Consuming more and polluting less today: intergenerationally efficient climate policy

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Abstract

Climate policy benefits future generations at the expense of current ones. We propose a system of transfers that allows future generations to compensate the current one for its mitigation effort and demonstrate the effects in an OLG model. When the marginal benefit to a—possibly distant—future generation is greater than the cost of compensating the current generation for its abatement effort, a Pareto improvement is possible by a combination of mitigation policy and transfer payments. However, the transfers are costly as they discourage saving. We derive the conditions for the Pareto frontier achievable through such policies: a Samuelson Rule modified for the real costs of the transfers and for the persistent effects on physical and natural capital.

1 Introduction

Most climate policy assessments model greenhouse gases (GHGs) as an externality the emission of which is beneficial at the date it is emitted, while its contribution to the atmospheric stock causes economic damages at all future dates. Policy appraisal under the infinitely lived representative agent paradigm will propose emission abatement to the point at which the abatement cost equals the sum of the benefits from the reduction in damages. If the infinitely-lived agent is taken to represent a sequence of non-overlapping generations, such 'optimal' policies have implications with respect to the welfare of different generations. Since almost all of the benefits accrue after the individuals who incurred the abatement cost have passed away, the abating generation will incur a net loss from such a policy. Indeed, it is often claimed that climate policy is 'unfair' precisely in this sense: those currently alive have to pay for reducing GHG emissions, with the benefits accruing to later generations.

Such arguments run against a very basic result from public economics: the presence of an uncorrected externality implies the possibility of Pareto improvements. What is required to realise these improvements is that those who benefit from the correction of the externality compensate those who bear the cost. In the case of climate change, the implication is that future generations have to compensate current generations. This is straightforward when generations overlap: a chain of transfers between a sequence of overlapping generations can transfer resources 'back in time'.

In this paper we use an overlapping-generations (OLG) model to explicitly consider the welfare of different generations while maintaining decentralised investment decisions. Emission abatement is interpreted as a public policy prescription which restricts current production possibilities in form of an abatement cost. In return, future production possibilities expand as less rapid accumulation of GHGs reduces future damages to output. Capital savings, on the other hand, are interpreted as the result of the private inter-temporal consumption decisions of *finitely* lived agents. Current abatement policy creates value well beyond the finite lives of the agents alive at the date of abatement. Therefore, abatement policy will be desirable to agents in the distant future, even once the current ones have done what is optimal amongst themselves. If these distant generations could somehow compensate the current ones for some additional abatement effort, and this compensation has a lower marginal cost *to the compensators* than the marginal benefit of an additional unit of abatement, a Pareto improvement is possible.

We achieve just such a compensation by taking advantage of the contemporaneity of old and young in an OLG model. The young could compensate the old at a date t, in return for costly abatement policy undertaken at $t-1$. This is essentially a pay-as-you-go pension. Furthermore, the young at $t+1$ could compensate the old at $t+1$, who themselves are the young at t and could pass on the compensation to the old at t for even more abatement at $t-1$. This system of reverse transfers can be achieved for an arbitrary number of generations.

Being a pay-as-you-go transfer, such policies have well-known deleterious effects on the incentives to save, and therefore on the capital accumulation process. Taking this into account we are able to demonstrate that even for arbitrarily distant generations it is beneficial to enter a contract with the current one (and all the intermediate ones who must act as conduits) in which the current generation abates in return for a pension at a later date. What is more, we can show that once a set of adjacent generations have exhausted all the mutual gains from such contracts, the possibility of adding one yet more distant generation to the contract results in Pareto improvements over the best that the subset of generations could achieve.

We characterise the set of policies that dominate the business-as-usual policy as well as the set of Pareto efficient policies. The condition for efficiency amongst any subset of adjacent generations takes a form that is similar to the Samuelson rule, where efficiency is determined by the policy level at which marginal cost (of abatement) is equal to the sum of the marginal benefits. However, because the beneficiaries are separated in time from the abaters, the compensation mechanism (the sequence of pay-as-you-go transfers) is itself costly, resulting in a modification of the rule whereby the marginal cost of compensating the abater *including the cost of compensation* is equalised to the marginal benefit. We derive an easily interpretable condition for this and use it to establish that such abatement-pension policies can provide significant mutual improvements in welfare.

We do not take a normative stance on which point on the Pareto frontier (or, indeed, outside it) would be preferable, but only point out that business-as-usual is likely to be Pareto-dominated by some set of policies mandating mitigation and compensatory transfers. The wider point of the present paper is that the conflict between generations, often seen to be one of the stumbling blocks for effective climate policy, may be an illusion. In this sense, the intragenerational conflict—free-riding between states—is the real problem in trying to tackle climate change.

1.1 Literature

Previous literature has covered related issues. In the context of a representative agent model, Rezai et al. (2012) and Foley (2007) propose that abatement effort come combined with a reduction in savings. If the reduction in savings is chosen correctly, the consumption in the period during which abatement takes place is not reduced and the benefit to subsequent generations is positive as the lower capital stock is compensated for by reduced climate damages. However, in such a model it is unclear whether a single agent lives across periods, in which case it is not necessary to ensure that consumption is greater in every period as long as the agent's total welfare is increased; or whether separate generations live just for one period, in which case the the savings behaviour is unmodeled since (non-altruistic) agents living for a single period would presumably not save of their own volition.

Bovenberg and Heijdra (1998) look at Pareto improving mitigation in a Blanchard-Yaari OLG model in which investment is decentralised and the compensation of mitigating generations is achieved contemporaneously by adjustment of public debt. However, they focus on a particular efficient allocation—one in which the gains from the policy are distributed 'evenly' across generations—and focus only on cases in which the economy starts from a steady state. We characterise the entire Pareto frontier in a very intuitive fashion, for any dynamic equilibrium.

Gerlagh and Keyzer (2001) expand a pure exchange OLG model (Gale, 1973) by adding a productive non-renewable natural resource with amenity value. They then show that allocating property rights to the natural resource to a trust fund, tasked with providing each generation a given income flow, yields a Pareto improvement over zero extraction: the productive resource is used, with some of the consumption gain transferred forward to compensate future generations for lower resource amenities. The authors again focus on a single point on the Pareto frontier. More importantly, as the model does not feature physical capital, the impact of intergenerational transfers on capital accumulation is not studied. We work in the standard Diamond OLG setup, and so can study such issues.

John and Pecchenino (1994) develop an OLG model with two assets, physical and natural capital, both of which create a positive externality on the subsequent generation. The share of output that is invested is exogenous, but the relative shares going into physical and natural capital are made endogenous. They find that the allocation of investment between the two capital stocks may be inefficient, since the agents don't live infinite lives. They also demonstrate, much as we do, that Pareto improvements are possible even if the economy is dynamically efficient (with respect to both types of capital stocks). However, they do not give conditions for efficiency. We do, while also endogenising the consumption-savings decision.

Karp and Rezai (2012) consider long-lived capital stocks in an OLG setup. The productivity of capital depends on the state of the environment, as well as current emissions. They show that an improvement in the future state of the environment, by increasing future productivity, leads to appreciation in the value of the physical assets held by the current old relative to the wage of the young. Climate policy then induces a cost on both generations alive at the date in question, but since the young must acquire the appreciated assets from the old, the old may achieve a net benefit with the entire cost of abatement falling on the young. Thus, a Pareto improving policy would involve the current beneficiaries of the climate policy—the old who hold the appreciated capital—to compensate the current young. Future generations benefit as well, but their marginal benefit is not exhausted, so that increasing abatement would yield further Pareto improvements. Thus, the paper tells a story about intergenerational distribution, but one which goes somewhat against the grain of the conventional thinking that current owners of capital stocks might lose out from mitigation policies. We tell a complementary story. The main difference in our paper is that we do not consider adjustment costs related to investment, so that the price of capital is always equal to one. Thus capital cannot appreciate, and the transfers we find run from the future towards the present. We also incorporate the benefit to distant future generations into the current abatement decision.

Howarth and Norgaard (1992) work in a general equilibrium OLG framework similar to ours, but only consider sustainability of consumption paths under social welfare maximisation, and do not consider indexation of the intergenerational transfers to the state of the environment. Instead of sustainability issues, we are more interested in policies which take the economy to points on the Pareto frontier which dominate the business-as-usual outcome.

Rangel (2003) and Boldrin and Montes (2005) consider whether non-altruistic overlapping generations are able to sustain the provision of a public good (education) by linking it to social security provision once agents are old. The present paper takes the policies as exogenous, and so does not consider strategic incentives the various generations have to renege on policies. Our framework allows private saving to perfectly substitute for public saving, and we consider a very persistent public good, so that the mechanisms employed in these papers cannot be directly transposed into our setup. Future work will extend the present model by considering mechanisms which allow the policy variables to be endogenised.

1.2 Organisation of the paper

In Section 2 we outline the OLG framework and the nature of climate externality. In Section 3 we state the main result of the paper, Theorem 1, and discuss its implications. In Section 4 we assume specific functional forms for the production and utility functions which allow us to explicitly solve OLG model for the equilibrium prices and quantities. This explicit solution is used throughout Sections 5 to 7 to demonstrate the intuition of the result, namely that Pareto improvements are possible by linking the magnitudes of pay-as-you-go pensions to the abatement cost. We show that including more generations into such an intergenerational contract in which pensions are used to compensate for the mitigation effort yields greater improvements. That is to say, accounting for the marginal damage to $N+1$ generations immediately succeeding the abatement policy in question results in greater abatement and a Pareto improvement over the policy that only accounts for first N succeeding generations. Finally, in Section 8 we generalise our results to more general functional forms than the ones assumed in Section 4. Theorem 3, which is the main result of the section, is the complete statement of Theorem 1.

2 The model

Our model consists of a production economy augmented by a persistent externality on output given by the atmospheric stock of greenhouse gases (GHGs). Capital is saved by overlapping generations (OLG) of consumers who live for two periods (see

Diamond (1965)). The carbon stock is accumulated as a byproduct of economic output the reduction of which is costly. Denoting gross output by F^t, the cost of emissions reduction (*abatement*) by Φ^t, and the economic damage from climate change by Ψ^t, our model is simply a modification of Diamond's OLG model in which net economic output is given by

$$Y^t = F^t - \Phi^t - \Psi^t. \tag{1}$$

2.1 Production and damage

In each period t, following date t, the economy is endowed with a gross productive capacity net of depreciation represented by a function, F^t, that has constant returns to scale (CRS) in physical capital and labour. More formally,

$$(K^t, L^t) \mapsto F^t(K^t, L^t). \tag{2}$$

For all functions we will omit the arguments when no confusion is possible as to what values these take. When a function is subscripted by an argument it denotes the derivative with respect to the argument in question.

Industrial emissions are modeled as an additional output of production, the quantity of which can be reduced by a costly abatement effort. The GHGs emitted during period t are given by

$$E^t = (\bar{e}^t - a^t) \cdot F^t, \tag{3}$$

where $\bar{e}^t$ is an exogenous parameter referred to as the business-as-usual *emission intensity* or *emission-to-output ratio*, and a^t is the abatement effort at date t. The actual emission-to-output ratio is $e^t = \bar{e}^t - a^t$. The cost of abatement during period t is given by

$$\Phi^t = B^t(a^t) \cdot F^t \tag{4}$$

where $B^t(a^t)$ is the *abatement cost per unit of output.* This is similar to the approach taken in the integrated assessment model RICE-2010. By varying $\bar{e}^t$ and the functional form of B^t this formulation is sufficiently flexible for the realistic modeling of the productivity, cost of extraction, substitutability and time value of fossil fuel energy inputs.

Greenhouse gas emissions add to the stock of atmospheric carbon. A proportion of the stock dissipates over time and the actual atmospheric stock is presumed to cause climate change and thereby an externality as a damage to *future output*. Following Golosov et al. (2011), the stock of carbon in the atmosphere at date t is modeled as

$$S^t = \sum_{i=1}^{t-T_0} (1 - d^i) E^{t-i}, \tag{5}$$

where T_0 is the date at which industrial GHG emission began, and the *dissipation parameters* have the properties $d^i \in (0, 1)$ and $d^i \leq d^{i+1}$. The interpretation of these is as follows. Of the emissions in period t a proportion $(1 - d^i)$ remains in the atmosphere in period $t + i$. The damage at date t is

$$\Psi^t = D^t(S^t) \cdot L^t, \tag{6}$$

where $D^t(S_t)$ is the *per capita climate damage*. At this stage it may be useful to point out that there are several ways in which damages can be modeled in such a framework. In a decentralised economy the labour wage is set to the marginal product of labour and the interest rate (return on capital) to the marginal product of capital. If the damage enters as proportional to labour, the climate damages result in a loss only to the wage, as can be seen from equations (11) and (12). Conversely, if the damages enter as proportional to capital, rather than labour, the damages result in a loss on the return on capital. The damage can be modeled as proportional to *any* CRS function of labour and capital without disrupting the decentralised logic that labour and capital shares exhaust output. Damages could thus be proportional to output itself, or any other function aggregating labour and capital in a CRS manner. The result would be that loss terms would appear both in the wage and interest rates ((11) and (12)) in some proportion. That is to say the way in which the damage function enters net output determines how the loss is distributed amongst the different parts of the population.

The choice of damage function also determines how the magnitude of the effect of a given stock of carbon changes over time. In this regard, modeling damages as proportional to capital or output may seem more desirable than to population. The magnitude of the damage at a given stock of atmospheric carbon will depend on what is affected (rather than who bears the loss). The cost of the loss of coral reefs and UNESCO world heritage sites can reasonably be thought of as proportional to the amount of people who will be around to enjoy them. There are, of course, damages that are more likely to be proportional to capital.

The choice to model damages as proportional to population is based on the incidence of the loss. Having the loss accrue exclusively to the wage earners without affecting the capital return simplifies the accounting of damages as each generation is only affected once, during the period it earns its wages. This allows for a clearer exposition of the main features, at the possible cost of misrepresenting the magnitude. If we relax the assumption of perfect competition, and impose that the complete loss – whether the damage affects capital productivity or labour productivity – falls on the wage share and none on the capital share (as in (11) and (12)), then the results of this chapter extend to any specification of the damage function.

2.2 Net output

Under assumptions (4) and (6) net economic output becomes

$$Y^t(K^t, L^t, S^t, a^t) = (1 - B^t(a^t))F^t(K^t, L^t) - D^t(S^t)L^t \tag{7}$$

The damage functions D^t and abatement costs B^t are taken to satisfy the following assumptions.

$$D^t_S(\cdot) > 0; \quad D^t_{SS}(\cdot) \geq 0 \tag{8}$$

$$B^t_a(\cdot) \geq 0; \quad B^t_{aa}(\cdot) > 0 \tag{9}$$

$$B^t(0) = B^t_a(0) = 0; \quad \lim_{a \to \bar{e}^t} B^t_a(a) = \infty \tag{10}$$

Conditions (8) state that the damage is increasing and the marginal damage is weakly increasing as a function of the carbon stock. The convexity assumption is not strictly necessary, but simplifies the exposition as the alternative *necessary* condition is more involved and has a less intuitive interpretation. In (9) we assume that abatement cost is weakly positive and *stricly* convex as a function of abatement. Furthermore, at zero abatement both the cost and the marginal cost are assumed zero. The presumption underlying the latter is that $\bar{e}^t$ is set at just the emission level that is considered optimal by the competitive factor markets at date t, and thus the first unit of emissions reduction is virtually costless. Condition (10) is akin to an Inada condition and guarantees that the first order conditions are satisfied in the interior, $a^t \in (0, \bar{e}^t)$.

2.3 Firms

Economic output is produced by firms that are modeled as perfectly competitive and profit maximising. This yields the well-known result that the *wage* and *interest rates* are equated to the marginal products of labour and capital respectively:

$$w^t = \left[1 - B^t(a^t)\right] F_L^t(K^t, L^t) - D^t(S^t) \tag{11}$$

$$r^t = \left[1 - B^t(a^t)\right] F_K^t(K^t, L^t). \tag{12}$$

The net production function (equation (7)) has constant returns to scale in capital and labour which ensures that output is exhausted by the labour and capital shares, i.e.

$$Y^t = W^t + R^t := w^t L^t + r^t K^t.$$

We refer to W^t as the total wage and R^t as the *capital rent*.

2.4 Overlapping generations of consumers

Consumers live for two periods, *youth* and *retirement*, and are grouped together into homogenous generations consisting of one unit of population – or labour. The generation born at date t is denoted by $\mathcal{G}_t$. It derives utility from consumption goods and saves capital to transfer them between youth and retirement. The savings of generation $\mathcal{G}_t$ constitute the entire capital stock at date $t+1$. Denoting $\mathcal{G}_t$'s youth and retirement consumption by C^{1t} and C^{2t+1} its maximisation problem can be written as

$$\max_{C^{1t},C^{2t+1}} U(C^{1t}, C^{2t+1}) : \tag{13}$$

$$C^{1t} + K^{t+1} = M^t \tag{14}$$

$$C^{2t+1} = \left[1 + r^{t+1}\right] K^{t+1} + Z^{t+1} \tag{15}$$

where the endowments of consumption goods during youth and retirement are denoted by M^t and Z^{t+1} and the rate at which wealth can be transferred between the two periods is given by the growth factor $(1 + r^{t+1})$.

Generations are born without assets and earn a wage during youth. Absent intergenerational transfers the entire wage is either consumed during youth or saved for retirement when the entire principal plus the capital rent is consumed; there are no bequests. In period t generation $\mathcal{G}_t$ pays $\mathcal{G}_{t-1}$ a (possibly nil) pension denoted P^t. Including these transfers the endowments in the budget constraints (14) and (15) become

$$M^t = w^t - P^t \tag{16}$$

$$Z^{t+1} = P^{t+1}. \tag{17}$$

In addition to the endowments M^t and Z^{t+1} the capital rent $R^{t+1} = r^{t+1}K^{t+1}$ is the remaining source of wealth to $\mathcal{G}_t$.

Consider abatement at date m. As we explain in greater detail later, this will have a detrimental effect on the welfare of the generations alive during period m, and a positive effect on all future generations. The former can be seen by the effect abatement has in reducing the wage of $\mathcal{G}_m$ (equation (11)) and the capital rent of $\mathcal{G}_{m-1}$ (equation (12)). The latter is a consequence of the fact that such an effort will reduce the carbon stock *in all future periods* and thereby the damages in all periods $t \geq m + 1$. Notice that climate damages enter only in the wage and not in the capital rent (equations (11) and (12)). Therefore, even though $\mathcal{G}_m$ will be alive at $m + 1$, when the first benefits from reduced damages appear, it will *not* benefit from the abatement at date m. The benefits will only accrue to generations from $\mathcal{G}_{m+1}$ onwards.

As a function of the wealth variables, the solution to optimisation (13) yields a *savings function* such that $K^{t+1} = s(M^t, Z^{t+1}, r^{t+1})$. Recall that given the factor market equilibrium (equations (11) and (12)) and pension transfers (equations (16) and (17)) the three arguments of the the savings function are given by

$$M^t = \left[1 - B^t(a^t)\right] F_L^t - D^t(S^t) - P^t, \tag{18}$$

$$Z^{t+1} = P^{t+1}, \tag{19}$$

$$r^{t+1} = \left[1 - B^{t+1}(a^{t+1})\right] F_K^{t+1}(K^{t+1}, L^t). \tag{20}$$

The equilibrium capital is therefore determined as the fixed point K_*^{t+1}:

$$K_*^{t+1} = s\left(M^t, Z^{t+1}, \left[1 - B^{t+1}\right] F_K^{t+1}(K_*^{t+1}, L^t)\right). \tag{21}$$

How these variables affect the welfare of $\mathcal{G}_t$ is measured by the *value function*, which is defined as the utility evaluated at the consumption levels that maximise (13), i.e.

$$V^t = U(M^t - K_*^{t+1}, K_*^{t+1} + r^{t+1}K_*^{t+1} + Z^{t+1}). \tag{22}$$

where the equilibrium capital, K_*^{t+1}, is defined by (21).

Defining the equilibrium capital rent, $R_*^{t+1} = r^{t+1}K_*^{t+1}$, we have that M^t, Z^{t+1}, and R_*^{t+1} are the sources of wealth of $\mathcal{G}_t$, therefore changes in the policies that increase these variables will be beneficial to $\mathcal{G}_t$'s welfare and changes that reduce them will be detrimental. The relative magnitudes of any gains or losses are essentially the content of the value function (22). However, due to the general equilibrium adjustment of capital – equation (21) – the relationship is not a straightforward one. The effect of policies on welfare will depend jointly on (18), (19), (20), (21) and (22).

3 Main result

Denote the initial period by m. Consider a policy vector

$$\mathcal{P} = (a^m, P^m, a^{m+1}, P^{m+1}, a^{m+2}, P^{m+2}, \ldots, a^{m+N}, P^{m+N}, \ldots) \in \mathbb{R}^\infty \tag{23}$$

At any date t, the policies a^t and P^t are implemented by the generations alive, $\mathcal{G}_{t-1}$ and $\mathcal{G}_t$. Therefore, any complete policy vector $\mathcal{P}$ must be implemented as an agreement or contract between all generations, starting at date m. This raises issues of credibility of such a 'commitment' policy from which we currently abstract. In the following we will analyse policy vectors, which improve the welfare of all generations, conditional on the credibility of the policy. Extensions analysing the strategic aspect of such policies are under consideration.

Assume that policies before date m were zero. A given history $(\ldots, K^{m-1}, K^m)$ combined with a vector $\mathcal{P}$ completely determines the equilibrium evolution of capital, $\{K^t\}_{t>m}$, and carbon, $\{S^t\}_{t>m}$ (See (5) and (21)). By (22) this also determines the sequence of values $\{V^t\}_{t\geq m}$. We will write $V^t(\mathcal{P})$ to denote the dependence of these on the policy, while suppressing the dependence on the history.

Loosely speaking our main result states that any policy $\mathcal{P}_N$ which which reverts to the zero policy ($a^t = 0, P^t = 0$) from date $N+1$ onwards can be Pareto improved upon by one which includes non-zero policies at date $N+1$. More formally,

Definition 1. *A policy $\mathcal{P}$ is efficient between the set of generations $\{\mathcal{G}_{m+i} : i = 0, 1, \ldots, N\}$ if there is no policy $\mathcal{Q}$ such that*

$$V^t(\mathcal{Q}) \geq V^t(\mathcal{P}), \quad \forall t = m+1, m+2, \ldots, m+N$$

with a strict inequality for at least one t.

Theorem 1. *Suppose*

$$\mathcal{P}_N = (a_N^m, P_N^m, a_N^{m+1}, P_N^{m+1}, a_N^{m+2}, P_N^{m+2}, \ldots, a_N^{m+N}, P_N^{m+N}, \ldots) \in \mathbb{R}_+^\infty$$

is efficient between $\{\mathcal{G}_{m+i} : i = 0, 1, \ldots, N\}$ and that $P_N^t = 0$ for all $t \geq m+N$. Then there is a policy

$$\mathcal{P}_{N+1} = (a_{N+1}^m, P_{N+1}^m, a_{N+1}^{m+1}, P_{N+1}^{m+1}, a_{N+1}^{m+2}, P_{N+1}^{m+2}, \ldots, a_{N+1}^{m+N}, P_{N+1}^{m+N}, \ldots)$$

such that

1. $a_{N+1}^m > a_N^m$

2. $P^t_{N+1} > P^t_N$ *for all* $m \leq t \leq N+1$

3. $V^t(\mathcal{P}_{N+1}) \geq V^t(\mathcal{P}_N)$ *for all* $m \leq t \leq N+1$

where strict inequality holds in 3 for at least one generation.

According to point *3*, policy $\mathcal{P}_{N+1}$ Pareto dominates $\mathcal{P}_N$ for the generations under consideration. Thus, any policy $\mathcal{P}_N$ that is efficient between the first N generations is Pareto dominated by a policy $\mathcal{P}_{N+1}$ which includes the $(N+1)$'th generation. The improvement involves a higher initial abatement level (point *1*, $a^m_{N+1} > a^m_N$), and greater pension transfers in between all generations from the current one to the $(N+1)$'th (point *2*, $P^t_{N+1} > P^t_N$). Since the damages are persistent, considering efficiency between any subset of generations will no exhaust all gains. The theorem shows that such additional gains can be distributed in such a way that the very first generation incurring the abatement cost is sufficiently compensated.

Consider the *business-as-usual* policy, denoted by $\mathcal{P}^m_0 \equiv \vec{0}$. This is the policy for which both abatement and pensions are always zero.[1] Note that $\mathcal{P}^m_0$ is efficient between the generations alive during period m. This is because for the generations alive at date m, $\mathcal{G}_{m-1}$ and $\mathcal{G}_m$, abatement is costly at no benefit as the benefit only starts to accrue to the next, currently unborn, generation, $\mathcal{G}_{m+1}$. Applied to $\mathcal{P}^m_0$, the theorem states that a Pareto improvement is possible by increasing the abatement level in period m, a^m and the pension levels P^m and P^{m+1}. The intuition behind this is that since $\mathcal{G}_{m+1}$ benefits from abatement at date m, it is possible to find a pension P^{m+1} it would be willing to pay which is sufficiently high to compensate $\mathcal{G}_{m-1}$ and $\mathcal{G}_m$ for the abatement costs they incur.

What is remarkable about our result is that the same logic holds for any future beneficiary of *current* abatement policy, no matter how distant. Provided the right sequence of intermediate pensions are implemented, Pareto improvements can be achieved by redistributing part of the benefit of the future beneficiary back to the current generation incurring the abatement cost.

[1]Pensions may actually be at some positive level reflecting the fact that pay-as-you-go pensions exist independently of abatement policy. The essential feature of the business as usual policy is not altered by that, since only differences in pensions matter.

Since policies that are not efficient between a subset of generations, $\{\mathcal{G}_{m+i} : i = 0, 1, \ldots, N\}$, are dominated by an efficient one, and for any efficient one, we can find a policy that is superior to it by including a further generation into the agreement, the theorem shows that *any* policy can be dominated by one which includes further generations into the agreement.

Furthermore, we derive a condition, similar to the Samuelson rule which determines whether a given policy is efficient for the inclusion of a given number, N, of generations. The condition tells us whether efficiency holds and the direction in which policies must be adjusted if efficiency doesn't hold. This makes a numerical policy assessment based on a calibration of the model easy to implement.

4 Specific functional forms

In this section we will make assumptions about the functional forms of the production and utility functions which allow us to get a simple and intuitive expression for the value function – equation (22) – as well as an analytical description of the endogenous capital accumulation process which highlights the key equilibrium effects – equation (21). These assumptions will be taken to hold in the analysis of Sections 6 through 7 before we state the more general result in Section 8.

4.1 Leontief utility and logarithmic production

Definition 2. *The Leontief utility for the two-period inter-temporal consumption problem is defined by*

$$(C^1, C^2) \mapsto \min\{\beta C^1, C^2\}. \tag{24}$$

The parameter β quantifies the relative preference of second-period over first-period consumption.

The solution to the optimisation (13) with Leontief utility is given by the savings function

$$K^{t+1} = \frac{\beta M^t - Z^{t+1}}{1 + \beta + r^{t+1}} \tag{25}$$

Setting $\beta C^{1t} = C^{2t+1}$ and solving (14) and (15) for K^{t+1} as a function of M^t, Z^{t+1} and r^{t+1} establishes this.

Definition 3. *The logarithmic production function in capital and labour inputs is defined by*

$$(K^t, L^t) \mapsto A^t L^t \ln\left(\frac{K^t}{L^t}\right) \tag{26}$$

Note that the function has constant returns to scale in capital and labour.

As is shown in the Appendix, in the acceptable domain ($K \geq L \cdot e$) the function has the requisite properties of decreasing and convex isoquants in (L, K)-space. That is, given the right choice of units, it is a well-behaved production function. Notice that

$$F_K^t = \frac{A^t L^t}{K^t} \tag{27}$$

and therefore the gross capital rent $F_K^{t+1} K^{t+1} = A^{t+1} L^{t+1}$. That is, the capital rent is independent of the amount of capital saved.[2] The term $R_*^{t+1} = r^{t+1} K_*^{t+1}$ in (22) is therefore *independent of the equilibrium capital* K_*^{t+1}. We can therefore drop the asterisk subscript in R^{t+1}. This fact, along with the simple savings function resulting from the Leontief utility provides for the great simplification of the value and capital accumulation equation that results from Proposition 1.

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[2]In this respect the logarithmic production technology can be seen as the limit of the Cobb-Douglas production technology with vanishing capital share. Recall the Euler identity for homogenous functions, $F(K, L) = F_L \cdot L + F_K \cdot K$. By differentiating both sides with respect to K, we get that for every CRS production function:

$$F_K = \frac{\partial}{\partial K}(F_L L) + \frac{\partial}{\partial K}(F_K K), \tag{28}$$

i.e. the increase in output due to an increase in the amount of productive capital – the productivity of capital – is shared between the wage and the capital rent. In our notation equation (28) becomes $r^t = W_K^t + R_K^t$. For the Cobb-Douglas function with capital share α we have that

$$W_K^t = (1-\alpha) r^t; \quad R_K^t = \alpha r^t$$

For the logarithmic production function it is as if $\alpha = 0$:

$$W_K^t = r^t; \quad R_K^t = 0$$

Another way to see that the second order properties mimic a Cobb-Douglas function with α going to zero is the limit below:

$$L \ln\left(\frac{K}{L}\right) = \lim_{\alpha \to 0} \frac{K^\alpha L^{1-\alpha} - L}{\alpha}$$

Proposition 1. *Suppose $U(C^{1t}, C^{2t+1})$ is Leontieff, $F^t(K^t, L^t)$ is logarithmic and $K^t \geq L^t \cdot e$. Then the equilibrium interest rate is*

$$r^{t+1} = \frac{[1+\beta]R^{t+1}}{\beta M^t - Z^{t+1} - R^{t+1}} \tag{29}$$

and the equilibrium consumption and savings of $\mathcal{G}_t$ are given by

$$C^{1t} = \frac{M^t + Z^{t+1} + R^{t+1}}{1+\beta} = \frac{C^{2t+1}}{\beta} \tag{30}$$

$$K_*^{t+1} = \frac{\beta M^t - Z^{t+1} - R^{t+1}}{1+\beta} \tag{31}$$

where

$$M^t = \left[1 - B^t(a^t)\right] F_L^t(K^t) - D^t(S^t) - P^t \tag{32}$$

$$Z^{t+1} = P^{t+1} \tag{33}$$

$$R^{t+1} = \left[1 - B^{t+1}(a^{t+1})\right] A^{t+1} \tag{34}$$

Proof. Note that (for $L^t = 1$) by (12) and (27)

$$r^{t+1} = \left[1 - B^{t+1}\right] \frac{A^{t+1}}{K^{t+1}} = \frac{R^{t+1}}{K^{t+1}} \tag{35}$$

Substituting (35) into the Leontief savings function (25) yields

$$K_*^{t+1} = \frac{\beta M^t - Z^{t+1}}{1 + \beta + R^{t+1}/K_*^{t+1}} \tag{36}$$

Therefore

$$K_*^{t+1}\left[1+\beta\right] + R^{t+1} = \beta M^t - Z^{t+1} \tag{37}$$

and thus,

$$K_*^{t+1} = \frac{\beta M^t - Z^{t+1} - R^{t+1}}{1+\beta}$$

This establishes (31). The interest rate is shown to be (29) by substituting the equilibrium capital into (35) and the consumptions are shown to be (30) by substituting the equilibrium capital into the budgets (14) and (15). □

With the analytic expression for the equilibrium capital (31) we will now drop the asterisk subscript and in subsequent sections and refer to the equilibrium capital simply by K^{t+1}. Furthermore, having assumed that the population is constant across time and fixed at $L^t = 1$, we will drop the population argument in the production function and its derivatives.

5 Value, policies, and states

This section summarises all the effects that policy and state variables have on the welfare of different generations as well as the effect on the evolution of the capital stock. These relationships are used repeatedly throughout the subsequent sections.

5.1 The effect of policy on different generations' welfare

In equilibrium $\mathcal{G}_t$'s youth and retirement consumptions satisfy $C_{2t+1} = \beta C_{1t}$ and therefore its equilibrium utility simplifies to

$$U(C^{1t}, C^{2t+1}) = \min\{\beta C^{1t}, C^{2t+1}\} = \beta C^{1t}.$$

Thus, the value of $\mathcal{G}_t$ is simply the equilibrium retirement consumption. For notational convenience we will renormalise the utility function by $(1+\beta)/\beta$ so by equation (30) we get the value

$$V^t = M^t + Z^{t+1} + R^{t+1}.$$

Replacing for M^t, Z^{t+1} and R^{t+1} with (32), (33) and (34) we get the value as a function of the state and policy variables $(K^t, S^t, a^t, P^t, a^{t+1}, P^{t+1})$:

$$V^t = \left[1 - B^t(a^t)\right] F_L^t(K^t) - D^t(S^t) - P^t + P^{t+1} + \left[1 - B^{t+1}(a^{t+1})\right] A^{t+1} \quad (38)$$

Consider the direct effects that changing policy variables in period m has on the welfare of $\mathcal{G}_m$ and $\mathcal{G}_{m-1}$. Abatement in period m has a negative effect on the wage of $\mathcal{G}_m$ and on the capital rent of $\mathcal{G}_{m-1}$ thus reducing their wealth and thereby welfare:

$$V^m_{a^m} = -B^m_a F^m_L; \quad V^{m-1}_{a^m} = -B^m_a A^m \tag{39}$$

A pension transfer P^m simply reduces the welfare of $\mathcal{G}_m$ and increases that of $\mathcal{G}_{m-1}$ by the same unit:

$$V^m_{P^m} = -1; \qquad V^{m-1}_{P^m} = 1 \tag{40}$$

5.2 The effect of policy on capital accumulation

As in (38) replace for M^t, Z^{t+1} and R^{t+1} in (31) to get

$$K^{t+1} = \frac{\beta\left(\left[1 - B^t(a^t)\right] F^t_L(K^t) - D^t(S^t) - P^t\right) - P^{t+1} - \left[1 - B^{t+1}(a^{t+1})\right] A^{t+1}}{1+\beta} \tag{41}$$

Consider the effect of abatement on the capital stock. If anticipated in period $m-1$ we have that

$$K^m_{a^m} = \frac{B^m_a A^m}{1+\beta}. \tag{42}$$

That is, a promise of future abatement increases the the incentive to save for $\mathcal{G}_{m-1}$ as the abatement reduces its capital rent ($R^m = [1 - B^m(a^m)]A^m$). A pension has the opposite effect, as it increases $\mathcal{G}_{m-1}$'s retirement budget, thus reducing the incentive to save:

$$K^m_{P^m} = \frac{-1}{1+\beta} \tag{43}$$

The effect a^m and P^m on K^{m+1} is negative as both reduce the endowment M^m.

$$K^{m+1}_{a^m} = -\frac{\beta}{1+\beta} B^m_a F^m_L \tag{44}$$

and

$$K^{m+1}_{P^m} = -\frac{\beta}{1+\beta} \tag{45}$$

5.3 The value of capital

The effect of a change in the capital stock at date m on the value of $\mathcal{G}_m$ is given by

$$V_K^m = \left[1 - B^t\right] F_{LK}^t \tag{46}$$

Since $F_{LK}^m = F_L^m$ for the logarithmic production function, by (12), (46) becomes

$$V_K^m = r^m \tag{47}$$

6 Abatement at date m

Consider the decision to abate at date m. As can be seen from the equations (39), a^m has a negative effect on $\mathcal{G}_m$ and $\mathcal{G}_{m-1}$, the two generations alive during that period. The benefits due to a reduction in the carbon stock only appear in the values of $\{\mathcal{G}_t; t > m\}$. Thus, it is not in the economic interest of those alive in period m to choose any positive level of abatement.

However, since $\mathcal{G}_m$ is alive in period $m+1$, it can take compensation from $\mathcal{G}_{m+1}$ in form of a pension P^{m+1} to cover the abatement cost (to $\mathcal{G}_m$ and $\mathcal{G}_{m-1}$). Without the dynamic features of the problem the Samuelson rule would state that Pareto improvements are possible if the marginal abatement costs to $\mathcal{G}_{m-1}$ and $\mathcal{G}_m$ are less than the marginal damage avoided by $\mathcal{G}_{m+1}$. However, such an improvement would require a side-payment, or it wouldn't be a Pareto improvement at all. Because the abaters and beneficiaries are separated in time additional effects come into play, and therefore efficiency is *not* characterised by marginal cost being equal to marginal benefit. In this section we propose the pay-as-you-go pension transfer as the mechanism for the side payment and characterise the efficient policies as those upon which no Pareto improvements are possible, *given that the side-payments are themselves costly as they have a negative effect on the accumulation of capital.* We establish that under general assumptions the business-as-usual policy of zero abatement is dominated by a set of policies involving non-zero abatement and pension levels.

6.1 Pareto improving abatement

Throughout the paper the notation dx will be used to denote a sufficiently small change in the variable x so that the effects on the related variables may be described by the first order approximation. In this section we determine the conditions on the relative magnitudes of such small changes in the policies a^m, P^m and P^{m+1} such that the result is an improvement to the welfares of $\mathcal{G}_{m-1}, \mathcal{G}_m$ and $\mathcal{G}_{m+1}$. Any point at which such a joint improvement is *not* possible will be said to be on the *efficiency frontier*.

6.1.1 Indifference of $\mathcal{G}_{m-1}$ and insensitivity of K^m

To better focus on the improvements possible to future generations we will restrict ourselves to policies that leave $\mathcal{G}_{m-1}$ indifferent. Such policies have the effect of leaving the incentives to save of $\mathcal{G}_{m-1}$ unchanged and therefore have no effect on the capital stock K^m *even if the policies were anticipated.* For this purpose, we will fix the pension P^m at

$$P^m = B^m(a^m)A^m \tag{48}$$

When this is the case the value of $\mathcal{G}_{m-1}$ (see equation (38)) is

$$V^{m-1} = F_L^{m-1} - D^{m-1} - P^{m-1} + A^m + \overbrace{B^m(a^m)A^m}^{P^m} - B^m(a^m)A^m$$

Thus, the cost of the abatement effort is compensated in full by the pension. Furthermore, if both a^m and P^m are anticipated and (48) holds, the effect on $\mathcal{G}_{m-1}$'s savings behaviour is neutralised. From equation (41) we get that

$$K^m = \frac{\beta\left(F_L^{m-1} - D^{m-1} - P^{m-1}\right) - A^m - B^m(a^m)A^m + B^m(a^m)A^m}{1+\beta}$$

6.1.2 The welfare of $\mathcal{G}_m$

The share of the abatement cost that accrues to $\mathcal{G}_m$ is proportional to the gross wage $B^m(a^m)F_L^m$. Setting the pension that $\mathcal{G}_m$ must pay $P^m = B^m(a^m)A^m$ results in a total cost to $\mathcal{G}_m$ of

$$B^m(a^m)[F_L^m + A^m] = B^m(a^m)[F_L^m + A^m] = B^m(a^m)F^m$$

Setting the pension that $\mathcal{G}_m$ pays $\mathcal{G}_{m-1}$ to just the level that makes $\mathcal{G}_{m-1}$ indifferent results in a combined (pension and abatement) cost to $\mathcal{G}_m$ that is exactly equal to the total abatement cost $\Phi^m = B^m(a^m)F^m$ (see (4)). Taking (48) to hold as an identity throughout the remaining analysis, we can rewrite the value of $\mathcal{G}_m$ to explicitly reflect this

$$V^m = F_L^m(K^m) - D^m(S^m) - B^m(a^m)F^m(K^m) + P^{m+1} + [1 - B^{m+1}(a^{m+1})]A^{m+1} \tag{49}$$

Thus, the effect of $\mathrm{d}a^m$ and $\mathrm{d}P^{m+1}$ on the welfare of $\mathcal{G}_m$ is

$$\mathrm{d}V^m = -\mathrm{d}a^m B_a^m F^m + \mathrm{d}P^{m+1}. \tag{50}$$

Therefore, $\mathrm{d}V^m \geq 0$ if and only if

$$B_a^m F^m \mathrm{d}a^m \leq \mathrm{d}P^{m+1}. \tag{51}$$

By a minor abuse of terminology will will refer to the curves in $(a^m, P^{m+1}) - space$ along which $\mathcal{G}_m$ has constant utility as *indifference curves*. The slope of these is given by

$$\left.\frac{dP}{da}\right|_{U^m} := B_a^m(a^m)F^m \tag{52}$$

Notice that the magnitude of the slope (52) depends on the level of abatement a^m, but *not* on the pension level P^{m+1}. This is because the marginal cost for which $\mathcal{G}_m$ must be compensated is an increasing function of a^m, but independent of the pension level. By (51), any (sufficiently small) *increase* in policy $(\mathrm{d}a^m, \mathrm{d}P^{m+1})$ such that

$$\mathrm{d}a^m \cdot \left.\frac{dP}{da}\right|_{U^m} < \mathrm{d}P^{m+1} \tag{53}$$

will lead to a welfare level for $\mathcal{G}_m$ that is greater than U_m.

6.1.3 The welfare of $\mathcal{G}_{m+1}$

Using equation (38), the change in value to $\mathcal{G}_{m+1}$ is given by

$$\mathrm{d}V^{m+1} = (1 - d^1)F^m D_S^{m+1}\mathrm{d}a^m - \mathrm{d}P^{m+1} + V_K^{m+1}\mathrm{d}K^{m+1} \tag{54}$$

The first term in $\mathrm{d}V^{m+1}$ contains the beneficial effect of abatement at date m on the welfare of $\mathcal{G}_{m+1}$: the reduction of damages through the lower carbon stock. The second term is simply the direct cost of the pension it must pay as a reduction of its wealth. In addition to those effects, *both* a^m and P^{m+1} have a negative effect on capital accumulation. By (43) and (44) we get that

$$\mathrm{d}K^{m+1} = \frac{-\beta B_a^m F^m \mathrm{d}a^m - \mathrm{d}P^{m+1}}{1+\beta} \tag{55}$$

The value to $\mathcal{G}_{m+1}$ of an additional unit of capital $\mathrm{d}K^{m+1}$ is $V_K^{m+1} = r^{m+1}$ (see (47)). Thus (54) becomes

$$\mathrm{d}V^{m+1} = (1 - d^1)F^m D_S^{m+1}\mathrm{d}a^m - \mathrm{d}P^{m+1} + r^{m+1}\left[\frac{-\beta B_a^m F^m \mathrm{d}a^m - \mathrm{d}P^{m+1}}{1+\beta}\right]$$

and $\mathrm{d}V^{m+1} \geq 0$ if and only if

$$\mathrm{d}P^{m+1}\left[1 + \frac{r^{m+1}}{1+\beta}\right] \leq \mathrm{d}a^m\left[(1 - d^1)F^m D_S^{m+1} - \frac{\beta r^{m+1} B_a^m F^m}{1+\beta}\right] \tag{56}$$

The left-hand-side of (56) contains the twofold negative effect on the welfare of $\mathcal{G}_{m+1}$ of paying a pension $\mathrm{d}P^{m+1}$. In addition to the direct cost of the pension, such an anticipated transfer of income in retirement will have the effect of reducing the previous generation's capital savings, and thus result in a lower capital stock at $m + 1$.[3] The right-hand-side contains the benefit to $\mathcal{G}_{m+1}$ from abatement due to reduced climate damages as well as the negative effect costly abatement policy has on the incentives to save and therefore the capital accumulation of $\mathcal{G}_m$.

In analogy to (52), the slope in $(a^m, P^{m+1}) - space$ of $\mathcal{G}_{m+1}$'s indifference curves

[3]This is the well known effect that pay as you go pensions have on the capital accumulation process.

is given by

$$\left.\frac{dP}{da}\right|_{U^{m+1}} := \frac{(1-d^1)F^m D_S^{m+1} - \frac{\beta r^{m+1} B_a^m F^m}{1+\beta}}{1+\frac{r^{m+1}}{1+\beta}} \tag{57}$$

By (56) any (sufficiently small) *increase* in the policies $(\mathrm{d}a^m, \mathrm{d}P^{m+1})$ such that

$$\mathrm{d}P^{m+1} < \mathrm{d}a^m \cdot \left.\frac{dP}{da}\right|_{\bar{U}^{m+1}} \tag{58}$$

leads to an outcome in which the welfare level of $\mathcal{G}_{m+1}$ is greater than U^{m+1}. Thus, by (53) and (58) if

$$\left.\frac{dP}{da}\right|_{\bar{U}^m} < \left.\frac{dP}{da}\right|_{\bar{U}^{m+1}} \tag{59}$$

there exists an increase in the policies (a^m, P^{m+1}) that constitutes an improvement to both $\mathcal{G}_m$ and $\mathcal{G}_{m+1}$.[4]

Proposition 2. *Suppose that the gross production function is logarithmic and each generation has Leontief utility. Then there exist policies* $(\mathrm{d}a^m, \mathrm{d}P^m, \mathrm{d}P^{m+1})$ *such that*

$$\mathrm{d}a^m > 0 \tag{60}$$

$$\mathrm{d}P^m = B_a^m A^m \mathrm{d}a^m \tag{61}$$

$$0 < \mathrm{d}P^{m+1} < \frac{(1-d^1)F^m D_S^{m+1}(S^{m+1})}{1+\frac{r^{m+1}}{1+\beta}}\mathrm{d}a^m \tag{62}$$

and if $\mathrm{d}a^m$ *is sufficiently small, such policies will keep* $\mathcal{G}_{m-1}$ *indifferent to and make* $\mathcal{G}_m$ *and* $\mathcal{G}_{m+1}$ *strictly better off than the business-as-usual (BAU) policy* $a^m = 0, P^m = 0, P^{m+1} = 0$.

Furthermore, under such a policy the capital stock K^{m+2} *is greater than and the carbon stock* S^{m+2} *is lower than under the (BAU), thus providing greater economic opportunities for the following generation.*

Proof. Firstly, every term in the fraction on the right hand side of (62) is positive, so policies satisfying (60), (61) and (62) exist. Denote $\mathcal{G}_m$ and $\mathcal{G}_{m+1}$'s utilities

[4] If the strict inequality in (59) goes the other way, there is a *decrease* in that is a mutual improvement.

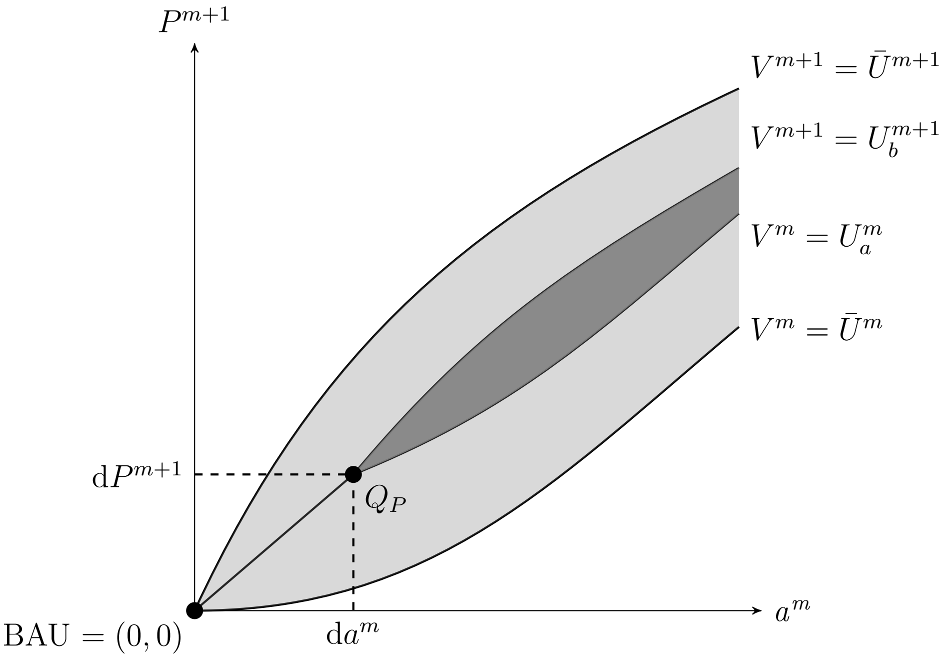

Figure 1: Pareto improvements

at the BAU by $\bar{U}^m$ and $\bar{U}^{m+1}$ respectively. Recall that at BAU the marginal abatement cost $B_a(0) = 0$. Therefore by (52) and (57)

$$\left.\frac{dP}{da}\right|_{\bar{U}^m} = 0$$

and

$$\left.\frac{dP}{da}\right|_{\bar{U}^{m+1}} = \frac{(1-d^1)F^m D_S^{m+1}(S^{m+1})}{1+\frac{r^{m+1}}{1+\beta}} > 0$$

and consequently

$$\left.\frac{dP}{da}\right|_{\bar{U}^m} < \left.\frac{dP}{da}\right|_{\bar{U}^{m+1}}$$

Policies $(\mathrm{d}a^m, \mathrm{d}P^{m+1})$ for which (60) and (62) hold satisfy

$$\left.\frac{dP}{da}\right|_{\bar{U}^m} < \frac{\mathrm{d}P^{m+1}}{\mathrm{d}a^m} < \left.\frac{dP}{da}\right|_{\bar{U}^{m+1}}$$

and by (53) and (58) are therefore strict improvements to $\mathcal{G}_m$ and $\mathcal{G}_{m+1}$. The condition (61) ensures that $\mathcal{G}_{m-1}$ is indifferent.

The policies in question have a twofold effect on the carbon stock S^{m+2}. By (3) and (5) we have that the change in the carbon stock relative to the business as usual is

$$\mathrm{d}S^{m+2} = -(1-d^2)F^m \mathrm{d}a^m + (\bar{e}^{m+1} - a^{m+1})F_K^{m+1}\mathrm{d}K^{m+1} \tag{63}$$

Since $\mathrm{d}K^{m+1}$ is given by (55) is must be negative and therefore $\mathrm{d}S^{m+2} < 0$. To see that the change in capital stock K^{m+2} is positive, first note that, due to (38) and (41) we have that, for all t

$$K^{t+1} = \frac{\beta}{1+\beta}V^t - P^{t+1} - \left[1 - B^{t+1}(a^{m+2})\right] A^{t+1} \tag{64}$$

Replacing $t = m+1$ and noting that $\mathrm{d}P^{m+2} = \mathrm{d}a^{m+2} = 0$ we get that

$$\mathrm{d}K^{m+2} = \mathrm{d}V^{m+1}\beta/(1+\beta)$$

Since $\mathrm{d}V^{m+1} > 0$ due to the fact that the policies were (strictly) Pareto improving, we get that $\mathrm{d}K^{m+2} > 0$. $\square$

Proposition 2 is illustrated in Figure 1. At the BAU the slope of the indifference curve of $\mathcal{G}_m$ is zero and the slope of the indifference curve of $\mathcal{G}_{m+1}$ is strictly positive. The slope of the segment BAU-Q_P is strictly in between the two, and therefore Q_P is inside the lightly grey shaded area, which contains all possible mutual improvements to $\mathcal{G}_m$ and $\mathcal{G}_{m+1}$. In the figure the slopes of the indifference curve that go through Q_P are drawn so that $\mathcal{G}_{m+1}$'s is steeper than $\mathcal{G}_m$'s. By the same logic the dark grey shaded area consists of the improvements over Q_P.

The indifference curves are drawn as convex and concave respectively. That this is in fact the case is shown in the Appendix.

6.2 Efficiency between $\mathcal{G}_{m-1}, \mathcal{G}_m$ and $\mathcal{G}_{m+1}$

From the preceding discussion it is clear that the points in $(a^m, P^{m+1}) - \textit{space}$ at which the slopes of the indifference curves are equal, i.e.

$$\left.\frac{dP}{da}\right|_{U^m} = \left.\frac{dP}{da}\right|_{U^{m+1}}$$

no improvement to both $\mathcal{G}_m$ and $\mathcal{G}_{m+1}$ is possible.[5] By equations (52) and (57) this is the case when

$$\left[1 + r^{m+1}(a^m, P^{m+1})\right] B_a^m(a^m)F^m - (1 - d^1)F^m D_S^{m+1}(S^{m+1}(a^m)) = 0 \qquad (65)$$

Condition (65) contains explicitly all the dependences on the policies a^m and P^{m+1}.[6] This makes it clear that the condition defines a locus in $(a^m, P^{m+1}) - \textit{space}$, which we draw as the dotted line going through in Q^* Figure 2. One way of understanding the locus is to consider a point Q_0 at which the welfare of $\mathcal{G}_m$ is U_0 and

$$\left.\frac{dP}{da}\right|_{U^m} < \left.\frac{dP}{da}\right|_{U^{m+1}}.$$

Moving north-eastwards along $\mathcal{G}_m$'s indifference curve defined by ensures that $\mathcal{G}_m$ remains indifferent. So long as the indifference curves of $\mathcal{G}_{m+1}$ are steeper than the indifference curve of $\mathcal{G}_m$ they are being crossed *in the direction of its preference* and $\mathcal{G}_{m+1}$'s welfare is being increased, i.e. $\bar{U}^{m+1} < U_1^{m+1} < U_2^{m+1}$. At Q^* the indifference curves are tangent and the mutual gains are exhausted. At that point (65) holds.

The efficiency locus is drawn with a negative *finite* slope. That it must be so is easily shown by implicitly differentiating (65). The logic behind the result is the following. The efficiency locus is determined by the amount of abatement that $\mathcal{G}_{m+1}$ can compensate $\mathcal{G}_m$ for, given that the compensation itself has an additional negative effect through the reduction of the $\mathcal{G}_m$'s incentive to save and the resulting capital accumulated at date $m+1$. This additional cost of the compensation mechanism is proportional to the interest rate at $m+1$, since the interest

[5] Note that $\mathcal{G}_{m-1}$ is being kept indifferent throughout by (48).

[6] The dependence on P^m is contained in a^m and the condition that $P^m = B^m(a^m)A^m$.

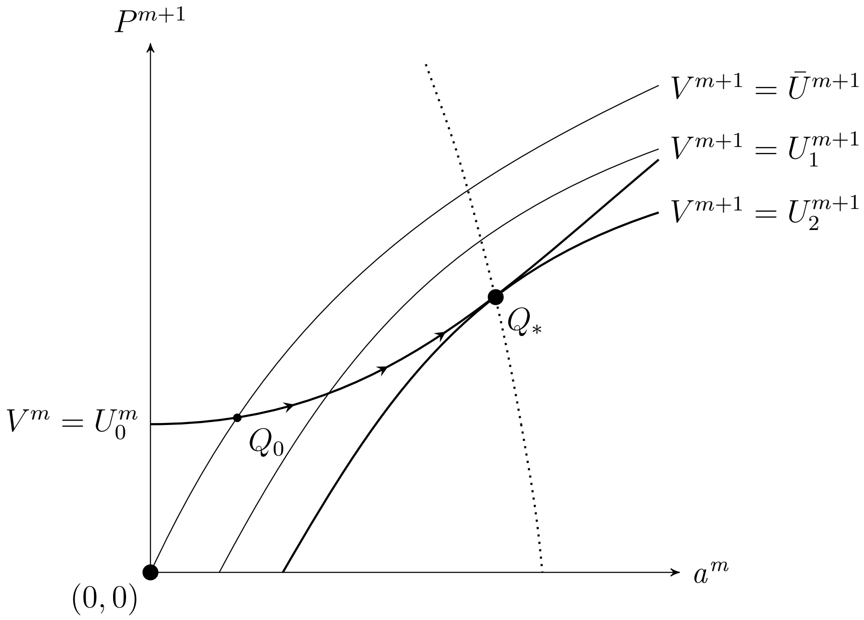

Figure 2: Efficiency frontier

rate determines the marginal value of capital. Since the equilibrium interest rate depends positively on the pension P^{m+1} (see equation (29)), compensation for an additional unit of abatement is greater at a higher pension level, and therefore the efficiency frontier has a lower abatement level at higher pension levels. The intuition is illustrated graphically in Figure 3. Consider a point $Q_\circ$ directly under Q_*. Both points have the same abatement level, but $P_*^{m+1} > P_\circ^{m+1}$. Therefore $r_*^{m+1} > r_\circ^{m+1}$ (see equation (29)). All the other terms in (65) are independent of P^{m+1} and therefore the left hand side of condition (65) must be less than zero at $Q_\circ$; the lower pension reduces the equilibrium interest rate which reduces the compensation cost to $\mathcal{G}_{m+1}$. More abatement can be compensated for, so the intersection of the frontier with the indifference curve of $\mathcal{G}_m$ going through $Q_\circ$ is to the northeast of $Q_\circ$. The above is summarised in Proposition 3.

Proposition 3. *Suppose that the gross production function is logarithmic and the each generation has Leontief utility. Then the efficiency frontier between $\mathcal{G}_{m-1}, \mathcal{G}_m$ and $\mathcal{G}_{m+1}$ is given by the monotonically decreasing locus in (a^m, P^{m+1}) space defined by* (65).

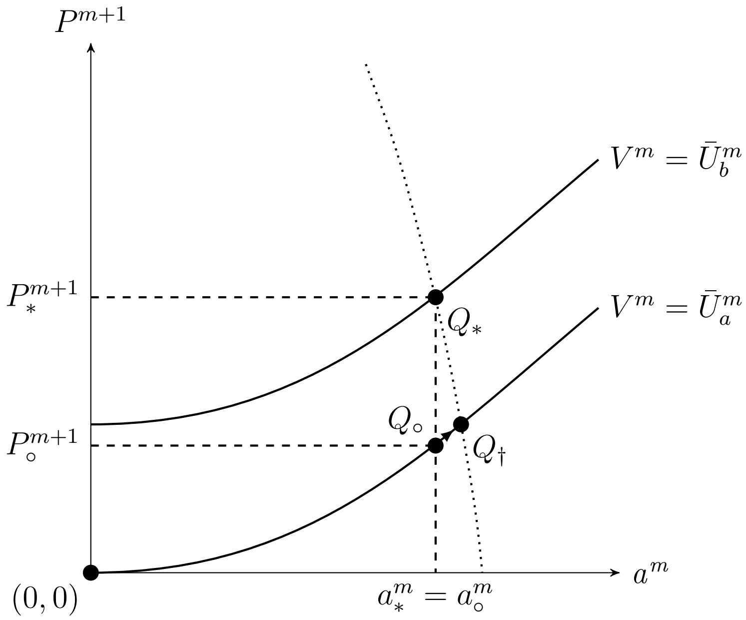

Figure 3: Higher cost at higher pension

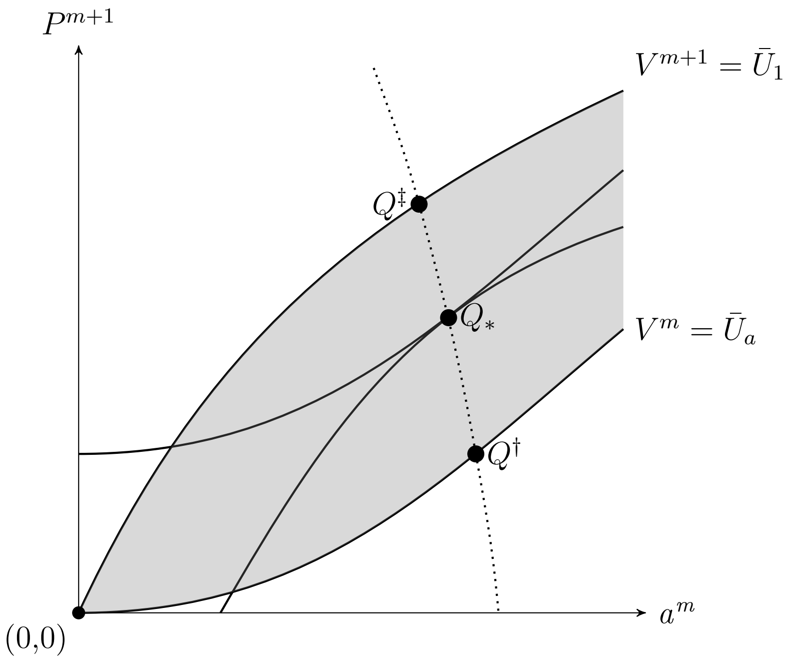

Figure 4: Pareto improvements and the frontier

Remark 1. *The left hand side of* (65) *embodies the cost to $\mathcal{G}_{m+1}$ of a unit of abatement conditional on paying $\mathcal{G}_m$ a pension that keeps it indifferent, accounting for the direct benefit and cost of abatement and the pension, as well as the indirect cost both abatement and pensions induce through the negative effect on the equilibrium capital stock at date $m+1$. When it is negative, it is not a cost, but a net benefit to $\mathcal{G}_{t+1}$.*

Figure 4 illustrates the results in Propositions 2 and 3. The grey shaded area consists of all the policy pairs that constitute Pareto improvements over the business-as-usual policy and the dotted line represents the efficiency frontier. The policies denoted by $Q^\dagger$ and $Q^\ddagger$ represent those policies on the frontier which make $\mathcal{G}_m$ and $\mathcal{G}_{m+1}$ respectively indifferent to the business-as-usual. The policy Q_* represents an outcome at which the surplus is shared between those two generations.

7 Even more abatement at date m

Throughout this section, when not otherwise stated we will be assuming that any Pareto improvements that are possible are those achievable by modifying *only* the abatement at date m and the pension payouts in subsequent periods. *The abatement levels at all future dates $t > m$ are assumed fixed at exogenous levels.*

This section generalises the results of Section 6 to an arbitrary number of generations, $N \in \mathbb{N}$. In Subsection 7.1 we derive the conditions that define the locus policies that are efficient between the set of generations $\{\mathcal{G}_{m+i} : i = 0, 1, \dots, N\}$. In Subsection 7.2 we use these conditions to establish that policy vectors that are efficient between $\{\mathcal{G}_{m+i} : i = 0, 1, \dots, N\}$ can be (Pareto) improved upon by the inclusion of $\mathcal{G}_{m+N+1}$ and a change in policy involving a greater level of abatement at date m and a positive pension P^{m+N+1}.

7.1 The N-pension frontier

We now consider the welfare of $\mathcal{G}_{m+2}$ in addition to that of $\mathcal{G}_{m-1}, \mathcal{G}_m$ and $\mathcal{G}_{m+1}$. Condition (65) defines policies at which mutual gains are exhausted from changes in policies a^m, P^m and P^{m+1}. We now allow for a further pension P^{m+2} to be paid to $\mathcal{G}_{m+1}$ by $\mathcal{G}_{m+2}$. Equation (50) still describes the change in $\mathcal{G}_m$'s value from the policy changes considered. By (38) the change in $\mathcal{G}_{m+1}$'s value differs from (54) by the inclusion of $\mathrm{d}P^{m+2}$,

$$\mathrm{d}V^{m+1} = D_S^{m+1}(1-d^1)F^m \mathrm{d}a^m - \mathrm{d}P^{m+1} + r^{m+1}\mathrm{d}K^{m+1} + \mathrm{d}P^{m+2}. \tag{66}$$

The change in the value of $\mathcal{G}_{m+2}$ is essentially analogous to that of $\mathcal{G}_{m+1}$ with the difference that there are two mechanisms whereby the carbon stock at $m+2$ is lower. From (3) and (5) it is easy to see that the direct effect on S^{m+2} from abatement in at date m is $(1-d^2)F^m$. There is an additional effect on S^{m+2} given by the fact that the capital stock K^{m+1} is changed, which (by (3)) leads to a change in E^{m+1} given by

$$E_K^{m+1}\mathrm{d}K^{m+1} := (\bar{e}^{m+1} - a^{m+1})F_K^{m+1}\mathrm{d}K^{m+1}, \tag{67}$$

where we denote by E_K^t the increase in period t emissions due to a unit increase in the stock of productive capital at date t. Thus, the total effect on the welfare of $\mathcal{G}_{m+2}$ is given by

$$\mathrm{d}V^{m+2} = D_S^{m+2}[(1-d^2)F^m \mathrm{d}a^m - E_K^{m+1}\mathrm{d}K^{m+1}] - \mathrm{d}P^{m+2} + r^{m+2}\mathrm{d}K^{m+2}. \tag{68}$$

By (50), $\mathcal{G}_m$ is made indifferent if the policies are such that

$$\mathrm{d}P^{m+1} = B_a^m F^m \mathrm{d}a^m \tag{69}$$

Notice that, by (55) this yields $\mathrm{d}K^{m+1} = -\mathrm{d}a^m B_a^m F^m = -\mathrm{d}P^{m+1}$. Conditional on $\mathcal{G}_m$ being kept indifferent then, by (66), $\mathcal{G}_{m+1}$ is made indifferent if

$$\mathrm{d}P^{m+2} = [(1+r^{m+1})B_a^m F^m - D_S^{m+1}(1-d^1)F^m]\mathrm{d}a^m \tag{70}$$

So when (69) and (70) hold we have that $\mathcal{G}_m$ *and* $\mathcal{G}_{m+1}$ are indifferent. The key step in the generalisation of Proposition 3 to $\mathcal{G}_{m+2}$ and more generations is the realisation that in this case, by (38) and (41), for all t,

$$K^{t+1} = \frac{\beta}{1+\beta}V^t - P^{t+1} - \left[1 - B^{t+1}(a^{m+2})\right]A^{t+1}. \tag{71}$$

Since $\mathrm{d}a^{m+2} = 0$ by assumption and $\mathrm{d}V^{m+1} = 0$ by construction (see (70)) we get that

$$\mathrm{d}K^{m+2} = -\mathrm{d}P^{m+2} \tag{72}$$

Define

$$H_1^m = (1 + r^{m+1})B_a^m F^m - D_S^{m+1}(1 - d^1)F^m$$

and

$$H_2^m = (1 + r^{m+2})H_1^m - D_S^{m+2}\left[(1 - d^2)F^m + (1 - d^1)E_K^{m+1}B_a^m F^m\right]$$

Then by (69), (70), (72) and (68) the policy changes that keep $\mathcal{G}_m$ and $\mathcal{G}_{m+1}$ indifferent increase the welfare of $\mathcal{G}_{m+2}$ if

$$H_2^m \mathrm{d}a^m \leq 0 \tag{73}$$

Notice that at the frontier between $\mathcal{G}_m$ and $\mathcal{G}_{m+1}$, $H_1^m = 0$ (see equation (65)). When $H_1^m > 0$ the marginal cost to $\mathcal{G}_{m+1}$ to compensating $\mathcal{G}_m$ for a unit of abatement is greater than the marginal benefit it experiences via the reduced damages. Thus, when $H_1^m > 0$ too much abatement has taken place from the point of view of just $\mathcal{G}_m$ and $\mathcal{G}_{m+1}$ and the policies must be to the East of the frontier drawn in Figure 4. For those two generations to be kept indifferent to an additional unit of abatement when policies are past the frontier, $\mathcal{G}_{m+1}$ requires a pension transfer $\mathrm{d}P^{m+2} = H_1^m \mathrm{d}a^m$ (see equation (70)). The net cost of such a policy to $\mathcal{G}_{m+2}$, accounting for the negative effect of the capital stock is $(1 + r^{m+2})H_1^m \mathrm{d}a^m$ (see equation (72)). The benefit to $\mathcal{G}_{m+2}$ stems from the lower damages resulting from the reduced carbon stock:

$$D_S^{m+2}\left[(1 - d^2)F^m + (1 - d^1)E_K^{m+1}B_a^m F^m\right]$$

When the cost equals the benefit (to $\mathcal{G}_{m+2}$) efficiency is achieved, i.e. at $H_2^m = 0$.[7]

[7] Notice that $H_1^m = 0$ (equation (65)) implicitly defined the Pareto frontier in $(a^m, P^{m+1}) - space$. Similarly, $H_2^m = 0$ implicitly defines the points in $(a^m, P^{m+1}, P^{m+2}) - space$ that are efficient between $\mathcal{G}_m, \mathcal{G}_{m+1}$ and $\mathcal{G}_{m+2}$. In addition to the dependence on a^m and P^{m+1} within H_1^m outlined in Figure 3, E_K^{m+1} depends on a^m and P^{m+1} and r^{m+1} depends on a^m, P^{m+1} and P^{m+2}.

For the generalisation to the inclusion of $\{\mathcal{G}_{m+i} : i = 0, 1, \dots, N\}$ for arbitrary $N \in \mathbb{N}$ we introduce the following notation. Denote by

$$\mathcal{P}^m = (a^t, P^{m+1}, P^{m+2}, \dots, P^{m+N}, 0, 0, \dots) \in \mathbb{R}_+^{\mathbb{N}}$$

a vector of policies in question.[8]

Lemma 1. *Define the functions $H_i^m(\mathcal{P}^m)$ recursively by*

$$\begin{aligned}
H_0^m(\mathcal{P}^m) &= B_a(a^m)F^m \\
H_1^m(\mathcal{P}^m) &= (1 + r^{m+1})H_0^m F^m - D_S^{m+1}(1 - d^1)F^m \\
H_i^m(\mathcal{P}^m) &= (1 + r^{m+i})H_{i-1}^m(\mathcal{P}^m) - D_S^{m+i}\left[(1 - d^i)F^m + \sum_{j=1}^{i-1}(1 - d^j)E_K^{m+i-j}H_{i-1-j}^m\right]
\end{aligned} \tag{74}$$

Suppose the gross production function is logarithmic and each generation has Leontief preferences. Then

$$H_N^m(\mathcal{P}^m) = 0 \tag{75}$$

is a necessary condition for the $\mathcal{P}^m$ to be on the efficiency frontier between the generations $\mathcal{G}_m, \mathcal{G}_{m+1}, \dots \mathcal{G}_{m+N}$

The formal proof of Lemma 1 is in the Appendix.

7.2 Pareto improvements

Denote by $V^t(\mathcal{P}^m)$ the welfare of generation $\mathcal{G}_t$ under the policy $\mathcal{P}^m$. Define the binary relation $\gg$ on Euclidian space by

$$a \gg b \iff a_i > b_i \quad \forall i = 1, 2, \dots, |a| \tag{76}$$

With this we can state our main result.

[8] Recall that the abatement levels in all future periods are assumed fixed at some non-negative level.

Theorem 2. *Suppose that the gross production function is logarithmic and each generation has Leontief preferences. Then, if* $\mathcal{P}_N^m = (a_N^m, P_N^{m+1}, \dots, P_N^{m+N}, 0)$ *is on the efficiency frontier between* $\{\mathcal{G}_{m+i}, i = 0, 1, \dots, N\}$*, there is a policy*

$$\mathcal{P}_{N+1}^m = (a_{N+1}^m, P_{N+1}^{m+1}, \dots, P_{N+1}^{m+N}, P_{N+1}^{m+N+1})$$

such that

$$\mathcal{P}_{N+1}^m \gg \mathcal{P}_N^m \tag{77}$$

and

$$V^t(\mathcal{P}_{N+1}^m) \geq V^t(\mathcal{P}_N^m), \forall t \geq m \tag{78}$$

with strict inequality for at least one $t \in \{m, m+1, m+2, \dots, m+N+1\}$*. Furthermore, under* $\mathcal{P}_{N+1}^m$ *the capital stock* K^{m+N+2} *is greater than and the carbon stock* S^{m+N+2} *is lower than under* $\mathcal{P}_N^m$*, thus endowing* $\mathcal{G}_{m+N+2}$ *with greater economic possibilities.*

The theorem states that any outcome that is on the efficiency frontier between $\{\mathcal{G}_{m+i} : i = 0, 1, \dots, N\}$ can be improved upon by the inclusion of $\mathcal{G}_{m+N+1}$.

Proof. By Lemma 1 any efficient point $\mathcal{P}_N^m$ must satisfy $H_N^m(\mathcal{P}_N^m) = 0$. If that is the case, then $H_{N-1}^m(\mathcal{P}_N^m) > 0$ and in fact $H_i^m(\mathcal{P}_N^m) > 0$ for all $i = 0, 1, \dots, N-1$. Consider the policy change

$$\mathrm{d}a^m > 0, \tag{79}$$

$$\mathrm{d}P^{m+i} = H_i^m(\mathcal{P}_N^m)\mathrm{d}a^m \quad \text{for } i = 0, 1, 2, \dots, N-1 \tag{80}$$

Such a sequence of policies changes will make $\{\mathcal{G}_{m+j} : j = 0, 1, \dots, i-1\}$ indifferent. Since $H_N^m(\mathcal{P}_N^m) = 0$ a pension $\mathrm{d}P^{m+N+1}$ such that

$$0 < \mathrm{d}P^{m+N+1}$$

will will lead to a strict improvement in the welfare of $\mathcal{G}_{m+N}$ (given (79) and (80)).

Furthermore

$$
\begin{aligned}
H^m_{N+1}(\mathcal{P}^m_N) =&(1+r^{m+N+1})H^m_N(\mathcal{P}^m_i) && (81)\\
&- D_S^{m+N+1}\left[(1-d^{N+1})F^m + \sum_{j=1}^{N}(1-d^j)E_K^{m+N+1-j}H^m_{N-j}\right] && (82)\\
=&- D_S^{m+N+1}\left[(1-d^{N+1})F^m + \sum_{j=1}^{N}(1-d^j)E_K^{m+N+1-j}H^m_{N-j}\right] < 0 && (83)
\end{aligned}
$$

So there is a strictly positive benefit to $\mathcal{G}_{m+N+1}$ to the policies defined by (79) and (80). Thus, $\mathrm{d}P^{m+N+1}$ such that

$$
\mathrm{d}P^{m+N+1} < D_S^{m+N+1}\left[(1-d^{N+1})F^m + \sum_{j=1}^{N}(1-d^j)E_K^{m+N+1-j}H^m_{N-j}\right]\mathrm{d}a^m \tag{84}
$$

will result in a strict improvement in the welfare of $\mathcal{G}_{m+N+1}$. Thus, defining

$$
\mathrm{d}\mathcal{P} = (\mathrm{d}a^m, \mathrm{d}P^{m+1}, \dots, \mathrm{d}P^{m+N+1}) \tag{85}
$$

for some $\mathrm{d}P^{m+N+1} > 0$ that satisfies (84) and

$$
\mathcal{P}^m_{N+1} = \mathcal{P}^m_N + \mathrm{d}\mathcal{P} \tag{86}
$$

we have that $\mathcal{P}^m_{N+1} \gg \mathcal{P}^m_N$ and $V^t(\mathcal{P}^m_{N+1}) = V^t(\mathcal{P}^m_N)$ for $t = m, m+1, \dots, m+N-1$ and $V^t(\mathcal{P}^m_{N+1}) > V^t(\mathcal{P}^m_N)$ for $t = m+N, m+N+1$.

To see that the capital stock is greater under $\mathcal{P}^m_{N+1}$ than under $\mathcal{P}^m_N$, notice that, by (64), it must be the case that

$$
\mathrm{d}K^{m+N+2} = \frac{\beta}{1+\beta}\mathrm{d}V^{m+N+1} - \mathrm{d}P^{m+N+2} \tag{87}
$$

So if the change in $\mathcal{G}_{m+N+1}$'s welfare is positive and there is no change in the pension level it receives, the change in the capital it accumulates in equilibrium must also be positive. That S^{m+N+2} decreases is as simple consequence of (3) and (5) and the fact that $\mathrm{d}a^m > 0$ and $\mathrm{d}K^{m+i} < 0$ for $i = 1, 2, \dots, m+N+1$. The latter is a consequence of (87) and the fact that $\mathrm{d}P^{m+i} > 0$ for $i = 1, 2, \ldots, m + N + 1$. This establishes the claim. $\square$

8 The general result

The assumptions of logarithmic production and Leontief utility allow for a very simple, albeit highly stylised solution to the inter-temporal choice problem. They are relaxed in this section. In what follows we will assume that the utility function to be a general monotonically increasing and twice differentiable function of consumption: $U(C_{1t}, C_{2t+1})$. As mentioned in Subsection 2.4, the solution to the inter-temporal optimisation problem (13) yields a savings function

$$(M^t, Z^{t+1}, r^{t+1}) \mapsto s(M^t, Z^{t+1}, r^{t+1}) = K^{t+1} \tag{88}$$

We make no assumptions on the gross production function other than that it have constant returns to scale. The wage and interest rates are then give by (11) and (12), and we will write

$$w_K^t = (1 - B^t)F_{LK}^t \quad \text{and} \quad r_K^t = (1 - B^t)F_{KK}^t$$

for their derivatives with respect to capital.

8.1 Policies and frontier

We can now state the generalisation of Lemma 1.

Lemma 2. *Suppose the gross production function has constant returns to scale in capital and labour and the each generation has a twice differentiable utility function resulting in a savings function* (88) *as the solution to each generations* *inter-temporal consumption problem. Define the functions $G_i^m(\mathcal{P}_t)$ recursively by*

$$G_0^m = (1 + r^{m+1})T^m B_a^m(a^m)F^m \tag{89}$$

$$G_1^m = (1 + r^{m+2})T^{m+1}\left[R^m G_0^m - D_S^{m+1}(S^{m+1})(1 - d^1)F^m\right] \tag{90}$$

$$G_i^m = (1 + r^{m+i+1})T^{m+i}\left[R^{m+i-1}G_{i-1}^m - D_S^{m+i}\frac{\partial S^{m+i}}{\partial a^m}\right] \tag{91}$$

with

$$J^t = \left(1 + \frac{r_K^{t+1}K^{t+1}s_M^t}{(1 + r^{t+1})(1 - r_K^{t+1}s_r^t)}\right) \tag{92}$$

$$N^t = \left(1 + \frac{r_K^{t+1}K^{t+1}s_Z^t}{1 - r_K^{t+1}s_r^t}\right) \tag{93}$$

$$Q^t = \frac{(s_M^{t-1} - s_Z^t)}{(1 + r^t)(1 - r_K^t s_r^t) + r_K^t K^t s_M^{t-1}} \tag{94}$$

$$R^t = 1 + w_K^{t+1}Q^t \tag{95}$$

$$T^t = \frac{J^t}{N^t} \tag{96}$$

$$\frac{\partial S^t}{\partial a^m} = (1 - d^{t-m})F^m + \sum_{j=1}^{t-m-1}(1 - d^j)E_K^{t-j}G_{t-m-j-1}^m Q^{t-j-1} \tag{97}$$

Then, provided the policies at date m are unanticipated by $\mathcal{G}_{m-1}$,

$$G_N^m(\mathcal{P}^m) = 0 \tag{98}$$

is a necessary condition for efficiency between $\mathcal{G}_m, \mathcal{G}_{m+1}, \ldots \mathcal{G}_{m+N}$

Note that apart from the factors (92) to (96), the functions G_i^m in Lemma 2 are identical to the functions H_i^m in Lemma 1. In particular, the qualitative dependence of G_{i+1}^m on G_i^m is the same as that of H_{i+1}^m on H_i^m. Since this is the only feature required for the proof of Theorem 2, we can state its generalisation without further proof.

Theorem 3. *Suppose the gross production function has constant returns to scale in capital and labour and the each generation has a twice differentiable utility function resulting in a savings function* (88) *as the solution to each generations intertempo-*

ral consumption problem. Suppose further that the policies implemented at date m were unanticipated by $\mathcal{G}_{m-1}$ at date $m-1$. Then, if $\mathcal{P}_N^m = (a_N^m, P_N^{m+1}, \dots, P_N^{m+N}, 0)$ is on the efficiency frontier between $\{\mathcal{G}_{m+i}, i = 0, 1, \dots, N\}$, there is a policy

$$\mathcal{P}_{N+1}^m = (a_{N+1}^m, P_{N+1}^{m+1}, \dots, P_{N+1}^{m+N}, P_{N+1}^{m+N+1})$$

such that

$$\mathcal{P}_{N+1}^m \gg \mathcal{P}_N^m \tag{99}$$

and

$$V^t(\mathcal{P}_{N+1}^m) \geq V^t(\mathcal{P}_N^m), \, \forall t \geq m \tag{100}$$

with strict inequality for at least one $t \in \{m, m+1, m+2, \dots, m+N+1\}$. Furthermore, under $\mathcal{P}_{N+1}^m$ the capital stock K^{m+N+2} is greater than and the carbon stock S^{m+N+2} is lower than under $\mathcal{P}_N^m$.

8.2 Qualitative and quantitative difference

Notice that the relative magnitude of the points on the Pareto frontiers defined by (75) and (98) is somewhat determined by the product of the coefficients $R^t \cdot T^t$. If $R^m \cdot T^m = 1$ the frontiers defined by (75) and (98) for $N = 1$ are *identical.* If that were the case, the different simplifications assumed in Section 4 would cancel out exactly.

For $T^m R^m \leq 1$ it is the case that the frontier defined by (75) has uniformly greater abatement levels than (98). This is essentially because it is less costly to $\mathcal{G}_{m+1}$ to compensate $\mathcal{G}_m$ for any unit of abatement. The converse is also true, i.e. if $T^m R^m \geq 1$ the frontier defined by (75) has uniformly lower abatement levels than (98).

The frontiers for $N > 1$ are not as easy to characterise. This is because the frontier is partially determined by the emissions that are reduced by virtue of the reduction in capital stock at the intermediate dates $t = m+1, m+2, \dots, m+N-1$ – the summation on the right hand side of (97). If $T^m R^m \leq 1$ and $Q^t \leq 1$ the effect on the capital stock is greater in equation (74) than in (91) so even though the cost of compensation is less in the general case than in the simplified case, the benefit is also slightly lower. However, if the coefficients E_K^t for $t = m+1, m+2, \dots, m+N-1$

are sufficiently small, one can also conclude that the frontiers defined by (75) have a uniformly greater abatement level than the ones defined by (98). In this case, the simpler model (75) would underestimate the amount of desirable abatement (at any given level of pensions).

Testing the range of values that can be taken by the product $R^t \cdot T^t$ with Mathematica we can conclude that if the production and decision parameters are restricted to the domains specified below

- Capital share: $\alpha \in [0, 1]$
- Generational interest rate: $r^t \in (0, 4)$
- First period wealth effect: $s_M^{t-1} \in (0, 1)$
- Second period wealth effect: $s_Z^t \in (-1, 0)$
- Interest elasticity of savings: $s_r^t \frac{r^t}{K^t} =: \mathcal{E}_{s,r} \in (-1, 1)$

the product must be in the domain $T^t \cdot R^t \in (0, 1)$. For the following specific values deemed as reasonable by the authors

- $\alpha = 0.3$
- $r_t = 1$
- $s_M^{t-1} = -s_Z^t = 0.4$
- $\mathcal{E}_{s,r}^t = 0.01$

we have that

$$T^t R^t \approx 0.89$$

and

$$Q^t \approx 0.46$$

For those parameter values it would mean that the approximation introduced in Section 4 underestimates the efficient levels of abatement for the contracts involving only two generations. Whether or not the contracts involving more generations under or overestimate the abatement levels depends on the magnitude of the parameters $E_K^t = e^t F_K^t$, i.e. the product of the future emission intensities with the gross productivity of capital.

Either way, the approximation will presumably result in slightly conservative prescriptions of the same order of magnitude as the more complete model for any reasonable parameters for the production technology and savings propensities.

9 Conclusion

We have derived the conditions for a policy vector involving abatement in a given period and compensating intergenerational transfers in all subsequent periods to be efficient in the sense that any further benefits to future generations from current abatement are smaller than the cost of compensating the abaters for the cost they incur. We use this result to establish that Pareto improvements are possible if the benefits to any generation have not been included into the agreement.

The theoretical exposition is self-contained and complete. This being said, further work is necessary to exhaust the full potential of this research project. It would be desirable to include population growth into the analysis. The damages have been modeled as proportional to the population rather than the more usual form that makes them proportional to output. As discussed in Section 2.1 this feature makes our claim of the existence of Pareto improving abatement more difficult to establish, as damages on output would create an incentive for every generation to abate simply for the benefit of increasing their capital rent, which would be affected by damages if they are modeled as proportional to output. Thus, adding the dependance on output will not yield any new further intuition, but with a view of calibrating the model to one of the widespread integrated assessment models it would be useful to model the damages in the standard way. Such a calibration would allow us to establish the orders of magnitude of the pensions involved and help determine the political feasibility of such a policy.

From a theoretical point of view there is one extension in particular that may be worth pursuing. We have used used endowment transfers (pay-as-you-go pensions) as the compensation mechanism to achieve the Pareto improvements. These have a well-known theoretical property of disincentivising capital accumulation, a feature which determines the location of the efficiency frontier. It may be possible to achieve further gains if the compensation mechanism used is a subsidy on the capital returns, since these could easily achieve the same amount of compensation at possibly nil disincentive to the savings decision and therefore capital accumulation.

Finally, the strategic credibility of such intergenerational contracts must be analysed. There is an existing literature looking at this in OLG models. Most notably, Rangel (2003) looks at a link between social security and environmental services in a strategic setting. Our models differ significantly, but an analysis similar to his could yield interesting results in our framework as well.

10 Appendix: proofs and mathematical detail

Lemma 3. *In the domain* $\mathcal{D} = \{(L, K) : L > 0, K > L \cdot e\}$ *the isoquants of*

$$F(K, L) = L \ln\left(\frac{K}{L}\right)$$

are downward sloping and convex.

Proof. By the implicit function theorem the slopes of the isoquants are given by

$$\frac{\partial K}{\partial L} = -\frac{F_L}{F_K} = -\frac{K}{L}\left[\ln\left(\frac{K}{L}\right) - 1\right] \tag{101}$$

which is negative in $\mathcal{D}$. Notice that

$$L\frac{\partial K}{\partial L} = -K\left[\ln\left(\frac{K}{L}\right) - 1\right] \tag{102}$$

Differentiating (102) with respect to L you get the left hand side

$$\text{LHS} = \frac{\partial K}{\partial L} + L\frac{\partial^2 K}{\partial L^2}$$

and right hand side

$$\begin{aligned}
\text{RHS} &= -\frac{\partial K}{\partial L}\left[\ln\left(\frac{K}{L}\right) - 1\right] - K\left[\frac{\partial K}{\partial L}\frac{1}{K} - \frac{1}{L}\right] \\
&= -\frac{\partial K}{\partial L}\left[-\frac{\partial K}{\partial L}\frac{L}{K}\right] - \frac{\partial K}{\partial L} + \frac{K}{L}
\end{aligned}$$

Equating LHS and RHS yields

$$\begin{aligned}
L\frac{\partial^2 K}{\partial L^2} &= \frac{L}{K}\left[\left(\frac{\partial K}{\partial L}\right)^2 - 2\frac{K}{L}\frac{\partial K}{\partial L} + \left(\frac{K}{L}\right)^2\right] \\
&= \frac{L}{K}\left[\frac{\partial K}{\partial L} - \frac{K}{L}\right]^2 \geq 0
\end{aligned}$$

Thus the isoquants are convex, which completes the proof. □

Lemma 4. *The indifference curves (in* $(a^m, P^{m+1}) - space$*) of* $\mathcal{G}_m$ *are convex and* *within the domain for which they are increasing those of $\mathcal{G}_{m+1}$ are concave.*

Proof. The slope of the indifference curves of $\mathcal{G}_m$ is given by

$$\left.\frac{dP}{da}\right|_{U^m} = B_a^m(a^m)F^m \tag{103}$$

The derivative of (103) with respect to a^m is

$$\left.\frac{d^2P}{da^2}\right|_{U^m} = B_{aa}^m F^m > 0$$

The slope of the indifference curves of $\mathcal{G}_{m+1}$ is given by

$$\left.\frac{dP}{da}\right|_{U^{m+1}} = \frac{(1-d^1)F^m D_S^{m+1} - \frac{\beta r^{m+1} B_a^m F^m}{1+\beta}}{1 + \frac{r^{m+1}}{1+\beta}} \tag{104}$$

The derivative of (104) with respect to a^m is

$$\left.\frac{d^2P}{da^2}\right|_{U^{m+1}} = \frac{-(1-d^1)(F^m)^2 D_{SS}^{m+1} - \frac{\beta r^{m+1}}{1+\beta} B_{aa}^m F^m}{1 + \frac{r^{m+1}}{1+\beta}} \tag{105}$$

$$- \frac{(1-d^1)F^m D_S^{m+1} - \frac{\beta r^{m+1} B_a^m F^m}{1+\beta}}{1 + \frac{r^{m+1}}{1+\beta}} \frac{1 + \frac{r_K^{m+1}}{1+\beta}\left[K_{a^m}^{m+1} + K_{P^{m+1}}^{m+1} \left.\frac{dP}{da}\right|_{U^{m+1}}\right]}{1 + \frac{r^{m+1}}{1+\beta}} \tag{106}$$

$$= \frac{-(1-d^1)(F^m)^2 D_{SS}^{m+1} - \frac{\beta r^{m+1}}{1+\beta} B_{aa}^m F^m}{1 + \frac{r^{m+1}}{1+\beta}} \tag{107}$$

$$- \left.\frac{dP}{da}\right|_{U^{m+1}} \frac{1 + \frac{r_K^{m+1}}{1+\beta}\left[K_{a^m}^{m+1} + K_{P^{m+1}}^{m+1} \left.\frac{dP}{da}\right|_{U^{m+1}}\right]}{1 + \frac{r^{m+1}}{1+\beta}} \tag{108}$$

Since $r_K^{m+1} < 0$ and $K_{a^m}^{m+1} > 0$, $K_{P^{m+1}}^{m+1} > 0$

$$\left.\frac{dP}{da}\right|_{U^{m+1}} \geq 0$$

is sufficient for

$$\left.\frac{d^2P}{da^2}\right|_{U^{m+1}} < 0$$

$\square$

Lemma 5. *Under the assumptions of Lemma 1 the effect on the welfare of generation $\mathcal{G}_t$ from small changes in past policies up to and including a^{t-1} and P^{t+1} is given by*

$$\begin{aligned}\mathrm{d}V^t = {} & D_S^t \sum_{i=1}^{t-m}(1-d^i)F^{t-i}\mathrm{d}a^{t-i} + \mathrm{d}P^{t+1} + r^t\frac{\beta}{1+\beta}\mathrm{d}V^{t-1} - (1+r^t)\mathrm{d}P^t \\ & - D_S^t \sum_{i=1}^{t-m-1}(1-d^i)E_K^{t-i}\left(\frac{\beta}{1+\beta}\mathrm{d}V^{t-i-1} - \mathrm{d}P^{t-i} + A^{t-i}B_a^{t-i}\mathrm{d}a^{t-i}\right)\end{aligned} \tag{109}$$

where period m is the first in which there is a change in the abatement policy.

Proof. The value V^t is a function of $K^t, S^t, a^t, a^{t+1}, P^t$ and P^{t+1} so

$$\mathrm{d}V^t = \frac{\partial V^t}{\partial K^t}\mathrm{d}K^t + \frac{\partial V^t}{\partial S^t}\mathrm{d}S^t + \frac{\partial V^t}{\partial a^t}\mathrm{d}a^t + \frac{\partial V^t}{\partial a^{t+1}}\mathrm{d}a^{t+1} + \frac{\partial V^t}{\partial P^t}\mathrm{d}P^t + \frac{\partial V^t}{\partial P^{t+1}}\mathrm{d}P^{t+1} \tag{110}$$

We are assuming $\mathrm{d}a^t = \mathrm{d}a^{t+1} = 0$

$$\frac{\partial V^t}{\partial P^t} = -1 \quad \text{and} \quad \frac{\partial V^t}{\partial P^{t+1}} = 1 \tag{111}$$

$$\frac{\partial V^t}{\partial S^t} = -D_S^t \quad \text{and} \quad \frac{\partial V^t}{\partial K^t} = r^t \tag{112}$$

It remains to be shown that

$$\mathrm{d}S^t = -\sum_{i=1}^{t-m}(1-d^i)F^{t-i}\mathrm{d}a^{t-i} + \sum_{i=1}^{t-m-1}(1-d^i)E_K^{t-i}\mathrm{d}K^{t-i} \tag{113}$$

and

$$\mathrm{d}K^t = \frac{\beta}{1+\beta}\mathrm{d}V^{t-1} - \mathrm{d}P^t + A^t B_a^t \mathrm{d}a^t \tag{114}$$

(113) is a direct consequence of (5). To see that (114) holds note that by combining (31) with (38) you get that

$$K^t = \frac{\beta}{1+\beta}V^{t-1} - P^t - A^t(1 - B^t(a^t)) \tag{115}$$

The conclusion follows directly. $\square$

Proof of Lemma 1. Consider the vector of policy changes

$$\mathrm{d}\mathcal{P}^m = (\mathrm{d}a^m, \mathrm{d}P^{m+1}, \ldots, \mathrm{d}P^{m+N})$$

Since only abatement in period m is being considered, the hypothesis of Lemma 5 holds for $t > m$, and by (109)

$$\begin{aligned}\mathrm{d}V^t &= D_S^t(1-d^{t-m})F^m\mathrm{d}a^m + \mathrm{d}P_{t+1} + r_t\frac{\beta}{1+\beta}\mathrm{d}V^{t-1} - (1+r_t)\mathrm{d}P_t \\ &\quad - D_S^t\sum_{i=1}^{t-m-1}(1-d^i)E_K^{t-i}\left(\frac{\beta}{1+\beta}\mathrm{d}V^{t-i-1} - \mathrm{d}P^{t-i}\right)\end{aligned}$$

Consider policies that leave $\{\mathcal{G}_i; i = m, m+1, \ldots, t-1\}$ indifferent. That is $\mathrm{d}V^i = 0$ for all $i = m, m+1, \ldots, t-1$. The welfare of $\mathcal{G}_t$ then changes by

$$\mathrm{d}V^t = D_S^t(1-d^{t-m})F^m\mathrm{d}a^m + \mathrm{d}P_{t+1} - (1+r_t)\mathrm{d}P_t + D_S^t\sum_{i=1}^{t-m-1}(1-d^i)E_K^{t-i}\mathrm{d}P^{t-i}$$

Such a policy will leave $\mathcal{G}_t$ indifferent if

$$\mathrm{d}P^{t+1} = (1+r^t)\mathrm{d}P^t + D_S^t(1-d^{t-m})F^m\mathrm{d}a^m + D_S^t\sum_{i=1}^{t-m-1}(1-d^i)E_K^{t-i}\mathrm{d}P^{t-i}$$

Therefore, when

$$\frac{\mathrm{d}P^{t+1}}{\mathrm{d}a^m} = (1+r^t)\frac{\mathrm{d}P^t}{\mathrm{d}a^m} - D_S^t(1-d^{t-m})F^m - D_S^t\sum_{i=1}^{t-m-1}(1-d^i)E_K^{t-i}\frac{\mathrm{d}P^{t-i}}{\mathrm{d}a^m} \tag{116}$$

it is the case that $\{\mathcal{G}_i; i = m, m+1, \ldots, t\}$ are all indifferent to the policy $\mathrm{d}\mathcal{P}$. Since $\mathcal{G}_{m+N+1}$ is *not* included, $\mathrm{d}P^{m+N+1} = 0$ by hypothesis. Thus, for $\mathcal{G}_N$ to be indifferent to the policy

$$(1+r^t)\frac{\mathrm{d}P^t}{\mathrm{d}a^m} - D_S^t(1-d^{t-m})F^m - D_S^t\sum_{i=1}^{t-m-1}(1-d^i)E_K^{t-i}\frac{\mathrm{d}P^{t-i}}{\mathrm{d}a^m} \stackrel{!}{=} 0$$

Thus, defining

$$H_i^m := \frac{\mathrm{d}P^{m+i+1}}{\mathrm{d}a^m}$$

and replacing for H_i^m in (116) yields the definitions in the hypothesis and the condition of the conclusion. □

Proof of Theorem 2. In this more general setting, the value function and equilibrium capital are defined by equations (18) to (22). Thus, the general form of the derivatives of the value function and the capital accumulation equation – equations (39) to (47) – are

$$V_{a^m}^m = -B_a^m F_L^m u'(C_{1m}) + K^{m+1} r_K^{m+1} K_{a^m}^{m+1} \beta u'(C_{2m+1}) \tag{117}$$

$$V_{P^m}^m = -u'(C_{1m}) + K^{m+1} r_K^{m+1} K_{P^m}^{m+1} \beta u'(C_{2m+1}) \tag{118}$$

$$V_{a^m}^{m-1} = -B_a^m F_K^m K^m \beta u'(C_{2m}) + K^m r_K^m K_{a^m}^m \beta u'(C_{2m}) \tag{119}$$

$$V_{P^m}^{m-1} = \beta u'(C_{2m}) + K^m r_K^m K_{P^m}^m \beta u'(C_{2m}). \tag{120}$$

The derivatives (135) and (136) are correct provided the respective change in policy was anticipated and thus allowed for an adjustment in the savings rate. If the change is unanticipated we get

$$V_{a^m}^{m-1} = -B_a^m F_K^m K^m \beta u'(C_{2m}) \tag{121}$$

$$V_{P^m}^{m-1} = \beta u'(C_{2m}). \tag{122}$$

The dependance of the values on the state variables is given by

$$V_{K^m}^m = w_K^m \left[u'(C_{1m}) - K^{m+1} r_K^{m+1} K_{P^m}^{m+1} \beta u'(C_{2m+1})\right] \tag{123}$$

$$V_{S^m}^m = -D_S^m \left[u'(C_{1m}) - K^{m+1} r_K^{m+1} K_{P^m}^{m+1} \beta u'(C_{2m+1})\right] \tag{124}$$

The first order effects on future capital savings are given by

$$K_{a^m}^{m+1} = \frac{-B_a^m F_L^m s_M^m}{1 - r_K^{m+1} s_r^{m+1}} \tag{125}$$

$$K_{P^m}^{m+1} = \frac{-s_M^m}{1 - r_K^{m+1} s_r^{m+1}} \tag{126}$$

and the effect on the current capital stock, if the the policy change was anticipated, is given by

$$K^m_{a^m} = \frac{-B^m_a F^m_K s^m_r}{1 - r^m_K s^m_r} \tag{127}$$

$$K^m_{P^m} = \frac{s_Z}{1 - r^m_K s^m_r} \tag{128}$$

Due to the fact that the Euler equation must hold in equilibrium for every generation we have

$$u'(C_{1t}) = (1 + r^{t+1})\beta u'(C_{2t+1}). \tag{129}$$

We will divide the derivatives of the value function of $\mathcal{G}_t$ by $u'(C_{1t})$ in order to get the change in value in first period consumption units. Using the Euler equation (129), the coefficient $\beta u'(C_{2t+1}$ will get replaced by $(1 + r^{t+1})^{-1}$. Replacing (125) through (128) into (133) through (136), defining

$$J^m = 1 - (1 + r^{m+1})^{-1} K^{m+1} r^{m+1}_K K^{m+1}_{P^m} \tag{130}$$

$$N^m = 1 + K^{m+1} r^{m+1}_K K^{m+1}_{P^{m+1}} \tag{131}$$

$$\Lambda^m = 1 + \frac{r^m_K s^m_r}{1 - r^m_K s^m_r} \tag{132}$$

and with a minor abuse of notation we get that the derivatives of the value function in first period consumption units become

$$V^m_{a^m} = -B^m_a F^m_L J^m \tag{133}$$

$$V^m_{P^m} = -J^m \tag{134}$$

$$V^{m-1}_{a^m} = -\frac{B^m_a F^m_K K^m}{1 + r^m} \Lambda^m \tag{135}$$

$$V^{m-1}_{P^m} = \frac{N^{m-1}}{1 + r^m}, \tag{136}$$

$$V^{m-1}_{a^m} = -B^m_a F^m_K K^m (1 + r^m)^{-1} \tag{137}$$

$$V^{m-1}_{P^m} = (1 + r^m)^{-1} \tag{138}$$

and

$$V_{K^m}^m = w_K^m J^m \tag{139}$$
$$V_{S^m}^m = -D_S^m J^m \tag{140}$$

Consider, as in Proposition 1, the vector of policy changes

$$\mathrm{d}\mathcal{P}^m = (\mathrm{d}a^m, \mathrm{d}P^{m+1}, \ldots, \mathrm{d}P^{m+N})$$

with the additional assumption that $\mathrm{d}P^{m+N+1} = 0$. The welfare change of $\mathcal{G}_{m-1}$ is given by

$$\mathrm{d}V^{m-1} = V_{a^m}^{m-1}\mathrm{d}a^m + V_{P^m}^{m-1}\mathrm{d}P^m \tag{141}$$

If the policy changes were unanticipated when $\mathcal{G}_{m-1}$'s savings decision was made, the condition on $\mathrm{d}a^m$ and $\mathrm{d}P^m$ that ensures $\mathcal{G}_{m-1}$ is indifferent to the business as usual is

$$\mathrm{d}P^m = B_a^m F_K^m K^m \mathrm{d}a^m \tag{142}$$

The condition for the changes to be welfare improving to $\mathcal{G}_m$,

$$\mathrm{d}V^m = V_{a^m}^m \mathrm{d}a^m + V_{P^m}^m \mathrm{d}P^m + V_{P^{m+1}}^m \mathrm{d}P^{m+1} \geq 0, \tag{143}$$

becomes

$$\mathrm{d}V^m = V_{a^m}^m \mathrm{d}a^m + V_{P^m}^m B_a^m F_K^m K^m \mathrm{d}a^m + V_{P^{m+1}}^m \mathrm{d}P^{m+1} \geq 0, \tag{144}$$

when $\mathrm{d}P^m$ is replaced with (142). At the time $\mathcal{G}_m$ makes it's savings decision the policy changes are assumed to be known and therefore $\mathrm{d}V^m \geq 0$ becomes

$$(1 + r^{m+1})^{-1} N^m \mathrm{d}P^{m+1} \geq B_a^m F^m J^m \mathrm{d}a^m \tag{145}$$

The condition for any future generation $\mathcal{G}_t$, for $t > m$ to remain (at least) indifferent is

$$\mathrm{d}V^t = V_{S^t}^t \mathrm{d}S^t + V_{K^t}^t \mathrm{d}K^t + V_{P^t}^t \mathrm{d}P^t + V_{P^{t+1}}^t \mathrm{d}P^{t+1} = 0 \tag{146}$$

Since abatement only happens during period m, for all $t > m$

$$\mathrm{d}V^{t-1} = \frac{s_M^{t-1} - s_Z^t}{1 + r^t s_M^{t-1}} \mathrm{d}P^t + \left[\frac{1 - r_K^t s_r^t}{s_M^{t-1}} + \frac{K^t r_K^t}{1 + r^t}\right] \mathrm{d}K^t \tag{147}$$

To see this notice that

$$V_{a^{t-1}}^{t-1} \mathrm{d}a^{t-1} - \left[\frac{1 - r_K^t s_r^t}{s_M^{t-1}} + \frac{K^t r_K^t}{1 + r^t}\right] K_{a^{t-1}}^t \mathrm{d}a^{t-1} = 0 \tag{148}$$

$$V_{P^{t-1}}^{t-1} \mathrm{d}P^{t-1} - \left[\frac{1 - r_K^t s_r^t}{s_M^{t-1}} + \frac{K^t r_K^t}{1 + r^t}\right] K_{P^{t-1}}^t \mathrm{d}P^{t-1} = 0 \tag{149}$$

and

$$V_{P^t}^{t-1} \mathrm{d}P^t - \left[\frac{1 - r_K^t s_r^t}{s_M^{t-1}} + \frac{K^t r_K^t}{1 + r^t}\right] K_{P^t}^t \mathrm{d}P^t = \left[\frac{1}{1 + r^t} - \frac{s_Z^t}{s_M^{t-1}}\right] \mathrm{d}P^t = \frac{s_M^{t-1} - s_Z^t}{(1 + r^t) s_M^{t-1}} \mathrm{d}P^t \tag{150}$$

Combining (148), (149) and (150) yields (147). Defining

$$Q^t = \frac{s_M^{t-1} - s_Z^t}{(1 + r^t)(1 - r_K^t s_r^t) + K^t r_K^t s_M^{t-1}} \tag{151}$$

we can rewrite (147) into

$$\mathrm{d}K^t = Q^t \left[\frac{(1 + r^t) s_M^{t-1}}{(s_M^{t-1} - s_Z^t)} \mathrm{d}V^{t-1} - \mathrm{d}P^t\right] \tag{152}$$

By (146) the condition that $\mathrm{d}V^t \geq 0$ *conditional* on $\mathrm{d}V^{t-1} = 0$ can be rewritten to

$$\frac{N^t}{1 + r^{t+1}} \mathrm{d}P^{t+1} = J^t \left[\left(1 + w_K^t Q^t\right) \mathrm{d}P^t + D_S^t \mathrm{d}S^t\right] \tag{153}$$

Finally, recall that, by (113)

$$\mathrm{d}S^t = -(1 - d^{t-m}) F^m \mathrm{d}a^m + \sum_{j=1}^{i-1} (1 - d^j) E_K^{t-j} \mathrm{d}K^{t-j} \tag{154}$$

Substituting (152) conditional on $\mathrm{d}V^t = 0$ for $t - m = 0, 1, \ldots, i - 1$ into (154) yields

$$\mathrm{d}S^t = -(1 - d^{t-m})F^m \mathrm{d}a^m - \sum_{j=1}^{i-1}(1 - d^j)E_K^{t-j}Q^{t-j}\mathrm{d}P^{t-j} \tag{155}$$

Thus, when

$$\frac{N^t}{1 + r^{t+1}}\mathrm{d}P^{t+1} = J^t\left[\left(1 + w_K^t Q^t\right)\mathrm{d}P^t - D_S^t\left((1 - d^{t-m})F^m \mathrm{d}a^m + \sum_{j=1}^{i-1}(1 - d^j)E_K^{t-j}Q^{t-j}\mathrm{d}P^{t-j}\right)\right] \tag{156}$$

holds for $t = m+1, \ldots, m+N-1$, the generations $\{\mathcal{G}_t, t = m, 1, 2, \ldots, m+N-1\}$ are indifferent to $\mathrm{d}\mathcal{P}$. Since $\mathrm{d}P^{m+N+1} = 0$, the condition for $\mathcal{G}_{m+N}$ to be indifferent is

$$\left(1 + w_K^t Q^t\right)\mathrm{d}P^t - D_S^t\left((1 - d^{t-m})F^m \mathrm{d}a^m + \sum_{j=1}^{i-1}(1 - d^j)E_K^{t-j}Q^{t-j}\mathrm{d}P^{t-j}\right) = 0 \tag{157}$$

Defining G_i^m as in the hypothesis, conditions (156) and (157) yield the conclusion.

□

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