Climate Change and Optimal Energy Technology R&D Policy

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Public policy response to global climate change presents a classic problem of decision making under uncertainty. Theoretical work has shown that explicitly accounting for uncertainty and learning in climate change can have a large impact on optimal policy, especially technology policy. However, theory also shows that the specific impacts of uncertainty are ambiguous. In this paper, we provide a framework that combines economics and decision analysis to implement probabilistic data on energy technology research and development (R&D) policy in response to global climate change. We find that, given a budget constraint, the composition of the optimal R&D portfolio is highly diversified and robust to risk in climate damages. The overall optimal investment into technical change, however, does depend (in a non-monotonic way) on the risk in climate damages. Finally, we show that in order to properly value R&D, abatement must be included as a recourse decision.

Key words: R&D portfolio, energy technology, climate change, stochastic programming, public policy

1. Introduction

Emissions of greenhouse gases have risen more than 30% over the past two decades, and a further 36% increase is estimated between 2006 and 2030 (DOE 2006). While scientists largely agree these emissions are changing the climate, there is a great deal of uncertainty about the degree to which global warming will cause economic, social, and environmental damages in the future. Public policy responses to climate change are being developed under this uncertainty.

Possible near term policy responses to global climate change include both restrictions on emissions (through emissions limits or taxes) and investment in environmentally friendly technologies. Overall, addressing climate change in a cost effective way will almost certainly require the development of better energy technologies (Hoffert et al. 1998). As an example of recent policy actions aimed in this direction, the U.S. Government has allocated \$16.8 billion to U.S. Department of Energy (DOE) as part of the 2009 American Recovery and Reinvestment Act to support research and development (R&D) in energy technologies.

It is clear that the optimal energy technology R&D policy will involve investing in a *portfolio* of technologies. It is not clear, however, what technologies the portfolio should contain. Answering this question involves a number of issues, and in particular requires explicitly incorporating uncertainty over multiple dimensions (Baker and Shittu 2008). The process of R&D is inherently uncertain – we cannot predict whether any particular program will be successful, or the degree to which it will meet or exceed goals. In the case of climate change, we also have deep uncertainty on the benefits side as there is considerable uncertainty about the damages that will be caused by climate change, and hence, the benefits from reducing emissions. This feeds back, to create uncertainty about the value of having any particular technology available.

A number of researchers have investigated the question of how the presence of uncertainty and learning impacts near term optimal climate policy (see Baker and Shittu (2008) and Baker (2009) for reviews). The answer to this question seems to be "it depends": optimal near term decision variables, such as R&D investment, may increase or decrease with increases in risk or increases in learning. Thus, the next step is to try to characterize the uncertainty that we are facing and implement this into policy models.

In this paper we combine economics and decision analysis to get insights about the optimal energy technology R&D portfolio under uncertainty, and how it changes with increasing risk in climate damages. More specifically, we try to answer the following policy question based on actual empirical data gathered through expert elicitations: How should government funding in climate change energy technology R&D be allocated to different technologies and projects?

To this end, we implement data collected from expert elicitations on how government funding

impacts the probability of success in three key climate change energy technologies – solar photovoltaics, nuclear power, and carbon capture and storage. We choose these three technologies based on the analysis and observation by Lewis and Nocera (2006) that solar, nuclear power, and carbon capture and storage are the three technologies with sufficient resources to provide the carbonneutral energy needed to address the climate change problem. It is important to note that a full portfolio would consider a range of other technologies, including a wider range of solar technologies such as solar thermal or liquid fuels directly from sunlight, enabling technologies such as batteries or fuel cells, and other renewable technologies such as energy from biomass and wind. However, the enabling technologies gain their value largely from the success of the three main technologies, while other renewables have limited resource bases. Therefore, a portfolio analysis based on the three major technologies above produce useful policy results.

Given the gathered empirical data, our approach to answering the above research question involves three phases. In the first phase, we combine the data with an economic model to derive stochastic marginal abatement cost curves that describe the cost of reducing emissions by one additional ton. In the second phase, we develop an energy technology R&D portfolio model that uses this probabilistic information. Noting that the model is highly nonconvex, and not amenable to structural analysis, as is the case for most portfolio models, we develop a convex reformulation of the model as a two-stage stochastic programming problem. In the third phase of the research, we analyze the structure of the optimal climate change energy technology R&D portfolio, and identify the resulting policy implications. The components of our analysis and their relationships are displayed in Figure 1.

In addition to the derivation of a convex energy technology R&D portfolio model and its policy implications, our approach also presents a framework for turning empirical data into a working stochastic model. This is significant because most studies, as we note in the literature review section below, either use purely theoretical probability distributions that are conveniently analyzed through a developed model, or they use simplified approaches to go with elicitations of specific



Figure 1 Components of the energy technology R&D portfolio analysis and their relationships.

variables. These procedures, however, typically result in some loss of accuracy and validity in the analysis.

The remainder of this paper is structured as follows. We complete this section with a review of the literature on general R&D portfolio management as well as on climate change energy technology policy. In Section 2 we describe some theoretical background for the problem. In Section 3 we combine expert elicitations with economic analysis and derive stochastic marginal abatement cost curves. Based on this information, we then develop a climate change energy technology R&D portfolio model in Section 4, and describe a stochastic programming formulation and solution procedure. In Section 5, we present our analysis and policy implications based on the results from the model. In Subsection 5.1 we show that, given our data, the composition of the optimal portfolio is robust to climate damage uncertainty. The value of technical change, however, does depend explicitly on uncertainty. In Subsection 5.2 we go on to show that the overall optimal investment in R&D changes in the riskiness of climate damages; however it changes in a non-monotonic way. Thus, there is real value in characterizing the uncertainty over climate damages. In Subsection 5.3, we investigate fixed abatement policies and draw conclusions about their efficiency. Finally, in Section 6 we summarize our conclusions.

1.1. Literature on R&D Portfolio Management

While there exists some significant research in technology portfolio management, a direct application of the proposed methods to the climate change R&D problem is not possible. This is due to the major differences that exist between the cost/return functions of traditional R&D investments and the energy technology investments under climate change uncertainty. More specifically, returns from climate change energy technology R&D are not calculated directly, but rather through the impact of successful technologies on an emission abatement function, which is further complicated by the uncertainty in damages due to climate change and interactions between different technologies. We describe this unique R&D structure in detail in Section 4.

Nonetheless, there are similarities between traditional R&D management and energy technology portfolio management. Thus, we first outline the existing literature in general R&D portfolio management and then discuss models specifically proposed for investing in energy technologies.

De Reyck et al. (2005) study the impact of R&D portfolio management techniques on performances of projects and the overall portfolios. The authors identify certain key components required for an effective R&D portfolio management approach such as capturing of returns and risks, modeling of interdependencies, as well as the determination of prioritization, alignment and selection of projects.

The models that have thus far been proposed for R&D portfolio management include capital budgeting models, which typically use accounting-based criteria, such as return on investment or internal rate of return. These models capture interdependencies between different projects, but fail to model the uncertainty in returns (Luenberger 1998). There are also several mathematical programming based deterministic models that have been proposed. Dickinson et al. (2001) present a deterministic nonlinear integer programming model to optimize project selection. Elfes et al. (2005) address the problem of determining optimal technology investment portfolios that minimize mission risk and maximize the expected science return of space missions. Lincoln et al. (2006) develop a method for prioritization of technology investments using a linear programming formulation to maximize an objective function subject to overall cost constraints. In a more general multistakeholder and multicriteria decision based study, Grushka-Cockayne et al. (2008) develop a framework for project valuation and selection in air traffic management system design, where project interactions and other complexities are explicitly modeled.

As a stochastic R&D portfolio model, April et al. (2003) describe a simulation optimization tool, which utilizes metaheuristics to search for good technology portfolios, but the model is limited in capturing the interdependencies among technologies. Other stochastic approaches include real options based methods. Bardhan et al. (2006) propose a multi-period optimization model where the objective is based on real options values of the portfolio calculated according to the results from Bardhan et al. (2004). Campbell (2001) and Lee et al. (2001) model project contingencies as real options to determine optimal startup dates for the projects.

In a dynamic programming based model, important analytical results under some limiting assumptions have been developed by Loch and Kavadias (2002) in the context of new product development. Further, Solak et al. (2007) develops a stochastic programming based method for R&D portfolio management with explicit consideration of project interactions and probabilistic return realizations. However, the model requires significant computational effort for portfolios with large number of projects.

In addition to these models, many strategic planners and project portfolio managers rely on decision tools such as Analytical Hierarchy Process and Quality Function Deployment, in planning the funding of technology development (Thompson 2006). Similar systematic evaluation methods are also proposed by Sallie (2002) and Utturwar et al. (2002), where the authors propose bilevel approaches in selecting technologies to fund. The latter study also contains an optimization procedure based on a genetic algorithm implementation. While these methods provide significant insights, they are also limited in their ability to fully quantify the complicated return and investment structure in a portfolio of energy technologies with combinatorial interactions, mainly due to their deterministic nature and other simplifying assumptions.

1.2. Literature on Climate Change and Energy Technology R&D

In terms of energy technology R&D, there is a growing body of work on endogenous technical advance in the context of climate change. This literature covers technical change that is in some way induced by policy, generally by the indirect effect on market actors, but also as a control variable. For surveys of the literature, the reader can refer to Clarke and Weyant (2002), Grubb et al. (2002), Loschel (2004), Sue-Wing (2006), Clarke et al. (2006a, 2006b) and Gillingham et al. (2007). While the papers covered in these surveys are largely deterministic, they indicate that technology development and deployment should be part and parcel of climate change policy evaluation.

There is some very recent literature investigating the optimal investment in energy technology R&D in the face of uncertainty. Some papers consider uncertainty in the climate damages (Farzin and Kort 2000, Baker et al. 2006, Baker and Shittu 2006, Baker 2009), while some consider uncertainty in technological change (Bosetti and Drouet 2005, Bosetti and Gilotte 2007, Goeschl and Perino 2009), and one paper considers both (Baker and Adu-Bonnah 2008). However, all of these studies consider investment in one technology at a time, rather than a portfolio of technologies. While the conclusions of these papers vary, it appears that uncertainty in technological change has a quantitatively larger impact on optimal actions than does uncertainty in climate damages, and that the optimal investment in R&D is often much higher when uncertainty is explicitly included.

A small number of papers have studied the impact of uncertainty on a *portfolio* of energy technologies. Gritsevskyi and Nakicenovic (2002) and Grubler and Gritsevskyi (2002) consider the question of how diversified the near term technology portfolio should be when the rate of technological learning is uncertain, and find that investment should be distributed across technologies that are in a cluster. Further, the second paper indicates that optimal diversification increases with uncertainty in damages as long as increasing returns to scale are present. However, they consider technical change through the avenue of learning by doing, rather than through R&D. Two studies that are closely related to this paper are Blanford and Weyant (2007) and Blanford (2009). They consider the question of the optimal R&D portfolio when there is uncertainty in both technological change and climate damages, with a focus primarily on the drivers of diversification in the portfolio. They show that it is not enough to just consider the potential value of new technologies, but that the uncertain relationship between program funding and effectiveness is just as important. They provide a framework for considering spillovers between technologies, but don't operationalize it. One benefit to our approach is that we can explicitly examine the impact of increasing uncertainty on optimal investment. The key difference between the two approaches, however, is that we build our model on empirical estimates of the probability of success based on expert judgments, whereas they propose a theoretical probability model in which they assume decreasing returns to scale. This assumption allows them more freedom in two directions. First, they model a sequential decision problem in which R&D investments can be made in two periods, whereas we focus on a single near term decision only. They find, however, that the effect of possible future R&D decisions on near term decisions is small. Second, their decision variable, R&D expenditures, is continuous, where ours is modeled as an integer yes-or-no problem to be consistent with our elicited data and current decision framework.

2. Theoretical Background and Motivation

In this section we start by providing our motivation for focusing on Marginal Abatement Cost Curves (MAC), i.e curves that reflect the cost of reducing emissions by an additional ton. We then provide a very simple example that illustrates the importance of explicitly including damage uncertainty when choosing an optimal portfolio.

2.1. The Marginal Abatement Cost Curve

The uncertainties in both climate damages and in technical change are dynamic, in that we expect to learn more about each as time goes on. We know that humans are changing the climate, but we are uncertain about the exact relationship between the stock of emissions in the atmosphere and the change in global mean temperature. Moreover, we are uncertain about how global mean temperature will translate to specific local climate effects such as drought and flooding, heat waves, and increases in intensity or quantity of storms such as hurricanes. Similarly, the pace and direction of technical change is also uncertain. Some technologies, such as nuclear fusion, have eluded breakthroughs for a very long time, while other technologies, such as wind turbines and natural gas combined cycle turbines, have been more successful than most people imagined.

The value of a particular R&D program for a given technology depends not only on whether the technology development is successful, but also on the severity of climate change damages in the future. Some technologies, such as improvements in fossil fuel efficiencies, may have the largest impact if climate change turns out to be mild and only small reductions in emissions are called for. On the other hand, at very high abatement levels society will tend to substitute away from fossil fuel, and thus improvements in those technologies will have less impact. Other technologies, such as electric vehicles, may have the most impact if climate change turns out to be very severe, calling for an almost total reduction in greenhouse gas emissions.

It is particularly important to understand how new technologies will impact the MAC. In Figure 2 we illustrate how the impact of technical change on optimal abatement varies with technology and with the severity of marginal damages. The solid upward sloping line represents the original MAC. The two dashed lines represent different types of technical change. The horizontal lines represent two levels of marginal damages (MD), i.e. high and low. On the horizontal axis we show the optimal level of abatement in each case, where μ_{ij} represents optimal abatement given damages i = H, L and MAC curve j = 0, 1, 2. Note that the technical change embodied by MAC₁ has no effect when marginal damages are low, but a significant effect when damages are high, while the impacts of MAC₂ on optimal abatement are nearly the reverse. By paying attention to the impact of technology all along the curve (rather than just a point estimate), we gain information about how optimal behavior will change with changes in marginal damages. However, both the impact of technology and the marginal damages involve significant uncertainty.

2.2. The Impact of Damage Uncertainty

Previous work has shown that the optimal level of investment in a particular technology depends on the probability distribution over climate change damages (Baker et al. 2006, Baker and Adu-Bonnah 2008). Here we illustrate with a simple example that the *choice* of technology also depends



Figure 2 Stylized representations of technical change impact on the MAC, and resulting optimal abatement levels.

on the probability distribution over damages.

For a given abatement level μ , let the baseline cost of abatement be $c(\mu) = \frac{\mu^2}{2}$ and the damages from climate change be $z(1-\mu)$, where z represents the level of damages. The baseline MAC is then $c'(\mu) = \mu$.

We observe that the effect of technology on the MAC is a combination of a downward pivot and a downward shift, as shown in Figure 3 through a stylized example. Without loss of generality, consider two technologies with the same R&D cost, one that *pivots* the MAC by $\alpha = 0.5$ to give a new MAC of $\tilde{c}'(\mu) = 0.5\mu$; and another that *shifts* the MAC by h = 0.125 to give a new MAC of $\bar{c}'(\mu) = \mu - 0.125$. Given this simple formulation, optimal abatement under the pivot technology is $\mu_{\alpha z} = \frac{z}{1-\alpha} = 2z$ and optimal abatement under the shift technology is $\mu_{hz} = z + h = z + 0.125$ (both limited to a maximum of 1). Let mean damages be $\bar{z} = 0.3$. At this damage level the total social cost is the same under either of the technologies, i.e. they have equivalent value. Now consider a mean-preserving spread (MPS) where the random damage parameter Z is equal to $z_l = 0.1$ or $z_m = 0.5$ with equal probability. In this case, the pivot technology is strictly preferred to the shift technology as it has a lower expected total social cost. However, consider a different MPS, where $Z = z_l = 0.1$ with probability 27/29 and $Z = z_h = 3$ with probability 2/29. In this case the shift technology is strictly preferred to the pivot. In Figure 4, we illustrate this example.



Figure 3 A stylized example of a shift and a pivot to the MAC.



Figure 4 Illustrative MACs and marginal damage curves.

The reason for the change in optimal choice is as follows. When damages are below the mean of $\bar{z} = 0.3$, the shift technology is better than the pivot, and above \bar{z} the opposite is true. Under low damages, $z_l = 0.1$, the optimal level of abatement is about the same under the two technologies, i.e. the MD low curve in Figure 4 crosses both MACs at about the same place. However, the cost of abatement is lower for the shift, i.e. the area under the shift curve is smaller than the area under the pivot curve. Under the medium damage case, $z_m = 0.5$, optimal abatement under the pivot is just equal to 1. The first MPS favors the pivot, because there is a small difference between the two technologies when damages are small, but a large difference when the damages (not shown in the other hand, the second MPS has a very small probability of very large damages (not shown in the figure). The pivot technology is much better in the high damage case, but the probability is so low that it favors the shift technology.

This analysis illustrates that in general (1) the optimal portfolio may depend on the risk in the climate damages, and (2) it does not change monotonically in risk. Therefore, it is crucial to do

sensitivity analysis over the probability distribution of damages to determine if such behavior holds under currently available technology and climate change information.

3. Deriving Uncertain Marginal Abatement Cost Curves

In this section we discuss how we combine data based on expert elicitations with economic modeling to derive probabilistic inputs for our R&D portfolio model. Specifically, we derive and parameterize uncertain Marginal Abatement Cost Curves which are then used to define the stochastic return structures of technologies in the portfolio optimization problem described in Section 4. Our analysis focuses on two questions: (1) How will different technologies impact the MAC?, and (2) What is the probability distribution over different outcomes of technical change? In the following subsections we provide answers to these questions. We first present the data that we elicited from experts, and then describe the methodology we used to combine this data with empirical MACs to generate stochastic parametric MACs defining the impact of technical change.

3.1. Elicitation Data

Past data on technological advance contains little information about future technological breakthroughs. In fact, a technological breakthrough, by its nature, is unique; and therefore we cannot use past data and relative frequencies to construct a probability distribution over success for future breakthroughs. Yet, current decisions depend on understanding the likelihood of such breakthroughs. For example, sound government technology R&D policy should consider the *likelihood of success* and the *impacts of success*, along with the total cost of a program, when making decisions (National Research Council 2007). When past data is unavailable or of little use, the alternative is to rely on subjective probability judgments (Apostolakis 1990). Expert elicitations are a formal method for gathering these judgments.

Decision analytic methods including expert elicitations (Howard 1988) have been applied productively to R&D in numerous industries, such as the automotive, pharmaceutical, and electronics industries (Sharpe and Keelin 1998, Clemen and Kwit 2001), as well as issues relating to societal decisions (Howard et al. 1972, Peerenboom et al. 1989, Morgan and Keith 1995). Most relevantly, National Research Council (2007) recommends that the U.S. Department of Energy use panel-based probabilistic assessment of R&D programs in making funding decisions.

Baker et al. (2008), Baker et al. (2009a) and Baker et al. (2009b) describe expert elicitations on the three major energy technologies: solar photovoltaic cells, carbon capture and storage, and nuclear power. Before discussing how we combined these elicitations with an economic model to derive stochastic MACs, we first describe each technology and potential research directions for these technologies in the coming years, based on expert opinions.

Solar photovoltaic (PV) cells turn the energy in sunlight into electricity. We consider three research directions for this technology. Purely organic solar cells use organic materials as semiconductors, with the advantage of easier manufacturing and a wide range of potential end uses. The second research direction is essentially a search for better inorganic semiconductors, to replace silicon or other less promising but well-studied alternatives. Finally, third generation concepts include highly efficient technologies involving new cell architectures, quantum dots and multi-junction cells.

Carbon capture and storage (CCS) refers to the process of capturing the CO₂ generated by fossilfuel electricity plants before it is released into the atmosphere and storing it either underground in aquifers or in the deep ocean. There are three main categories of CCS corresponding to three points in the process: Pre-combustion carbon capture, alternative combustion, and post-combustion removal. Pre-combustion capture works in conjunction with combined cycle power plants to remove CO_2 from syngas generated from fossil fuels or biomass. Challenges are to make this process energy efficient and robust. Chemical looping, the alternative combustion technology we consider, uses fine solid particles to carry oxygen to react with the fuel and then carry CO_2 away from the reaction without release into the air. This technology is at the early stages of research and faces some daunting challenges, but if successful, it is a very attractive technology, with much lower energy and non-energy demands. Post-combustion CO_2 separation, which removes CO_2 from flue gases, is the most mature of the technologies we consider, and mainly faces challenges related to cost reduction.

Technology	Project	NPV of Funding (000,000)	Probabiltiy of success	Alpha	Shift
Solar		¢116	0.0%	0.050	0.017
	Organic	Φ 110	13.0%	0.022	0.007
		\$830	3.9%	0.050	0.017
			24.8%	0.022	0.007
	Inconcorio	\$39	26.7%	0.022	0.007
	morganic	\$77	44.3%	0.022	
	3rd Generation	\$386	2.0%	0.050	0.017

 Table 1
 Summary of assessment results for solar.

For nuclear power, we consider improvements on the current Light Water Reactors (LWR); and also two more radical directions: High Temperature Reactors (HTR) and Fast Burner Reactors (FR). Both of these have the advantage of higher efficiencies and potentially lower waste.

The products of the expert elicitations, which are described in detail in Baker et al. (2008, 2009a, 200b), include explicit definitions of endpoints for each technology, and probabilities of achieving those endpoints for given funding trajectories. In Tables 1 - 3 we report the relevant results. The first column in each table identifies the technology category and the second column lists the sub-categories we consider for each technology. The third column gives the NPV of the funding trajectory considered. The funding trajectories themselves varied by yearly amount and by the number of years. We have used a discount rate of 5% to calculate the NPVs, and considered multiple funding trajectories for some technologies. The fourth column reports the average probability of success elicited from the experts. In some cases, we defined two different levels of success. In these cases, the probability on the top is the probability for a high level of success, and on the bottom for a moderate level of success. For example, organic solar cells have two levels of success for each funding trajectory, while inorganic solar cells have only one level of success. The fifth and sixth columns represent the pivoting and shifting impacts on the MAC, respectively. The derivation of these impact parameters is described in Section 3.2 below.

Technology	Project	NPV of Funding (000,000)	Probabiltiy of success	Alpha	Shift
		\$39	2.7%		0.004
	Pre-Com	\$154	11.0%	0.347	
		\$386	22.3%		
	Chem-Loop	\$19	8.0%		0.020
CCS		\$38	29.5%	0.380	
		\$56	42.0%		
		\$52	59.0%		-0.008
	Post-Com	\$224	70.0%	0.319	
		\$519	78.5%		

 Table 2
 Summary of assessment results for CCS.

Technology	Project	NPV of Funding (000,000)	Probabiltiy of success	Alpha	Shift	
		\$173	21.3%			
	LWR	\$260 33		0.325	0.118	
		\$346	60.0%			
		\$777	0.3%	0.327	0.111	
	HTR	\$/12	0.9%	0.111	0.028	
		¢1 544	17.0%	0.327	0.111	
		\$1,544	9.2%	0.111	0.028	
Nuc		£2.000	30.2%	0.327	0.111	
		\$3,089	10.1%	0.111	0.028	
	FR	¢1 150	0.1%	0.332	0.115	
		\$1,138	7.4%	0.115	0.029	
		\$4,622	0.5%	0.332	0.115	
		\$4,055	32.0%	0.115	0.029	
		\$15.442	16.3%	0.332	0.115	
		\$15,445	43.8%	0.115	0.029	

 Table 3
 Summary of assessment results for nuclear.

3.2. Computational MACs using MiniCAM

Our next step is to determine how the technologies would impact the MAC, if they achieve the defined endpoints. Specifically, we derive MACs for the year 2050 under different assumptions about technological pathways. We consider each of the technologies on their own, as well as all combinations of technologies to model interactions. Our baseline MAC assumes no CCS, solar PV

at 35 cents/kWh, and current nuclear technology at about 4.7 cents/kWh in 2050.

The analysis was conducted using the MiniCAM integrated assessment model, which integrates an economic model with a climate model. It looks out to 2095 in 15-year timesteps through a partial-equilibrium model with 14 world regions that includes detailed models of land-use and the energy sector (Brenkert et al. 2003, Edmonds et al. 2005). Assumptions for technologies other than the specific ones considered were based on the version of MiniCAM used in the Climate Change Technology Program (CCTP) MiniCAM reference scenario (Clarke et al. 2008).

Here we briefly address some additional complexities encountered in modeling each of the technologies. First, since solar is an intermittent resource, i.e. it cannot be turned off and on, it potentially poses problems for integration onto the electricity grid. The baseline assumption in MiniCAM is that when the penetration of solar into the electricity grid reaches 20%, every additional kW of solar installed requires the installation of a kW of gas-fired backup generation. In future work, we will also model the other extreme, simply assuming that there is no problem with grid integration. These two scenarios will give an envelope of the impact that solar might have.

We also faced a range of challenges in modeling nuclear power. Many of the advantages of new technologies, such as high-temperature reactors and fast reactors, are not easily modeled or valued. These include a reduction in proliferation concerns, a reduction in radioactive waste, and a reduction in the complexity of the technology. Moreover, the nuclear science experts that we worked with provided relatively low costs for the technological endpoints, of \$1500/kW or \$1000/kW, while nuclear economists have commented that these costs may be too low. The baseline assumption in MiniCAM is that LWR will have a cost of \$2100/kW in 2020. Our results focus mainly on improved LWR, and should be interpreted as reducing their cost by more than 50% below what otherwise would occur. We do not explicitly model limits to the penetration of nuclear power due to political-economy reasons.

Finally, there is concern about the widespread implementation of CCS. The Department of Energy Carbon Sequestration and Technology Roadmap lists a number of challenges, including



Figure 5 Representative MACs

permanence; monitoring, mitigation and verification; permitting and liability; and public acceptance (DOE 2007). Our elicitations did not consider these issues explicitly. A National Academy of Sciences study, however, considers public opposition based on the risk of sequestration; regulatory issues; and physical siting requirements (National Research Council 2007). They report that the "average panel probability that the large-scale sequestration would be allowed is 0.66 without DOE's research support, and increases to 0.77 with DOE's support." We use a baseline value of 70% as the likelihood that CCS will be allowed.

In Figure 5 we present four representative MACs plus the baseline. Besides the baseline, we show the MACs generated assuming (1) success in organic solar cells only; (2) success in chemical looping CCS only; (3) success in LWR only; and (4) success in all three of these technologies simultaneously. The left panel shows the impacts on low abatement levels and the right panel for high abatement levels. Note that solar only has a small impact on the MAC, even at a cost of \$0.03/kWh, due to the assumptions about grid integration. Nuclear and CCS have different types of impacts on the MAC. At low abatement levels, nuclear has the greatest impact. In particular, success in LWR implies that carbon emissions would drop by about 10% even in the absence of a carbon policy. At high abatement levels, however, CCS begins to dominate, significantly reducing the MAC at abatement levels above 70%. Finally, the combined MAC shows that the technologies are substitutes to a large degree.

If we combine these empirical curves with the elicited probabilities above, we have random MACs

– a probability distribution over a discrete number of curves. However, working with random functions is challenging theoretically and computationally. So, in the next subsection we parameterize these functions to make them more tractable to work with and analyze the impacts based on these parameter values.

3.3. Parameterization of the MAC

In this section we discuss how we produce a probability distribution over MACs for different levels of funding of different projects. We use the data generated by MiniCAM to estimate a smooth relationship between technical change and the impacts on the MAC. We noted in Section 2.2 that the effect of technology on the MAC could be parameterized by two parameters, α measuring the pivot and h measuring the shift:

$$\widetilde{MAC}(\mu;\alpha,h) = (1-\alpha) \left[MAC(\mu) - h * MAC(0.5)\right]$$
(1)

where the tilde represents the MAC after technical change parameterized by α and h, and $MAC(\cdot)$ is the original MAC before technical change. The first term on the right hand side pivots the MAC down. The second term in the square brackets shifts the MAC downward by a fixed amount. The constant h differs for each individual technology and technology combination. In order to make the parameterization portable to multiple models, we anchored the shift to the marginal cost of 50% abatement. For the individual technologies, we estimated the values for α and h from the empirical MAC curves using a least squares method. Based on experimental analysis, we concluded that the pivot for the combined technologies is best represented by a multiplicative combination of the individual technologies: $\alpha_{CSN} = 1 - (1 - \alpha_C) (1 - \alpha_S)(1 - \alpha_N)$. The values for h for combinations of technologies were again estimated using the least square method. The values of α and h for each single technology are given in Tables 1 - 3. In Figure 6 we graph the values of h and α for each individual technology and technology combinations. Nuclear, solar, and their combinations have relatively weaker pivots and stronger shifts than portfolios that include CCS. This matches what can be seen in Figure 5. CCS has mostly a pivot effect, with virtually no impact when the carbon



Figure 6 The shift and pivot of all technology combinations.

price is very low, and a strong impact when it is high. Nuclear, on the other hand, shifts the MAC downward, but has a lower pivot effect, as seen from the right panel in Figure 5.

4. The Portfolio Model

Given a probabilistic representation of the MAC based on the distribution of the parameters α and h, we next consider the portfolio of technologies that would minimize the expected costs in this stochastic setting. We start this section by presenting the conceptual model and discussing some of the challenges to implementing this model. In Subsection 4.1 we present our initial non-convex model and discuss the calibration of this model. In Subsection 4.2 we present the convexification of the model, and in Subsection 4.3 we discuss our solution procedures.

The traditional Decision Analysis (DA) R&D model is represented as an influence diagram in the upper panel of Figure 7. In this model, a firm decides which portfolio of projects to invest in, which in turn impacts the eventual portfolio of technologies that are successful. The market value of each successful portfolio can be estimated, but is also uncertain. The profits are based on the market value of the successful portfolio. The objective is to choose the investment portfolio to maximize expected profits. Climate change, however, is better represented as a dynamic decision problem, represented in the lower panel of Figure 7. In this model, the portfolio of successful technologies results in an abatement cost *curve*; and similarly, damages are represented by a damage *curve* that



Figure 7 Influence diagrams of R&D decision problems.

depends on the stock of greenhouse gases in the atmosphere, which in turn depends on abatement. The future decision about how much to abate will be made based on knowledge about the set of technologies available and about climate damages.

The overall goal of the climate change energy technology model is to minimize the sum of expected abatement costs and expected damages for a given R&D budget. The initial decision is which set of R & D projects to fund. Each potential funded portfolio leads to a probability distribution over successful portfolios, based on our expert elicitations. Each successful portfolio will determine a MAC, based on our parameterizations as described above.

As noted earlier, climate change damages are also uncertain. To model this uncertainty, we develop three-point probability distributions over the damages using estimates based on an expert elicitation in Nordhaus (1994). Part of our analysis is to perform sensitivity analysis over these probability distributions to understand the role of increasing risk in climate change.

The second-stage decision is *how much to abate*, for a given damage and abatement cost curve. In the absence of a corner point, abatement will be chosen so that the marginal cost of abatement, after technical change, is equal to the marginal damages. We will consider corner points where the marginal cost of abatement is less than marginal damages and full abatement is optimal. An analytical approach is intractable for this model due to the combinatorial structure of the problem and the difficulty of evaluating the expectation in the cost function. In the absence of an analytical approach, we consider dynamic programming and stochastic mathematical programming as two methods to solve this problem. Our problem presents challenges for both of these approaches. For traditional dynamic programming or decision trees the imposition of constraints and a large number of choices leads to a problem of intractable size. Stochastic programming, on the other hand, allows us to apply convex optimization methods to solve the problem numerically. However, the natural structure of our problem involves an endogenous process as higher investment in a particular technology increases the probability of success in that technology. In particular, our experts have given us probabilities conditional on funding trajectories. This endogenous structure results in nonconvexities in the portfolio model, preventing direct application of convex optimization methods.

Despite these challenges, we approach this problem using stochastic programming and develop methods to deal with the difficulties mentioned above. In the next three subsections, we first describe the general non-convex structure of the problem, and then develop a procedure to reformulate and solve the problem as an equivalent convex problem.

4.1. Initial Non-convex model

We let the indices *i* and *j* represent the technology category (solar, CCS, nuclear) and the specific project within the category, respectively. Further, the index *k* represents the investment level. The key binary decision variables are x_{ijk} , which equal 0 if there is no investment in project *ij* at funding level *k*, and 1 otherwise. The second stage continuous decision variable is abatement $\mu \in [0, 1]$, i.e. the fraction of emissions reduced below a business-as-usual level. This variable is conditioned on the state of climate damages, represented by a random multiplier *Z*; and by the state of the invested technologies, represented by the random vector $\vec{\alpha}$. The objective is to minimize the expectation of the sum of abatement costs and damage costs as follows:

$$\min_{x,\mu\left(\overrightarrow{\alpha},Z\right)} E\left[c\left(\mu;\overrightarrow{\alpha}\right) + ZD\left(\mu\right)\right] \tag{2}$$

Note that the investment in a technology is made without information on technical success or climate damages, while abatement is chosen conditional on technical success and damages, i.e. it is a second stage decision. The investment decisions are constrained by the R&D budget B, and by the fact that a project can be invested in only at one level:

$$\sum_{i} \sum_{j} \sum_{k} f_{ijk} x_{ijk} \le B \tag{3}$$

$$\sum_{k} x_{ijk} \le 1, \quad \forall i, j \tag{4}$$

where f_{ijk} is the required level of investment for funding level k of project ij.

We assume that the probability of technical success in any technology is independent of other technologies (and of the damages of climate change). Thus, the probability of any realization of the random vector $\vec{\alpha}$ is simply the product of the probability of the individual components of that realization.

According to our elicitation, the probabilities of success for individual projects depend on whether that project has been invested in or not, as well as the level of investment. In parallel with the general stochastic modeling framework, we perform the following steps to define the input distributions of the model. First, we calculate the probability of each realization of $\vec{\alpha}$ exogenously, using the probability of success if funded. For each funded project ijk there are three potential outcomes: failure, moderate success, or high success. We index these by l = -1, 0, 1. Then, for example, the probability of the event that there is high funding, i.e. k = 2, in organic solar cells (i = S, j = 1) and that we get high success in organic cells and no success in anything else is:

$$p_{S12,1} * \prod_{(i,j,k)\neq(S,1,2)} p_{ijk,-1} \tag{5}$$

where $p_{ijk,l}$ represents the probability that funded project ijk will result in outcome l.

Second, we define the outcomes so that they correctly match with the probabilities. Specifically, the outcome of each realization of $\vec{\alpha}$ is a vector with entries α_i , i = S, C, N, representing the amount of technical change in each category. We assume that only the best technology project in each category will diffuse in the economy. For example, if all solar projects are highly successful, we assume that the lowest cost technology will take over the market, giving solar a cost of 0.029/kWh and $\alpha_s = 0.05$. Let $\vec{\omega}$ be the state of the world, a vector containing the realized outcome of each project. Then we define the components of $\vec{\alpha}$ vector as follows:

$$\alpha_i(x;\omega) = \max_{j,k} \{ x_{ijk} \alpha_{ijkl} \}$$
(6)

where α_{ijkl} is a parameter taken from Tables 1-3. In this formulation, if we do not invest in technology ij, then $x_{ijk} = 0$. For example, consider again the event that there is high funding in organic solar cells and that we get high success in organic cells and no success in anything else. The realization of $\vec{\alpha}$ associated with this event depends on whether organic solar is funded at the high funding level or not. If $x_{512} = 0$ then the outcome will be $\vec{\alpha} = (0,0,0)$; if $x_{512} = 1$ the outcome will be $\vec{\alpha} = (.05,0,0)$. The outcome depends on the decision variable x_{ijk} , while the probability does not.

Between the technology categories, we assume that the pivots are multiplicative, but that the shifts are defined according to dependency relationships between the technologies, as mentioned in Section 3.1 above.

Based on the relationships established through simulations, the total shift in the MAC, h can then be defined as:

$$h = K(\mathbf{x}, \overrightarrow{\alpha}) \tag{7}$$

where $K(\mathbf{x}, \vec{\alpha})$ represents the shift value for the realized combination of technologies. For each possible combination of α values, these mappings are generated exogenously and included in the optimization model. This is further discussed in Section 4.2.

Given h, the abatement cost is:

$$c(\mu; \overrightarrow{\alpha}) = \prod_{i} (1 - \alpha_i) [c(\mu) - hc(0.5) \mu]$$
(8)

where $c(\mu)$ is the cost before technical change. Notice that the shift is multiplied by μ . This is because the parameterization above was done on the MAC and now we are working with the cost. We have based our baseline cost on the DICE 2007 model (Nordhaus 2008):

$$c(\mu) = b_0 \mu^{b_1} \tag{9}$$

Zh	1 3		14.6	14.6	3	14.6	
	(no risk)	(med.risk)	(high risk)	(baseline)	(hi.dmg. no risk)	(hi.dmg. high risk)	
P(Z=0)	-	0.666	0.931	0.245	-	0.795	
P(Z=1)	-	-	-	0.737	-	-	
P(Z=Zh)	1	0.334	0.068	0.018	1	0.205	
μ^* if Z=Zh	46%	80%	100%	-	-	-	
			Table 4 D	amaa ma Umaamt	ainstaa		

Table 4
 Damage Uncertainty

The damage function is assumed to be quadratic, a common assumption in the literature (Tol 1995, Nordhaus 2008):

$$M_0(S - M_1\mu)^2 \tag{10}$$

Calibration of the Model. We calibrated b_0, b_1, M_0, M_1 and S to DICE 2007. The stock of emissions in the atmosphere S is set equal to stock of emissions in 2185 under the Business As Usual (BAU) scenario in DICE, equal to 2.5 trillion metric tons of carbon. The damage constants M_0, M_1 are set so that the damages equal the net present value of damages between 2005 and 2185 in DICE under the BAU and "optimal" scenarios. We used the BAU scenario to calculate that $M_0 = 2.74$, and took the optimal level of abatement (with no technical change) to be the average of the optimal abatement in DICE 2007 over the period 2005 to 2185, or 0.46. Given this, M_1 was determined to be 0.597. The value of b_1 was set as 2.8, the value in DICE. Further, we set b_0 so that the optimal abatement is 0.46, which leads to a value of $b_0 = 10.4$.

We consider multiple cases for uncertainty over climate damages which are based on Nordhaus (1994) and are represented in Table 4. High damages, where Z = 14.6, are equivalent to a 20% loss in GDP given a 2.5°C increase in mean temperature. Each risk scenario in columns 2 - 5 has a mean of 1. The high risk case has the highest possible probability for the high damages without allowing negative damages (i.e. benefits). The medium risk case is an MPS of the no risk case (Rothschild and Stiglitz 1970), while the high risk case is an MPS of both the no-risk and medium risk cases. The last two columns have a higher mean of $\overline{Z} = 3$.

4.2. Stochastic Programming Formulation

In order to formulate the problem as a two-stage stochastic programming model, we first expand our definition of ω and let $\omega \in \Omega$ represent a scenario consisting of possible values of the parameters α_{ijkl} and Z, and define p^{ω} as the probability of the scenario ω , calculated as described in Section 4.1. Since the scenario definition involves both the vector $\vec{\alpha}$ and the random parameter Z, we refer to the realized value α_{ijkl} as α_{ijk}^{ω} for consistency in the description of the formulation. Note, our convention is that realizations of random variables have ω as a superscript; whereas decision variables that are conditional on the realization have ω as a subscript. The overall stochastic optimization problem can then be expressed as follows,

$$\min_{\mathbf{x}\in\mathcal{X}} \sum_{\omega\in\Omega} p^{\omega} \{ \prod_{i} (1 - \max_{j,k} \{\alpha_{ijk}^{\omega} x_{ijk}\}) (b_0 \mu_{\omega}^{b_1} - c_{0.5} h_{\omega} \mu_{\omega}) + Z^{\omega} M_0 (S - M_1 \mu_{\omega})^2 \}$$
(11)

s.t.
$$h_{\omega} = K(\mathbf{x}, \overrightarrow{\alpha}) \quad \forall \omega$$
 (12)

$$0 \le \mu_{\omega}, h_{\omega} \le 1 \quad \forall \omega \tag{13}$$

where \mathcal{X} represents the set of feasible investment decisions, as defined by (3)-(4). Note that problem (11)-(13) is the deterministic equivalent of the stochastic optimization problem (2). On the other hand, the multiplicative nature of the pivot terms in the cost function, i.e. the product $\prod_i (1 - \max_{j,k} \{\alpha_{ijk}^{\omega} x_{ijk}\})$, results in the model being highly nonconvex. Thus, convex optimization approaches are not applicable to the model, and a convex approximation or reformulation is necessary. The nonconvex product term is an integral part of the overall model, and results in a set of bilinear and trilinear components, approximation of which are typically not tight. However, we show below that an equivalent convex reformulation of the problem can be developed by defining some new variables and revising the definition of some parameters.

To develop an equivalent convex formulation, we first let ϕ_i be a nonnegative variable such that it is equal to the value of $-\ln(1 - \max_{j,k}\{\alpha_{ijk}\})x_{ijk}$ for $j,k \in \arg\max_{j,k}\{\alpha_{ijk}\}$. Note that these variables are defined for each scenario, but we leave out the index ω in these definitions for the clarity of presentation. Further, we define a new nonnegative variable $w = h + \mu$, and binary indicator variables δ_{ijk} and β_i to represent the modified problem structure. β_i corresponds to the case with no investment in technology i, while δ_{ijk} is an auxiliary variable used to indicate whether the corresponding set of constraints holds in the model. Further, for technology category i, δ_{ijk} identifies the funded project determining the value of α_i , which is the highest realized value among all funded project returns in that category. In addition, we let the random parameter $\bar{\alpha}_{ijk}^{\omega}$ represent $\ln(1 - \alpha_{ijk}^{\omega})$, which is calculated exogenously. Finally, we define the set of variables $y_{i,i',i''}^{\pi}$ for all $i, i', i'' \in \{C, N, S\}$, where π corresponds to a distinct combination of possible α_{ijk} values for the three technology categories. The variables y are used to denote the dependency relationships that apply to the shift parameter h in a given solution to the problem. We will refer to the combined set of y variables as y_I^{π} , and assume that a constant K_I^{π} is calculated exogenously for each possible combination.

With these definitions and modifications, the following equivalent formulation of the climate change energy technology R&D problem can be developed:

Minimize
$$\sum_{\omega} p^{\omega} [(e^{-\sum_{i} \phi_{i\omega} + \ln(b_0 \mu_{\omega}^{b_1} - \frac{1}{2}c(0.5)(w_{\omega}^2 - h_{\omega}^2 - \mu_{\omega}^2))}) + Z^{\omega} M_0 (S - M_1 \mu_{\omega})^2]$$
(14)

subject to
$$\sum_{i} \sum_{j} \sum_{k} f_{ijk} x_{ijk} \le B$$
 (15)

$$\sum_{k} x_{ijk} \le 1, \quad \forall i, j \tag{16}$$

$$\phi_{i\omega} + \bar{\alpha}^{\omega}_{ijk} x_{ijk} + M \delta_{ijk\omega} \le M \quad \forall i, j, k, \omega$$
(17)

$$\phi_{i\omega} + \bar{\alpha}^{\omega}_{ijk} x_{ijk} + m\delta_{ijk\omega} \ge m \quad \forall i, j, k, \omega$$
(18)

$$\sum_{i} \sum_{k} \delta_{ijk\omega} + \beta_i = 1 \quad \forall i, \omega \tag{19}$$

$$\phi_{i\omega} + M\beta_i \le 1 \quad \forall i, \omega \tag{20}$$

$$h_{\omega} - \sum_{I} y_{I\omega}^{\pi} K_{I}^{\pi} = 0 \quad \forall \omega$$
⁽²¹⁾

$$y_{I\omega}^{\pi} = 1 \Leftrightarrow \sum_{i \in I} \left(\sum_{j} \sum_{k} \delta_{ijk\omega} \alpha_{ijk}^{\omega} \right) = \alpha_{I}^{\pi} \quad \forall I, \pi$$
(22)

$$w_{\omega} = h_{\omega} + \mu_{\omega} \quad \forall \omega \tag{23}$$

$$\delta_{ijk\omega} - x_{ijk} \le 0 \quad \forall i, j, k, \omega \tag{24}$$

$$\mathbf{x}, \mathbf{y}, \delta, \beta \in \{0, 1\} \tag{25}$$

$$0 \le \mu, h \le 1; w, \phi \ge 0. \tag{26}$$

where α_I^{π} refers to the sum of the α values for the combination π , and M and m are upper

and lower bounds based on the corresponding constraints. The objective function (14) in the above formulation is based on two reformulation steps. First, the bilinear term $h\mu$ is expressed as a function of the new variable w, as by definition $w^2 = h^2 + 2h\mu + \mu^2$. Then, we describe the product terms using the corresponding natural logs. The constraints (15) and (16) are the first stage constraints (3)-(4). The inequalities (17) and (18) ensure that the value of ϕ_i is equal to $\bar{\alpha}_{ijk}x_{ijk}$ if project ijk is selected and $j,k \in \arg\max_{j,k}\{\alpha_{ijk}\}$, while (19) is used to define β_i such that $\beta_i = 1$ if no investment is made in technology *i*. Similarly, (20) ensures that $\phi_i = 0$, if no investment is made in the technology. Based on exogenous parameters K_I^{π} , constraints (21)-(22) define the variable *h* as described in (12). The relationships enforced through constraint set (22) are not stated explicitly for the sake of clarity, but these relations are modeled using standard integer programming methods (Nemhauser and Wolsey 1999). Constraint (23) defines the variable w, and finally the inequality (24) ensures that a project can contribute to the portfolio only if it is selected.

Problem (14)-(26) is an integer program with a nonlinear objective function and linear constraints. Further the objective function is convex as we show below:

THEOREM 1. Problem (14)-(26) is convex.

Proof: Since the problem contains linear constraints, it suffices to show that the objective function (14) is convex in the decision variables. Note that this function consists of two components, an exponential term and a quadratic function of the variable μ . It is trivial to show that the quadratic component is convex.

For the exponential term, we know that the exponentiation of a convex function is convex. Thus, the problem reduces to showing that $g(h_{\omega}, \mu_{\omega}) = -\ln(b_0\mu_{\omega}^{b_1} - \frac{1}{2}c(0.5)(w_{\omega}^2 - h_{\omega}^2 - \mu_{\omega}^2)) = -\ln(b_0\mu_{\omega}^{b_1} - \frac{1}{2}c(0.5)((h_{\omega} + \mu_{\omega})^2 - h_{\omega}^2 - \mu_{\omega}^2))$ is convex. Note that $g(h_{\omega}, \mu_{\omega})$ is twice differentiable, and the Hessian $H_g(h_{\omega}, \mu_{\omega})$ is given by:

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

where we use the values listed in Section 4.1 for b_0, b_1 and c(0.5) to obtain

$$\begin{aligned} a_{11} &= \frac{2.22\mu_{\omega}^{2}}{(10.4\mu_{\omega}^{2.8} - 0.745(h_{\omega} + \mu_{\omega})^{2} + 0.745h_{\omega}^{2} + 0.745\mu_{\omega}^{2})^{2}} \\ a_{12} &= a_{21} = \frac{1.49(10.4\mu_{\omega}^{2.8} - 0.745(h_{\omega} + \mu_{\omega})^{2} + 0.745h_{\omega}^{2} + 0.745\mu_{\omega}^{2}) - \mu_{\omega}(43.39\mu_{\omega}^{1.8} - 2.22h_{\omega})}{(10.4\mu_{\omega}^{2.8} - 0.745(h_{\omega} + \mu_{\omega})^{2} + 0.745h_{\omega}^{2} + 0.745\mu_{\omega}^{2})^{2}} \\ a_{22} &= \frac{(29.12\mu_{\omega}^{1.8} - 1.49h_{\omega})^{2} - 52.42\mu_{\omega}^{0.8}(10.4\mu_{\omega}^{2.8} - 0.745(h_{\omega} + \mu_{\omega})^{2} + 0.745h_{\omega}^{2} + 0.745\mu_{\omega}^{2})}{(10.4\mu_{\omega}^{2.8} - 0.745(h_{\omega} + \mu_{\omega})^{2} + 0.745h_{\omega}^{2} + 0.745\mu_{\omega}^{2})^{2}} \end{aligned}$$

Clearly, $a_{11} \ge 0$, as all of its components are nonnegative. Further, it can be shown through algebraic manipulation that $a_{22} \ge 0$ holds for the ranges $0 < \mu_{\omega} \le 1$ and $0 < h_{\omega} \le 1$. Similarly, $|H_g(\mu_{\omega}, h_{\omega})| \ge 0$, as the determinant of the matrix is given by

$$\frac{61.92\mu_{\omega}^{4.8}(\frac{0.08h_{\omega}^{2}}{\mu_{\omega}^{2.8}} - \frac{0.31h_{\omega}}{\mu_{\omega}} - 1.71\mu_{\omega}^{0.8})}{(1.49h_{\omega}\mu_{\omega} - 10.4\mu_{\omega}^{2.8})^{4}}$$
(27)

Hence, $H_g(\mu_{\omega}, h_{\omega})$ is positive semidefinite, and $g(h_{\omega}, \mu_{\omega})$ is convex. It follows that problem (14)-(26) is convex. \Box

Given the above result, the problem (14)-(26) can be solved using any nonlinear integer programming solver or through a branch and bound implementation, provided that the number of considered scenarios is not large. For large number of scenarios, which is the case for the climate change energy technology portfolio model, sampling based procedures based on solving randomly sampled small scale instances can be used to determine good or near-optimal solutions, which we describe in the next subsection.

4.3. Solution Approach

To solve problem (14)-(26), we make use of the sample average approximation (SAA) method (also known as the sample path method), a Monte Carlo simulation technique that approximates a stochastic program by a set of smaller problems based on a random sample from the set of possible scenarios (Shapiro 2003, Linderoth et al. 2006). Letting $\omega^1, ..., \omega^N$ be an i.i.d. random sample of Nrealizations of the random vector ω , the SAA problem for (14)-(26) can be defined as:

$$\min_{\mathbf{x}\in\mathcal{X}} \{ \hat{g}_N(\mathbf{x}) = \frac{1}{N} \sum_{l=1}^N G(\mathbf{x}, \omega^l) \}$$
(28)

where $G(\mathbf{x}, \omega^l)$ is the objective function (14) for realization ω^l . If v^* and \hat{v}_N represent the optimal values of the "true" and SAA problems respectively, Kleywegt et al. (2002) show that \hat{v}_N converges to v^* at an exponential rate as sample size N is increased. However, given that the computational complexity of the SAA problem increases exponentially with the value of N, it is typically more efficient to select a smaller sample size N, and solve several SAA problems with i.i.d. samples.

We solve M SAA problems with N samples in each, and use \hat{v}_N^m and $\hat{\mathbf{x}}_N^m$, $m = 1, \ldots, M$, to refer to the optimal objective value and solution of the mth replication, respectively. Once a feasible solution $\hat{\mathbf{x}}_N^m \in \mathcal{X}$ is obtained by solving the SAA problem, the objective value $g(\hat{\mathbf{x}}_N^m)$ needs to be determined. While we determine these values exactly for problem (14)-(26), in general the value of a given solution can be approximated by the estimator

$$\hat{g}_{N'}(\hat{\mathbf{x}}_{N}^{m}) = \frac{1}{N'} \sum_{l=1}^{N'} G(\hat{\mathbf{x}}_{N}^{m}, \omega^{l})$$
(29)

where N' is typically larger than N, as the computational effort required to estimate the objective value for a given solution is generally less than that required to solve the SAA problem. The quality of a solution $\hat{\mathbf{x}}_N^m$ is then computed through the optimality gap estimator $v^* - g(\hat{\mathbf{x}}_N^m)$, where $g(\hat{\mathbf{x}}_N^m)$ can be calculated exactly or estimated by (29), and v^* is approximated by

$$\bar{v}_{N}^{M} = \frac{1}{M} \sum_{m=1}^{M} \hat{v}_{N}^{m}$$
(30)

The sampling procedure can be terminated once the optimality gap estimate is sufficiently small or after performing all M replications, and the best solution among the SAA solutions can be selected using an appropriate criterion.

Effective implementation of the above sampling procedure requires that the SAA problems can be solved efficiently for relatively large values of the sample size N, and the candidate solutions are evaluated accurately. Problem (14)-(26) is especially suitable for such implementation, as it is relatively easy to evaluate the second stage objective function for given values of the **x** vector.

In the computations performed, depending on the instance, the values of N and M varied between 100-1000 and 100-250, respectively. Furthermore, as noted above, we calculated the values of candidate portfolios exactly through an algorithmic procedure, without the need for sampling. Hence, we could show numerically that the results obtained from the SAA method corresponded to true optimal portfolios.

5. Results and Policy Implications

Our analysis of the optimal climate change energy technology portfolio under different configurations resulted in several interesting implications from a policy perspective. As part of our analysis, we first considered different R&D budget levels and observed the impact of damage uncertainty on the value and composition of the optimal portfolio. Then, we investigated the impact of risk and of assumptions about opportunity costs on the overall optimal investment in energy technology R&D in the presence of climate change. Finally, we analyzed the structure of the optimal portfolio if a fixed planned abatement policy was used in response to climate change. We summarize our findings in the next three subsections.

5.1. Composition of Optimal Energy Technology R&D Portfolio

Our first finding is that the composition of the optimal portfolio is robust to different levels of damage risk, conditional on a budget. In Figure 8 we show the composition of the optimal portfolio at budget levels ranging between \$200 million and \$2000 million. These portfolios did not change under any of the risk scenarios in Table 4. On the other hand, we know from previous research (Baker et al. 2006, Baker and Adu-Bonnah 2008), as well as from the results in Section 2.2, that in general damage risk can impact the optimal investment in technology. Thus, our result shows the value of incorporating actual data in the portfolio analysis. Specifically, in this case, the data leads to projects that are fairly differentiated – some projects (such as chemical looping and LWR) have high probabilities and high payoffs, and therefore get funded regardless of risk, and even regardless of the mean of damages. Hence, based on currently available data and expert opinion, it can be concluded that the optimal R&D investment is robust to uncertainty in climate damages.

Second, we see the effects of the problem having a "knapsack" structure. We see that solar, in particular, goes in and out of the portfolio at different budget levels. The solar projects (under our



Figure 8 Optimal portfolios.



Figure 9 Expected total social costs normalized according to the no-risk case.

assumption of grid integration limits) are less efficient than some of the other projects, but also less costly. Thus, for example, we see a significant investment in solar at the \$200 million budget level; but this investment is reduced in favor of nuclear when the budget increases. We do see strong diversification – all three technology categories come in to the optimal portfolio even at a fairly low budget. At higher budget levels, not shown in Figure 8, nuclear dominates the portfolio, since it has the highest budgets.

In Figure 9 we show how the expected total social cost (damages plus abatement) is impacted by R&D investment, in the four risk cases in columns 2-5 in Table 4. The curves in the figure

	Marginal Value (\$)					
Budget (\$ millions)	No Risk	Baseline	Medium Risk	High Risk		
200	1632	1388	2508	707		
400	1247	1024	1572	403		
600	660	536	822	191		
800	119	98	162	38		
1000	81	67	112	27		
1200	42	34	53	12		
1500	37	30	47	11		
2000	3	2	3	1		
5000	28	21	35	8		

 Table 5
 Marginal value of R&D at different budget levels

are normalized so that all cases appear on the same scale as the no-risk case.¹ In addition, Table 5 shows the non-normalized approximations of the marginal value of R&D for each budget level. The table shows the additional value of the portfolio divided by the amount of R&D investment, in billions. Notice that R&D is very efficient. The lowest budget we considered has an NPV of \$0.2 billion and reduces the expected total social cost by over \$300 billion in the no-risk case, a marginal value of \$1,632 for every dollar spent. An additional \$0.4 billion investment leads to an additional \$370 billion reduction in costs. Even in the high risk case, the additional \$0.4 billion reduces costs by about \$120 billion, leading to a marginal value of \$403 for every dollar spent. Also note that there is an "elbow point" in each of the curves in Figure 9, where the cost savings from a bigger portfolio slows down considerably. This happens at a budget of \$600 million, and consists of a portfolio including a high investment in chemical looping, LWR, and purely inorganic PVs, along with medium level investments in the other two CCS technologies.

We pointed out above that the composition of the optimal portfolio at given budget levels is constant over a variety of different risk configurations. However, in Figure 9 and Table 5 we show that the *value* of R&D is impacted by the level of risk. First, R&D has the least value in the high risk case. This is because in that scenario we either have no damages and no abatement, or we have very high damages that lead to full abatement regardless of the technology. Thus, the technology reduces the cost of abatement, but does not change the optimal level of abatement – it

¹ Total expected cost is lower under risk, since abatement is increased when damages are high (Baker 2009).

has no environmental-side effect. As a contrast, in the no risk case, the presence of technology not only lowers the costs of abatement for a given level of abatement, but also leads to optimally more stringent abatement. In fact, when there is no risk our results show that the overall expected cost of abatement increases as the R&D budget increases – the optimal level of abatement increases enough that it outweighs the reduced cost of abating any given level. That is, the technology has a significant environmental-side benefit. Thus, it has overall more value.

We see, however, that the value of R&D is non-monotonic in risk, increasing significantly in the medium risk case, as we get both cost-side and environmental-side benefits. In this case, our results show that both expected damages and the overall cost of abatement decrease at higher budget levels.

In Figure 10 we illustrate this point. The figures show the baseline MAC, the expected MAC when the budget is \$600 million, and the marginal damages when the damage parameter Z = 1 and Z = 3. The left-hand chart shows the impact of technical change when Z = 1. In this case, optimal abatement increases from 46% to about 65%, thus there is environmental-side benefit. The total cost of abatement is the area under the curve, and it can be seen that the total abatement cost is slightly higher after technical change. The right-hand panel shows the impact of technical change when Z = 3. Optimal abatement increases from 80% to 100%, thus again there is an environmental-side benefit. Overall abatement cost also decreases in this case, as can be seen by comparing the lightest wedge (cost saved after technical change) with the darkest trapezoid (costs added after technical change because of higher abatement). Thus, overall, technical change has more value in the second case than the first.

5.2. Optimal Level of R&D Investment

In this section we calculate the overall optimal investment in R&D under different assumptions about the actual cost of R&D. In Tables 1-3 we report the NPV of the funding levels from earlier expert elicitations (Baker et al. 2008, 2009a,b). These funding trajectories represent an amount of money going into the hands of high quality researchers in the appropriate areas. The funding



Figure 10 Optimal abatement and total cost of abatement.

trajectories do not account for administrative costs of awarding the funding, nor do they account for the possibility of "pork" – money awarded by earmark for political reasons rather than based on scientific merit. Moreover, money spent on R&D is considered to have a particularly high opportunity cost in the economy, perhaps up to 4 times as much as the out-of-pocket expense (Nordhaus 2002, Pizer and Popp 2008).

Although the exact nature and amount of these opportunity costs are still an open question, we perform an analysis over a range of opportunity costs. We show results for the cases of no opportunity costs as well as total costs equal to 2, 4, and 8 times the net costs. The lower assumption would hold if "pork" was minimal and only about 50% of new energy R&D was replacing other kinds of R&D (Popp 2006), while the highest assumption would hold if "pork" doubled the cost of R&D and all energy R&D replaced other kinds of R&D. We find the optimal portfolio under three risk cases and these four assumptions about cost. The results are shown in Tables 6 - 8. Columns 2-10 in each table show the optimal investment in each specific technology; column 11 shows the overall optimal net investment in R&D (that is, not including opportunity costs); and the last column shows the total expected social cost (including the opportunity cost of investment). It can be seen from the three tables that the optimal investment level varies in the risk of climate damages. These results are summarized in Figure 11. The pattern that emerges is consistent with the results in Figure 10, in which R&D has the highest value in the medium risk case and the lowest value in the high risk case. We see here that the optimal net investment in R&D is highest $\mathbf{2}$

Coef		Investments (\$ million) Tot. Inv. Tot. Cost									
	$\operatorname{Pre} C$	Chem L	Post C	LWR	HTR	\mathbf{FR}	Org.	Inorg.	3rd g	(\$bil)	(\$ tri)
1	386	56	519	346	3089	15443	830	77	386	21.132	13.863
2	386	56	519	346	3089	15443	830	77	386	21.132	13.885
4	386	56	519	346	3089	0	830	77	0	5.303	13.915
8	386	56	519	346	3089	0	116	77	0	4.589	13.936
	Ta	ble 6 Oj	ptimal por	tfolio as	a functi	on of the	e oppor	tunity co	st multip	olier for no r	isk
Coef			Inve	estment	ts (\$ m	illion)				Tot. Inv.	Tot. Cost
	$\operatorname{Pre} \mathcal{C}$	Chem L	Post C	LWR	HTR	FR	Org.	Inorg.	3rd g	(\$bil)	(\$ tri)
1	386	56	519	346	3089	15443	830	77	386	21.132	11.849
2	386	56	519	346	3089	15443	830	77	386	21.132	11.870
4	386	56	519	346	3089	4633	830	77	0	9.936	11.912
8	386	56	519	346	3089	0	830	77	0	5.303	11.935
	Table 7 Optimal portfolio as a function of the opportunity cost multiplier for medium risk										
~ ^			-		(.	· · ·					
Coet	Investments (\$ million) Tot. Inv. Tot. Cost							Tot. Cost			
	Pre C	Chem L	Post C	LWR	HTR	FR	Org.	Inorg.	3rd g	(\$bil $)$	(\$ tri)
1	386	56	519	346	3089	4633	830	77	0	9.936	$10.3\overline{39}$

4.589

4.589

10.344

10.354



Figure 11 Bar chart showing optimal level of investment as a function of opportunity cost

in the medium risk case and lowest in the high risk case. Notice that when the opportunity cost is low, the entire portfolio is funded under the no- and medium-risk cases.

Consistent with our findings that the composition of the portfolio is robust to risk, it appears that the value of the individual technologies is not strongly effected by risk. If we read each table from



Figure 12 Optimal levels of R&D expenditure under fixed abatement, compared with optimal expenditure under recourse for the three risk cases

top to bottom, we can see which technologies get reduced funding or leave the optimal portfolio as the opportunity cost gets higher. It appears that the first technology to be reduced is 3rd generation solar, followed by the Fast Reactor, followed by organic solar cells, and finally the HTR reactor.

5.3. Fixed Abatement Levels

As noted, R&D has the least value when technical change has no impact on abatement. Moreover, the most common type of analysis in the climate change policy literature consists of determining the value of investments in technology for fixed stabilization levels – this is equivalent to a fixed abatement path over time. Thus, we investigate how the optimal investment level changes if we fix the second stage decision, i.e. assume a fixed abatement level. Specifically, we consider three target abatement levels of 46%, 62%, and 80%. These values respectively correspond to the optimal fixed abatement in the absence of uncertainty or technical change, the optimal fixed abatement in our R&D model without recourse, and a commonly discussed target abatement level. Note that the optimal R&D investment is not impacted by uncertainty in damages when abatement is fixed, as the payoff function is linear in Z when there is no recourse. In Figure 12, we show the optimal level of R&D expenditure under these three fixed abatement levels (assuming a 4x opportunity cost),



Figure 13 Example case demonstrating the impact of fixed level of abatement

and compare this to the optimal level of R&D expenditures in the model with recourse, under our three risk assumptions. We see from this figure that if the fixed abatement level is low, then technology may be undervalued compared to the true optimal case, whereas if the fixed abatement level is high, technology may be overvalued.

In Figure 13, we illustrate the opposing factors that determine whether technology is under- or over-valued. We present an example involving a single technology and no uncertainty in damages. The two upward sloping lines are a baseline MAC and a MAC after success in a CCS technology. The marginal damages are assumed to be 0.2, and optimal abatement based on the baseline MAC is about 44.5%; while it is about 57% based on the MAC after technical change. If we assume that there is a 60% chance of technical success, the expected amount of abatement is about 52% as shown. If we use this expected abatement level instead of optimal abatement to evaluate technical change, there are two countervailing effects. On the one hand, we will miss the fact that optimal abatement should be higher when the technology is successful. This effect will cause technical change to be undervalued with fixed abatement. On the other hand, if the fixed abatement level is high compared to the optimal level given the baseline, technology will lead to a significant cost savings in meeting this level of abatement. This effect will cause technical change to be over-valued with fixed abatement. In Figure 13, the two shaded triangles represent the relative impacts of these two effects. Specifically, the smaller triangle shows the benefits from technical change that we fail to recognize when abatement is fixed. The larger triangle shows the extra benefits from technical change when abatement is fixed at 52%. With the parameters as given, the benefit from the R&D appears greater under fixed abatement than under optimal abatement. R&D is over-valued in this case.

Consider a different case, in which the probability of success of the technology is lower, say 30%. In this case the expected abatement level would also be lower, around 49%, and the sizes of the two triangles would be reversed. Thus, ignoring the 2nd stage decision would cause R&D to be under-valued.

If society focusses on a fixed abatement level that ignores the possibility of technical change, then R&D will be undervalued. On the other hand, if society focusses on a fixed abatement level that assumes technical breakthroughs will occur, then R&D will be over-valued. In order to properly and accurately value the role of R&D, abatement must be considered as a recourse decision.

6. Conclusions

In this paper we have gone beyond the previous theoretical analyses to present results from a data-based climate change energy technology R&D portfolio model. Our R&D portfolio model has provided a number of insights. First, while it is easy to show theoretically that the optimal portfolio can depend on the level of risk, we have found in our data-based model that the optimal portfolio is robust to climate damage risk. This is good news, since determining the probability distribution over climate change damages is very difficult. Moreover, the optimal portfolio is even robust to the mean of the distribution. Second, we do see a high level of diversification, with even less-promising technologies included in the portfolio, although this is partly a result of it being a knapsack problem.

Third, while the portfolio at any given budget level is robust to risk, this is not true for the value of R&D. We go on to show that the optimal level of spending depends explicitly on the probability distribution around climate damages. Fourth, we see that R&D and technical change will not be valued correctly if future emissions levels are fixed, which is a commonly discussed option in the climate change literature. Finally, the value of technology is non-monotonic in risk,

with the maximum value being in cases where technology leads to higher abatement and significant reductions in abatement costs.

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