

Probabilistic Stabilization Targets*

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Abstract

We study stabilization targets: common environmental policy recommendations that specify a maximum probability of an environmental variable exceeding a fixed target (e.g. limit climate change to at most two degrees C above preindustrial). An emissions policy (e.g. greenhouse gas emissions abatement) affects the environmental variable. Previous work generally considers stabilization targets under certainty equivalence. Using an integrated assessment model with uncertainty about the sensitivity of the temperature to greenhouse gas (GHG) concentrations (the climate sensitivity), learning, and random weather shocks, we calculate the optimal GHG emissions policy with and without stabilization targets. We characterize the range of feasible targets and show that in general, climate change has far too much uncertainty and inertia to be controlled with the precision implied by stabilization targets.

We calculate the welfare cost of stabilization targets. We find that the stabilization targets have three welfare costs. First, the targets are inflexible and do not adjust to new information about the climate system. Second, the target forces the emissions policy to overreact to transient shocks. Third, the commonly proposed target temperature is lower than the unconstrained optimum under certainty. Total welfare costs are on the order of 5%, of which one quarter is caused by inflexibility and overreaction, effects present only in a model with uncertainty.

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1 Introduction

Stabilization targets are common environmental policy recommendations that specify the maximum allowable change of some environmental variable tied to pollution emissions. The most common application is in climate change policy. For example, many atmospheric scientists and policy makers recommend limiting greenhouse gas (GHG) concentrations so that the global mean temperature rises by at most 2°C above its preindustrial level. Stabilization targets call for limiting temperature changes to reduce the likelihood of irreversible and catastrophic climate change, or to prevent the temperature from changing more than an amount deemed harmful to society.¹ Once the target is specified, economists calculate the least-cost way to stay under the target.

However, a stabilization target is not feasible when the environmental variable is subject to random shocks or is a function of uncertain parameters. Indeed, random weather shocks and uncertain parameters, such as the sensitivity of the climate to GHG concentrations (the climate sensitivity), will cause the target to be exceeded in any period with positive probability. Of course, learning reduces uncertainties over time, which allows some fine tuning of the emissions policy as the variable approaches the target. Nonetheless, slow learning and strong inertia in the climate makes fine tuning difficult. We show that even if GHG emissions are immediately and permanently reduced to zero, enough inertia exists in the climate so that the global mean temperature will exceed 2°C with 15% probability. An alternative is probabilistic stabilization targets, which instead require that the environmental variable stay beneath the target with a given probability.² Probabilistic stabilization targets are feasible for a given temperature if the allowable probability of exceeding the target is sufficiently high. In general, however, we show that climate change has far too much uncertainty and inertia to be controlled to the degree assumed by probabilistic stabilization targets: uncertain parameters cause the temperature to drift to levels for which the target is either non-binding, or infeasible.

Using an integrated assessment model of the climate and economy with Bayesian

¹Stabilization targets are also relatively straightforward to communicate to the public.

²One can think of a standard stabilization target as a special case of a probabilistic target where the probability of exceeding the target is set to zero. Therefore, we will henceforth without loss of generality consider only probabilistic targets.

learning and random weather shocks,³ we calculate the welfare cost of stabilization targets. We show that probabilistic stabilization targets cause three welfare losses. First, the target may be set lower than the average temperature resulting from the optimal emissions policy without the target. This welfare loss is present even if the temperature is certain. For example, Nordhaus (2007) calculates the welfare loss of restricting the temperature to 2°C under certainty, relative to an optimal unconstrained policy of a little over 3°.

The second welfare loss is new to this paper and is present only when the temperature is a function of uncertain and/or random variables such as the climate sensitivity and weather shocks, and the model does not assume certainty equivalence. As new information arrives, the optimal temperature changes. For example, if the temperature turns out to be very sensitive to GHG concentrations, then achieving a given temperature requires more abatement expenditures. Since the cost of meeting a given temperature rises, and the benefit is unchanged, the optimal temperature rises. By definition, however, the target temperature does not change with a stabilization target. Therefore, an additional welfare cost ensues because the stabilization target is *inflexible*.

The third welfare loss is also new and present only with uncertainty. We show that stabilization targets force overly stringent policy responses to transient shocks. For example, when a random weather fluctuation causes the environmental variable to exceed the target, stabilization expenditures must rise in response. However, the environmental variable naturally reverts back to the target as the shock dies out. Therefore, the stabilization target forces a costly policy response that provides only a relatively mild benefit of causing the environmental variable to return to the target more quickly than would occur naturally.

We show that stabilization targets cause a welfare loss of about 5%, of which 25% is due to inflexibility and overly stringent responses to transient shocks. Further, this result is relatively insensitive to the maximum probability of exceeding the target.

There is a long history in the environmental economics literature of evaluating the welfare costs of sub-optimal policies. However, most previous work either compares market with inefficient, non-market based regulation (see for example, Stavins 1993), or evaluates whether or not a given sub-optimal policy improves welfare relative to

³See Kelly and Kolstad (1999a) for a survey of integrated assessment models.

a baseline without the policy (see for example, Portney, Parray, Gruenspecht, and Harrington 2003). In contrast, the welfare losses we focus on arise solely due to uncertainty. Indeed, under certainty one could always choose a temperature target high enough so that no welfare loss ensues. However, with uncertainty stabilization target become inflexible, causing welfare loss.

Stabilization targets are ubiquitous in climate change policy. Policy makers recommending a 2°C stabilization target include the European Commission (2007), the Copenhagen Accord (2009), and the German Advisory Council on Global Change (Schubert et al. 2006). Many atmospheric scientists (Hansen 2005, O’Neill and Oppenheimer 2002) also advocate for the 2°C limit; Hansen in particular is a vocal advocate. Other climate-related stabilization targets are also common. For example, the German Advisory council recommends limiting sea level rise to at most 1 meter and ocean acidification to at most 0.2 units of pH below its preindustrial level.⁴

Economists (Nordhaus 2007, Richels, Manne, and Wigley 2004) then compute the least cost GHG emissions path which stabilizes the climate at 2°C under certainty. However, it is well known that parameters of the climate system are uncertain. For example, the climate sensitivity, which measures the elasticity of the global mean temperature with respect to GHG concentrations, is notoriously uncertain (Intergovernmental Panel on Climate Change 2007, Kelly and Kolstad 1999b). Therefore, following the least cost pathway calculated under certainty can, under uncertainty actually result in climate change which exceeds the target by a considerable margin.⁵ Indeed, a branch of the literature focuses on the likelihood of meeting current targets for various emissions scenarios proposed by policy makers, or what emissions paths satisfy the target for various values of the climate sensitivity. For example, Hare and Meinshausen (2006) and Keppo, O’Neill, and Riahi (2007) compute temperature changes for various emissions scenarios; Harvey (2007) proposes allowable

⁴Some authors refer to a policy which stabilizes GHG concentrations at a particular level as a stabilization target (see for example den Elzen, Meinshausen, and van Vuuren 2007). Policy groups such as 350.org also favor a target GHG concentration. Since carbon cycle uncertainties are smaller than uncertainty in climate models, our results are less applicable to GHG concentration targets. Nonetheless, our results make clear that a fixed GHG target is not optimal because optimal GHG concentrations change with the resolution of uncertainty.

⁵Paradoxically, stabilization targets evolved as method of dealing with uncertainty in integrated assessment models. The idea was to propose limits on temperature changes which, if exceeded, would cause damages high enough so that uncertainties in the cost of abatement and other parameters are less relevant.

CO₂ emissions paths for different ranges of the climate sensitivity. One robust result from the aforementioned studies is that emission paths following the upper boundary of emissions scenarios are less likely to meet targets than those which follow the lower boundary. The above research provides an important first step in estimating the range of feasible probabilities of exceeding the target, given current information. Here we take the next step and consider stabilization targets with uncertainty and learning.

Implementing a stabilization target with uncertainty is non-trivial. For any given emissions policy time path, a possibility exists such that the climate sensitivity will be high enough so that the temperature exceeds the target. We show that controlling temperature to the degree implied by a stabilization target often requires a very high emissions abatement rate. For example, we find that if the climate sensitivity is such that a doubling of GHGs causes a 3.9°C temperature change, rather than the prior estimate of 2.8°C, the planner must raise the abatement control rate from 33% to 68% as early as 2015.

Uncertainty makes controlling the temperature more difficult. Learning, by reducing uncertainty, allows the planner to more easily stay within the target by quickly reducing emissions if new information indicates the climate sensitivity is higher than previously thought. Therefore, learning allows the planner to move closer to the target, and still remain below the target with the same probability. However, Kelly and Kolstad (1999b), Leach (2007), and Roe and Baker (2007) show that learning about the climate sensitivity is a slow process, due in part to the random weather fluctuations. Therefore, the optimal near term policy is similar to the case without learning. Our model has Bayesian learning about the climate sensitivity, and random weather fluctuations. We find that learning moves the optimal target under uncertainty closer to the target under certainty, but the effect is marginal since learning is slow.

Lorenz, Schmidt, Kriegler, and Held (2012) and Schmidt, Lorenz, Held, and Kriegler (2011) debate the magnitude of the value of information which allows more precise control of the climate with probabilistic stabilization targets. These studies use frameworks where learning occurs once and emissions paths are adjusted once. Here learning is Bayesian and incremental: beliefs about unknown parameters and decisions based on beliefs adjust each period as in Kelly and Kolstad (1999b). We

show that a stabilization target increases the value of information, because the planner avoids accidentally exceeding the target, which is costly since once the target is exceeded, abatement expenditures must rise to bring the temperature back to the target.

Our integrated assessment model computes both the cost of emissions abatement and the damages from higher temperatures. Therefore, we can calculate the welfare cost of a stabilization target. We find that the expected welfare cost of stabilization targets is about 5%, and can increase to 14% or more depending on how the uncertainty resolves. The welfare cost is not sensitive to the choice of the maximum probability of staying under the target. The probability of exceeding the target in any period depends mostly on factors such as the resolution of the uncertainty and the climate inertia, and is affected very little by the current emissions. Therefore, the probability of staying under the target tends to be either zero or one and the maximum allowable probability has little effect.

Other authors compute optimal emissions paths under certainty which keep temperatures below a threshold, beyond which specific irreversible and disastrous consequences occur. Keller et al. (2005) propose emissions paths which prevent coral bleaching or the disintegration of ice sheets. Kvale et al. (2012) propose emission paths which limit ocean level rise and acidification. Additionally, Bruckner and Zickfeld (2009) compute emission paths that reduce the likelihood of a collapse of the Atlantic thermohaline circulation. The aforementioned studies employ the tolerable windows approach, an inverse modeling method that asks: in order to limit GHG concentrations or warming below a threshold at all future dates, how should emissions be controlled in every period moving forward?⁶ Models with irreversibilities that compute emissions paths under certainty overestimate optimal emissions if the climate system is uncertain, because under certainty no insurance motive exists. While we do not specifically model irreversibilities, our model combines convex damages with uncertainty. Therefore, the planner insures against very high damages by pursuing a conservative emissions policy.

⁶An exception is Lemoine and Traeger (2013), who consider various climate tipping points in an environment with uncertainty and learning.

2 Model

We consider an infinite horizon version of the Nordhaus DICE model (Nordhaus 2007). In the DICE model, economic production causes GHG emissions, which raise the global mean temperature. Higher temperatures reduce total factor productivity (TFP). The social planner chooses capital investment and an emissions control rate to maximize welfare. Our model has four differences from the DICE model. First, we use an annual time step rather than the 10 year step in DICE. Second, we use the simplified model of the atmosphere/climate due to Traeger (2012), in which the ocean temperature changes exogenously and GHG concentrations immediately mix evenly in the atmosphere. Third, the model is stochastic, with an uncertain climate sensitivity and random weather shocks that obscure the effect of GHGs on temperature. The planner learns about the uncertain climate sensitivity over time by observing temperature changes. Fourth, we impose stabilization targets to ascertain their effects on welfare, temperature, and economic growth. Sections 2.1-2.2 describe the economic and climate models briefly (refer to Traeger 2012, for a detailed discussion).

2.1 Economic system

The global economy produces gross output, Q , from capital K and labor L according to:

$$Q_t = A(t) K_t^\gamma L(t)^{1-\gamma}. \quad (1)$$

Here variables denoted as a function of t , such as $L(t)$ and TFP, $A(t)$, grow exogenously. Appendix A.1 gives the growth rates for all variables which change exogenously over time. Variables with a t subscript are endogenous.

An emissions abatement technology exists which can reduce emissions by a fraction x_t at a cost of $\Lambda(x_t) = \Psi(t) x_t^{A_2}$ fraction of gross output. Here Λ is the cost function and $\Psi(t)$ is the exogenously declining cost of a backstop technology which reduces emissions to zero. Further, increases in global mean temperatures above preindustrial levels, T_t , reduce TFP by a factor $1/(1 + D(T_t))$, where $D(T_t) = b_1 T^{b_2}$ is the damage function. Therefore, output net of abatement spending and climate

damages, Y_t , is:

$$Y_t = \frac{1 - \Psi(t) x_t^{a_2}}{1 + b_1 T_t^{b_2}} A(t) K_t^\gamma L(t)^{1-\gamma}. \quad (2)$$

Let C_t be consumption and let capital depreciate at rate δ_k . Then the resource constraint is:

$$Y_t = C_t + K_{t+1} - (1 - \delta_k) K_t. \quad (3)$$

Period utility is constant relative risk aversion:

$$u = \frac{c^{1-\eta} - 1}{1 - \eta}. \quad (4)$$

The discount factor for future utility is $\exp(-\delta_u)$, where δ_u is the pure rate of time preference.

2.2 Climate System

Current period GHG emissions, E_t , from production depend on the planner's choice of control rate x_t , the emissions intensity of output $\sigma(t)$, exogenous emissions from land use changes, $B(t)$, and gross global output:

$$E_t = (1 - x_t) \sigma(t) Q_t + B(t). \quad (5)$$

The stock of GHG equivalents, M_t , depends on current period emissions and the natural decay rate of GHGs into the biosphere and ocean. Let $\delta_m(t)$ denote the decay rate (which changes exogenously) and MB denote the stock of GHGs during pre-industrial times. Then M_t accumulates according to:

$$M_{t+1} - MB = (1 - \delta_m(t)) (M_t - MB) + E_t. \quad (6)$$

We normalize GHG stocks relative to pre-industrial. Let $m_t \equiv \frac{M_t}{MB}$, then:

$$m_{t+1} - 1 = (1 - \delta_m(t)) (m_t - 1) + \frac{E_t}{MB}. \quad (7)$$

Radiative forcing of GHGs, F_t , increases the temperature:

$$F_{t+1} = \Omega \log_2(m_{t+1}) + EF(t). \quad (8)$$

Here $EF(t)$ is forcing from other sources, which grows exogenously.

The global mean temperature evolves according to:

$$\hat{T}_{t+1} = \hat{T}_t + \frac{1}{\alpha} \left(F_{t+1} - \frac{\hat{T}_t - \Gamma}{\tilde{\lambda}} + \xi (\hat{O} - \hat{T})(t) - \Gamma \right) + \tilde{v}_{t+1} \quad (9)$$

Here \hat{T} and \hat{O} denote the absolute global atmospheric and oceanic temperatures in °C, respectively; α is the thermal capacity of the upper oceans; Γ is the pre-industrial atmospheric temperature; ξ is a coefficient of heat transfer from the upper oceans to the atmosphere; $\tilde{v}_t \sim N(0, 1/\rho)$ is the random weather shock; $\tilde{\lambda}$ is the uncertain climate sensitivity. We assume the ocean-atmosphere temperature differential changes exogenously.

The climate sensitivity $\tilde{\lambda}$ describes how sensitive the atmospheric temperature is to GHG concentrations, and is the subject of great uncertainty. Not only is the climate sensitivity unknown, but the shape of its uncertainty is unknown.

Let $\Delta T_{2\times}$ be the steady state atmospheric temperature deviation from pre-industrial time resulting from a doubling of GHG concentrations, also relative to pre-industrial levels. Then:

$$\Delta T_{2\times} = \Omega \tilde{\lambda}. \quad (10)$$

Since the climate sensitivity is uncertain, $\Delta T_{2\times}$ is also uncertain. Stocker, Dahe, and Plattner (2013) estimates that $\Delta T_{2\times}$ is most likely to lie somewhere between 1.5°C and 4.5°C. The initial mean of the prior distribution is 2.8, taken from the mean of estimates in the atmospheric science literature (Roe and Baker 2007).

Let $T_t = \hat{T}_t - \Gamma$ and $O_t = \hat{O}_t - \Gamma$ be the current deviations from pre-industrial temperatures, $\tilde{\beta}_1 = 1 - 1/\tilde{\lambda}\alpha$ denote the climate feedback parameter, $\beta_2 = \frac{1}{\alpha}$, and $\beta_3 = \xi/\alpha$. The climate system simplifies to:

$$T_{t+1} = \tilde{\beta}_1 T_t + \beta_2 F_{t+1} + \beta_3 (O - T)(t) + \tilde{v}_{t+1}. \quad (11)$$

Since $\tilde{\lambda}$ is uncertain, the climate feedback parameter is also uncertain. The climate feedback parameter is increasing in $\tilde{\lambda}$. For example, if GHG induced warming reduces ice cover, which reduces the amount of sunlight reflected back into space (the albedo effect), causing still higher temperatures, we have a positive feedback (higher $\tilde{\beta}_1$ and

$\tilde{\lambda}$).⁷ The climate feedbacks and therefore the climate sensitivity is highly uncertain (Stocker, Dahe, and Plattner 2013).

2.3 Learning

Assume the planner has prior beliefs that the climate feedback parameter is drawn from a normal distribution with mean μ_t and precision η_t . The weather shock ν occurs at the beginning of each period, before the control rate is chosen. We combine the two uncertain terms in equation (11) and denote the sum \tilde{H} :

$$\tilde{H}_{t+1} = \tilde{\beta}_1 T_t + \tilde{\nu}_{t+1}. \quad (12)$$

Since \tilde{H}_t is the sum of two normally distributed random variables, it is also normally distributed with mean $\mu_t T_t$ and variance $\sigma_{\tilde{H}}^2 = T_t^2/\eta_t + 1/\rho$. The planner observes $H_{t+1} = T_{t+1} - \beta_2 F_{t+1} - \beta_3 (O - T)(t)$ at the beginning of $t + 1$ and updates beliefs of $\tilde{\beta}_1$. Bayes' Rule implies that the posterior distribution of $\tilde{\beta}_1$ is also normal, with mean and precision:

$$\mu_{t+1} = \frac{\eta_t \mu_t + \rho H_{t+1} T_t}{\eta_t + \rho T_t^2}, \quad (13)$$

$$\eta_{t+1} = \eta_t + \rho T_t^2. \quad (14)$$

Perfect information implies that $\mu = \tilde{\beta}_1$ and $\eta = \infty$. The information set used by the planner to select x_t includes μ_t .

2.4 Recursive Problem

The planner chooses emission abatement each period to maximize social welfare.

$$W = \max_{k_{t+1}, x_t} \mathbb{E} \left[\sum_{t=0}^{\infty} \exp(-\delta_u t) L_t u \left(\frac{C_t}{L_t} \right) \right]. \quad (15)$$

Let $k \equiv K / \left(LA^{\frac{1}{1-\gamma}} \right)$ denote normalized capital per productivity adjusted person (Kelly and Kolstad 2001, Traeger 2012) and the same for y , and $s = [k, T, m, t, \mu, \eta]$.

The recursive version of the social planning problem is:

$$V(s) = \max_{k', x \in [0,1]} \left\{ u(c) + \beta(t) \int_{-\infty}^{\infty} V[s'] N \left(\mu T, \frac{T^2}{\eta_t} + \frac{1}{\rho} \right) d\tilde{H} \right\}, \quad (16)$$

⁷Other feedbacks include changes in cloud cover and water vapor.

subject to:

$$T' = \beta_2 F' + \beta_3 (O - T)(t) + \tilde{H}', \quad (17)$$

$$F' = \Omega \log_2(m') + EF(t), \quad (18)$$

$$m' = 1 + (1 - \delta_m(t))(m - 1) + \frac{E}{MB}, \quad (19)$$

$$E = (1 - x) \sigma(t) A(t)^{\frac{1}{1-\gamma}} L(t) k^\gamma + B(t). \quad (20)$$

$$\mu' = \frac{\eta\mu + \rho\tilde{H}'T}{\eta + \rho T^2}, \quad (21)$$

$$\eta' = \eta + \rho T^2. \quad (22)$$

$$t' = t + 1. \quad (23)$$

Equation (16) condenses the double expectation over $\tilde{\beta}_1$ and $\tilde{\nu}_{t+1}$ into one expectation over the random variable H_{t+1} . Appendix A.1 gives the equations which govern the evolution of variables that change exogenously over time. Therefore time, t , is a state variable. The discount factor accounts for growth in population and productivity. Because the growth rates change over time, the normalized discount factor $\beta(t)$ is not constant, but is exogenous.

In the model, two state variables, t and η , are non-stationary. Therefore, the computational solution replaces the precision η with the variance $1/\eta$, and replaces time with a bounded, monotonic increasing function.⁸ Table 1 gives parameter values and definitions for the above problem.

2.5 Stabilization Targets

The probabilistic stabilization target is a constraint on temperature which satisfies:

$$Pr(T_{t+1} \geq T^*) \leq \omega, \quad \forall t. \quad (24)$$

A pure stabilization target is a special case of equation (24) with $\omega = 0$, if $\omega = 1$ the constraint is always satisfied. In period t a single constraint exists which restricts the probability that $T_{t+1} \geq T^*$.

⁸See for example, Kelly and Kolstad (1999b) or Traeger (2012).

The stabilization target is effectively a constraint on the control rate x , since x affects T_{t+1} . Rewriting the left hand side of (24) gives:

$$Pr(T_{t+1} \geq T^*) = Pr\left(\beta_2 F_{t+1} + \beta_3 (O - T)(t) + \tilde{H}_{t+1} \geq T^*\right), \quad (25)$$

$$= Pr\left(\tilde{H}_{t+1} \geq T^* - \beta_2 F_{t+1} - \beta_3 (O - T)(t)\right), \quad (26)$$

$$= 1 - \text{NCDF} \left[\frac{(T^* - \beta_2 F_{t+1} - \beta_3 (O - T)(t) - \mu_t T_t)}{\sqrt{\frac{T_t^2}{\eta_t} + \frac{1}{\rho}}} \right]. \quad (27)$$

Here NCDF is the cumulative distribution function of the standard normal distribution. Let:

$$P_{t+1} \equiv \beta_2 F_{t+1} + \beta_3 (O - T)(t) + \mu_t T_t, \text{ and} \quad (28)$$

$$\sigma_{H,t} \equiv \sqrt{\frac{T_t^2}{\eta_t} + \frac{1}{\rho}} \quad (29)$$

be the mean and standard deviation of T_{t+1} , respectively, then:

$$Pr(T_{t+1} \geq T^*) = 1 - \text{NCDF} \left[\frac{T^* - P_{t+1}}{\sigma_{H,t}} \right]. \quad (30)$$

Constraint (24) is therefore equivalent to:

$$\text{NCDF} \left[\frac{T^* - P_{t+1}}{\sigma_{H,t}} \right] \geq 1 - w, \quad (31)$$

$$\frac{T^* - P_{t+1}}{\sigma_{H,t}} \geq \text{NCDF}^{-1}[1 - w], \quad (32)$$

$$F_{t+1} \leq \frac{1}{\beta_2} (T^* - \beta_3 (O - T)(t) - \mu_t T_t - \sigma_{H,t} \cdot \text{NCDF}^{-1}[1 - w]), \quad (33)$$

$$x_t \geq 1 + \frac{MB}{\sigma(t) L(t) A(t)^{\frac{1}{1-\gamma_t}} k_t^\gamma} \left(1 + (1 - \delta_m(t))(m_t - 1) - \exp \left\{ \frac{\log(2)}{\Omega \beta_2} \left[T^* - \beta_3 (O - T)(t) - \mu_t T_t - \sigma_{H,t} \cdot \text{NCDF}^{-1}[1 - w] \right] \right\} \right) \quad (34)$$

$$x_t \geq PC(s_t, T^*, \omega). \quad (35)$$

Here PC is the right hand side of (34). A stabilization target is therefore equivalent to a minimum control rate.

Let θ denote the Lagrange multiplier on the probabilistic constraint. The recursive version of the problem, which includes the probabilistic constraint, is then:

$$V(s) = \max_{k', x \in [0,1]} \left\{ u(c) + \theta \left[x - PC(s, T^*, \omega) \right] + \beta(t) \int_{-\infty}^{\infty} V[s'] N(\mu T, \sigma_H) d\tilde{H} \right\}, \quad (36)$$

subject to equations (17)-(23). In period t , the planner anticipates facing constraints in periods $t+i$, which restrict the probability that $T_{t+i+1} \geq T^*$ for all $i = 1, 2, \dots$. Therefore, constraints in period t on the probability that $T_{t+i+1} \geq T^*$ are not necessary, since the planner anticipates choosing a control rate in $t+i$ such that these constraints are satisfied.

3 Feasibility

3.1 A Feasible Constraint

Assuming emissions cannot be negative, constraint (35) may not be feasible; the set of $x \in [0, 1]$ which also satisfy (35) may be empty, violating a necessary condition for the existence of a maximum. The problem is not feasible when $PC(s, T^*, \omega) > 1$, since the maximum control rate is one. This occurs when the temperature rises close to or above T^* . In this case, given the inertia of the climate, even a control rate of 1 cannot reduce the temperature enough to satisfy the constraint. Feasibility is also affected by ω : as $\omega \rightarrow 1$, the planner is allowed to exceed the target with high probability, and so the model has a feasible solution even if the temperature is relatively high.

To solve infeasibility problem while keeping with the spirit of a stabilization

target, we assume that $x = 1$ is always feasible.⁹ That is, we use:

$$V(s) = \max_{k', x \in [0,1]} \left\{ u(c) + \theta \left[x - \min \{ PC(s, T^*, \omega), 1 \} \right] + \beta(t) \int_{-\infty}^{\infty} V[s'] N(\mu T, \sigma_H) d\tilde{H} \right\}, \quad (37)$$

subject to equations (17)-(23).

Problem (37) always has a feasible solution. Further, when the probabilistic constraint is accidentally exceeded due to a large random weather shock or an unexpectedly high realization of β_1 , the planner must return as quickly as possible (by setting $x = 1$) to the range where $PC < 1$. This is in keeping with the idea that exceeding the target carries risk of high damages and should be avoided.

3.2 Tightness of the Constraint

Here we calculate probabilities in period $t + i$, ω_{t+i}^{\min} and ω_{t+i}^{\max} , which are the probabilities of exceeding the target when the control rate is set to 0 and 1, respectively, for all i , conditional on current information. Values of ω between ω_{t+i}^{\min} and ω_{t+i}^{\max} are feasible policies from periods t to $t + i$. For $\omega > \omega_{t+i}^{\max}$, the constraint is non-binding at period $t + i$, since the planner can set $x_{t+i} = 0$ for all periods up to $t + i$ and still expect to satisfy the constraint. Conversely, any value of $\omega < \omega_{t+i}^{\min}$ implies $PC(s_{t+i}) > 1$, even if the control rate is set to one immediately for all periods up to $t + i$. Further, by doing this exercise we glean intuition about how ω affects the control rate, which helps to explain the results in the next section. The tightness of the constraint may equivalently be controlled by altering T^* , but we assume here T^* is a given policy.

The highest possible probability for which $T_{t+1} \geq T^*$ occurs with a zero control rate. In this case, we have from (19):

$$E_t^{\max} = \sigma(t) A(t)^{\frac{1}{1-\gamma}} L(t) k_t^\gamma + B(t), \quad (38)$$

⁹Other options are possible, but less attractive. Ignoring infeasible constraints is not attractive because the planner would have an incentive to push the climate change as close as possible to T^* in the hope that the climate will go over the target so that the constraint can be ignored. The other option would be to include a penalty function for going over T^* . However, a penalty for high temperatures already exists (the damage function), so it is unclear what the penalty represents.

$$m_{t+1}^{\max} - 1 = (1 - \delta_m(t)) (m_t - 1) + \frac{E_t^{\max}}{MB}, \quad (39)$$

$$F_{t+1}^{\max} = \Omega \log_2 (m_{t+1}^{\max}) + EF(t). \quad (40)$$

Next, from (17):

$$T_{t+1}^{\max} = \tilde{H}_{t+1} + \beta_2 F_{t+1}^{\max} + \beta_3 (O - T)(t), \quad (41)$$

We can then calculate ω^{\max} as:

$$\omega_{t+1}^{\max} = Pr (T_{t+1}^{\max} \geq T^*), \quad (42)$$

$$\omega_{t+1}^{\max} = Pr \left\{ \tilde{H}_{t+1} \geq T^* - \beta_2 F_{t+1}^{\max} - \beta_3 (O - T)(t) \right\}, \quad (43)$$

$$\omega_{t+1}^{\max} = 1 - \text{NCDF} \left(\frac{T^* - P_{t+1}^{\max}}{\sigma_{H,t}} \right), \text{ where} \quad (44)$$

$$P_{t+1}^{\max} \equiv \mu_t T_t + \beta_2 F_{t+1}^{\max} + \beta_3 (O - T)(t). \quad (45)$$

Since the difference between the target and the expected temperature in period $t + 1$ under maximum emissions is not infinite, we have immediately from (44) that $\omega^{\max} < 1$.

Analogously, the minimum probability of exceeding the constraint occurs when the control rate is one.

$$E_t^{\min} = B(t), \quad (46)$$

$$m_{t+1}^{\min} - 1 = (1 - \delta_m(t)) (m_t - 1) + \frac{E_t^{\min}}{MB}, \quad (47)$$

$$F_{t+1}^{\min} = \Omega \log_2 (m_{t+1}^{\min}) + EF(t), \quad (48)$$

$$T_{t+1}^{\min} = \tilde{H}_{t+1} + \beta_2 F_{t+1}^{\min} + \beta_3 (O - T)(t), \quad (49)$$

$$\omega_{t+1}^{\min} = 1 - \text{NCDF} \left(\frac{T^* - \mu_{t+1}^{\min}}{\sigma_{H,t}} \right), \quad (50)$$

$$\mu_{t+1}^{\min} = \mu_t T_t + \beta_2 F_{t+1}^{\min} + \beta_3 (O - T)(t). \quad (51)$$

From (44) and (50), current emissions have only a small effect on the probability of exceeding the target. First, current emissions are small relative to concentrations

that have built up over centuries. Second, the forcing equation is logarithmic, further limiting the effect of current emissions on the current temperature. Indeed, near term temperatures are largely a function of inertia in the climate and total GHG concentrations.

The set of probabilities achievable with an interior control rate expands over time, since the planner can lower future temperatures via a sustained reduction in emissions. On the other hand, the uncertain climate feedback parameter has a multiplicative effect over time. Therefore, future temperatures are more uncertain and therefore are more difficult to control. We can calculate ω_t^{\min} and ω_t^{\max} over time for the given parameter values via Monte Carlo simulation. Figure 1 plots the results.

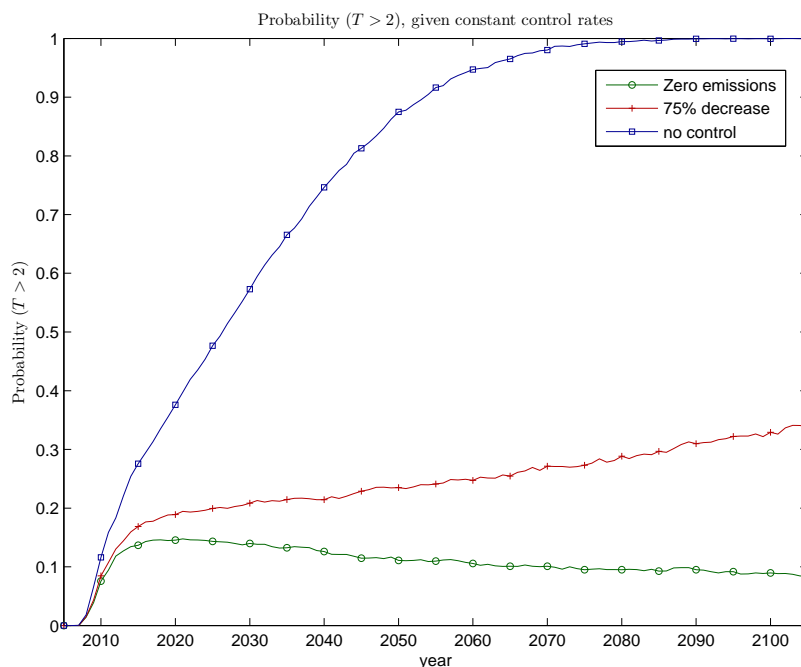


Figure 1: Probability of exceeding $T^* = 2^\circ\text{C}$, given various emissions policies. The graph is the fraction of 1000 simulations, each of which draw random realizations of β_1 (from the prior distribution) and ν_t , for which the temperature exceeds the target in the given year, for the given control policy.

For the first period, the probability of exceeding 2°C is nearly zero. Hence the current probabilistic constraint is non-binding for almost any ω . Given current in-

formation, however, there exists an approximately 15% chance that the 2°C target will eventually be exceeded, even with an immediate, permanent drop to zero emissions. Therefore, values of $\omega < 0.15$ are infeasible given today's information. An immediate, permanent, 75% drop in endogenous emissions below 2005 levels violates the constraint for any $\omega < 0.36$.¹⁰

Figure 1 also shows that a zero emissions policy can eventually overcome the inertia in the climate and reduce the probability of exceeding 2°C to near zero. A policy of zero emissions over time will slowly return the GHG concentrations to preindustrial levels (see equation 7).

The planner can choose an emissions policy over a period of decades such that most probabilistic constraints may be eventually satisfied with interior control rates, given current information. Nonetheless, the planner has little control over the climate on a year-to-year basis. Therefore, regardless of the emissions policy, in any given period the planner will likely find the probabilistic constraint to be either non-binding or infeasible. To see this, we compute the optimal unconstrained policy, and then simulate the unconstrained solution assuming the prior is correct. Figure 2 plots the unconstrained solution, along with the probabilistic constraint.

¹⁰An immediate 75% drop below 2005 emissions is a far stricter policy than, for example, a 350 ppm GHG target or the Kyoto agreement of 7% below 1990 levels.

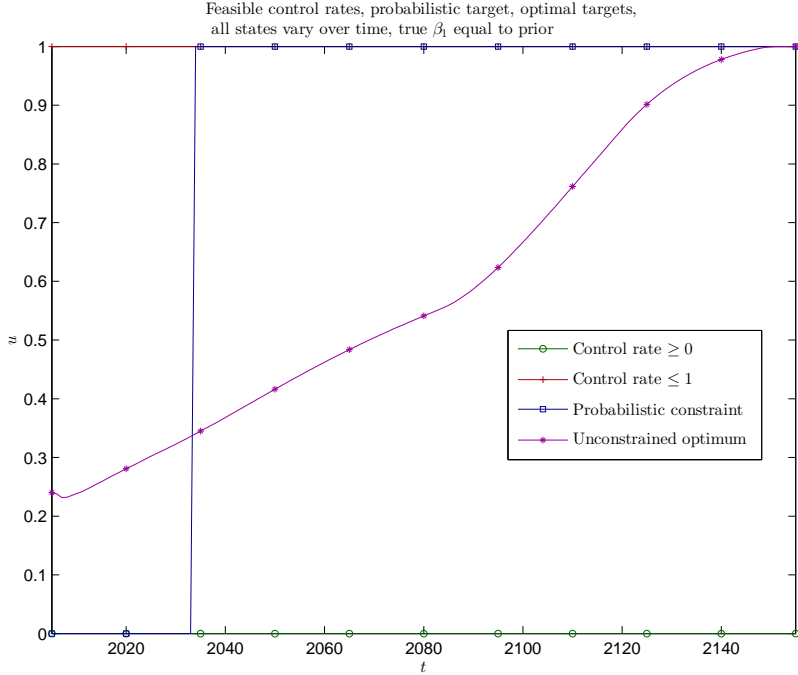


Figure 2: Probability of exceeding $T^* = 2^\circ\text{C}$, and unconstrained optimal policy, true $\Delta T_{2\times} = 2.8$. The unconstrained optimal policy is the average of 1000 simulations using the solution to (16). The probabilistic constraint sets $\omega = 0.5$.

Figure 2 shows the probabilistic constraint will bind in 2035 if the unconstrained optimal policy is implemented and the true climate sensitivity equals the prior. Notice, however, that the probability of exceeding the constraint is essentially zero in years up to 2034, and then nearly one after. Once the temperature is sufficiently close to the 2°C maximum, climate inertia causes the temperature to exceed the target with probability one regardless of the abatement policy.

Further, higher values of the climate sensitivity imply the temperature has more inertia. If instead the true value of the climate sensitivity turns out to be relatively high, then it is very likely that climate inertia will cause the temperature to exceed the constraint, even if emissions immediately and permanently drop to zero.

It is also clear that altering the probability of exceeding the constraint, ω , will have little effect: for most of the state space, the probability of exceeding the target is either zero or one, so it is irrelevant if the constraint allows the target to be exceeded with probability 0.5 or 0.6. This intuition provides a foundation for many of the

results in the next section.

4 Results

4.1 Optimal policy and uncertainty

Appendix A.3 details the solution method. Here we analyze how the optimal abatement policy varies with the probabilistic target and the uncertainty. Figure 3 plots the optimal abatement policy for two different true values of β_1 : a high value in which a doubling of GHGs causes a steady state temperature change of $\Delta T_{2\times} = 3.9^\circ\text{C}$, and the prior for which $\Delta T_{2\times} = 2.8$.

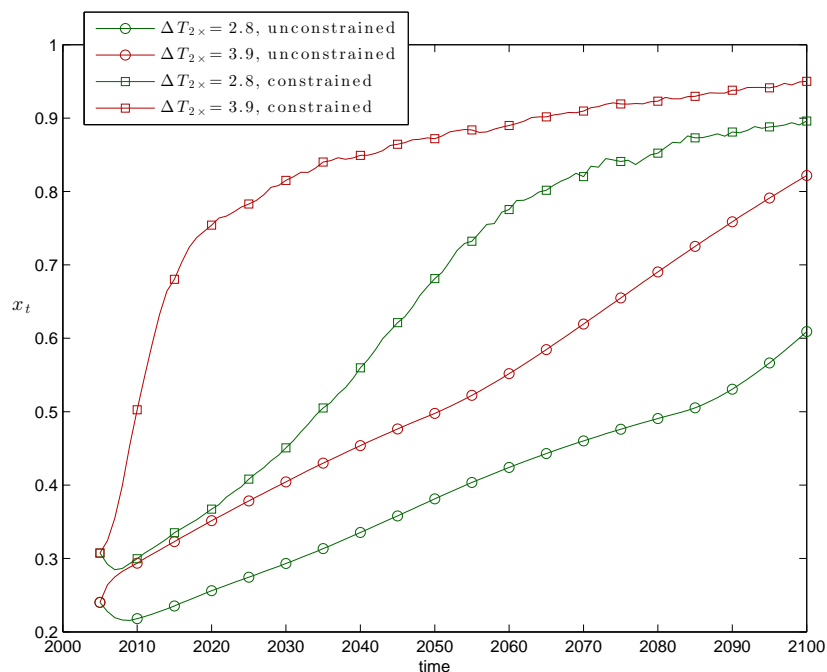


Figure 3: Emissions control rate for $\Delta T_{2\times} = [2.8, 3.9]$, unconstrained and constrained with $\omega = 0.5$. Mean of 1500 simulations.

In both cases, the probabilistic constraint increases the abatement rate. The constraint increases the abatement rate initially as the planner must begin reducing emissions immediately to prevent the target from eventually being exceeded. The initial abatement rate with the probabilistic constraint is 31%, in contrast to the

unconstrained initial abatement rate of 24%. As new information arrives, when $\Delta T_{2\times} = 3.9$, the planner updates the prior and therefore must increase the abatement rate to meet the target. With $\Delta T_{2\times} = 3.9$, the constrained planner must dramatically increase the abatement rate to 68% by 2015. In contrast, when $\Delta T_{2\times} = 2.8$, the constrained planner has more time to slow the climate inertia, and the abatement rate is only 33% in 2015.

The difference between the constrained and unconstrained control rates in 2010-2060 is much greater when $\Delta T_{2\times} = 3.9$. When $\Delta T_{2\times} = 3.9$, the optimal steady state temperature rises. Since the planner estimates the climate is more sensitive to GHG concentrations, the expected future temperature increases. Therefore, the abatement expenditure required to keep the temperature at a given level increases, but the benefits are unchanged. In contrast, by definition, the probabilistic constraint employs a fixed target. Therefore, the difference between the unconstrained and constrained policies rises with $\Delta T_{2\times}$ because the constraint is inflexible: T^* and ω cannot adjust as new information arrives.

Figure 4 plots the average temperature changes for the above two cases.

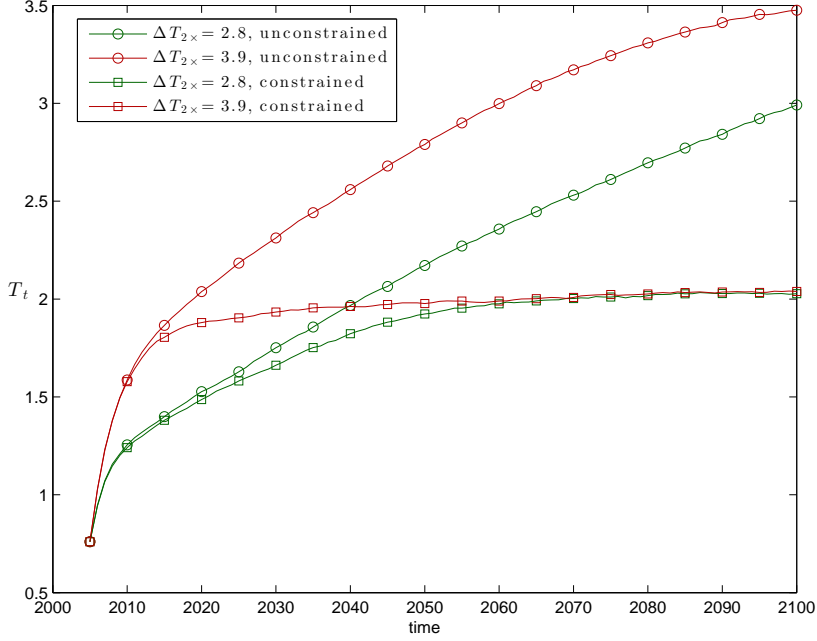


Figure 4: Temperature change for $\Delta T_{2\times} = [2.8, 3.9]$, unconstrained and constrained with $\omega = 0.5$. Mean of 1500 simulations.

When $\Delta T_{2\times} = 2.8$, the unconstrained optimal temperatures cross the target in about 30 years. Therefore, the planner has more time to slow climate change and can spread out the increase in the control rate. In contrast, when $\Delta T_{2\times} = 3.9$, the unconstrained temperature crosses the target in only 14 years. Therefore, in the constrained case, the abatement rate must rise more quickly to keep the temperature below the target. Notice the unconstrained optimal temperature rises when $\Delta T_{2\times}$ is higher as the planner responds to the higher required expenditure to keep the temperature at a given level by letting the temperature rise more. However, the target stays fixed at 2°C .

Next, we examine how abatement policy and the temperature respond to changes in the probability of exceeding the target, ω . We solve the model (37) for various values of ω , and simulate each solution 1500 times. Figure 5 reports the mean abatement control rate for each solution, assuming the true value equals the prior.

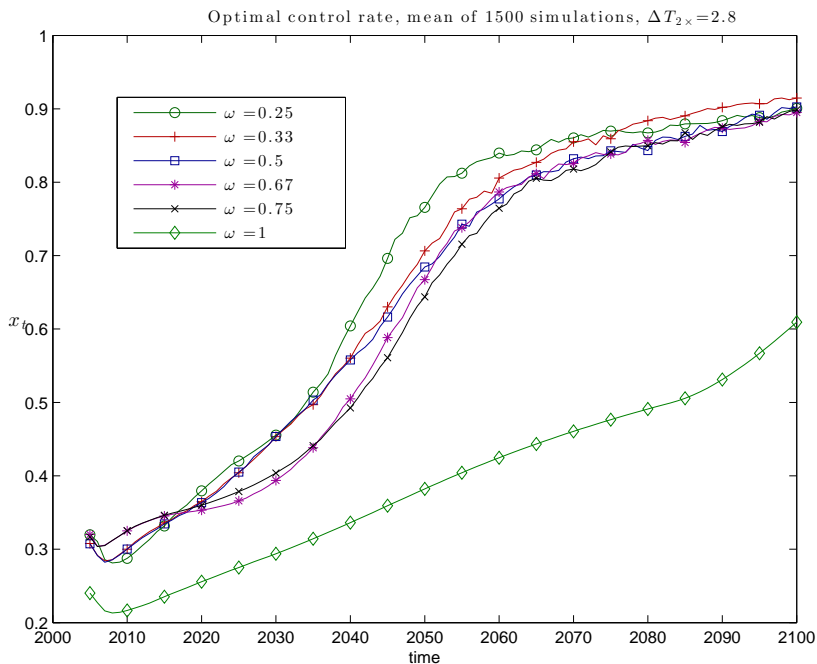


Figure 5: Optimal constrained abatement policy. Each curve is the mean of 1500 simulations, with true $\Delta T_{2\times} = 2.8$ and the reported value of ω .

In Figure 5, $\omega = 1$ corresponds to the unconstrained optimum. The abatement rate drops over the period 2005-2010. Kelly and Tan (2013) show that, while overall learning the climate sensitivity is a slow process, the planner can rule out extreme values of $\Delta T_{2\times}$ relatively quickly if the true value is close to the prior. Therefore, learning quickly eliminates one motivation for early abatement- to insure against potentially extreme values of the climate sensitivity. The control rate then rises over time as abatement becomes less expensive, uncontrolled emissions rise due to economic growth, and wealthier future households are more willing to purchase abatement.

The constraint forces x significantly higher, and the control rate rises as the maximum probability of exceeding the target (ω) becomes smaller. Nonetheless, the maximum probability of exceeding the constraint has only a small effect on the optimal abatement rate. From Figure 2, the probability of exceeding the constraint is zero or one over most of the state space, so the constraint changes little with ω .

Figure 6 shows the temperature change for the same simulations as Figure 5.

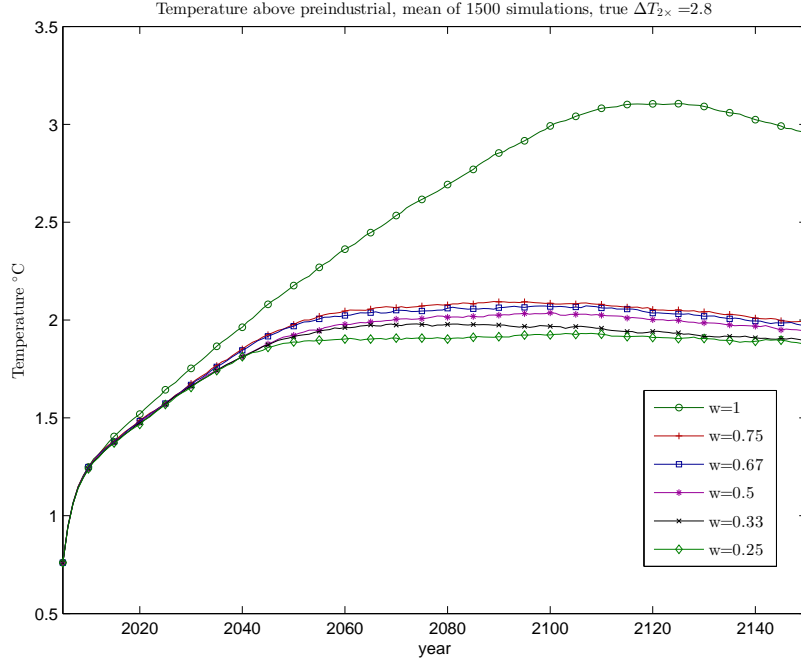


Figure 6: Optimal constrained temperature. Each curve is the mean of 1500 simulations, with true $\Delta T_{2\times} = 2.8$ and the reported value of ω .

The unconstrained optimal maximum mean temperature change is about 3.1°C , so the 2° target is about 1.1 degrees too stringent, on average.¹¹ Smaller values of ω imply a smaller maximum mean temperature change. At $\omega = 0.25$, the maximum mean temperature change is 1.8°C . The temperature is changing randomly with each simulation, so to keep the probability of exceeding 2° below 25% requires a temperature below 2° . The weather shock is mean zero, therefore the probability of exceeding 2° is 50% when the temperature is at the target.

However, the maximum mean temperature in general is not very sensitive to ω . As shown in Figure 2, for most of the temperature range, the realization of the uncertain climate sensitivity causes the temperature to exceed the target with probability one or zero irrespective of ω . If the climate sensitivity is sufficiently high, the temperature exceeds the target with probability one, causing zero emissions regardless of ω . If the climate sensitivity is sufficiently low, the cost of reducing the temperature becomes

¹¹Our model is based on the Nordhaus DICE model, which has no tipping points, irreversibilities, etc. Other models with these features may feature smaller optimal maximum temperature changes.

inexpensive and the unconstrained optimum falls below 2°C. In this case, emissions equal the unconstrained optimum regardless of ω . Therefore, much of the distribution of uncertainty over the climate sensitivity results in emissions which are independent of ω . Anticipating that future emissions and utility will be likely independent of ω , the planner sets near term policy also largely independent of ω .

In addition, learning narrows the uncertainty eventually and the weather shocks have a standard deviation of only 0.11°C. Therefore, if all uncertainty is resolved, $\omega = 0.25$ corresponds to a maximum mean temperature of 1.93°C, whereas $\omega = 0.5$ corresponds to a maximum mean temperature of 2°C, a small difference.

Figure 6 represents an ideal case where learning confirms the prior. Choosing the true $\Delta T_{2\times}$ randomly from the prior distribution results in 25% of simulations exceeding 2° when $\omega = 0.25$. For example, Figure 7 plots average temperature change over time as a function of ω when $\Delta T_{2\times} = 5$.¹²

¹²Weitzman (2009) using IPCC data assigns prior probability that $\Delta T_{2\times} \geq 5 = 0.07$.

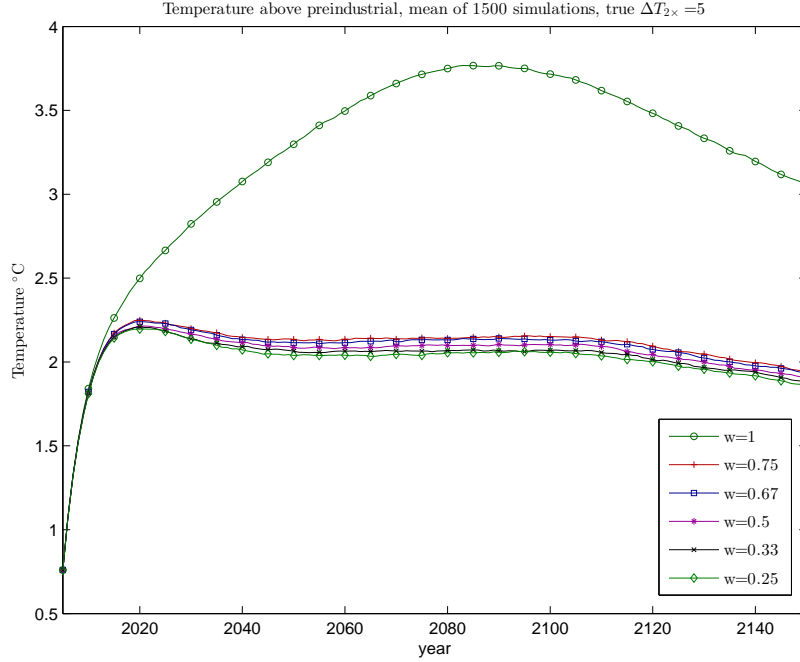


Figure 7: Optimal constrained temperature. Each curve is the mean of 1500 simulations, with true $\Delta T_{2\times} = 5$ and the reported value of ω .

The mean optimal unconstrained temperature change rises to 3.77°C , indicating the optimal unconstrained policy is sensitive to climate sensitivity. The mean optimal unconstrained temperature change increases because when the climate is more sensitive to GHG concentrations, the abatement expenditures required to keep the temperature at a given level rises, but the benefits are unchanged. In contrast, the target is by definition inflexible and does not vary with the resolution of uncertainty. The mean maximum temperature change falls with ω as in Figure 6, but is not very sensitive to ω . Notice that when $\Delta T_{2\times} = 5$, on average the temperature exceeds the target for about 15 years, regardless of ω . From the planner's point of view, $\Delta T_{2\times} = 5$ is unexpectedly high, so this case is one realization in which the 2°C limit is crossed. Integrating over all possible realizations of $\Delta T_{2\times}$ yields simulations which exceed the target with probability equal to ω . Once the target is exceeded, the planner must set $x = 1$ until the temperature returns to the target. Therefore, the planner incurs an additional welfare cost here in that the planner must return to the target faster than is optimal.

4.2 Welfare Loss

The probabilistic constraint at least weakly restricts the choice set for the planner, and therefore must result in a welfare loss.¹³ Our interest is how the welfare loss varies with ω and how uncertainty affects the welfare loss.

Figure 8 graphs the welfare loss as a percentage of the welfare of the unconstrained problem, $\omega = 1$.

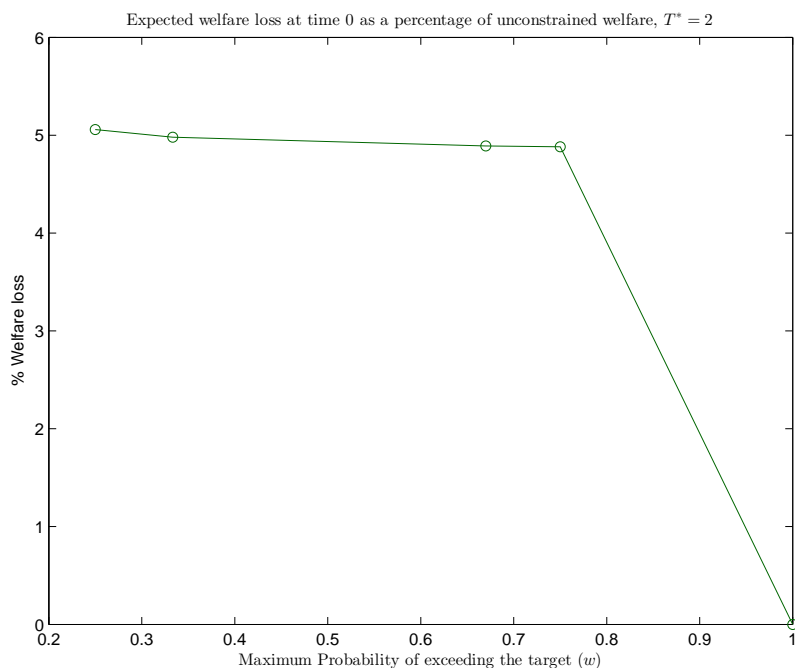


Figure 8: Welfare loss as a function of ω . The graph plots $1 - v(s_0; \omega) / v(s_0, 1)$, where v is the solution to (36), as a function of ω .

From the graph, the unconstrained problem has no welfare loss, and welfare loss is nearly constant in ω . Welfare loss is about 5% of the unconstrained policy, regardless of ω . For most of the state space, the probability of exceeding the target is either zero or one regardless of ω , and so the optimal temperature paths do not vary much

¹³We are following, for example, Nordhaus (2007), who imposes a 2°C constraint a version of the model with no uncertainty, and then calculates the welfare loss. Other authors use CEA and replace the damage function with the probabilistic constraint. The motivation for reducing emissions then arises from the constraint, rather than the damage function. To the extent that these are different, using CEA would only add to the welfare loss calculated here.

with ω . Nonetheless, the welfare loss increases slightly as the constraint becomes more restrictive (ω falls).

Figure 9 plots the welfare loss for various true values of $\Delta T_{2\times}$.

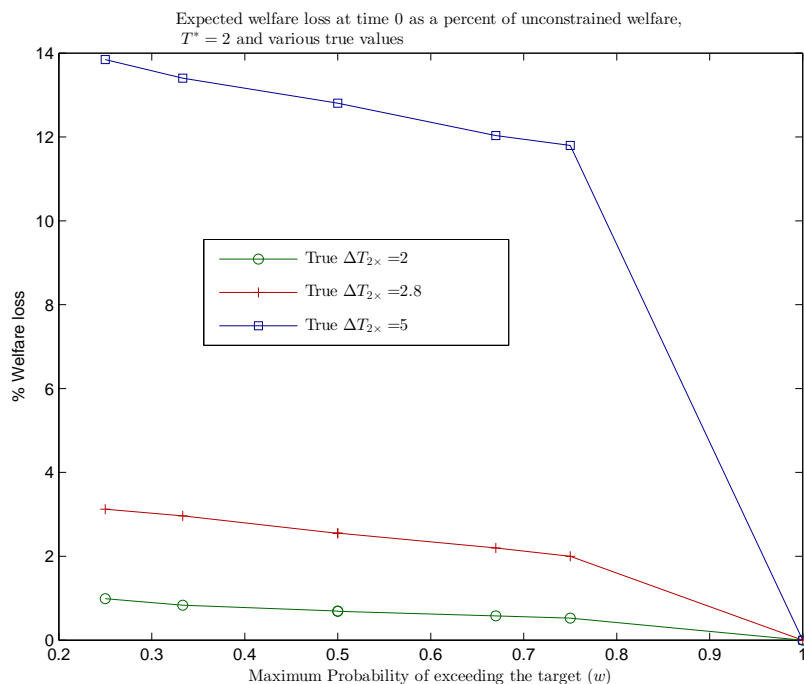


Figure 9: Welfare loss as a function of ω and the true $\Delta T_{2\times}$.

Welfare loss increases with the true value of $\Delta T_{2\times}$, rising to 14% or more when $\Delta T_{2\times} = 5$.

When the arrival of new information indicates the climate sensitivity is higher than expected, the unconstrained planner learns that the cost of keeping the temperature at a given level has risen. The planner increases abatement in response to the higher expected damages. Nonetheless, the high climate sensitivity implies temperature is on a higher trajectory. In contrast, the constrained planner must increase abatement still further, to keep the temperature on the same trajectory despite the high climate sensitivity, because the constraint does not change. Therefore, for high $\Delta T_{2\times}$, the constraint has a welfare loss because the constraint is inflexible.

We can decompose the total welfare loss into a welfare loss caused by the target being set too low under certainty (as in Nordhaus 2007), a welfare loss caused from

inflexibility of the constraint (new here), and a welfare loss caused by the probabilistic constraint over-reacting to transient shocks (new here).

Let $\hat{x}(s)$ be the optimal unconstrained abatement policy derived from $v(s, 1, T^*)$. Note that \hat{x} is independent of T^* since $\omega = 1$. Further, let $T(\hat{x}(s_t), s_t, \beta_1, \nu_t)$ be a temperature path associated with the optimal unconstrained policy. Finally, let:

$$\hat{T}_t = \text{E}[T(\hat{x}(s_t), s_t, \beta_1, \nu_t)], \quad (52)$$

$$\hat{T} = \max \hat{T}_t. \quad (53)$$

So \hat{T} is the maximum over time of the mean temperature path. If the target was $T^* = \hat{T}$, then the constraint just binds on average, and any remaining welfare loss is due to inflexibility and over-reaction. That is:

$$\text{Loss} = \frac{V(s_0; 1, T^*) - V(s_0; \omega, T^*)}{V(s_0; 1, T^*)}, \quad (54)$$

$$= \frac{V(s_0; 1, T^*) - V(s_0; \omega, \hat{T})}{V(s_0; 1, T^*)} + \frac{V(s_0; \omega, \hat{T}) - V(s_0; \omega, T^*)}{V(s_0; 1, T^*)}, \quad (55)$$

$$= \text{Loss, target too low} + \text{Loss, inflexibility and over-reaction}. \quad (56)$$

Calculating the above losses, we find that about 25% of the loss is due to inflexibility of the constraint and over-reaction to transient shocks. Therefore, uncertainty exacerbates the welfare cost of probabilistic targets.

5 Conclusions

In this paper, we evaluated the common policy recommendation of a 2°C temperature limit in the context of an integrated assessment model with uncertainty and learning about the climate sensitivity. Because climatic processes are still highly uncertain and subject to inertia, it is difficult to envision controlling the climate to the precise degree implied by the 2°C target. Indeed, we show that even with an immediate reduction of all GHG emissions to zero, the temperature eventually exceeds 2°C

with probability 0.15. Further, we show that as uncertain climatic processes evolve, the temperature randomly moves to a place where the 2° target is either impossible to satisfy or is satisfied even with no abatement. Our results therefore cast doubt on how workable a stabilization target is in an uncertain environment.

Further, we show that stabilization targets induce 3 welfare losses: first, the stabilization target may be too low on average, inducing a welfare loss even in a model with certainty. Second, with uncertainty, as new information arrives the optimal temperature path adjusts. Because the stabilization target is by definition inflexible, adhering to a stabilization target causes welfare loss. Third, as the climate randomly evolves over time, the temperature will exceed the target for at least some periods. In this case, the planner must set an excessively high control rate to immediately return the temperature back to the target. We show that the welfare loss from all the causes is on the order of 5%, at least 25% of which occurs only with uncertainty.

Our model may be extended in a number of ways. We could consider other targets such as limit GHG concentrations to 350 ppm or limiting sea level rise or ocean acidification. Our results will also likely extend to regulations other than climate change. For example, some fisheries regulations try to achieve a particular stock of fish, even though fish stocks are affected by many uncertain processes other than the size of the catch. One may envision stabilization targets as providing some welfare benefits outside the current model. For example, they could serve as a commitment device to induce firms to invest in irreversible abatement capital.

Stabilization targets are easy to convey to the public in that a 2°C temperature limit is easier to understand than, for example, a particular carbon tax. Since damages are a function of temperature, it is also easier to understand the impacts of 2°C temperature limit. However, one must use caution in that a stabilization target may convey the false impression that we have precise control over an uncertain climate and that our understanding of the optimal temperature change will not change as new information arises.

A Appendix

A.1 Exogenous variables

The DICE model includes many variables which change exogenously over time. Further, unlike DICE, we assume the ocean temperature also changes exogenously to reduce the state space. Reducing the state space from seven to six variables significantly reduces the computation time. Traeger (2012) presents a deterministic DICE model with exogenous ocean temperature. Therefore, we take the evolution of the exogenous variables directly from that study. For completeness, they are listed below. We refer the reader to Traeger (2012) for details.

Population growth:

$$L(t) = L_0 + (\bar{L} - L_0) (1 - \exp[-g_L t]). \quad (57)$$

TFP growth:

$$A(t) = A_0 \exp \left[g_{A,0} \frac{1 - \exp[-\delta_A t]}{\delta_A} \right]. \quad (58)$$

Backstop cost:

$$\Psi(t) = \frac{\sigma(t)}{a_2} a_0 \left(1 - \frac{1 - \exp[g_\Psi t]}{a_1} \right). \quad (59)$$

Emissions intensity of output:

$$\sigma(t) = \sigma_0 \exp \left[g_{\sigma,0} \frac{1 - \exp[-\delta_\sigma t]}{\delta_\sigma} \right]. \quad (60)$$

Exogenous emissions:

$$B(t) = B_0 \exp[-\delta_B t]. \quad (61)$$

Decay rate of GHGs:

$$\delta_m(t) = \bar{\delta}_m + (\delta_{m,0} - \bar{\delta}_m) \exp[-\delta_m^* t]. \quad (62)$$

Exogenous forcing:

$$EF(t) = EF_0 + 0.01 (EF(100) - EF_0) \cdot \min \{t, 100\}. \quad (63)$$

Heat uptake from ocean:

$$O(t) = \max \{ \Delta_{T1} + \Delta_{T2}t + \Delta_{T3}t^2, 0 \}. \quad (64)$$

Discount factor:

$$\beta(t) = \exp \left[-\delta_u + (1 - \eta) \log \left(\frac{A(t+1)}{A(t)} \right) + \log \left(\frac{L(t+1)}{L(t)} \right) \right]. \quad (65)$$

A.2 Tables

Parameter	Description	Value
L_0	Initial population	6514
\bar{L}	Steady state population	8600
g_L	decline rate in population growth	0.035
A_0	Initial TFP	0.0058
g_{A0}	Initial TFP growth rate	0.0131
δ_A	Decline rate in TFP growth rate	0.001
γ	Capital share	0.3
δ_k	Depreciation rate of capital	0.1
η	Coefficient of risk aversion	2
δ_u	Pure rate of time preference	0.015
g_Ψ	Backstop cost growth rate	-0.005
a_0	Initial backstop cost	1.17
a_1	Backstop cost parameter	2
a_2	Cost convexity	2
b_1	Damage function parameter	0.00284
b_2	Damage function convexity	2
σ_0	Initial emissions intensity	0.1342
$g_{\sigma,0}$	Initial growth rate in σ	-0.0073
δ_σ	Decline rate in emissions intensity growth	0.003
B_0	Initial exogenous emissions	1.1
δ_B	decay rate in exogenous emissions	0.0105
MB	Pre-industrial GHG stock (gigatons)	590
δ_m^*	decay rate in GHG decay rate	0.0083
$\bar{\delta}_m$	steady state GHG decay rate	0.01
$\delta_{m,0}$	initial decay rate	0.014
EF_0	Initial exogenous forcing	-0.06
EF_{100}	Exogenous forcing at $t = 100$	0.3
Ω	Forcing coefficient	4.39
α	Ocean heat uptake	0.2295^{-1}
ξ	Heat transfer coefficient from the ocean	0.07
Δ_{T1}	Ocean Temperature Parameter	0.7
Δ_{T2}	Ocean Temperature Parameter	0.02
Δ_{T3}	Ocean Temperature Parameter	-0.00007
k_0	Initial capital per TFP adjusted person	3.6261
T_0	Current air warming above pre-industrial	0.76
m_0	Current GHG stock, relative to pre-industrial	1.371
μ_0	Mean of climate feedback prior distribution	0.65
η_0	Precision of climate feedback prior distribution	0.13^{-2}
ρ	Precision of weather shock	0.11^{-2}

Table 1: Description and values of model parameters.

A.3 Solution

We solve the model by forming a grid of values for the state space, and then assuming v takes the form of a cubic spline with continuous first and second derivatives. The model can then be solved by assuming an initial spline, optimizing at each grid point, and then using the solution to update the parameters of the spline.¹⁴

Once the model converges, we obtain the optimal decision rules, $x(s)$ and $k'(s)$. We then simulate the model using the decision rules and the transition equations (17)-(23). The algorithm is:

1. Draw a true value of the climate feedback parameter β_1^* from $N[\mu_0, \frac{1}{\eta_0}]$.
2. Given s_0 compute $x(s_0) = x_0$.
3. Given x_0, s_0, β_1^* , and a randomly drawn weather shock $\nu_{0,,}$, compute s_1 from transition equations.
4. Repeat steps (2)-(3) for np years.
5. Repeat steps (2)-(4) ns times with different draws for ν and β_1 .
6. Compute means over all simulations to get the expected value of each variable in each time period.

The above algorithm gives the expected value of each variable conditional on the prior distribution for β_1 . In some cases, we fix a value for β_1 and vary on ν across simulations. In this case, we obtain the expected value of each variable conditional on β_1 .

¹⁴Kelly and Kolstad (1999b) give a detailed explanation of this solution method, except they use neural networks rather than splines.

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