Measuring Welfare in Dynamic Models with Externalities: Towards Money Metric Measures*

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Abstract

This paper is concerned with welfare measurement in multisector dynamic general equilibrium models with externalities. We start with the utility metric theory under different settings, and then transfer them into money metric measures. With ideal accounting prices for all externalities, we show that a money measure of dynamic welfare should encompass both the comprehenstive NNP and consumer surpluses. Under externalities, a forward-looking term reflecting the present value of the future externalities has to be taken into account. For a local-in-time welfare comparison, we show that growth in conventionally measured NNP would work, provided that an externality-adjusted genuine rate of return is positive.

Keywords: Growth, welfare, externalities, and money metrics [JEL: D61; D91; Q01].

1 Introduction

A series of recent papers cover welfare measurement in dynamic general equilibrium models and, in particular, welfare measurement in imperfect market economies[1]. It turns out that the presence of market imperfections result in welfare measures that,

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[1] See e.g., Aronsson and Löfgren (1997, 1999), Arrow et. al. (2003), Dasgupta (2001), and Dasgupta and Mäler (2000).

in relation to the corresponding welfare measures in perfect market economies, generate extra forward-looking terms which contain entities that are not properly priced, or not priced at all. Working exclusively in a utility metrics, Aronsson et. al. (2004) measure the relative welfare losses (in comparison to first best) resulting from different market imperfections.

However, empirically meaningful measures are important if one attempts to do practical green accounting. Since the measurement of utility is not practically feasible, a money metrics is required. However, there are at least three complications: Firstly, externalities in consumption add an autonomous time dependence that makes the utility from a given consumption vector a function of the magnitude of the externalities. Secondly, the marginal utility of income will change over time, implying that the relationship between monetary and utility measures changes over time through a changed yardstick. This makes exact money metrics comparisons over time difficult. Finally, the imperfections are typically not priced or incorrectly priced.

The second problem can be solved by an index idea invented by Weitzman (2001), though the solution entails a rather empirically demanding price index. The third problem can be solved partly by measures of willingness to pay, and partly by estimates of marginal losses in production (cf. Aronsson and Löfgren, 1999). However, this still leaves out the first problem, but as we will show it can be handled by assuming that the instantaneous utility function can be separated into two components, one containing the externalities and the other containing consumption.

In an attempt to investigate in what sense growth in NNP works as a local welfare indicator with externalities, we introduce an exact local welfare indicator for an imperfect market economy. The measure shows what the time derivative of the value functions should look like under market imperfections. This "genuine saving" measure[2] is useful not only in itself, but also because it can help us to develop a criterion that tells us when growth in NNP (nota bene, not Green NNP) will work as a local welfare criterion.

To gain insight into the problem, we use a multisector growth model with externalities.To keep the exposition as brief as possible, we will to a large extent draw on results from previous work, sometimes without introducing rigorous proofs. The remaining part of the paper is structured as follows. Section 2 presents the model and introduces the wealth-like dynamic welfare measure, and section 3 gives an interpretive overview on the utility-based stationary equivalent welfare measures for three typical cases. This includes the first-best case with no externalities as a benchmark, the second best case with externalities but with perfect non-market valuation (ideal green accounting), and the case with incomplete accounting (some

[2]Possibly first developed in Aronsson and Löfgren (1998), Proposition 7.

externalities are accounted afterwards while the rest not). In section 4, we transform these measures into money metrics for global welfare comparisons.In section 5, we show that growth in conventionally measured NNP works as a local welfare indicator, provided that an externality adjusted genuine rate of return is positive. Section 6 sums up the study.

2 A multisector growth model with externalities

The model used here is a hybrid of the Weitzman (1976, 2001) multisector growth model and the Brock (1977) model with externalities. Both models have the standard Ramsey growth theory as the core, but with extensions along different directions. While Weitzman generalized the basic model to accommodate for heterogenous goods and services in a first-best setting, Brock developed it with an environmental externality in terms of a pollution stock. For the purpose of this paper, we will need both of these extensions. We consider ordinary consumption at time t as a n_1-dimensional vector $\mathbf{C}(t) = (C_1(t), C_2(t),...,C_{n_1}(t))$, and ordinary capital stocks such as infrastructure, machines and inventories, natural resources, and human resources, as a n_2-dimensional vector $\mathbf{K}(t) = (K_1(t), K_2(t),...,K_{n_2}(t))$. In addition, there is a $m$dimensional vector of stock externalities $\mathbf{X}(t) = (X_1(t), X_2(t), ..., X_m(t))$, which both affects utility and production in the economy. Initially, we have $\mathbf{K}(0) = \mathbf{K}_0 \geq 0$ and $\mathbf{X}(0) = \mathbf{X}_0 \geq 0$ with at least some element of $\mathbf{K}_0$ strictly positive. For simplicity, we consider a separable instantaneous utility function

$$U(\mathbf{C}(t), \mathbf{X}(t)) = \bar{U}(\mathbf{C}(t)) + V(\mathbf{X}(t)) \tag{1}$$

which satisfies the regularity conditions, i.e. concave, increasing in $\mathbf{C}(t)$ and the elements of $\mathbf{X}(t)$ with positive welfare contributions, whereas decreasing in the remaining elements of $\mathbf{X}(t)$ with negative welfare effects. We also assume $\{\mathbf{C}(t), \mathbf{X}(t)\}$ contain all variables relevant for human welfare, and $\{\mathbf{K}(t), \mathbf{X}(t)\}$ exhaust all possible stocks relevant for production in the economy. The vector of net investments in the ordinary capital goods is denoted by $\mathbf{I}_k(t) = \dot{\mathbf{K}}(t)$, and the corresponding expressions for the stock externalities by $\mathbf{I}_x(t) = \dot{\mathbf{X}}(t)$. Together, we have in the model a complete or comprehensive list of consumption goods and services as well as for net investments and capital stocks.

For any sequence $\{\mathbf{C}(t), \mathbf{X}(t)\}_0^\infty$ resulting from a given resource allocation mechanism, we consider the intertemporal welfare at time $t \geq 0$ to be of the Ramsey-Koopmans form

$$W(t) = \int_t^\infty U(\mathbf{C}(s), \mathbf{X}(s)) \exp(-\theta(s-t))ds, \tag{2}$$

where $\theta > 0$ is the the rate of pure time preference. Note that the resource allocation mechanism needs not be optimal here but is has to satisfy the feasibility constraint $(\mathbf{C}(t), \mathbf{I}_k(t), \mathbf{K}(t), \mathbf{I}_x(t), \mathbf{X}(t) \in A$, where A is a convex attainable possibility set. Now, given a sequence of the values of the objective function $W(t)\}_0^\infty$, it would be straightforward to make dynamic welfare comparisons. Welfare is improving over a period $[t', t"]$ if $W(t'') > W(t')$, whereas welfare is deteriorating if $W(t'') < W(t')$. For an infinitesimal time interval $[t, t + dt]$, it is possible to use time derivatives to infer welfare changes, i.e. $\dot{W}(t) > 0$ indicates a local welfare improvement, whereas $\dot{W}(t) < 0$ indicates a deterioration of welfare.

Although the welfare comparison rules above are straightforward, there are two major problems associated with such direct measurement of the intertemporal welfare. First, welfare as expressed in (2) depends on the whole stream of future consumption goods and services, and a prediction of which would entail enormous information requirement. Second, the metrics of welfare in (2) is utility, which is not observable in practice. To overcome these difficulties, we seek in the first place a measure of welfare, which can, at least in principle, be estimated using current statistics. We require, however, a suitable price index to transform the measure from utility into money metrics.

3 The utility-metric theory under different settings

In this section, we briefly cover the utility-metric theory of dynamic welfare measurement under three different settings. We begin with the first-best setting of an benevolent optimizing representative agent, a benchmark case without externalities or when all externalities are perfectly internalized (Weitzman, 1976, 2001). We show how the theory can, through the use of ideal accounting prices, be extended to imperfect economies where the dynamics of stock externalities are ignored by the optimizing agents. Finally, we deal with the other extreme case with absence of accounting prices for stock externalities[3].

Let us start with the benchmark case with no externalities, when the utility component $V(\mathbf{X})$ in (1) is, for simplicity, modeled as if identically zero. The dynamic problem is to maximize the intertemporal welfare $W(0)$ in (2) at $t = 0$, subject to the initial condition $\mathbf{K}(0) = \mathbf{K}_0$, the stock dynamics $\dot{\mathbf{K}}(t) = \mathbf{I}_k(t)$, and the feasibility constraint $(\mathbf{C}(t), \mathbf{I}_k(t), \mathbf{K}(t)) \in A$. Suppose that the sequence $\{\mathbf{C}^*(t), \mathbf{I}_k^*(t), \mathbf{K}^*(t)\}_0^\infty$ is the unique solution to this problem. Then, the following

[3]In reallity, however, we expect an intermediate case with some stock externalities valued through their accounting prices while others not. Although green or comprehensive accounting aims at valuing all externalities in the framework of national accounts, such an ultimate goal may never be achived.

maximized current-value Hamiltonian

$$H^*(t) = \bar{U}(\mathbf{C}^*(t)) + \mathbf{\Psi}_k(t)\mathbf{I}_k^*(t) \tag{3}$$

will serve as a stationary equivalent measure of dynamic welfare (Weitzman, 1976, 2001), where $\mathbf{\Psi}_k(t)$ denotes utility shadow price vector of the capital stocks $\mathbf{K}$, satisfying the Euler equation

$$\dot{\mathbf{\Psi}}_k(t) - \theta\mathbf{\Psi}_k(t) = -\partial H^*(t)/\partial\mathbf{K} \ . \tag{4}$$

More precisely, the maximized Hamiltonian in (3) $H^*(t)$ and the maximized intertemporal welfare $W^*(t)$ along the optimal sequence $\{\mathbf{C}^*(t), \mathbf{X}^*(t)\}_t^\infty$ are proportional to each other such that

$$H^*(t) = \theta W^*(t). \tag{5}$$

Since the utility rate of discount θ is positive and constant, it is obvious that $H^*(t)$ and $W^*(t)$, the optmal value function, will always rank welfare in the same manner. In the literature, the maximized current-value Hamiltonian in (3) has been termed comprehensive utility NNP, i.e. the sum of "consumption value" plus the value of net "investments" when all goods and services are taken into account. Thus, we have the following proposition due to Weitzman (1976):

Proposition 1 *In the absence of externalities, the maximized current-value Hamiltonian given in (3) is a correct measure of dynamic welfare. If $H^*(t'') > H^*(t')$ for $t" > t'$, then welfare is improving over the period $[t', t'']$. In other words, growth in utility NNP over a discrete time period indicates welfare improvement.*

Now, let us examine how the first-best benchmark result can be extended to imperfect economies where the stock externalities are present but ignored in the marketplace. Here, the externality stocks will affect human well-being and interact with the other capital stocks, but these effects are not taken into account by private agents in the market economy. The objective here is to maximize the intertemporal welfare $W(0)$ subject to the initial conditions $\mathbf{K}(0) = \mathbf{K}_0$, $\mathbf{X}(0) = \mathbf{X}_0$, the stock dynamics $\dot{\mathbf{K}}(t) = \mathbf{I}_k(t)$, the feasibility constraint $(\mathbf{C}(t), \mathbf{I}_k(t), \mathbf{K}(t), \mathbf{I}_x(t), \mathbf{X}(t) \in A$, but with no regard to the dynamics of stock externalities $\dot{\mathbf{X}}(t) = \mathbf{I}_x(t)$. Obviously, the resulting sequence $\{\mathbf{C}^0(t), \mathbf{I}_k^0(t), \mathbf{K}^0(t)\}_0^\infty$ will be suboptimal in this case as the associated externality sequence $\{\mathbf{I}_x^0(t), \mathbf{X}^0(t)\}$ is imposed rather than optimized.

The underlying (quasi) Hamitonian can still be expressed in a similar form $H^0(t) = U(\mathbf{C}^0(t), \mathbf{X}^0(t)) + \mathbf{\Psi}_k^0(t)\mathbf{I}_k^0(t)$, satisfying the Euler equation $\dot{\mathbf{\Psi}}_k^0(t) - \theta\mathbf{\Psi}_k^0(t) = -\partial H^0(t)/\partial\mathbf{K}$, but the proportionality relationship between $H^0(t)$ and the resulting dynamic welfare $W^0(t)$ in (5) will not hold here due to the ignorance of externalities. A closer examination reveals that

$$H^0(t) + \mathbf{\Psi}_x^0(t)\mathbf{I}_x^0(t) = \theta \int_t^{\infty} U(\mathbf{C}^0(s), \mathbf{X}^0(s))e^{-\theta(s-t)}ds \equiv \theta W^0(t), \tag{6}$$

where the vector product $\mathbf{\Psi}_x^0(t)\mathbf{I}_x^0(t)$ denotes the shadow value of growth in the externality stocks with

$$\mathbf{\Psi}_x^0(t) = \int_t^{\infty} \left. \frac{\partial U(\mathbf{C}(s), \mathbf{X}(s))}{\partial \mathbf{X}(\mathbf{t})} \right|_{0(t)} e^{-\theta(s-t)}ds. \tag{7}$$

where $\big|_{0(t)}$ denotes that the stream of future utilities are evaluated along the imperfect market solution path. Following Arrow et al (2003), we know that the vector of accounting prices $\mathbf{\Psi}_x^0(t)$ along the market solution path $\{\mathbf{C}^0(s), \mathbf{X}^0(s)\}_t^{\infty}$ satisfies the following system of differential equations[4]

$$\dot{\mathbf{\Psi}}_x^0(t) = \theta \mathbf{\Psi}_x^0(t) - \left. \left\{ \frac{\partial U(\mathbf{C}(t), \mathbf{X}(t))}{\partial \mathbf{X}(t)} + \frac{\partial \mathbf{I}_k(t)}{\partial \mathbf{X}(t)} \mathbf{\Psi}_k^0(t) + \frac{\partial \mathbf{I}_k(t)}{\partial \mathbf{X}(t)} \mathbf{\Psi}_x^0(t) \right\} \right|_{0(t)}, \tag{8}$$

This expression would reduce to the standard Euler equation if the externalities were internalized and optimized. Since these prices are not available from market transactions, they have to be revealed in some other ways. One may, for example, use contingent valuation to ask people about their maximum willingness to pay for reducing (increasing) a unit of each of the negative (positive) externality stock. Suppose that the accounting prices were all known, then the augmented Hamiltonian

$$\tilde{H}^0(t) = U(\mathbf{C}^0(s), \mathbf{X}^0(s)) + \mathbf{\Psi}_k^0(t)\mathbf{I}_k^0(t) + \mathbf{\Psi}_x^0(t)\mathbf{I}_x^0(t), \tag{9}$$

which might be called "green NNP", becomes proportional to the intertemporal welfare according to (6). Thus, we have

Proposition 2 *If the accountings prices of all externalities were perfectly known, then the adjusted current-value Hamiltonian (the green utility NNP) given in (9) is a correct measure of dynamic welfare evaluated along the market solution path. If $\tilde{H}^0(t'') > \tilde{H}^0(t')$ for $t'' > t'$, then welfare is improving over the period $[t', t'']$.*

[4] Note that Arrow et al. (2003) provide no formal proof of the Euler-like equations, but they are correct. The formula can be obtained by taking the cross derivative of the value function with respect to $\mathbf{X}$ and t, and invoking the dynamic envelope theorem.

The central message here is that the principle of dynamic welfare theory in the first-best setting carries over to imperfect economies, provided that the externalities were perfectly priced afterwards. There is no doubt that the resulting welfare level would be lower than in the first-best setting, but the point is that the externality adjusted green (utility) NNP still serves as an ideal surrogate measure of the suboptimal welfare (cf. Arrow et al, 2003). This implies that even if we do not live in a first-best world, we can still use the same principles to measure dynamic welfare involving externalities.

A natural question is thus to what extent can we value the various externalities through ideal accounting prices. In the present green accounting practice, only a subset of externality-like stocks are included such as forest, oil and carbon sequestration while many others are largely left to the side (Hamilton, 1994; Hamilton and Clements, 1999). As pointed out by Dasgupta (2001), estimating accounting prices of certain categories of resources may simply be impossible. Thus, there will always exist unaccounted residual externalities outside any green account. In the extreme case where accounting prices are unknown for all externalities, the expression corresponding to (9) would take the form

$$H^0(t) + \int_t^{\infty} V_x\left(\mathbf{X}^0\right)\dot{\mathbf{X}}^0(s)e^{-\theta(s-t)}ds = \theta\int_t^{\infty} U(\mathbf{C}^0,\mathbf{X}^0)e^{\theta(s-t)}ds, \qquad (10)$$

where the integral expression on the left hand side measures the current value of the marginal externality along the future path of the market economy (cf. Kemp and Long, 1982; Löfgren, 1992; Aronsson et al., 2004). The forward-looking integral expression provides another way of taking into account the effect of externalities in green national accounting, i.e. to elicit the future stream of people's willingness to pay for avoiding the marginal externality. When the value of this integral is added to $H^0(t)$ i.e. the part of green utility NNP adjusted only for the externality damages on the value of consumption, the sum would constitute a complete green NNP measure of dynamic welfare, although in utility metrics.

Corollary 1 *The integral term on the left hand side of (10) would vanish in steady states with* $\dot{\mathbf{X}}(\mathbf{t}) = \mathbf{0}$.

4 Towards money-metric measures of dynamic welfare

Since utility is not directly observable in the market, it is desirable to move from utility to money-metric welfare measures. For this purpose, we need to factor out the marginal-utility-of-income term from the utility function, and then use this factor to rescale the Hamiltonian expressions. To pave the way for this idea, let us define $\mathbf{p}(t)$ and $\mathbf{q}(t)$ as the nominal price vectors of consumption and investment goods, respectively, at time t. We can then establish the relationship between utility and money prices through the marginal utility of income $\lambda(t)$, such that $\nabla\bar{U}(\mathbf{C}(t)) = \lambda(t)\mathbf{p}(t)$ and $\mathbf{\Psi}_k(t) = \lambda(t)\mathbf{q}(t)$ where ∇ is a gradient operator.

We start with the benchmark case without externalities. Along the first-best path, the utility derived from the consumption at time t can be written as

$$\bar{U}(\mathbf{C}^*(t)) = \int_0^{\mathbf{C}^*(t)} \nabla\bar{U}(\mathbf{C})d\mathbf{C} = \lambda^*(t)\left\{\mathbf{p}^*(t)\mathbf{C}^*(t) + \int_{\mathbf{p}^*(t)}^{\hat{\mathbf{p}}} \mathbf{D}(\mathbf{p},\lambda^*(t))d\mathbf{p}\right\} \quad (11)$$

where the star superscript indicates that the variables are evaluated along the first-best path. The expression $\mathbf{D}(\mathbf{p},\lambda^*(t))$ denotes the short-run demand function with respect to counterfactual prices $\mathbf{p}$ for a given marginal utility of income $\lambda^*(t)$, and $\hat{\mathbf{p}}$ stands for the choke-off prices at which all consumption would cease. While the first equality in (11) simply follows the definition of a utility function, the second one is derived by integration-by-parts using the duality between direct and inverse demand functions $\mathbf{C} = \mathbf{D}(\mathbf{p}, \lambda)$ and $\mathbf{p} = \mathbf{p}(\mathbf{C},\lambda)$. The last integral in (11) represents the standard Dupuit-Marshallian consumer surplus[5] corresponding to the area to the left of the demand curve integrated from the actual to the choke-off prices.

By inserting the utility expression in (11) into (3) and substituting $\lambda^*(t)\mathbf{q}^*(t)$ for $\mathbf{\Psi}_k(t)$, we obtain

$$H^*(t) = \lambda^*(t)\left\{\mathbf{p}^*(t)\mathbf{C}^*(t) + \mathbf{q}^*(t)\mathbf{I}_k^*(t) + \int_{\mathbf{p}^*(t)}^{\hat{\mathbf{p}}} \mathbf{D}(\mathbf{p},\lambda^*(t))d\mathbf{p}\right\} \quad (12)$$

where the expression $\mathbf{p}^*(t)\mathbf{C}^*(t) + \mathbf{q}^*(t)\mathbf{I}_k^*(t)$ represents comprehensive NNP in nominal prices, i.e. the sum of consumption and investment values. When augmented to include the consumer surplus term, the whole expression in braces might, following Li and Löfgren (2002), be called the "generalized" comprehensive NNP (GCNNP), because consumption is valued according to the marginal price of each unit rather than that of the last unit. In other words, the total consumption value reinterpreted as the income derived from selling the consumption goods using a first order price discrimination scheme.

[5]It is worth mentiong that, in order to ensure the existence of the consumer surplus, we need to normalize utility by $U(\mathbf{0}) = 0$. As utility is of a cardinal nature for comprehensive national accounting, this should be a sound normalization. Whoever wants to hear about a negative utility NNP, or even worse, a utility NNP of minus infinity? Moreover, even if the yardstick of the total consumer surplus might not be defined, the change in consumer suplus for a finite change in prices still exists. This is similar to the case of comparing the intertemporal welfare under the undiscounted utilitarinism. Although the present discounted value of future utilities is not defined, it is till possible to compare the difference between the intertemporal welfare along two different utility streams.

If the positive marginal utility of income $\lambda^*(t)$ were constant over time, then GCNNP differs from the maximized Hamiltonian only up to a scale factor, and can thus serve as an indicator of dynamic welfare. However, this is in general not the case since marginal utility of income typically varies over time in response to price change. In quest for an ideal money measure, we have thus to trace the dynamics of the marginal utility of income, or, at least, its relative change over time. The Euler equation in (4) implies that

$$\dot{\lambda}^*(t) = [\theta - r^*(t)]\,\lambda^*(t) \tag{13}$$

where

$$r^*(t) = \frac{\partial H^*(t)/\partial k_i}{\lambda^*(t) q_i(t)} + \frac{\dot{q}_i^*(t)}{q_i^*(t)}$$

represents the nominal interest rate evaluated for any element of the investment vector i, $i = 1, 2, ..., n_2$. In the standard Ramsey model with a homogenous consumption and investment good, the nominal price remains constant over time, and the interest rate $r^*(t)$ simply reduces to the scalar marginal productivity of capital. Solving the differential equation in (11), we obtain the following solution

$$\lambda^*(t) = \lambda^*(0) e^{\int_0^t (\theta - r^*(s))ds} \tag{14}$$

where $\lambda^*(0)$ denotes the reference marginal utility of income at time $t = 0$. It is seen that the change in the marginal utility depends on the relative strength of the rate of time preference and the interest rate over time. For a given interest path, a lower rate of time preference is expected to lead to relatively smaller future marginal utilities such that $\lambda^*(t) < \lambda^*(0)$. As a thought experiment, consider that the same basket of goods is consumed at time t and time 0, yielding the same instantaneous utility, then the general price level at time t would be higher than that at time 0. The trends would be the opposite for a higher rate of time preference. If the two rates cancel at each point in time, then marginal utility $\lambda^*(t)$ would remain constant over time, and the same consumption vector consumed at two instants in time would be consumed at the same price vector . Although we know that this may not be the case in reality, the thought experiment gives us a clue how to standardize the marginal utility of money and the associated prices. Let

$$\pi^*(t) = e^{-\int_0^t (\theta - r^*(s))ds} \tag{15}$$

be the standardizing factor at time t to deflate the nominal prices $\mathbf{p}^*(t)$ and $\mathbf{q}^*(t)$. Then, by using the expressions in (14) and (15), we can rewrite the Hamiltonian in (12) as

$$H^*(t) = \lambda^*(0) \left\{ \mathbf{P}^*(t)\mathbf{C}^*(t) + \mathbf{Q}^*(t)\mathbf{I}_k^*(t) + \int_{\mathbf{P}^*(t)}^{\hat{\mathbf{P}}} \mathbf{D}(\mathbf{P},\lambda^*(0)) d\mathbf{P} \right\} \tag{16}$$

where $\mathbf{P}^*(t) = \mathbf{p}^*(t)/\pi^*(t)$ and $\mathbf{Q}^*(t) = \mathbf{q}^*(t)/\pi^*(t)$ represent real prices for consumption and investment goods, respectively. What is important in the move from (12) to (16) is that the marginal utility of income at any time t is now standardized to its base year reference level $\lambda^*(0)$, which have three important implications. First, the short-run demand functions $\mathbf{D}(\mathbf{P},\lambda^*(0))$ are time-invariant, and thus may be estimated using pooled data.Second, the constant margnal utility level indicates that there is no income effect so that the consumer surplus integral is path-independent. Third, the deflated, real GCNNP in braces in (16), differs from the maximized Hamiltonian only up to a positive factor, and can thereby serve as money-metric measure for dynamic welfare comparisons. Thus, we have following money-metric version of Proposition 1:

Proposition 3 *In the absence of externalities, the generalized comprehensive net national product (GCNNP) in real prices*

$$Y^*(t) = \mathbf{P}^*(t)\mathbf{C}^*(t) + \mathbf{Q}^*(t)\mathbf{I}_k^*(t) + \int_{\mathbf{P}^*(t)}^{\hat{\mathbf{P}}} \mathbf{D}(\mathbf{P},\lambda^*(0))d\mathbf{P} \tag{17}$$

is a correct measure of dynamic welfare. If $Y^*(t'') > H^*(t')$ for $t" > t'$, then welfare is improving over the period $[t', t'']$. In other words, growth in real GCNNP over a discrete time period indicates welfare improvement.

What does the standardizing factor (15) stand for? According to (14), the factor equals the ratio between marginal utilities $\lambda^*(0)/\lambda^*(t)$, but it can be interpreted in a manner that conveys the underlying economic intuition. Since we assume a stationary utility functional form, the same consumption vector would always generate the same level of total as well as marginal utility, i.e. $\bar{U}(\mathbf{C}(t) = \bar{U}(\mathbf{C}(0))$ and $\nabla\bar{U}(\mathbf{C}(t)) = \nabla\bar{U}(\mathbf{C}(0))$ if $\mathbf{C}(t) = \mathbf{C}(0)$. With $\mathbf{C}(0)$ as the base basket, this implies that $\lambda^*(t)\tilde{\mathbf{p}}^*(t)\mathbf{C}(0) = \lambda^*(0)\mathbf{p}^*(0)\mathbf{C}(0)$, where $\mathbf{p}^*(0)$ represents the actual consumption prices at time $t = 0$, and $\tilde{\mathbf{p}}^*(t)$ the imputed market-clearing prices at time t at which $\mathbf{C}^*(0)$ would be demanded. With these prices, the standardizing factor in (15) can be shown to be .

$$\pi^*(t) \equiv \frac{\lambda^*(0)}{\lambda^*(t)} = \frac{\tilde{\mathbf{p}}^*(t)\mathbf{C}^*(0)}{\mathbf{p}^*(0)\mathbf{C}^*(0)} \tag{18}$$

which is termed an "ideal" consumer price index by Weitzman (2001).

It is worth mentioning that to directly impute the counterfactual prices $\tilde{\mathbf{p}}^*(t)$ would be extremely difficult , except in rather special cases such as when the utility function is homothetic Fortunately, such an index may be obtained indirectly through the use of the standardizing factor idea in (14). Since the interest rate path is readily available, the main problem is to assess the rate of time preference θ either by econometric estimation based on historical data (cf. Hall, 1978; Lawrance, 1991; Attanasio and Low, 2004) or through controlled laboratory experiments (Houser and Winter, 2004). Although there are practical difficulties in estimating this parameter due to uncertainties in consumption and interest rate, the idea provides, at least, a possible way to assess the ideal price index measure through a scalar rate of time preference.

After having developed the money-metric theory under the first-best setting, we are now ready to extend the theory to include externalities. First, we have to decompose this subutility function $V(\mathbf{X})$ as in (11) for $\bar{U}(\mathbf{C})$. Then, we can follow the procedure from (12) through (16) to rescale the extended Hamiltonian in (9), the externality adjusted comprehensive NNP. This leads to the following expression

$$\begin{aligned} \tilde{H}^0(t) &= \lambda^0(0)\Bigg\{\mathbf{P}^0(t)\mathbf{C}^0(t) + \mathbf{Q}^0(t)\mathbf{I}_k^0(t) + \int_{\mathbf{P}^0(t)}^{\hat{\mathbf{P}}} \mathbf{D}(\mathbf{P},\lambda^0(0))d\mathbf{P} \\ &\qquad + \mathbf{P}_x^0(t)\mathbf{X}^0(t) + \mathbf{Q}_x^0(t)\mathbf{I}_x^0(t) + \int_{\mathbf{P}_x^0(t)}^{\hat{\mathbf{P}}_x} \mathbf{D}_x(\mathbf{P}_x,\lambda^0(0))d\mathbf{P}_x\Bigg\} \qquad (19) \end{aligned}$$

where $\mathbf{P}_x^0(t) = \nabla V(\mathbf{X})/\lambda^0(0)$ represents the shadow price, in real terms, of consumption externalities with $t = 0$ as the base year. Lossely speaking, if an element of the price vector $\mathbf{P}_x^0(t)$ is positive, then it may be interpreted as the annual maximum willingness-to-pay for year t, in year zero's price, for enjoying the "amenity" from a marginal externality unit. When an element is negative, it would be the willingness-to-pay for avoiding experiencing the marginal stock disutility. The other price vector $\mathbf{Q}_x^0(t) = \mathbf{\Psi}_x^0(t)/\lambda^0(0)$ represents the (known) real shadow price for for a marginal unit of the externality. Note that both price vectors are evaluated along the imperfect market solution path.

While the first three terms in the braces in (19) corresponds to the conventionally measured GCNNP, the other three terms represent the corresponding green adjustment. The contents of the last integral deserves more explaination. The integrand represents a system of demand functions, i.e. the solution of inverse of $V'(\mathbf{D}) = \lambda(0)\mathbf{P}_x$, and thus the integral corresponds to a kind of net surplus value. To fix ideas, consider that all elements of $\mathbf{X}$ represent negative externalities. Then, $\mathbf{D}_x(\mathbf{P}_x,\lambda^0(0))$ can be interpreted as demand functions for externality stock reductions. The prices $\mathbf{P}_x^0(t)$ are the marginal value (willingness-to-pay) for reducing the first unit of externality stock from the market solution $\mathbf{X}^0$, and $\hat{\mathbf{P}}_x$ the marginal value for reducing the last unit. As the integration bounds satisfy $\hat{\mathbf{P}}_x \leq \mathbf{P}_x^0(t)$ by the property of $V(\mathbf{X})$, the net surplus value in this case is negative. For positive externalities, on the other hand, the integral becomes positive. In reality, with mixed externalities, the sign of the vector integral is a priori indeterminate. Since $\lambda^0(0) > 0$, the expression in braces in (19) will go along each other over time up to a positive constant. Thus, we have

Proposition 4 *In the presence of externalities, the adjusted GCNNP in real prices as expressed in the braces in (19) is a correct meeasure of dynamic welfare, the growth of which over any time interval* $[t', t'']$ *always indicates a local welfare improvement.*

The adjusted GCNNP is a theoretically correct money measure of dynamic welfare under externalities. However, to calculate this measure is empirically rather intricate. One does not only need a measure of the consumer surplus for all goods that are priced in the market, but also the net surplus for goods that are not priced in the market. With suitable data, it is not unreasonable to assume that one can come up with an acceptable measure of the marginal willingness to pay for avoiding a unit of current pollution, or gaining a unit of a welfare improving externality stock. However, to calculate the accounting prices $\mathbf{Q}_x^0(t)$ that also handles the future consequences of the net accumulation of externality stocks proves to be difficult. As expressed in (7), in a utility metrics, the valuation work involves a complete assessment of all future general equilibrium effects. Corollary 2 introduces another way of assessing the the value of future changes in the stocks of externalities.

Corollary 2 *Under the assumption of separabale utility functions as in (1), the externality adjustment term* $\mathbf{Q}_x^0(t)\mathbf{I}_x^0(t)$ *can be evaluated by the forward-looking integral* $\int_t^\infty \mathbf{P}_x^0(t)\dot{\mathbf{X}}^0(s)e^{-\theta(s-t)}ds$*, i.e. the present value of future externality stock changes.*

Using Corollary 2, we need the forcasts of future willingness-to-pays, but confined to the externality stocks only, without any involvement of the general equilibrium effects. The information requirement with this alternative is smaller than valuing the whole path of general equilibrium effects, but still enormous[6]. Another complication we have not dealt with is the one that emerges if the utility function is not separable. The problem that surfaces is that, as the externality is not a part of the optimization problem, we cannot derive a full set of demand functions containing consumption goods as well as the externalities that supports the equilibrium path of the economy. If we could find the current marginal willingness to

[6] Aronsson et.al. (2004) chapter 4 provides a numerical example showing that, when today's current marginal willingness to pay is used in the future, but updated after a certain time span, the measurement error is not overwhelmingly large. Using a "relative utility metrics", the error size is typically less than 3%.

pay for pollution along an optimal path, we would be able to introduce Pigouvian taxes, but then we are back to the first best analysis. Hence introducing additivity seems convenient, since otherwise the demand functions for consumption goods would contain the stock of pollution as an extra argument.

5 NNP growth as a local welfare indicator

The externality adjusted GCNNP developed in the previous section is a global money-metric measure of dynamic welfare. However, the concept is empirically rather demanding. One does not only have to estimate a complete demand system for market and nonmarket commodities, but also to impute the standardizing factor, the ideal price index, which is a challenging task. Although knowledge on the rate of time preference gives a potential practical method for calculating the index number, the empirical estimation problem is non-trivial. As a consequence, applied welfare analysis is usually based on crude approximations. Marginal utility of income, for example, has been almost always treated as a constant in the literature.

While the externality adjusted GCNNP concept is a universal money-metric welfare measure, it seems to be too heavily artillery for making simpler welfare analyses. For assessing, for example, whether or not welfare improves over an infinitesamal period of time, it may suffice with an lighter gauge as shown by Asheim and Weitzman (2001) and Li and Löfgren (2006). The main results from these studies are that growth in real NNP can indicate welfare improvement if some side condition is satisfied. In Asheim and Weitzman, a Divisia consumer price index with changing market baskets is used to deflate money NNP, and the side condition is a positive real consumption rate of interest. In Li and Löfgren (2006), we show that growth in constant price NNP, or, equivalently, growth in NNP at variable prices, deflated by the standard NNP price index indicates a welfare improvement, if a genuine rate of return measure is positive.

In the first-best setting with all externalities internalized, the Li and Löfgren result will would apply[7]. For the nominal comprehensive NNP expressed by $\mathbf{p}^*(t)\mathbf{C}^*(t) + \mathbf{q}^*(t)\mathbf{I}_k^*(t) + \mathbf{p}_x^*(t)\mathbf{X}^*(t) + \mathbf{q}_x^*(t)\mathbf{I}_x^*(t)$, the real counterpart becomes

$$y_c^*(t) = \bar{\mathbf{p}}^*(t)\mathbf{C}^*(t) + \bar{\mathbf{q}}^*(t)\mathbf{I}_k^*(t) + \bar{\mathbf{p}}_x^*(t)\mathbf{X}^*(t) + \bar{\mathbf{q}}_x^*(t)\mathbf{I}_x^*(t) \tag{20}$$

where the subscript "c" denotes that the measure is comprehensive, and $\bar{\mathbf{p}}^*(t) = \mathbf{p}^*(t)/\pi^*(t)$ and $\bar{\mathbf{q}}^*(t) = \mathbf{q}^*(t)/\pi^*(t)$ denote, respectively, the real consumption and investment prices deflated by any NNP deflator $\pi(t)$. Smilarly, $\bar{\mathbf{p}}_x^*(t) = \mathbf{p}_x^*(t)/\pi^*(t)$ and $\bar{\mathbf{q}}_x^*(t) = \mathbf{q}_x^*(t)/\pi^*(t)$ denote the corresponding shadow prices of the internalized

[7] The Asheim and Weitzman results will hold as well, but we leave the proof to the reader.

externalities for consumption and "investment" services. Following Li and Löfgren (2006), we know that growth in real comprehensive NNP can be expressed as

$$\dot{y}_c^*(t) = \rho^*(t)\left[\bar{\mathbf{q}}^*(t)\mathbf{I}_k^*(t) + \bar{\mathbf{q}}_x^*(t)\mathbf{I}_x^*(t)\right] \tag{21}$$

where the expression in brackets represents the value of genuine saving[8] (cf Hamilton, 1994), and

$$\rho^*(t) = r^*(t) - \frac{\dot{\mathbf{q}}^*(t)\mathbf{I}_k^*(t) + \dot{\mathbf{q}}_x^*(t)\mathbf{I}_x^*(t)}{\mathbf{q}^*(t)\mathbf{I}_k^*(t) + \mathbf{q}_x^*(t)\mathbf{I}_x^*(t)} \tag{22}$$

denotes the genuine rate of return, i.e. the nominal interest rate minus the inflation rate of all investment goods broadly defined to even include the internalized externality stocks. Since the genuine rate of return is locally proportional to change in intertemporal welfare, $\dot{W}^0(t)$, we can restate the Li and Löfgren (2006) result as

Proposition 5 *Growth in real comprehensive NNP including the value of internalized externalities in (20) indicates a local-in-time welfare improvement if the genuine rate of return measure given in (22) is positive.*

Under imperfect markets, however, one would like to know whether similar results can be accomplished, and in particular, whether growth in the conventionally measured NNP would indicate welfare improvement. This turns out to be the case, though conditional on a slightly modified genuine rate of return measure. The counterpart of (21) in imperfect economies is

$$\dot{y}^0(t) \equiv \bar{\mathbf{p}}^0(t)\dot{\mathbf{C}}^*(t) + \bar{\mathbf{q}}^0(t)\dot{\mathbf{I}}_k^0(t) = \tilde{\rho}^0(t)\left[\frac{\dot{W}^0(t)}{\lambda^0(t)\pi^0(t)}\right] \tag{23}$$

where $\dot{W}^0(t)$ denotes the intertemporal welfare change, and $\tilde{\rho}^0(t)$ is an adjusted genuine rate of return measure given by

$$\tilde{\rho}^0(t) = \frac{1}{1+\alpha(t)}\left\{r^0(t) - \frac{\dot{\mathbf{q}}^0(t)\mathbf{I}_k^0(t)}{\mathbf{q}^0(t)\mathbf{I}_k^0(t)}\right\} \tag{24}$$

with

$$\alpha(t) = \frac{\mathbf{q}_x^0(t)\mathbf{I}_x^0(t)}{\mathbf{q}^0(t)\mathbf{I}_k^0(t)} = \frac{\int_t^\infty \mathbf{p}_x^0\dot{\mathbf{X}}^0(s)\exp\left(-\int_t^s r(\tau)d\tau\right)ds}{\mathbf{q}^0(t)\mathbf{I}_k^0(t)} \tag{25}$$

[8] Weitzman (1976) is, to our knowledge, the first to understand that genuine saving is a local welfare indicator in a perfect market economy. This knowledge seems to be a kind of Folk-Theorem. It pops up everywhere in dynamic growth theory. Hamilton should, however, have the lion's share of the credit, since he also uses the concept empirically.

as the ratio between the shadow value of changes in the externality stocks and that of the productive capital. Since $\lambda^0(t)\pi^0(t) > 0$, it is now obvious that growth in NNP, $\dot{y}^0(t)$, and change in welfare in (23) will locally have the same sign, if the adjusted genuine rate of return measure $\tilde{\rho}^0(t)$ in (24) is positive. What role would the α-ratio measure in (25) play? The answer is this: as long as $\alpha(t) > -1$, i.e. the value of net accumulation of externalities does not dominate the value in net investments in capital stocks that do not produce externalities, then the welfare significance of the conventionally measured NNP derived in Li and Löfgren still holds. For simplicity, consider the case of $\mathbf{q}^0(t)\mathbf{I}_k^0(t) > 0$, i.e. the net investment value in productive capital is positive. For positive externalities with $\mathbf{q}_x^0(t)\mathbf{I}_x^0(t) > 0$, the welfare significance of conventional NNP would be reinforced. If growth in conventional NNP indicates a welfare improvemnt, then growth in comprehensive NNP would imply a even larger welfare improvement. For negative externalities with $\mathbf{q}_x^0(t)\mathbf{I}_x^0(t) < 0$, then the welfare significance of conventional NNP would be weaker. Growth in this case can indicate welfare improvement, provided that the future net externality effect is smaller than the future net production effect, i.e. $|\mathbf{q}_x^0(t)\mathbf{I}_x^0(t)| < \mathbf{q}^0(t)\mathbf{I}_k^0(t)$. We now summarize the result in the following proposition:

Proposition 6 *Growth in conventionally measured real NNP in (23) indicates a local-in-time welfare improvement if the adjusted genuine rate of return measure in (24) is positive. Moreover, the welfare significance of conventional NNP is reinforced by positive externalities, while its validity requires an extra sufficiency condition*

$$\left|\mathbf{q}_x^0(t)\mathbf{I}_x^0(t)\right| < \mathbf{q}^0(t)\mathbf{I}_k^0(t) \tag{26}$$

under negative externalities.

This proposition seems to offer an important insight into qualitative comparisons of dynamic welfare in the presence of externalities. In principle, we can base our local welfare analysis merely on the welfare significance of conventionally measured NNP, provided that the net effect from accumulating externality producing stocks does not dominate the net investment in stocks that do not cause externalities. In other words, externalities can be ignored if they are not dominant enough to render genuine investment change from positive to negative values. Another implication of this proposition is that whenever the negative externality effect dominates such that $\alpha(t) < -1$ and $\tilde{\rho}^0(t) < 0$, then a good policy is to invest less and consume more. The reason would be simply that the market economy is "dynamically inefficient" with over accumulation of capital.

6 Conclusions

This paper has shown how utility based welfare measures in dynamic general equilibrium under imperfect markets can be transferred into a money metrics. The sufficient conditions are, however, rather demanding. To start with, we need a standardizing factor, or an ideal price index, that is independent of the market basket, or we have to assume that the marginal utility of income is constant over time. The latter assumption is implicit in all practical applications of index theory, but nevertheless dubious. It can be remedied theoretically by using the index approach presented in this paper. Nevertheless, it may not not be easy to empirically estimate the index number in practice.

Secondly, we need to assess the accounting prices of all externalities or inquivalently a forward looking integral measuring the present value of external effects in the future. It is difficult to see how this can be avoided. Under perfect market conditions and perfect foresight, the forward looking information is buried in the current market prices of consumer and investment goods. The reason is that the perfect market economy supports the optimal growth path. Under imperfect market conditions, corresponding current shadow prices are not available in the marketplace; either for externalities in consumption or for externalities in production. However, as shown by Aronsson et. al. (2004), in numerical examples, current willingness to pay or current prices may be good approximations. A more radical way out, is to assume that the economy is in a steady state.

Thirdly, the typical comprehensive quasi-static welfare measure, the externality adjusted real GCNNP will contain a core that looks like an extended green NNP component, as well as consumer surplus terms for both consumption goods and the externality and, in addition, a forward looking component with the discounted marginal externality as the function to be integrated over time.

Finally, with respect to local welfare measures, growth in traditional NNP will surprisingly work, provided that one conditions on a positive average marginal return of investment. However, unlike a previous result in Li and Löfgren (2006), the rate of return concept has to be augmented with the current value of the future marginal externality.

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