called Armington price is multiplied by a sector and commodity specific indirect tax, τ^{cp}.

$$XAp_{k,j} = a_{k,j} ND_j \qquad \text{(P-5)}$$

$$PND_j = \sum_k a_{k,j}\left(1+\tau_{k,j}^{cp}\right) PA_k \qquad \text{(P-6)}$$

The second branch decomposes the aggregate value added bundle, *VA*, into three components: aggregate demand for capital and labor, *KL*, aggregate land demand, TT^d, and a sector-specific resource, *NR*,[155] see equations (P-7) through (P-9). The relevant component prices are *PKL*, *PTT* and *PR*, respectively, and the substitution elasticity is given by σ^v. Equation (P-9) allows for the possibility of factor productivity changes as represented by the λ parameter. The price of value added, *PVA*, is the CES aggregation of the three component prices, as defined by equation (P-10).

$$KL_i = \alpha_i^{kl}\left(\frac{PVA_i}{PKL_i}\right)^{\sigma_i^v} VA_i \qquad \text{(P-7)}$$

$$TT_i^d = \alpha_i^{tt}\left(\frac{PVA_i}{PTT_i}\right)^{\sigma_i^v} VA_i \qquad \text{(P-8)}$$

$$NR_i^d = \alpha_i^{nr}\left(\lambda_i^{nr}\right)^{\sigma_i^v-1}\left(\frac{PVA_i}{PR_i}\right)^{\sigma_i^v} VA_i \qquad \text{(P-9)}$$

$$PVA_i = \left[\alpha_i^{kl} PKL_i^{1-\sigma_i^v} + \alpha_i^{tt} PTT_i^{1-\sigma_i^v} + \alpha_i^{nr}\left(\frac{PR_i}{\lambda_i^{nr}}\right)^{1-\sigma_i^v}\right]^{1/(1-\sigma_i^v)} \qquad \text{(P-10)}$$

Third-level production nest

The third-level nest decomposes the aggregate capital-labor bundle, *KL*, into two components. The first is the aggregate demand for unskilled labor, *UL*, with an associated price of *PUL*. The second is a bundle composed of skilled labor and capital, *KSK*, with a price of *PKSK*. Equations (P-11) and (P-12) reflect the standard CES optimality conditions for the demand for these two components, with a substitution elasticity given by σ^{kl}. The price of capital-labor bundle, *PKL*, is defined in equation (P-13).

155 The latter will typically be zero in most sectors.

$$UL_i = \alpha_i^u \left(\frac{PKL_i}{PUL_i} \right)^{\sigma_i^{kl}} KL_i \qquad \text{(P-11)}$$

$$KSK_i = \alpha_i^{ksk} \left(\frac{PKL_i}{PKSK_i} \right)^{\sigma_i^{kl}} KL_i \qquad \text{(P-12)}$$

$$PKL_i = \left[\alpha_i^u PUL_i^{1-\sigma_i^{kl}} + \alpha_i^{ksk} PKSK_i^{1-\sigma_i^{kl}} \right]^{1/(1-\sigma_i^{kl})} \qquad \text{(P-13)}$$

Fourth-level production nest

The fourth-level nest decomposes the capital-skilled labor bundle into a capital component, KT^d, and a skilled labor component, *SKL*. Equations (P-14) and (P-15) represent the optimality conditions where the relevant component prices are *PKT* and *PSKL*, and the substitution elasticity is given by σ^{ks}. Equation (P-16) determines the price of the *KSK* bundle, *PKSK*.

$$SKL_i = \alpha_i^s \left(\frac{PKSK_i}{PSKL_i} \right)^{\sigma_i^{ks}} KSK_i \qquad \text{(P-14)}$$

$$KT_i^d = \alpha_i^{kt} \left(\frac{PKSK_i}{PKT_i} \right)^{\sigma_i^{ks}} KSK_i \qquad \text{(P-15)}$$

$$PKSK_i = \left[\alpha_i^s PSKL_i^{1-\sigma_i^{ks}} + \alpha_i^{kt} PKT_i^{1-\sigma_i^{ks}} \right]^{1/(1-\sigma_i^{ks})} \qquad \text{(P-16)}$$

Demand for labor by sector and skill

Equations (P-17) and (P-18) decompose the demands for aggregate unskilled and skilled labor, respectively, across their different components. The variable L^d represents labor demand in sector *i* for labor of skill level *l*. The relevant wage is given by *W* which is allowed to be both sector and skill-specific. The respective cross-skill substitution elasticities are σ^u and σ^s. Both equations (P-17) and (P-18) incorporate sector and skill specific labor productivity, represented by the variable λ^l. The aggregate unskilled and skilled price indices are determined in equations (P-19) and (P-20), respectively *PUL* and *PSKL*.

$$L_{i,ul}^{d} = \alpha_{i,ul}^{l}\left(\lambda_{i,ul}^{l}\right)^{\sigma_i^u - 1}\left(\frac{PUL_i}{W_{i,ul}}\right)^{\sigma_i^u} UL_i \qquad \text{for } ul \in \{\text{Unskilled labor}\} \qquad \text{(P-17)}$$

$$L_{i,sl}^{d} = \alpha_{i,sl}^{l}\left(\lambda_{i,sl}^{l}\right)^{\sigma_i^s - 1}\left(\frac{PSKL_i}{W_{i,sl}}\right)^{\sigma_i^s} SKL_i \qquad \text{for } sl \in \{\text{Skilled labor}\} \quad \text{(P-18)}$$

$$PUL_i = \left[\sum_{ul \in \{\text{Unskilled labor}\}} \alpha_{i,ul}^{l}\left(\frac{W_{i,ul}}{\lambda_{i,ul}^{l}}\right)^{1-\sigma_i^u}\right]^{1/(1-\sigma_i^u)} \qquad \text{(P-19)}$$

$$PSKL_i = \left[\sum_{sl \in \{\text{Skilled labor}\}} \alpha_{i,sl}^{l}\left(\frac{W_{i,sl}}{\lambda_{i,sl}^{l}}\right)^{1-\sigma_i^s}\right]^{1/(1-\sigma_i^s)} \qquad \text{(P-20)}$$

Demand for capital and land across types

The aggregate land and capital bundles, KT^d and TT^d respectively, are disaggregated across types, leading to type- and sector-specific capital and land demand, K^d and T^d. The decomposition is represented in equations (P-21) and (P-23), where the respective prices are R and PT which are both type- and sector-specific. The equations also incorporate productivity factors. Equations (P-22) and (P-24) represent the price indices for aggregate capital and land, respectively PKT and PTT.

$$K_{i,kt}^{d} = \alpha_{i,kt}^{k}\left(\lambda_{i,kt}^{k}\right)^{\sigma_i^k - 1}\left(\frac{PKT_i}{R_{i,kt}}\right)^{\sigma_i^k} KT_i^{d} \qquad \text{(P-21)}$$

$$PKT_i = \left[\sum_{kt} \alpha_{i,kt}^{k}\left(\frac{R_{i,kt}}{\lambda_{i,kt}^{k}}\right)^{1-\sigma_i^k}\right]^{1/(1-\sigma_i^k)} \qquad \text{(P-22)}$$

$$T_{i,lt}^{d} = \alpha_{i,lt}^{t}\left(\lambda_{i,lt}^{t}\right)^{\sigma_i^t - 1}\left(\frac{PTT_i}{PT_{i,lt}}\right)^{\sigma_i^t} TT_i^{d} \qquad \text{(P-23)}$$

$$PTT_i = \left[\sum_{lt} \alpha_{i,lt}^{k}\left(\frac{PT_{i,lt}}{\lambda_{i,lt}^{k}}\right)^{1-\sigma_i^t}\right]^{1/(1-\sigma_i^t)} \qquad \text{(P-24)}$$

Commodity aggregation

Each activity produces a single commodity, *XP*, indexed by *i*. Consumption goods, indexed by *k*, are a combination of one or more produced goods. Aggregate domestic supply of good *k*, *X*, is a CES combination of one or more produced goods *i*. In many cases, the CES aggregate is of a single commodity, i.e. there is a one-to-one mapping between a consumed good and its relevant production. There are cases, however, where it is useful to have consumed goods be an aggregation of produced goods, for example when combining similar goods with different production characteristics (e.g. public versus private, commercial versus small-scale, etc.) Equation (P-25) represents the optimality condition of the aggregation of produced goods into commodities. The producer price is *PP*, and the price of the aggregate supply is *P*. The degree of substitutability across produced commodities is σ^c. Equation (P-26) determines the aggregate supply price, *P*. The model allows for perfect substitutability, in which case the law of one price holds and the produced commodities are simply aggregated to form aggregate output.[156]

$$\begin{cases} XP_i = \alpha_{i,k}^c \left(\dfrac{P_k}{PP_i} \right)^{\sigma_k^c} X_k & \text{if} \quad \sigma_k^c \neq \infty \\ PP_i = P_k & \text{if} \quad \sigma_k^c = \infty \end{cases} \tag{P-25}$$

$$\begin{cases} P_k = \left[\sum_{i \in K} \alpha_{i,k}^c PP_i^{1-\sigma_k^c} \right]^{1/(1-\sigma_k^c)} & \text{if} \quad \sigma_k^c \neq \infty \\ X_k = \sum_{i \in K} XP_i & \text{if} \quad \sigma_k^c = \infty \end{cases} \tag{P-26}$$

Income distribution

The prototype model has a rich menu of income distribution channels—factor income and intra-household, government and foreign transfers (i.e. remittances). The prototype also includes corporations used as a pass-through account for channeling operating surplus.

Factor income

There are four broad factors—a sector specific resource, land, labor and capital—the latter three which can be sub-divided into various types. Equations (Y-1) through (Y-3) determine aggregate net-income from labor, *LY*, capital, *KY*, and land, *TY*, each indexed by its sub-types. The fourth equation determines aggregate income from the sector-specific resource. These are net incomes because the model incorporates factor taxes designated by τ^{fl}, τ^{fk}, τ^{ft} and τ^{fr} respectively.[157]

[156] Electricity is a good example of a homogeneous output but which could be produced by very different production technologies, e.g. hydro-electric, nuclear, thermal, etc.

[157] The factor taxes are type- and sector-specific. Note as well that the relevant factor prices represent the perceived cost to employers, not the perceived remuneration of workers.

$$LY_l = \sum_i \frac{W_{i,l} L^d_{i,l}}{1+\tau^{fl}_{i,l}} \quad \text{(Y-1)}$$

$$KY_{kt} = \sum_i \frac{R_{i,kt} K^d_{i,kt}}{1+\tau^{fk}_{i,kt}} \quad \text{(Y-2)}$$

$$TY_{lt} = \sum_i \frac{PT_{i,lt} T^d_{i,lt}}{1+\tau^{ft}_{i,lt}} \quad \text{(Y-3)}$$

$$RY = \sum_i \frac{PR_i R^d_i}{1+\tau^{fr}_i} \quad \text{(Y-4)}$$

Distribution of profits

All of labor, land and sector-specific factor income is allocated directly to households.[158] Profits (aggregated with income from the sector-specific resouce), on the other hand, are distributed to three broad accounts, enterprises, households, and the rest of the world (ROW). Equation (Y-5) determines the level of profits distributed to enterprises, TR^E. Equation (Y-6) represents the level of profits distributed directly to households, TR^H. And, equation (Y-7) determines the level of factor income distributed abroad, TR^W. Note that the three share parameters, φ^E, φ^H, and φ^W sum to unity.

$$TR^E_{k,kt} = \varphi^E_{k,kt} KY_{kt} \quad \text{(Y-5)}$$

$$TR^H_{k,kt} = \varphi^H_{k,kt} KY_{kt} \quad \text{(Y-6)}$$

$$TR^W_{k,kt} = \varphi^W_{k,kt} KY_{kt} \quad \text{(Y-7)}$$

Corporate income

Corporate income, TR^E, is split into four accounts. First, the government receives its share through the corporate income tax, κ^c. The residual is split into three: retained earnings, and income distributed to households and the rest of the world. Equation (Y-8) determines corporate income of enterprise e, CY. It is the sum, over possible capital types, of shares of distributed profits (to corporations).[159] Equation (Y-9) determines retained earnings, i.e. corporate savings, S^c, where the rate of retained earnings is given by s^c. Equations (Y-10) and (Y-11) determine the overall transfers to households and to ROW. Note that the two share parameters, φ^H and φ^W, and the retained earnings rate, s^c, sum to unity.

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[158] Depending on the structure of the final SAM, land and or income from the sector-specific resource may also pass through corporate accounts.

[159] The share parameters, φ^e, sum to unity.

$$CY_e = \sum_{kt} \varphi^e_{kt,e} TR^E_{k,kt} \quad \text{(Y-8)}$$

$$S^c_e = s^c_e \left(1 - \kappa^c_e\right) CY_e \quad \text{(Y-9)}$$

$$TR^H_{c,e} = \varphi^H_{c,e} \left(1 - \kappa^c_e\right) CY_e \quad \text{(Y-10)}$$

$$TR^W_{c,e} = \varphi^W_{c,e} \left(1 - \kappa^c_e\right) CY_e \quad \text{(Y-11)}$$

Household income

Aggregate household income, *YH*, is composed of eight elements: labor, land and sector-specific factor remuneration, distributed capital income and corporate profits, transfers from government and households, and foreign remittances, equation (Y-12).[160] Government transfers, in the standard closure, are fixed in real terms and are multiplied by an appropriate price index to preserve model homogeneity. Remittances, are fixed in international currency terms, and are multiplied by the exchange rate, *ER*, to convert them into local currency terms.[161]

$$\begin{aligned} YH_h = & \underbrace{\sum_l \varphi^h_{l,l} LY_l}_{\text{Labor}} + \underbrace{\sum_{kt} \varphi^h_{kt,h} TR^H_{k,kt}}_{\text{Capital}} + \underbrace{\sum_{lt} \varphi^h_{lt,h} TY_{lt}}_{\text{Land}} + \underbrace{\varphi^h_{nr,h} RY}_{\text{Sector-specific factor}} \\ & + \underbrace{\sum_e \varphi^h_{e,h} TR^H_{c,e}}_{\text{Enterprise}} + \underbrace{PLEV.TR^h_{g,h}}_{\text{Transfers from government}} + \underbrace{\sum_{h'} TR^h_{h,h'}}_{\text{Intra-household transfers}} + \underbrace{ER \sum_r TR^h_{r,h}}_{\text{Foreign remittances}} \end{aligned} \quad \text{(Y-12)}$$

$$YD_h = \left(1 - \lambda^h \kappa^h_h\right) YH_h - TR^H_h \quad \text{(Y-13)}$$

$$TR^H_h = \varphi^H_{h,h} \left(1 - \lambda^h \kappa^h_h\right) YH_h \quad \text{(Y-14)}$$

$$TR^h_{h,h'} = \varphi^h_{h,h'} TR^H_h \quad \text{(Y-15)}$$

$$TR^w_{h,r} = \varphi^w_{h,r} TR^H_h \quad \text{(Y-16)}$$

Disposable income, *YD*, is equal to after-tax income, less household transfers, equation (Y-13), where the household tax rate is κ^h. It is multiplied by an adjustment factor, λ^h, which is used for model closure. In the standard closure, government savings (or deficit), is held fixed, and the household tax schedule adjusts (uniformly) to achieve the given government fiscal balance. In other words, under this closure rule, the relative tax rates across households remain constant.[162]

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160 All share parameters within the summation signs sum to unity.

161 *ER* measures the value of local currency in terms of the international currency.

162 An alternative would be to use an additive factor, which would adjust the average tax rates, not the marginal tax rates.

Aggregate household transfers, TR^H, is a share of after tax income, equation (Y-14). This is transferred to individual households and abroad, respectively TR^h and TR^w, using constant share equations, (Y-15) and (Y-16).

Domestic final demand

Domestic final demand is composed of two broad agents—households and other domestic final demand. The model incorporates multiple households. Household demand has a uniform specification, however, with household-specific expenditure parameters. The other domestic final demand categories, in the standard model, include government current expenditures, *Gov*, private and public investment expenditures, *ZIp* and *ZIg*, exports of international trade and transport services, *TMG*, and changes in stocks, *DST*. The other domestic final demand categories, indexed by *f*, are also assumed to have a uniform expenditure function, but with agent-specific expenditure parameters. Demand at the top-level, reflects demand for the Armington good. The latter are added up across all activities in the economy and split into domestic and import components at the national level.[163]

Household expenditures

Households have a tiered demand structure, see figure 2. At the top-level, households save a constant share of disposable income, with the savings rate given by s^h. At the next level, residual income is allocated across goods and services, *XAc*, using the linear expenditure system (LES).[164] Equation (D-1) represents the LES demand function. Household consumption is the sum of two components. The first, θ, is referred to as the subsistence minimum. The second is a share of real supernumerary income. Supernumerary income is equal to residual disposable income, subtracting savings and aggregate expenditures on the subsistence minima from disposable income. The next level, undertaken at the national level, is the decomposition of Armington demand, *XAc*, into its domestic and import components, see below. Equation (D-2) determines household saving, S^h, by residual. The consumer price index, *CPI*, is defined in equation (D-3). Note that the consumer price is equal to the economy-wide Armington price, *PA*, multiplied by a household and commodity specific ad valorem tax, τ^{cc}.

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163 There are few SAMs, which would allow for agent-specific Armington behavior.

164 This class of models often uses the so-called extended linear expenditure system, which integrates household savings directly in the utility function. However, this can create calibration problems for households without savings.

$$XAc_{k,h} = \theta_{k,h} + \frac{\mu_{k,h}}{(1+\tau^{cc}_{k,h})PA_k}\left((1-s^h_h)YD_h - \sum_{k'}(1+\tau^{cc}_{k',h})PA_{k'}\theta_{k'} \right) \quad \text{(D-1)}$$

$$S^h_h = YD_h - \sum_k (1+\tau^{cc}_{k,h})PA_k XAc_{k,h} \quad \text{(D-2)}$$

$$CPI_h = \frac{\sum_k (1+\tau^{cc}_{k,h})PA_k XAc_{k,h,0}}{\sum_k (1+\tau^{cc}_{k,h,0})PA_{k,0} XAc_{k,h,0}} \quad \text{(D-3)}$$

Other domestic demand accounts

The other domestic final demand accounts all use a CES expenditure function (with the option of having fixed volume or value expenditure shares with an elasticity of 0 or 1, respectively). Equation (D-4) determines the expenditure share on goods and services, *XAf*. Equation (D-5) defines the expenditure price index, *PF*. And equation (D-6) defines the value of expenditures, *YF*. Model closure is discussed below.

$$XAf_{k,f} = \alpha^f_{k,f}\left(\frac{PF_f}{(1+\tau^{cf}_{k,f})PA_k}\right)^{\sigma^f_f} XF_f \quad \text{for} \quad f \in \{\text{Other final demand}\} \quad \text{(D-4)}$$

$$PF_f = \left[\sum_k \alpha^f_f \left((1+\tau^{cf}_{k,f})PA_k\right)^{1-\sigma^f_f}\right]^{1/(1-\sigma^f_f)} \quad \text{for} \quad f \in \{\text{Other final demand}\} \quad \text{(D-5)}$$

$$YF_f = PF_f XF_f \quad \text{for} \quad f \in \{\text{Other final demand}\} \quad \text{(D-6)}$$

Trade equations

This section discusses the modeling of trade. There are three sections—import demand, and export supply and demand. The first two use a tiered structure. Import demand is decomposed in two steps. The top tier disaggregates aggregate Armington demand into two components—demand for the domestically produced good and aggregate import demand. At the second tier, the aggregate import demand is allocated across trading partners. Both of these tiers assume that goods indexed by *k* are differentiated by region of origin, i.e. the so-called Armington assumption. A CES specification is used to model the degree of substitutability across regions of origin. The level of the elasticities will often be determined by the level of aggregation. Finely defined goods, such as wheat, would typically have a higher elasticity than more broadly defined goods, such as clothing. At the same time, non-price barriers may also inhibit the degree of substitutability, for example prohibitive transport barriers (inexistent or few transmission lines for electricity), or product and safety standards. Export supply is similarly modeled using a two-tiered constant-elasticity-of-transformation specification. This permits imperfect supply responses to changes in relative prices. Finally, the small-country assumption is relaxed for exports with the incorporation of export demand functions.

Top-level Armington nest

National demand for the Armington good, *XA*, is the sum of Armington demand over all domestic agents: intermediate demand, household and other domestic final demand, and demand generated by the internal trade and transport sector, *XAmg*, equation (T-1). Aggregate Armington demand is then allocated between domestic and import goods using a nested CES structure. Equation (T-2) represents demand for the domestically produced good, XD^d, where the top-level Armington elasticity is given by σ^m. Note that the price of the domestic good is equal to the producer price, *PD*, adjusted by the internal trade and transport margin, τ^{mg}. Demand for aggregate imports, *XMT*, is determined in equation (T-3). The price of aggregate imports is given by *PMT*.[165] The Armington price, *PA*, is defined in equation (T-4), using the familiar CES dual price aggregation formula.

$$XA_k = \sum_j XAp_{k,j} + \sum_h XAc_{k,h} + \sum_f XAf_{k,f} + \sum_m \sum_{k'} XAmg_{k,k',m} \qquad \text{(T-1)}$$

$$XD_k^d = \alpha_k^d \left(\frac{PA_k}{(1+\tau_{k,D}^{mg})\, PD_k} \right)^{\sigma_k^m} XA_k \qquad \text{(T-2)}$$

$$XMT_k = \alpha_k^m \left(\frac{PA_k}{PMT_k} \right)^{\sigma_k^m} XA_k \qquad \text{(T-3)}$$

$$PA_k = \left[\alpha_k^d \left((1+\tau_{k,D}^{mg})\, PD_k \right)^{1-\sigma_k^m} + \alpha_k^m PMT_k^{1-\sigma_k^m} \right]^{1/(1-\sigma_k^m)} \qquad \text{(T-4)}$$

Second-level Armington nest

At the second level, aggregate import demand, *XMT*, is allocated across trading partners using a CES specification. Equation (T-5) defines the domestic price of imports, *PM*.[166] It is equal to the world price (in international currency), *WPM*, multiplied by the exchange rate, and adjusted for by the import tariff, τ^m, i.e. *PM* represents the port-price of imports, tariff-inclusive. The tariff rate is both sector- and region of origin-specific. Equation (T-6) represents the import of commodity *k* from region *r*, *XM*, where the inter-regional substitution elasticity is given by σ^w. The relevant consumer price includes the internal trade and transport margin, τ^{mg}. The aggregate price of imports, *PMT*, is defined in equation (T-7).

165 It includes the trade and transport margins, sales tax, and import tariffs.

166 *PM* and *WPM* are indexed by both commodity, *k*, and trading partner, *r*.

$$PM_{k,r} = ER.WPM_{k,r}(1+\tau^{m}_{k,r}) \qquad \text{(T-5)}$$

$$XM_{k,r} = \alpha^{w}_{k,r}\left(\frac{PMT_k}{(1+\tau^{mg}_{k,M})\,PM_{k,r}}\right)^{\sigma^{w}_{k}} XMT_k \qquad \text{(T-6)}$$

$$PMT_k = \left[\sum_r \alpha^{w}_{k,r}\left((1+\tau^{mg}_{k,M})\,PM_{k,r}\right)^{1-\sigma^{w}_{k}}\right]^{1/(1-\sigma^{w}_{k})} \qquad \text{(T-7)}$$

Top-level CET nest

Domestic production is allocated across markets using a nested CET specification. At the top nest, producers allocate production between the domestic market and aggregate exports. At the second nest, aggregate exports are allocated across trading partners. The model allows for perfect transformation, i.e. producers perceive no difference across markets. In this case, the law-of-one-price holds. Equation (T-8) represents the link between the domestic producer price, *PE*, and the world price, *WPE*. Export prices are both sector- and region-specific. The FOB price, *WPE*, includes domestic trade and transport margins, τ^{mg}[167], as well as export taxes/subsidies, τ^{e}. Equations (T-9) and (T-10) represent the CET optimality conditions. The first determines the share of domestic supply, *X*, allocated to the domestic market, XD^{s}. The second determines the supply of aggregate exports, *XET*. *PET* represents the price of aggregate export supply. The transformation elasticity is given by σ^{x}. The model allows for perfect transformation. In this case, the optimal supply conditions are replaced by the law-of-one price conditions. Equation (T-11) represents the CET aggregation function. In the case of finite transformation, it is replaced with its equivalent, the CET dual price aggregation function. In the case of infinite transformation, the primal aggregation function is used, where the two components are summed together since there is no product differentiation.

[167] Note that the domestic trade and transport margins are differentiated for three different goods: domestically produced goods sold to the domestic market, exported goods, and imported goods.

$$PE_{k,r}\left(1+\tau^{mg}_{k,X}\right)\left(1+\tau^{e}_{k,r}\right)=ER.WPE_{k,r} \qquad \text{(T-8)}$$

$$\begin{cases} XD^s_k=\gamma^d_k\left(\dfrac{PD_k}{P_k}\right)^{\sigma^x_k}X_k & \text{if} \quad \sigma^x_k\neq\infty \\ PD_k=P_k & \text{if} \quad \sigma^x_k=\infty \end{cases} \qquad \text{(T-9)}$$

$$\begin{cases} XET_k=\gamma^e_k\left(\dfrac{PET_k}{P_k}\right)^{\sigma^x_k}X_k & \text{if} \quad \sigma^x_k\neq\infty \\ PET_k=P_k & \text{if} \quad \sigma^x_k=\infty \end{cases} \qquad \text{(T-10)}$$

$$\begin{cases} P_k=\left[\gamma^d_k PD^{1+\sigma^x_k}_k+\gamma^e_k PET^{1+\sigma^x_k}_k\right]^{1/(1+\sigma^x_k)} & \text{if} \quad \sigma^x_k\neq\infty \\ X_k=XD^s_k+XET_k & \text{if} \quad \sigma^x_k=\infty \end{cases} \qquad \text{(T-11)}$$

Second-level CET nest

The second-level CET nest allocates aggregate export supply, *XET*, across the various export markets, *XE*. Equation (T-12) represents the optimal allocation decision, where σ^z is the transformation elasticity. Equation (T-13) represents the CET aggregation function, where again, the CET dual price formula is used to determine the aggregate export price, *PET*. As above, the model allows the transformation elasticity to be infinite.

$$\begin{cases} XE_{k,r}=\gamma^x_{k,r}\left(\dfrac{PE_{k,r}}{PET_k}\right)^{\sigma^z_k}XET_k & \text{if} \quad \sigma^z_k\neq\infty \\ PE_{k,r}=PET_k & \text{if} \quad \sigma^z_k=\infty \end{cases} \qquad \text{(T-12)}$$

$$\begin{cases} PET_k=\left[\displaystyle\sum_r \gamma^x_{k,r}PE^{1+\sigma^z_k}_{k,r}\right]^{1/(1+\sigma^z_k)} & \text{if} \quad \sigma^z_k\neq\infty \\ XET_k=\displaystyle\sum_r XE_{k,r} & \text{if} \quad \sigma^z_k=\infty \end{cases} \qquad \text{(T-13)}$$

Export demand

Export, *ED*, demand is specified using a constant elasticity function, equation (T-14). If the elasticity, η^e, is finite, demand decreases as the international price of exports, *WPE*, increases. The numerator contains an exogenous export price competitive index. If the latter increases relative to the domestic export price, market share of the domestic exporter would increase. The model allows for infinite demand elasticity. This represents the small-country assumption. In this case, the domestic price of exports (in international currency units) is constant. If the two

CET elasticities are likewise infinite, then the domestic producer price is also equal to the world price of exports (adjusted for taxes and trade and transportation margins).

$$\begin{cases} ED_{k,r} = \alpha^e_{k,r}\left(\dfrac{\overline{WPE}_{k,r}}{WPE_{k,r}}\right)^{\eta^e_{k,r}} & \text{if} \quad \eta^e_{k,r} \neq \infty \\ WPE_{k,r} = \overline{WPE}_{k,r} & \text{if} \quad \eta^e_{k,r} = \infty \end{cases} \qquad \text{(T-14)}$$

Domestic trade and transportation margins

The marketing of each good—domestic, imports, and exports—is associated with a commodity specific trade margin.[168] Equations (M-1) through (M-3) define the revenues associated with the domestic trade and transport margins. Domestically produced goods sold domestically generate $Y^{mg}_{.,D}$. Imported goods generate $Y^{mg}_{.,M}$. And exported goods generate $Y^{mg}_{.,X}$. Equation (M-4) defines the volume of margin services. The production of the trade and transport services follows a Leontief technology. Equation (M-5) defines the demand for goods and services. In other words, to deliver commodity *k'* (in either sector *D*, *M*, or *X*) requires an input from commodity *k*, the level of which is fixed in proportions to the overall volume of delivering commodity *k'* in the economy, $XT^{mg}_{k'}$. Equation (M-6) is the expenditure deflator, $PT^{mg}_{k'}$, for individual trade margin activities.

$$YT^{mg}_{k,D} = \tau^{mg}_{k,D} PD_k XD^d_k \qquad \text{(M-1)}$$

$$YT^{mg}_{k,M} = \sum_r \tau^{mg}_{k,M} PM_{k,r} XM_{k,r} \qquad \text{(M-2)}$$

$$YT^{mg}_{k,X} = \sum_r \tau^{mg}_{k,X} PE_{k,r} XE_{k,r} \qquad \text{(M-3)}$$

$$XT^{mg}_{k,m} = YT^{mg}_{k,m} / PT^{mg}_{k,m} \qquad \text{(M-4)}$$

$$XAmg_{k,k',m} = \alpha^{mg}_{k,k',m} XT^{mg}_{k',m} \qquad \text{(M-5)}$$

$$PT^{mg}_{k',m} = \sum_k \alpha^{mg}_{k,k',m} PA_k \qquad \text{(M-6)}$$

Goods market equilibrium

There are three fundamental commodities in the model—domestic goods sold domestically, imports (by region of origin), and exports (by region of destination). All other goods are bundles (i.e. are defined using an aggregation function) and do not require supply/demand balance. The

[168] The model does not include international trade and transport margins. A change in the latter could be simulated by a change in the relevant world price index, *WPM* or $\overline{WPE}$.

small-country assumption holds for imports, and therefore any import demand can be met by the rest of the world with no impact on the price of imports. Therefore, there is no explicit supply/demand equation for imports.[169] Equation (E-1) represents equilibrium on the domestic goods market, and essentially determines, *PD*, the producer price of the domestic good. Equation (E-2) defines the equilibrium condition on the export market. With a finite export demand elasticity, the equation determines *WPE*, the world price of exports. With an infinite export demand elasticity, the equation trivially equates export demand to the given export supply.

$$XD_k^d = XD_k^s \quad \text{(E-1)}$$

$$ED_{k,r} = XE_{k,r} \quad \text{(E-2)}$$

Macro closure

Macro closure involves determining the exogenous macro elements of the model. The standard closure rules are the following:

- Government fiscal balance is exogenous, achieved with an endogenous direct tax schedule
- Private investment is endogenous and is driven by available savings
- The volume of government current and investment expenditures is exogenous
- The volume of demand for international trade and transport services is exogenous
- The volume of stock changes is exogenous
- The trade balance (i.e. capital flows) is exogenous. The real exchange rate equilibrates the balance of payments.

These are detailed further below.

Government accounts

Equation (C-1) defines total government revenues, *GY*. There are 10 components: revenues from the production tax, sales tax, import tax, export tax, land, capital and wage tax, corporate and household direct taxes, and transfers from the rest of the world. Equation (C-2) defines the government's current expenditures, *GEXP*. It is the sum of three components: expenditures on goods and services, transfers to households, and transfers to ROW. Government savings (on current operations), S^g, is defined in equation (C-3), as the difference between revenues and current expenditures. Real government savings, *RSg*, is defined in equation (C-4). It is this latter which essentially determines the level of direct household taxation since *RSg* is exogenous in the standard closure.

169 One could rather easily add an import supply equation and an equilibrium condition.

$$
\begin{aligned}
GY \quad = \quad & \underbrace{\sum_k \sum_j \tau^{cp}_{k,j} PA_k XAp_{k,j}}_{\text{Sales tax on intermediate demand}} + \underbrace{\sum_k \sum_h \tau^{cc}_{k,h} PA_k XAc_{k,h}}_{\text{Sales tax on household demand}} + \underbrace{\sum_k \sum_f \tau^{cf}_{k,f} PA_k XAf_{k,f}}_{\text{Sales tax on other final demand}} \\
+ \quad & \underbrace{ER \sum_k \sum_r \tau^m_{k,r} WPM_{k,r} XM_{k,r}}_{\text{Import tariff revenues}} + \underbrace{\sum_k \sum_r \tau^e_{k,r} (1+\tau^{mg}_{k,X}) PE_{k,r} XE_{k,r}}_{\text{Export tax revenues}} \\
+ \quad & \underbrace{\sum_{lt} \sum_i \frac{\tau^{ft}_{i,lt} PT_{i,lt} T^d_{i,lt}}{1+\tau^{ft}_{i,lt}}}_{\text{Land tax}} + \underbrace{\sum_{kt} \sum_i \frac{\tau^{fk}_{i,kt} R_{i,kt} K^d_{i,kt}}{1+\tau^{fk}_{i,kt}}}_{\text{Capital tax}} + \underbrace{\sum_l \sum_i \frac{\tau^{fl}_{i,l} W_{i,l} L^d_{i,l}}{1+\tau^{fl}_{i,l}}}_{\text{Wage tax}} + \underbrace{\sum_i \frac{\tau^{fr}_i PR_i NR^d_i}{1+\tau^{fr}_i}}_{\text{Resource tax}} \\
+ \quad & \underbrace{\sum_i \tau^p_i PX_i XP_i}_{\text{Production tax}} + \underbrace{\sum_e \kappa^c_e CY_e}_{\text{Corporate tax}} + \underbrace{\lambda^h \sum_h \kappa^h_h YH_h}_{\text{Income tax}} + \underbrace{ER \sum_r TR^g_{W,r}}_{\text{Transfers from ROW}}
\end{aligned}
\qquad \text{(C-1)}
$$

$$GEXP = YF_{Gov} + PLEV \sum_h TR^H_{g,h} + ER \sum_r TR^W_{g,r} \qquad \text{(C-2)}$$

$$S^g = GY - GEXP \qquad \text{(C-3)}$$

$$RSg = S^g / PLEV \qquad \text{(C-4)}$$

Investment and macro closure

Equation (C-5) defines the investment savings balance. In the standard closure, it determines the level of private investment since public investment and stock changes are exogenous. These three components are financed by aggregate savings defined over corporations, households, and the government, and adjusted by foreign savings. The latter is fixed (in international currency terms). Equations (C-6) through (C-9) define the exogenous volumes of public current and investment expenditures, exports of international trade and transport services and stock changes. The aggregate price level, *PLEV*, is the average absorption (Armington) price, equation (C-10). Equation (C-11) represents the balance of payments (in international currency terms). It can be shown to be redundant, and is dropped from the model specification.

$$YF_{ZIp} + YF_{ZIg} + YF_{DST} = \sum_e S_e^c + \sum_h S_h^h + S^g + ER.\sum_r S_r^f \quad \text{(C-5)}$$

$$XF_{Gov} = \overline{XF}_{Gov} \qquad \text{(C-6)}$$

$$XF_{ZIg} = \overline{XF}_{ZIg} \quad \text{(C-7)}$$

$$XF_{TMG} = \overline{XF}_{TMG} \qquad \text{(C-8)}$$

$$XF_{DST} = \overline{XF}_{DST} \qquad \text{(C-9)}$$

$$PLEV = \frac{\sum_k PA_k XA_{k,0}}{\sum_k PA_{k,0} XA_{k,0}} \qquad \text{(C-10)}$$

$$\begin{aligned} BoP \quad &= \quad \sum_r \sum_k WPE_{k,r} XE_{k,r} + YF_{TMG} + \sum_h TR_{W,h}^h + TR_W^g + S^f \\ &- \quad \sum_r \sum_k WPM_{k,r} XM_{k,r} - \frac{\sum_{kt} TR_{k,kt}^W + \sum_e TR_{c,e}^W + \sum_h TR_h^w}{ER} - TR_g^W \quad \text{(C-11)} \\ &\equiv \quad 0 \end{aligned}$$

Factor market equilibrium

The following sections describe the standard factor market equilibrium conditions.[170]

Labor markets

Labor markets are assumed to clear. Equation (F-1) sets aggregate demand, by skill-level, to aggregate supply, L^s. This equation determines the equilibrium wage, W^e.[171] Equation (F-2) equates sectoral wages to the equilibrium wage, i.e. the model assumes uniform wages across sectors.[172]

$$L_l^s = \sum_i L_{i.l}^d \qquad \text{(F-1)}$$

$$W_{i,l} = W_l^e \qquad \text{(F-2)}$$

[170] More detailed analysis may require more market segmentation, e.g. rural versus urban labor markets, though so of this segmentation can be picked up by the data itself.

[171] Market structure can emulate perfect market segmentation by an appropriate definition of labor skills. For example, unskilled rural labor can assume to be only employed in rural sectors, whereas unskilled urban labor is only employed in urban sectors. Perfect market segmentation, as modeled here, does not allow for migration.

[172] Quite a few alternatives could be used allowing for sector-specific wages, for example union wage bargaining models, efficiency wages, etc.

Capital market

Equilibrium on the capital market allows for both limiting cases—perfect capital mobility and perfect capital immobility, or any intermediate case. Aggregate capital, K^s, is allocated across sectors and type according to a nested CET system. At the top-level, the aggregate investor allocates capital across types, according to relative rates of return. Equation (F-3) determines the optimal supply decision, where TK^s is the supply of capital of type *kt*, with an average return of *PTK*. *PK* is the aggregate rate-of-return to capital. If the supply elasticity is infinite, the law-of-one-price holds. Equation (F-4) represents the top-level aggregation function, replaced by the CET dual price function in the case of a finite transformation elasticity. Perfect capital mobility is represented by setting ω^{kt} to infinity. Perfect immobility is modeled by setting the transformation elasticity to 0.

$$\begin{cases} TK_{kt}^{s} = \gamma_{kt}^{tks}\left(\dfrac{PTK_{kt}}{PK}\right)^{\omega^{kt}} K^{s} & \text{if} \quad \omega^{kt} \neq \infty \\ PTK_{kt} = PK & \text{if} \quad \omega^{kt} = \infty \end{cases} \qquad \text{(F-3)}$$

$$\begin{cases} PK = \left[\sum_{kt} \gamma_{kt}^{tks} PTK_{kt}^{1+\omega^{kt}}\right]^{1/(1+\omega^{kt})} & \text{if} \quad \omega^{kt} \neq \infty \\ K^{s} = \sum_{kt} TK_{kt}^{s} & \text{if} \quad \omega^{kt} = \infty \end{cases} \qquad \text{(F-4)}$$

At the second level, capital by type, TK^s, is allocated across sectors using another CET function. Equation (F-5) determines the optimal allocation of capital of type *kt* to sector *i*, K^s, where the transformation elasticity is ω^k. Equation (F-6) represents the CET aggregation function. The equilibrium return to capital, *R*, is determined by equation capital supply to demand, equation (F-7).[173]

$$\begin{cases} K_{i,kt}^{s} = \gamma_{i,kt}^{k}\left(\dfrac{R_{i,kt}}{PTK_{kt}}\right)^{\omega^{k}} TK_{kt}^{s} & \text{if} \quad \omega^{k} \neq \infty \\ R_{i,kt} = PTK_{kt} & \text{if} \quad \omega^{k} = \infty \end{cases} \qquad \text{(F-5)}$$

$$\begin{cases} PTK_{kt} = \left[\sum_{i} \gamma_{i,kt}^{k} R_{i,kt}^{1+\omega^{k}}\right]^{1/(1+\omega^{k})} & \text{if} \quad \omega^{k} \neq \infty \\ TK_{kt} = \sum_{i} K_{i,kt}^{s} & \text{if} \quad \omega^{k} = \infty \end{cases} \qquad \text{(F-6)}$$

$$K_{i,kt}^{d} = K_{i,kt}^{d} \qquad \text{(F-7)}$$

173 If the transformation elasticity is infinite, equation (F-5) determines the sector- and type-specific rate of return using the law-of-one price, and equation (F-7) trivially sets capital supply equal to capital demand.

Land market

Land market equilibrium is specified in an analogous way to the capital market with a tiered CET supply system. The first tier allocates total land across types. This could have a zero transformation elasticity if for example land used for rice production could not be used to produce other commodities. Their respective prices are *PLAND* and PTT^s.

$$\begin{cases} TT_{lt}^{s} = \gamma_{lt}^{tts}\left(\dfrac{PTT_{lt}^{s}}{PLAND}\right)^{\omega^{tl}} LAND & \text{if} \quad \omega^{tl} \neq \infty \\ PTT_{lt}^{s} = PLAND & \text{if} \quad \omega^{tl} = \infty \end{cases} \qquad \text{(F-8)}$$

$$\begin{cases} PLAND = \left[\sum_{lt} \gamma_{lt}^{tts}\left(PTT_{cl}^{s}\right)^{1+\omega^{tl}}\right]^{1/(1+\omega^{tl})} & \text{if} \quad \omega^{tl} \neq \infty \\ LAND = \sum_{lt} TT_{lt}^{s} & \text{if} \quad \omega^{tl} = \infty \end{cases} \qquad \text{(F-9)}$$

Equations (F-10) and (F-11) determine the optimality conditions at the second and final tier, determining land supply (by type and) by sector of use. Land market equilibrium is represented by equation (F-12).

$$\begin{cases} T_{i,lt}^{s} = \gamma_{i,lt}^{t}\left(\dfrac{PT_{i,lt}}{PTT_{lt}^{s}}\right)^{\omega_{lt}^{t}} TT_{lt}^{s} & \text{if} \quad \omega_{lt}^{t} \neq \infty \\ PT_{i,lt} = PTT_{lt}^{s} & \text{if} \quad \omega_{lt}^{t} = \infty \end{cases} \qquad \text{(F-10)}$$

$$\begin{cases} PTT_{lt}^{s} = \left[\sum_{i} \gamma_{i,lt}^{t} PT_{i,lt}^{1+\omega^{t}}\right]^{1/(1+\omega_{lt}^{t})} & \text{if} \quad \omega_{lt}^{t} \neq \infty \\ TT_{lt}^{s} = \sum_{i} T_{i,lt}^{s} & \text{if} \quad \omega_{lt}^{t} = \infty \end{cases} \qquad \text{(F-11)}$$

$$T_{i,lt}^{s} = T_{i,lt}^{d} \qquad \text{(F-12)}$$

Natural resource market

The market for natural resources differs from the others in the sense that there is no inter-sectoral mobility, i.e. this is a sector specific resource. There is therefore a sector specific supply curve (eventually flat).[174] Equation (F-13) describes the sector-specific supply function, or NR^s. Equation (F-14) then determines the equilibrium price, *PR*.

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[174] More realistic models allow for kinked supply curves. It is typically easier to take resources out of production than to bring them online—the latter requiring new investments and/or new exploration. Thus a so-called down supply elasticity would be higher than a so-called up supply elasticity.

$$\begin{cases} NR_i^s = \gamma_i^{nr}\left(\dfrac{PR_i}{PLEV}\right)^{\omega^{nr}} & \text{if} \quad \omega^{nr} \neq \infty \\ PR_i = PLEV.PR_{i,0} & \text{if} \quad \omega^{nr} = \infty \end{cases} \qquad \text{(F-13)}$$

$$NR_i^d = NR_i^s \qquad \text{(F-14)}$$

Macroeconomic identities

The macroeconomic identities are not normally needed for the model specification, i.e. they could be calculated at the end of a simulation. In the case of dynamic scenarios, one or more of them could be used to calibrate dynamic parameters to a given set of exogenous assumptions. For example, the growth of GDP could be made exogenous. In this case, a growth parameter, typically a productivity factor, would be endogenous and set to target the given growth path of GDP.

Equations (I-1) and (I-2) define nominal and real GDP, respectively, at market prices. Equation (I-3) is the GDP at market price deflator. Similarly, equations (I-4) and (I-5) define nominal and real GDP at factor cost. Note that real GDP at factor cost is evaluated in efficiency units.[175] Equation (I-6) defines the GDP at factor cost deflator.

$$\begin{aligned} GDPMP \quad = \quad & \sum_k \sum_h (1+\tau_{k,h}^{cc}) PA_k XAc_{k,h} + \sum_k \sum_f (1+\tau_{k,f}^{cf}) PA_k XAf_{k,f} \\ + \quad & ER \sum_k \sum_r WPE_{k,r} XE_{k,r} - \sum_k \sum_r PM_{k,r} (1+\tau_{k,M}^{mg})\ XM_{k,r} \end{aligned} \qquad \text{(I-1)}$$

$$\begin{aligned} RGDPMP \quad = \quad & \sum_k \sum_h (1+\tau_{k,c,0}^{cc}) PA_{k,0} XAc_{k,h} + \sum_k \sum_f (1+\tau_{k,f,0}^{cf}) PA_{k,0} XAf_{k,f} \\ + \quad & ER_0 \sum_k \sum_r WPE_{k,r,0} XE_{k,r} - \sum_k \sum_r PM_{k,r,0} (1+\tau_{k,M,0}^{mg})\ XM_{k,r} \end{aligned} \qquad \text{(I-2)}$$

$$PGDPMP = GDPGMP / RGDPMP \qquad \text{(I-3)}$$

$$GDPFC = \sum_l \sum_i W_{i,l} L_{i,l}^d + \sum_{kt} \sum_i R_{i,kt} K_{i,kt}^d + \sum_{lt} \sum_i PT_{i,lt} T_{i,lt}^d + \sum_i PR_i NR_i^d \quad \text{(I-4)}$$

$$\begin{aligned} RGDPFC \quad = \quad & \sum_l \sum_i W_{i,l,0} \lambda_{i,l}^l L_{i,l}^d + \sum_{kt} \sum_i R_{i,kt,0} \lambda_{i,kt}^k K_{i,kt}^d \\ + \quad & \sum_{lt} \sum_i PT_{i,lt,0} \lambda_{i,lt}^t T_{i,lt}^d + \sum_i PR_{i,0} \lambda_i^r NR_i^d \end{aligned} \qquad \text{(I-5)}$$

$$PGDPFC = GDPGFC / RGDPFC \qquad \text{(I-6)}$$

175 So is nominal GDP at factor cost, but the efficiency factors cancel out in the equation since the nominal wage is divided by the efficiency factor to derive the efficiency wage.

Growth equations

In a simple dynamic framework, equation (G-1) defines the growth rate of GDP at market price. Equation (G-2) determines the growth rate of labor productivity. The growth rate has two components, a uniform factor applied in all sectors to all types of labor, gl, and a sector- and skill-specific factor, xl. In defining a baseline, the growth rate of GDP is exogenous. In this case, equation (G-1) is used to calibrate the gl parameter. In policy simulations, gl is given, and equation (G-1) defines the growth rate of GDP. Other elements of simple dynamics include exogenous growth of labor supply, exogenous growth rates of capital and land productivity (typically 0), and investment driven capital accumulation, equation (G-3).[176]

$$RGDPMP = (1 + g^y)\ RGDPMP_{-1} \qquad \text{(G-1)}$$

$$\lambda^l_{ip,l} = (1 + \gamma^l + \chi^l_{ip,l})\ \lambda^l_{ip,l,-1} \qquad \text{(G-2)}$$

$$K^s = (1 - \delta)K^s_{-1} + XF_{ZIp,-1} \qquad \text{(G-3)}$$

176 Note that public investment, in this version of the model, has no impact on production technology.

Figure 1: Nested structure of production

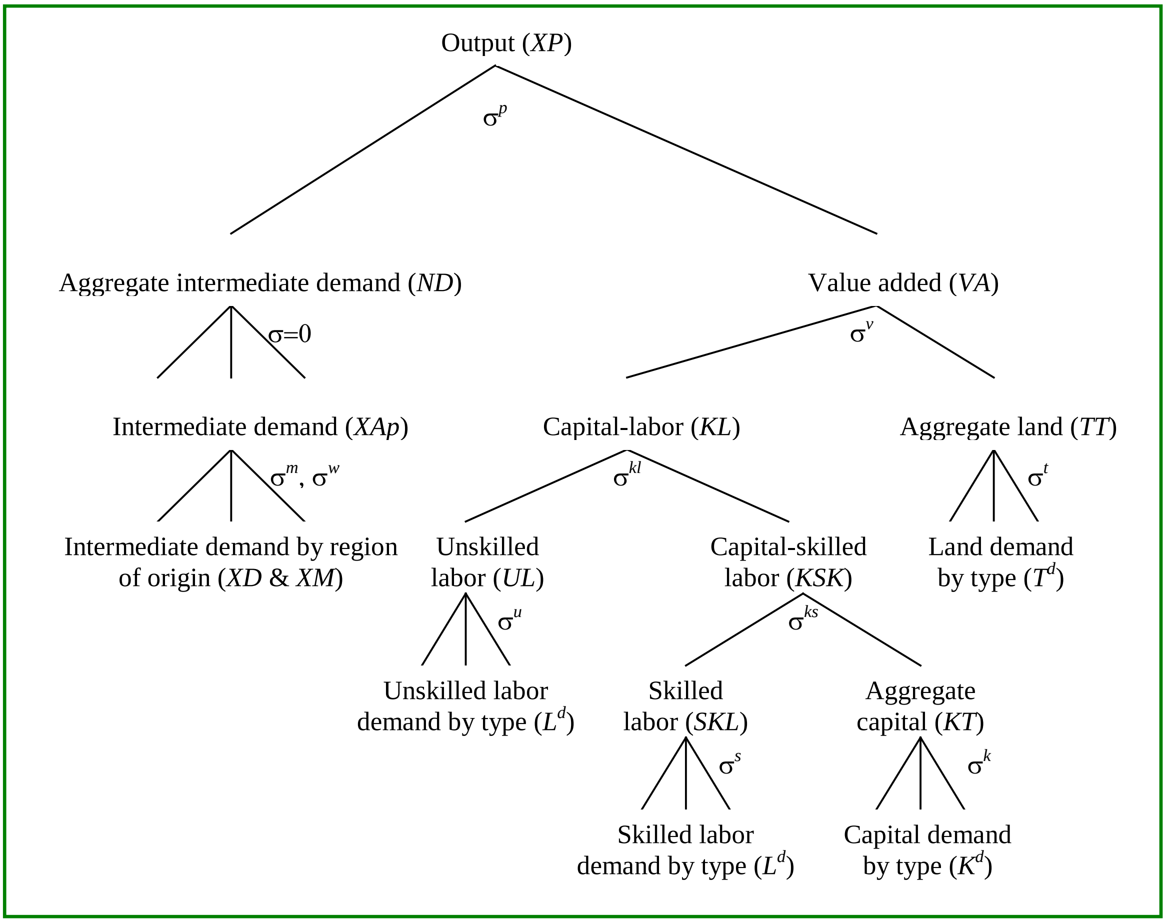
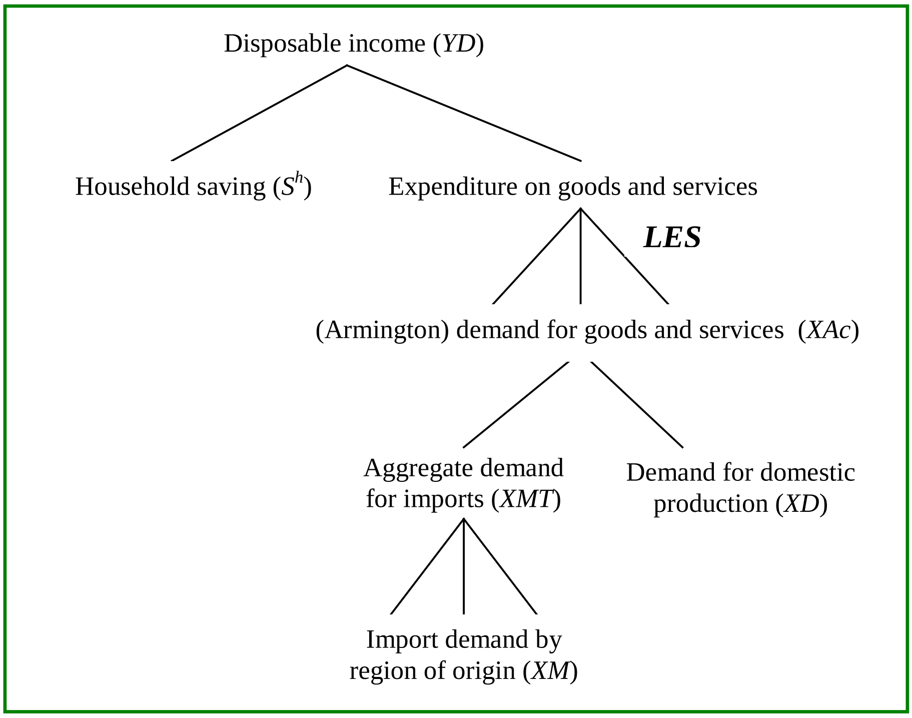


Figure 2: Nested structure of consumer demand



13. A Simplified Multilateral GE Model

Part IV Applications

14. Trade Policy and Poverty Alleviation

15. Labor Markets and Migration

16. Energy and Environment

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Annex
17. GAMS Output and Excel Pivot Tables

Saving GAMS Results in Database Format

The GAMS "put" facility enables virtually unlimited possibilities for saving GAMS results to text (i.e. ASCII) files. Later versions of Excel (and perhaps other spreadsheet programs) include a very powerful feature for structuring database type data in tabular format—these are known as pivot tables. Pivot tables are particularly useful for analyzing data of greater than two dimensions. Examples of these abound in GAMS programs. For example, macro data might have three dimensions if it is defined by variable name and has an index for time and scenario. Sectoral data may have three dimensions or more. Sectoral data will be defined by variable name, and indexed by sector, time, and scenario. Multi-regional models add an extra dimension. The best way to transfer this from GAMS is in database format. A database in text format has one record (i.e. line) per data item. The first line in a data base file contains the name of the fields of the record. For example, to save the macro results from a dynamic scenario would require GAMS code such as:

```
* Define the output file
 file report / 'BaU.csv' / ;
 put report ;
* Define the reporting years
 set tr(t) reporting years ;
 tr(t) = yes ;
*----- Write the header
 put "Title,Variable,Sector,Year,Value" / ;
*----- Loop over the reporting years
 loop(tr,
*----- Output the macro data
    put system.title, ",rgdp,,"      , tr.tl:4",",rgdpT(tr)     / ;
    put system.title, ",TCons,,"     , tr.tl:4",",TConsT(tr)    / ;
    put system.title, ",TInv,,"      , tr.tl:4",",TInvT(tr)     / ;
    put system.title, ",TGov,,"      , tr.tl:4",",TGovT(tr)     / ;
    put system.title, ",TExp,,"      , tr.tl:4",",TExpT(tr)     / ;
    put system.title, ",TImp,,"      , tr.tl:4",",TImpT(tr)     / ;
 ) ;
```

After opening the file, the first put statement writes the data base fields. In the example above, there are five fields: the scenario title, the name of the output variable, and the corresponding sector, year, and value. The output routine loops over the number of reporting years. In this case, it is all years of the simulation, but the subset *tr* can be defined to be less than the full number of years. (N.B. This structure can also be used for a sequence of comparative static experiments where the index time is replaced by simple ordinal indices rather than by defining calendar years.) Macro variables have no sector definitions, so in the output of each record, the sector definition is blank, and the consecutive commas indicate this. The reason why the sector field may be included is that often the output will mix both sectoral data and macro data. These could be separated into different files, but it is also possible to filter the data when being read into

Excel which makes this unnecessary (see below). Sectoral data can be saved into the same file by GAMS code such as:

```
*   Output the sectoral data

  loop(i, put system.title,",rent,",  i.tl:10,",",tr.tl,",", rentT(i,tr)  / ; ) ;
  loop(i, put system.title,",wage,",  i.tl:10,",",tr.tl,",", wageT(i,tr)  / ; ) ;
  loop(i, put system.title,",xp,",    i.tl:10,",",tr.tl,",", XPT(i,tr)    / ; ) ;
  loop(i, put system.title,",ES,",    i.tl:10,",",tr.tl,",", EST(i,tr)    / ; ) ;
  loop(i, put system.title,",XD,",    i.tl:10,",",tr.tl,",", XDT(i,tr)    / ; ) ;
  loop(i, put system.title,",XM,",    i.tl:10,",",tr.tl,",", XMT(i,tr)    / ; ) ;
```

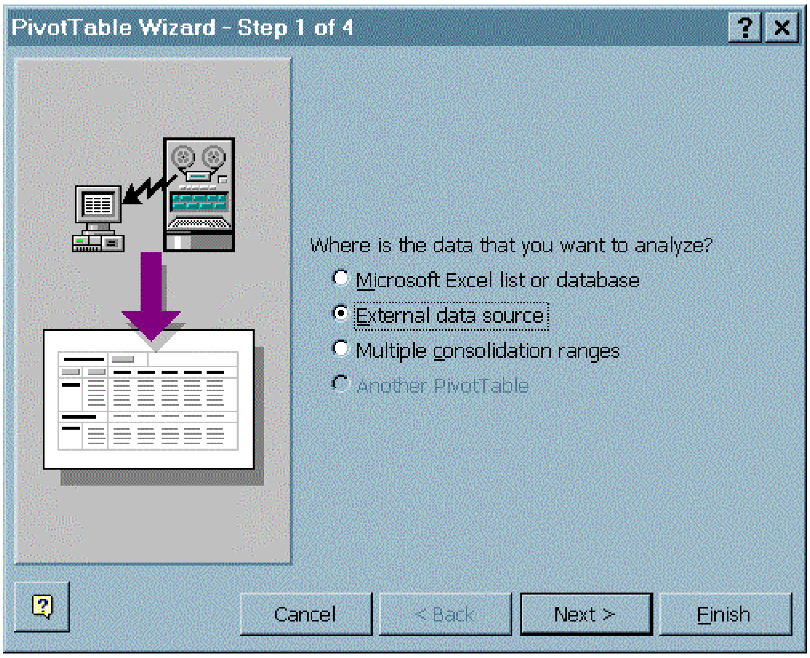
The inner loops ranging over the sectoral indices *i*, are contained in the outer loop over the reporting years *tr*.

In many cases it is desirable to compare the results of different simulations. In this case, all one has to do is to concatenate the different output files into a single file. Only the first file should contain the field descriptors. All subsequent files should only contain data records. There are a variety of ways to concatenate files. The DOS copy command works well, as does cutting and pasting from any editor or word processor (remembering to save the file as text only if that is necessary). (N.B. Some of the result files may have spurious blank lines which will show up as blank records in the pivot table. These can either be deleted in an editor, or else the blank records can be hidden from within the pivot table.)

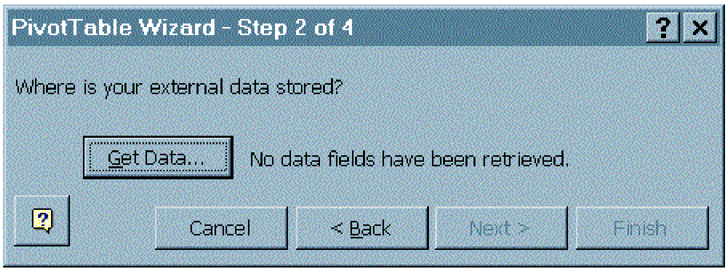
Excel Pivot Tables

To create a pivot table from a text (or CSV) file, start Excel.[177] Typically one initiates the process from a blank work sheet, but this is not strictly necessary. Under the Data Menu, choose the item PivotTable Report... which starts the PivotTable Wizard and brings up the following screen:

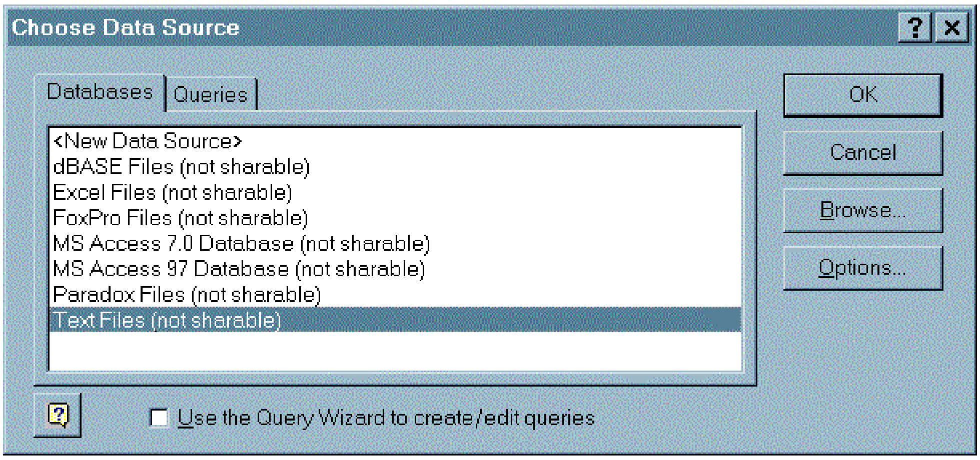
[177] This assumes that the pivot table function, with all of its options have been fully installed during the installation of Microsoft Office. In particular, reading pivot tables from external databases requires the installation of Microsoft Query, plus the pre-defined data base types available with Microsoft Query. For our purposes, these pre-defined databases include text and csv files. While the windows and examples in this document have been prepared with Office 97 and Windows NT Version 4, the functions should be similar using other versions of Excel (starting with Version 5 and other operating systems).



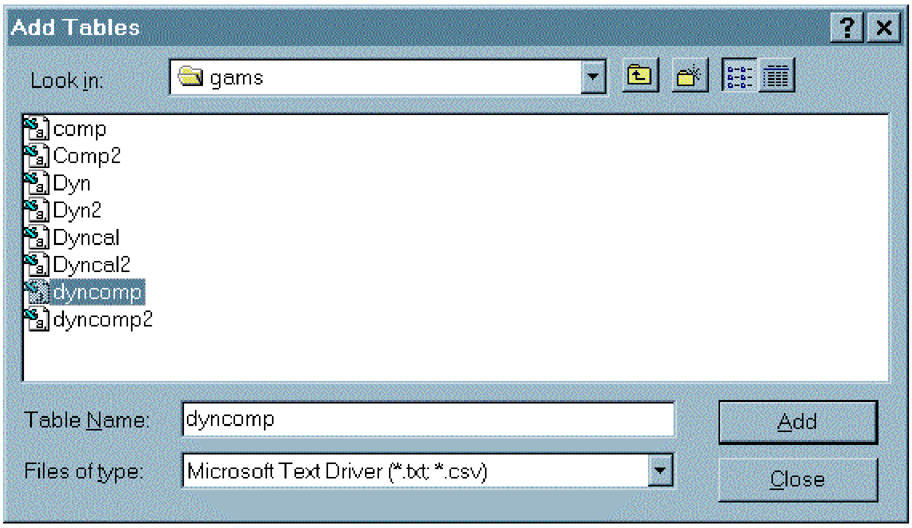
Since the data is located in an external text file click on the button indicated by External data source and then click on the Next > button. This will bring up Step 2 of the PivotTable Wizard which is indicated by the following screen:



This step proceeds to retrieve the data from the external data source, it actually builds a link between the Excel pivot table and the data in the text file. To retrieve the data, click on the button Get Data... This action will start a new procedure known as Microsoft Query which is built into MS Office, and starts with the following screen:

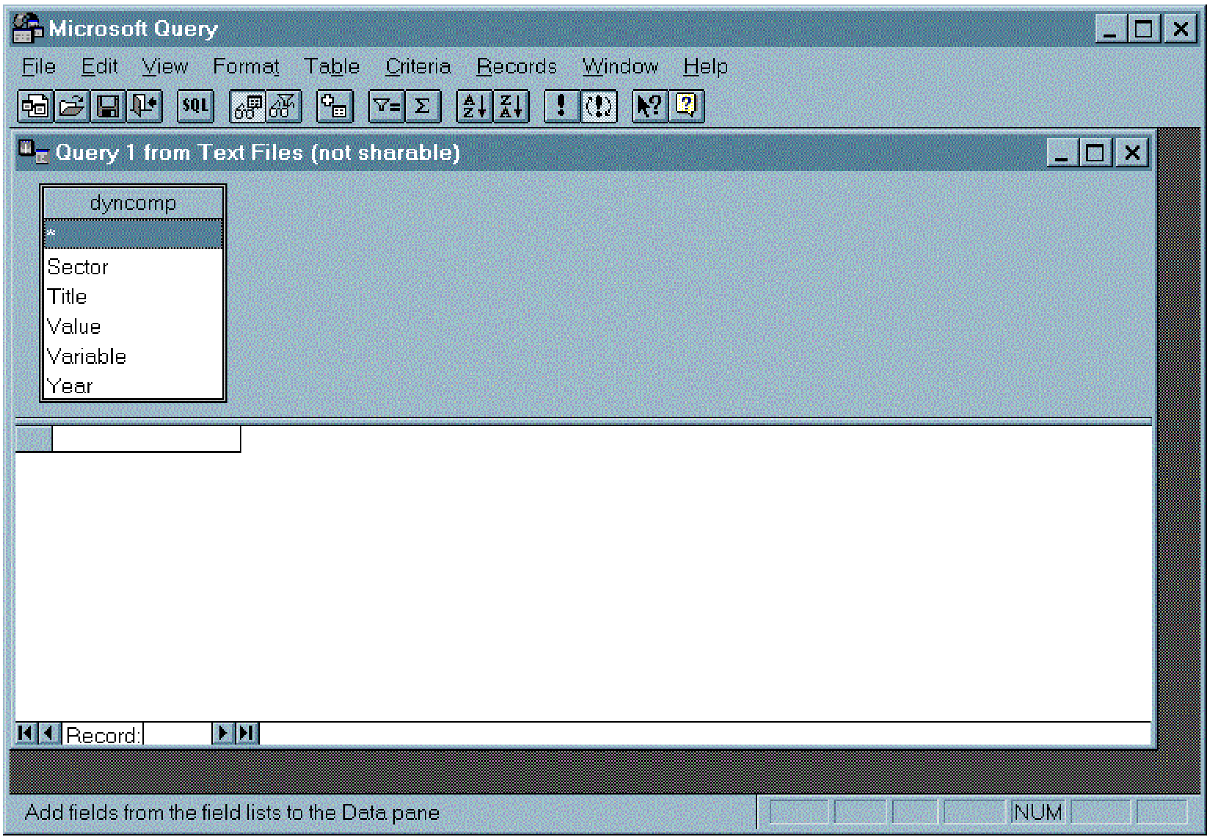


Click once on Text Files (not sharable) and then on the OK button, or double click on the text choice. Microsoft Query will then bring up the following screen:

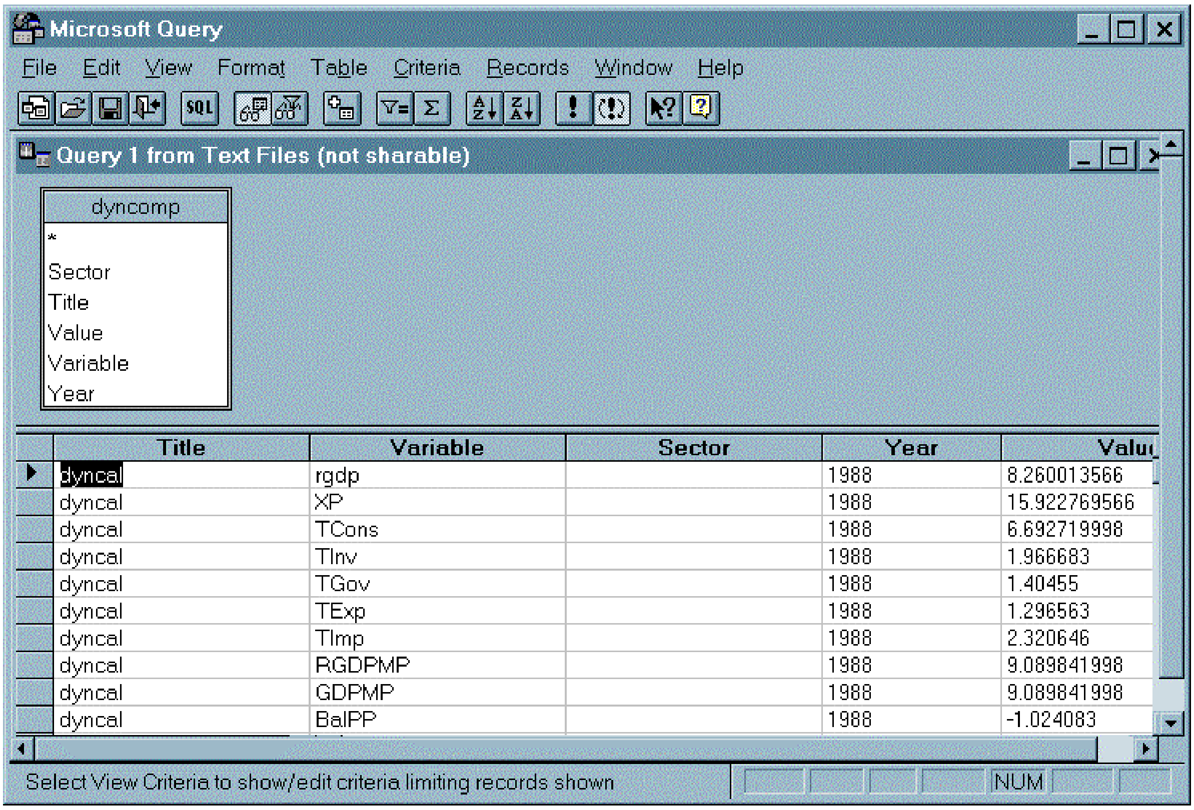


At this point, choose the name of the file containing the GAMS results in database format. The file extension should be either **txt** or **csv**. It may be necessary to navigate through some folders to find the right file. Click (once) on the appropriate file name, and then click once on the Add, followed by clicking once on the Close button.

Assuming a link has been made with the external file (called dyncomp in this case), Microsoft Query will parse the first line of the database into the different fields. The next window will look like the following:

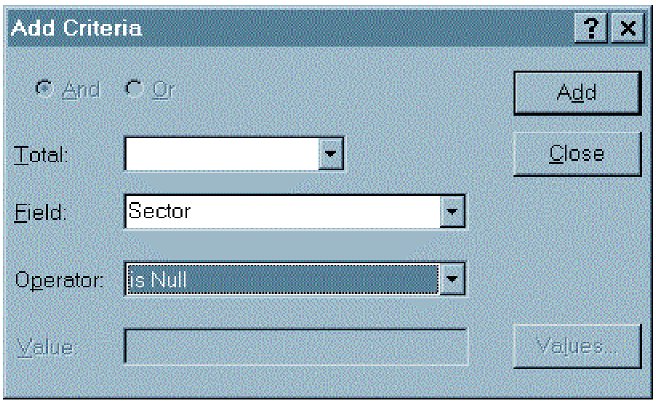


Drag the highlighted rectangle containing the asterisk into the blank rectangle in the bottom half of the screen. This indicates to Microsoft Query that all fields will be passed forward to the pivot table. Upon successful completion of this step, the main Microsoft Query window will have the following appearance:

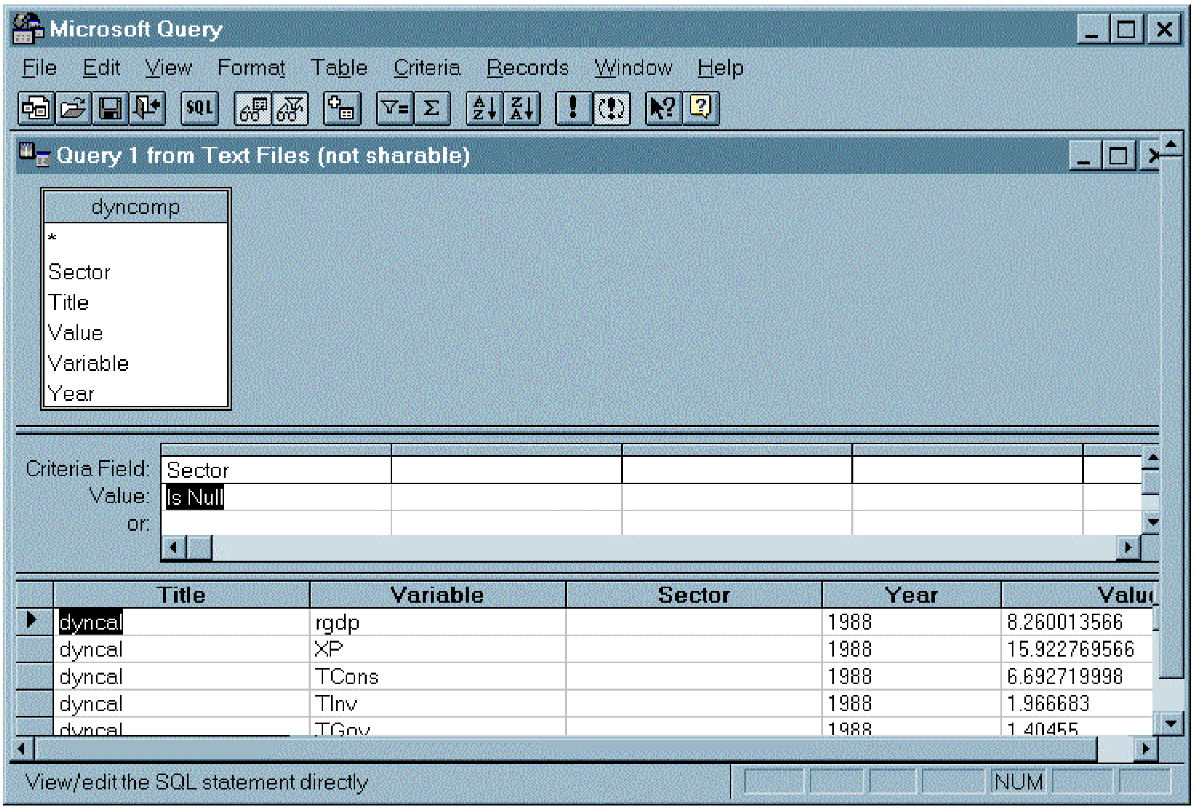


where each field is already laid out in tabular format, with the field headers at the top of the data matrix, and each data record on a corresponding line of the data matrix. One of the more powerful features of Microsoft Query is the ability to filter the data before transferring it to Excel. This is done by using the Add Criteria... item under the Criteria Menu. This document is only meant to be an introduction to the use of pivot tables, so it will not go into all the many different uses of the add criteria function. Some examples may help in providing some initial insights into its usefulness.

The Add Criteria... menu item brings up the following window:

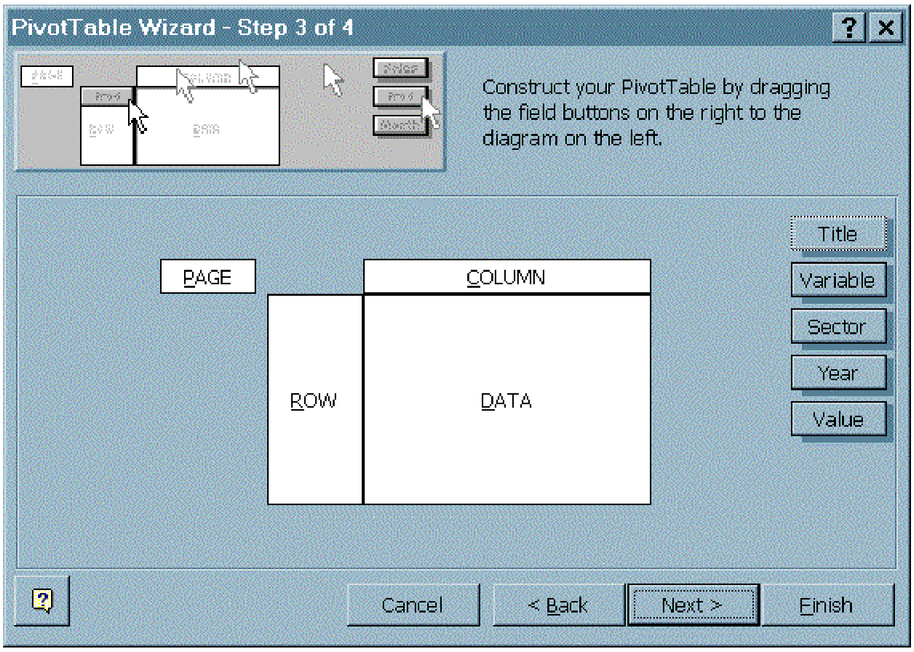


The criteria can be defined over any field and can be filtered in many ways. For example, to select only macro data, i.e. records with no data in the Sector field simply choose the field Sector in the text box designated by Field and choose the filter "is Null" in the text box designated by Operator. When finished with the first criteria, click on the Add button. This will return the user back to the main Microsoft Query window:

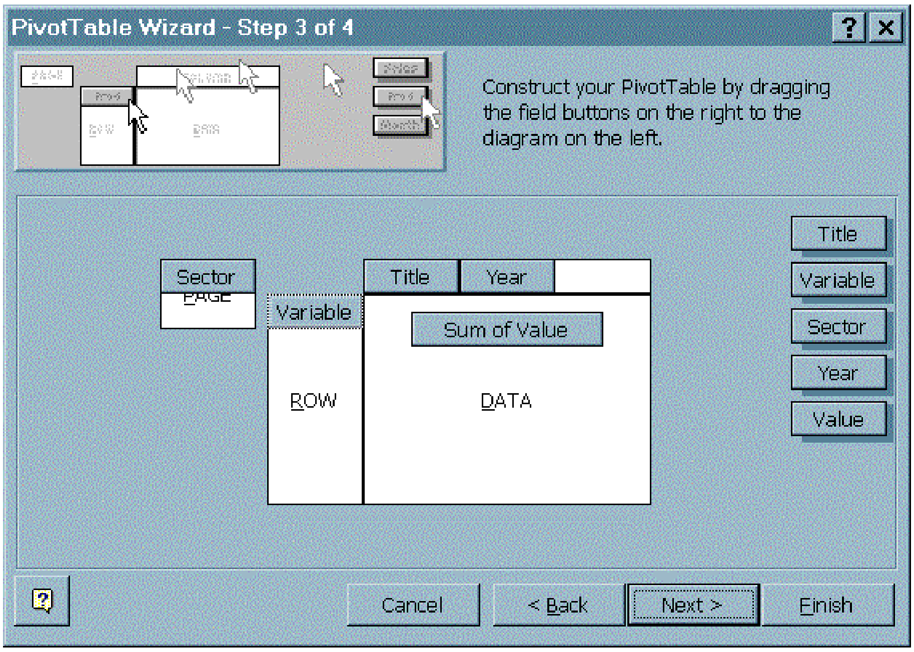


More criteria can be added and can be conditioned by the logical operators 'and' and 'or'. For example, it is possible to select only the macro data and for specific years.

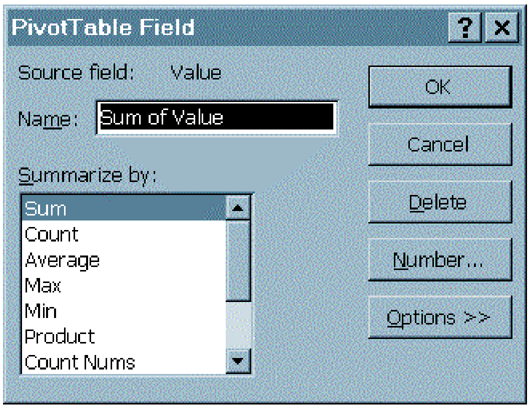
After selecting the data and optionally adding any filtering criteria, the next step is to send the data back to the Excel PivotTable Wizard. From the Microsoft Query File Menu, click on the item Return Data to Microsoft Excel. This sends the user back to step 2 of the Pivot Table Wizard. Click on the Next > button to start step 3 of the PivotTable Wizard which presents the following screen:



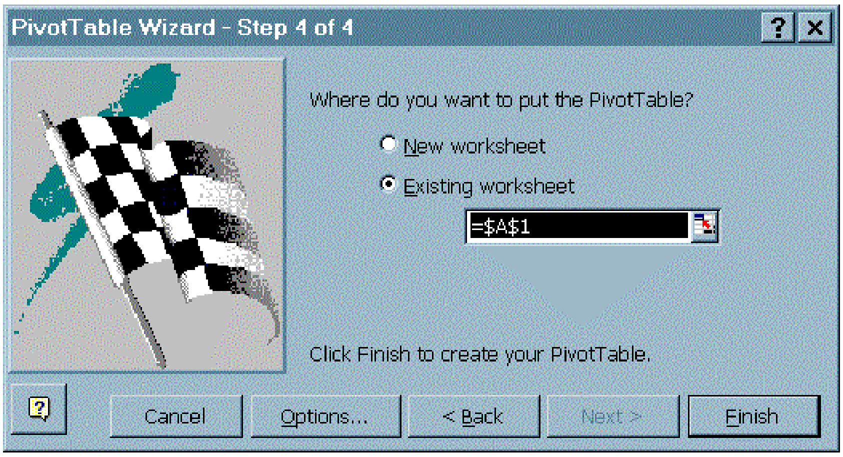
This step allows the user to designate the structure of the pivot table. Note that the structure is very easy to modify later once the initial pivot table is constructed. Pivot tables have four dimensions, though in this example, there are five fields. Therefore, one of the dimensions of the pivot table will actually contain two fields. The ‘Value’ field virtually always goes in the Data section of the pivot table. The other four fields can be arranged in a variety of ways. If the file contains data from only one scenario (i.e. the ‘Title’ field is uniform), and the data only contains macro results, than it is traditional to put the ‘Title’ and ‘Sector’ fields in the Page dimension, let ‘Year’ represent the Columns, and put the ‘Variables’ in the Rows. For results from more than one scenario, it might be practical to have the ‘Title’ and ‘Year’ side by side in the Columns. This latter structure is depicted by the next window:



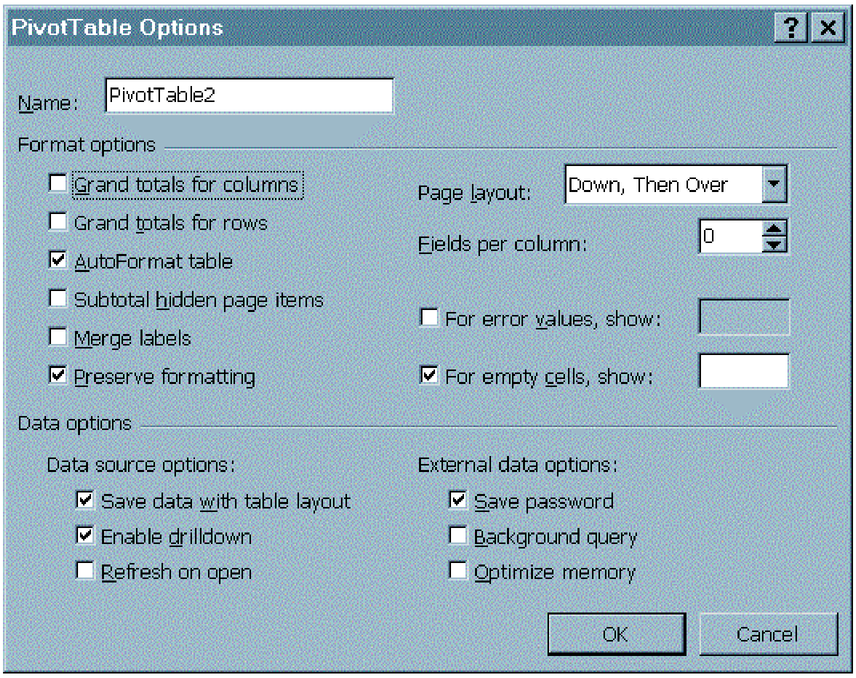
There are still a variety of options which can be set from the window above (i.e. in Step 3), though some can also be done at anytime once the pivot table is finished. One option is to define the format for the data. Doubling clicking on the Sum of Value button brings up the following window:



Double clicking on the Number... button brings up the usual Format Cells Excel dialogue. Choose any appropriate format and then click on the OK button twice to return to the PivotTable Wizard Step 3 window. After having chosen all the options available under step 3 (try experimenting by double clicking on the other fields and choosing options), click on the Next > button to get to Step 4 of the PivotTable Wizard. The following window appears:

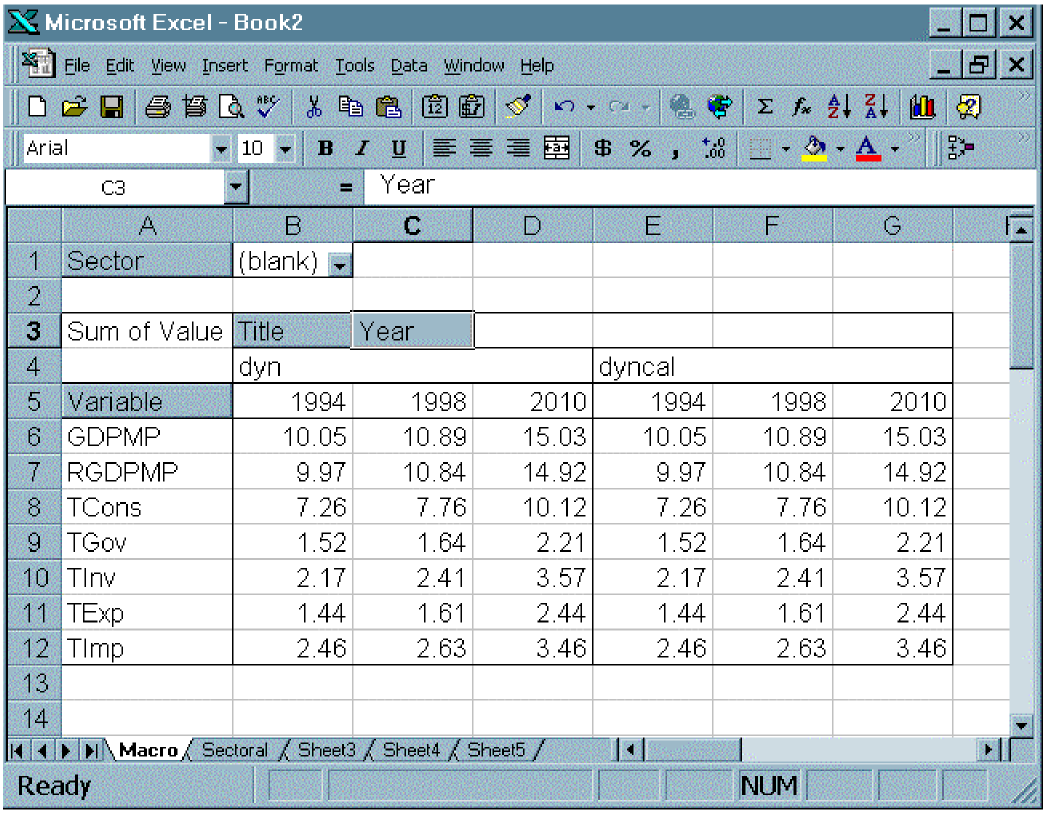


The primary role of this step is to designate the location of the pivot table. If you have started from a new worksheet the default is to put the pivot table in the upper left corner of the worksheet, i.e. cell A1. You can designate another location if you so desire. There are some more options which can be set from this step. On pressing the Options button, the following screen will appear:



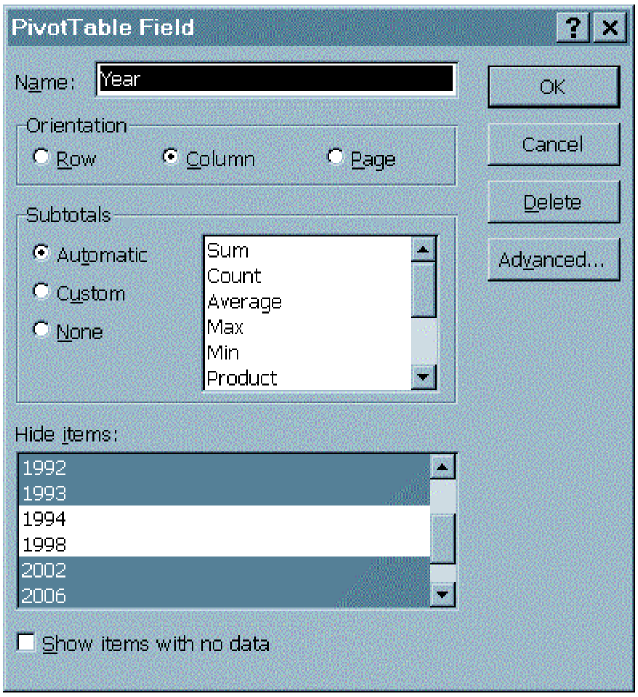
Typically, given the nature of the data in the pivot table, it does not make sense to add the data in the columns or in the rows. The fields Grand totals for columns and Grand totals for rows should be unchecked in this case. (There are clearly times when the totals make sense and can be useful. For example, it is possible to save a sequence of SAMs—indexed by either time or region or both for example—as a data base and to load it into a pivot table.) After clicking on the OK button, the final step is click on the Finish button from the Step 4 window of the PivotWizard.

The appearance of the final pivot table will have a form similar to the following window:



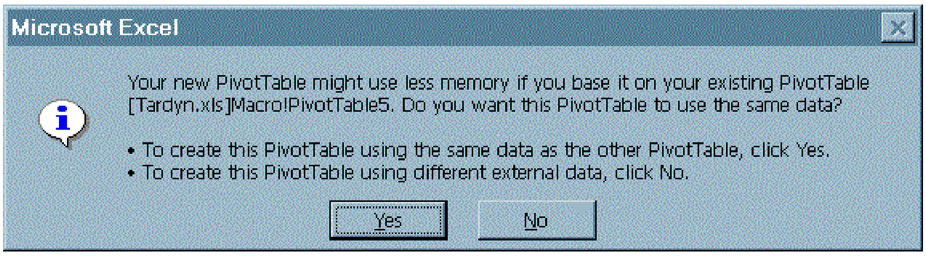
	A	B	C	D	E	F	G
1	Sector	(blank)					
2							
3	Sum of Value	Title	Year				
4		dyn			dyncal		
5	Variable	1994	1998	2010	1994	1998	2010
6	GDPMP	10.05	10.89	15.03	10.05	10.89	15.03
7	RGDPMP	9.97	10.84	14.92	9.97	10.84	14.92
8	TCons	7.26	7.76	10.12	7.26	7.76	10.12
9	TGov	1.52	1.64	2.21	1.52	1.64	2.21
10	TInv	2.17	2.41	3.57	2.17	2.41	3.57
11	TExp	1.44	1.61	2.44	1.44	1.61	2.44
12	TImp	2.46	2.63	3.46	2.46	2.63	3.46
13							
14							

Once the pivot table is finished, it is possible to re-structure the pivot tables in different ways. For example, it is possible to move fields from one location to another. The title field can be moved into the top left corner where the sector field is, etc. It is also possible to change some options for specific fields. For example, it is possible to hide certain variables or years, or to have sums calculated or suppressed for specific fields. Double clicking on any one of the field names will bring up the following screen:



By clicking on specific field names it is possible to hide items. Other options are also available from this dialogue. From within the pivot table it is possible to re-arrange the ordering of rows and columns by dragging and moving specific records.

It is also possible for the same Excel file to contain more than one pivot table. For example, one worksheet may contain macro data only. A second worksheet may be loaded with a pivot table which contains all of the sectoral data, etc. Creating a second pivot table simply requires going through all of the same steps as defined above, but using different criteria to provide a different lens for analyzing the output. (The PivotTable Wizard will at some point bring up the following dialogue when creating multiple pivot tables in the same Excel file:



Our experience has shown that it is better to answer no since it relies on the criteria specified in Microsoft Query. Unless you desire to see the same data (with the same criteria) with a different view, the correct answer is no.)