The Economics of Competiton Between Individuals in Biological Populations

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Abstract

Describing biological growth where the changes in time and in the size of individuals of a biological population together with the fact that individuals compete for scarce resources are considered give rise to a partial integrodifferential equation. With respect to the determination of the optimal forest management regime this set up leads to a distributed optimal control problem. The paper presents a numerical technique that allows transforming the original distributed control problem into an ordinary control problem. In contrast to the previous literature this procedure does not require to program complex numerical algorithms but can be implemented by standard optimization software. In an empirical study the optimal selective logging regime is compared with the optimal clear cutting regime for the case of the Scots Pine (pinus sylvestris).

Key words: distributed optimal control; forest management; partial differential equations; numerical methods

JEL Classification: C610, C630, Q230

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1 Introduction

The economic literature about the optimal management of renewable natural resources has recognized that the assumption of non-structured populations does not allow to address correctly the two key issues of the management of renewable natural resources - the determination of the optimal replacement periods, and the optimal long-run allocation of the total population among the different values of the structuring variable. Economic analysis based on non structured populations provides the optimal forest rotation of the entire stand in the case of forest management or the optimal catch of total fishes in the case of fishery. However, the optimal replacement period is not calculated correctly since the underlying growth process does not take account of the fact that the entire forest or the entire biomass of the fishes varies respectively with individual characteristics of each tree or each fish. For example, consider the case where the structuring variable is age. Given the same amount of biomass, the rate of growth of a population formed by young individuals is higher than the rate of growth of a population formed by old individuals. Thus, non-structured population models are not able to model the biological growth process correctly and therefore the optimal replacement period cannot be determined correctly either. Obviously, the second key issue of the management of renewable natural resources - the optimal long-run allocation of the total population with respect structuring variable - cannot be resolved by non-structured population models.

Several empirical studies introduced the structuring variable diameter of the tree in order to resolve the problem of optimal management of the forest. Initially, Adams and Ek (1974) determined only the optimal long-run allocation of all trees among the different tree vintages, but not the optimal replacement periods. Haight, Brodie and Adams (1985) solved simultaneously for the optimal long-run allocation of all trees among the different tree vintages and the optimal replacement periods. These two empirical studies were formulated in a discrete framework. i. e. time and diameter can only take on certain values. Later studies for example by Sedjo and Lyon (1990), and Adams, Alig, McCarl, Callaway and Winnet (1996) utilize discrete time and a continuous structuring variable to analyze the problem of forest management. The same modelling framework was used by several theoretical studies that analyzed the properties of the transient path and of the steady state (Mitra and Wan (1985),Mitra and Wan (1986), Wan (1994) and Salo and Tahvonnen (2002)). The authors of the latter study show that the steady state distribution is cyclical. However, as the period length of discrete time goes to zero the cycle vanishes and the steady state distribution is given by a forest where all tree vintages are evenly presented. Such a forest is called a normal forest.

While these theoretical and empirical studies were very helpful to address both key issues of forest management economists have not yet presented and analyzed a theoretical model which is continuous in time and the structuring variable. Looking at reality it seems difficult to justify that the trees can be cut only in certain moments of time and the structuring variable takes on only particular values. Most importantly, however is the fact none of the previous studies take account of the competition between the individuals belonging to the same population. In the case of forest the trees compete for space, light and nutrients. In order to capture this situation biologist simply use the term environment in order to express in general biotic or abiotic factors that influence the life cycle of the individual. For example in predatory prey models the environment for the population of the prey is usually modelled as the population of the predatory. In the absence of a population of a predatory the life cycle of a single individual is mostly influenced by the other individuals of the population. Thus, in order to model biological growth correctly one does not only have to take account of the individual characteristics but also the distribution of the individual characteristics over the entire population. As an example we can refer again to the structuring variable age. Given the same initial amount of biomass, the rate of growth of a population will be the higher the more young individuals form part of this population. In other words the more young individuals form part of a population the less competition the single individual faces, and therefore the growth rate of the entire population increases with the share of young individuals of the entire population.

The distribution of the individual characteristics over the entire population is not only important in order to model correctly biological growth but most importantly in order to take account of the multiple services biological resources offer. For example, forests produce besides timber a large variety of services such that: amenity and recreational values, natural habitat of wildlife, mushrooms, protection of watersheds, carbon sequestration etc. (Rojas, 1996). The production of most of these services does depend to a large extent on the distribution of the individual characteristics over the entire forest and not on the aggregate measure of the trees or the particular value of the individual characteristics. A simple example is the production of wildlife habitat for deers. They need young trees to feed on and old trees to take refuge. Thus, a particular distribution of the individual characteristics over the entire forest is necessary to favor the habitat of deers.

The methodological approach we propose in this paper, however is not only applicable to the case of biological resources but also to other fields of economics where the state variable of a dynamic system is structured. For instance in the case of optimal economic growth or replacement of capital at the firm level this approach allows to incorporate network effects. Usually, recently introduced capital is more productive than capital that has been introduced some time ago. However, the productivity of capital does not only depend on its vintage but also on the distribution of the vintages of the entire capital. Recently introduced capital may be based on a different technical standard than the capital introduced before. Thus, it may well be that the full potential of the new capital cannot be achieved until a sufficient quantity of new capital is acquired which is compatible with itself but not with the preexisting capital. Thereby leading to a positive network effect. Another possibility to increase the potential of new capital is to incur in adjustment costs for instance in form of additional training of the human resources or the acquisition of specific capital to bridge the gap between different technical standards. The migration from relatively new capital to the very late capital generates most likely lower adjustment costs than the migration from fairly obsolete capital to the very late capital, Consequently, adjustment costs depend on the distribution of the vintage of the capital. Most likely, they increase with the share of obsolete capital of the entire stock of capital. This relationship can be considered as a negative network effect. Negative and positive network effects may be presented simultaneously. While the economic literature recognized the importance of taking account of the vintage of capital (Boucekkine, Germain and Licandro (1997), Boucckkine, Germain, Licandro and Magnus (2001), Feichtinger, Hartl, Kort and Veliov (forthcoming 2005), and Feichtinger, Hartl, Kort and Veliov (forthcoming 2004)), it has yet not considered the accompanying network effects.

Although the approach presented in this paper can be applied within very distinct contexts, we restrict ourself to the case of forest management in order to enhance the clarity of the exposition. In particular we present a theoretical model to determine the optimal selective-logging regime of a size-distributed forest. The law of motion of the economic model is governed by a partial integrodifferential equation that describes the evolution of the forest stock over time. Given the complexity of the problem it is not possible to obtain an analytical solution. In contrast to the existing literature where distributed control problems are solved numerically by a gradient projection method (Feichtinger, Prskawetz and Veliow, 2004) we employ a different technique known as the "Escalator Boxcar Train". This technique has the advantage that it does not require to program numerical algorithm but can be implemented with standard optimization packages such as GAMS. The empirical part of the paper determines the optimal selective-logging regime of a forest that consists of *pinus sylvestris* from a private perspective. The analysis allows to compare the optimal selective-logging regime with the optimal clear-cutting regime of the Faustmann model. The results show that the clear-cutting regime leads to lower benefits than the selective-logging regime. This is due to the rigidity of the Faustmann model with respect to logging and planting.

The paper is organized as follows. The following section describes the features of the economic model. As such it is divided into a subsection that describes the underlying biological processes and a subsection that outlines the economic decision problem in form of a distributed optimal control problem. Section 3 derives and presents the employed numerical technique to solve the distributed optimal control problem and section 4 determines and compares the optimal selective and the clear cutting regime of a forest. Finally, section 5 presents the conclusions.

2 The economic model

Before presenting the complete economic model that allows to determine the optimal selective logging regime we characterize the underlying biological model that describes the growth process of the trees.

2.1 The growth process of the trees

In the previous economic literature the age of the tree is regarded as the structuring variable of the biological population (Wan (1994) and Salo and Tahvonnen (2002). However, from an economic point of view it is not the age but size of the tree that is important. The price of lumber changes with the size of the tree but not with age. As established by forest scientists the age of a tree can be only be considered as a poor proxy for its size (Björklund, 1999). A large genetic variety between the trees, and of the different local conditions of each tree makes it difficult to establish a functional relationship between size and age. Consequently, the coefficient of determination (R^2) resulting from an econometric estimation of this relationship is not greater than 0.3.

In forestry, the size of a tree, and consequently the size of a forest, is usually measured by the diameter at breast height, that is, the diameter of the trunk at a height of 1.30 m above the ground. We denote the diameter of a tree by $l \in \Omega$, $\Omega \equiv [l_b, l_m)$, where l_b and l_m indicate the biological minimum and maximum size of a tree. The exogenous variable l together with calender time t form the domain of the control and state variables. We assume that a diameter-distributed forest can be fully characterized by its number of trees and by the distribution of the diameter of the trees. In other words, space and the local environmental conditions of the trees are not accounted for. Given that the value of diameter of a tree lies in the interval $[l_b, l_m)$, and that the number of trees is large by assumption, the distribution of the trees of the forest can be represented by a density function. It is denoted by $x(t, l)$, and indicates the population density with respect to the structuring variable, l, at time t. Therefore, the number of trees in the forest at time t is given by

$$X(t) = \int_{l_b}^{l_m} x(t, l)\, dl. \tag{1}$$

The dynamics of the forest is driven by the following four processes: growth, reproduction, mortality and environmental characteristics. Let define $g(E(l), l)$ the rate of change in the diameter of a tree as a function of its current diameter l where $E(l)$ presents a collection of environmental characteristics, that affect the rate of growth of the individual tree. In the absence of a predator these environmental characteristics are given by the local conditions where the tree is growing, and by the neighboring trees. The local conditions and the competition between individuals for space, light and nutrients affect the life cycle of each individual. Since our model does not consider space, the variable $E(l)$ reflects exclusively the competition between individuals. Environmental characteristics $E(l)$ for a tree with diameter $\hat{l}$, can be captured for example by the total number of trees, or the basal area[1] of all trees with $l \geq \hat{l}$. A large basal

[1] The basal area is the area of the cross section of a tree at a height of 1.30 m above the ground. The basal area is often used to measure and describe the density of trees in the forest, where the sum of the basal area of all trees is expressed per unit area of land (e.g., square meters per hectare).

area of these trees signify a high pressure of competition on the life cycle of an individual tree with diameter $\hat{l}$. Thus, the change in the diameter over time of a single tree is given by

$$\frac{dl}{dt} = g(E(l), l), \tag{2}$$

where the functional relationship between diameter and basal area is employed to determine $E(l)$ which is given by

$$E(l) = \int_l^{l_m} \pi \left(\frac{s}{2}\right)^2 x(t, s) \, d\, s. \tag{3}$$

The instantaneous death rate is denoted by $\delta(E(l), l)$. It describes the rate at which the probability of survival of a tree with diameter l, given the environmental characteristics $E(l)$, decreases with time.

The reproduction of the forest can either be modelled as a biological or as a man made reproduction. In the former case we would obtain a boundary condition for the partial integrodifferential equation that reflects the reproduction process. In the case of man made reproduction we have a completely managed forest in mind, where the young trees with diameter l_b are planted and no biological reproduction takes place. Hence, the control variables for the management of the forest are given by $u_1(t, l)$ and $u_2(t, l_b)$, and denote cutting density at time t with diameter l, and planting density at time t with diameter l_b respectively. Thus, the optimal management forest problem is a distributed optimal control problem where the time dependent control variable $u_1(t, l)$ is distributed over l, and the time dependent boundary control variable $u_2(t, l_b)$ for the initial diameter l_b of the tree (Feichtinger and Hartl, 1986). Based on the well known McKendrick equation for age structured populations (McKendrick, 1926) the dynamics of the diameter distributed forest can be described by the following partial integrodifferential equation discussed by de Roos (1997), or by Metz and Diekmann (1986)

$$\frac{\partial x(t, l)}{\partial t} + \frac{\partial g(E(l), l)\, x(t, l)}{\partial l} = -\delta(E(l), l) x(t, l) - u_1(t, l) \tag{4}$$

subject to the boundary condition $g(E(l_b), l_b) x(t, l_b) = u_2(t, l_b)$.

2.2 The Distributed Optimal Control Problem

We assume that the forest is privately owned and managed over a planning horizon of t_1. Using the definitions given in the preceding section, the formal decision problem of the forest owner can be stated as:

$$\begin{aligned} \max_{u_1(t,l),u_2(t,l_b)} \quad & \int_0^{t_1}\int_{l_b}^{l_m} V_1(x(t,l),u_1(t,l),l)e^{-rt}\,dl\,dt \\ & -\int_0^{t_1} V_2(x(t,l),u_2(t,l_b))e^{-rt}\,dt \\ & +\int_{l_b}^{l_m} V_3(x(t_1,l))e^{-rt_1}\,dl \\ & +\int_0^{t_1} V_4(x(t,l_m))e^{-rt}\,dt, \end{aligned} \tag{P}$$

subject to the constraints

$$\begin{aligned} & \frac{\partial x(t,l)}{\partial t} \equiv f(E(l),t,l) \equiv -\frac{\partial (g(E(l),l)\,x(t,l))}{\partial l} - \delta(E(l),l)x(t,l) - u_1(t,l), \\ & x(t_0,l) = x_0(l), \quad g(E(l_b),l_b)x(t,l_b) = u_2(t,l_b), \quad u_1,u_2 \geq 0 \quad u_1(t,l) \leq x(t,l), \end{aligned}$$

where $E(l)$ is given by equation (3) and r denotes the discount rate. The function $V_1(\cdot)e^{-rt}$ presents the discounted net margin of the timber, i.e. the revenue of the timber sale minus cutting and maintenance costs. The function $V_2(\cdot)e^{-rt}$ captures the discounted cost of planting trees with diameter l_b, the function $V_3(\cdot)e^{-rt_1}$ the value of the standing trees at the final point in time of the planning horizon, and $V_4(\cdot)e^{-rt}$ the value of the standing trees that have not been cut and have reached the maximum diameter l_m. The term $x_0(l)$ denotes the initial diameter distribution of the trees, and the restriction $g(E(l_b),l_b)x(t,l_b) = u_2(t,l_b)$ requires that the planted density coincides with the density of the stock variable with diameter l_b. Finally, it is required that the control variables are nonnegative, and cutting density does not exceed the tree density of the forest.

Using Pontryagins Maximum Principle the current value Hamiltonian is given by

$$\begin{aligned}\mathcal{H} &\equiv \int_{l_b}^{l_m} \big[V_1(\cdot) + \lambda(t,l) f(\cdot)\big] dl - V_2(\cdot) + \lambda_b(t)\big[u_2(t,l_b) - g(E(l_b),l_b)x(t,l_b)\big] \\ &\equiv \int_{l_b}^{l_m} \mathcal{H}_1\, dl + \mathcal{H}_2,\end{aligned}$$

where $\mathcal{H}_1$ stands for $V_1(\cdot)+\lambda(t,l)f(E(l),t,l)$, and $\mathcal{H}_2$ for $-V_2(\cdot)+\lambda_b(t)\big[u_2(t,l_b)-g(E(l_b),l_b)x(t,l_b)\big]$. The variables $\lambda(t,l)$ and $\lambda_b(t)$ denote the costate variable and the Lagrange multiplier respectively. The term $\mathcal{H}_1$ is associated with the distributed part of the optimal control problem, and the term $\mathcal{H}_2$ with the boundary part of the control problem (Feichtinger and Hartl, 1986), and (Muzicant, 1980). That is why $\mathcal{H}_1$ is integrated over the range of l but not $\mathcal{H}_2$. In other words $\mathcal{H}_2$ is similar to a standard optimal control problem (lumped optimal control) since it is valid for all moments of time but only for a single value of l, i.e. l_b (lumped). However, it is not a proper optimal control problem as the constraint $u_2(t,l_b) - g(E(l_b),l_b)x(t,l_b) = 0$ is constant over time. As a result, the first order conditions associated with this part of the problem do not involve a system of canonical differential equations. Taking the constraints of the control variables into account, leads to the Langrangian $\mathcal{L}$ given by

$$\mathcal{L} \equiv \int_{l_b}^{l_m} \mathcal{H}_1\, dl + \mathcal{H}_2 + \mu_1 u_1 + \mu_2 u_2 + \mu_3(x - u_1),$$

where $\mu_i, i = 1, 2, 3$ are the corresponding Langrange multipliers.

The following necessary conditions (Sage, 1968) and (Feichtinger, Tragler and Veliov, 2003) are obtained.

$$\begin{aligned}
\frac{\partial \mathcal{H}_1}{\partial u_1} &\equiv V_{1_{u_1}} - \lambda(t,l) + \mu_1 - \mu_3 = 0, \ \forall t, \ \forall l && (5)\\
\frac{\partial \mathcal{H}_2}{\partial u_2} &\equiv -V_{2_{u_2}} + \lambda_b(t) + \mu_2 = 0, \ \forall t && (6)\\
\frac{\partial \mathcal{H}_2}{\partial \lambda_b} &\equiv g(E(l),l)u_2(t,l_b) - x(t,l_b) = 0, \ \forall t && (7)\\
\frac{\partial \lambda(t,l)}{\partial t} &= (r + \delta(E(l),l))\lambda(t,l) - \frac{\partial\big(g(E(l),l)\lambda(t,l)\big)}{\partial l} && (8)\\
\frac{\partial x(t,l)}{\partial t} &= -\frac{\partial\big(g(E(l),l)\, x(t,l)\big)}{\partial l} - \delta(E(l),l)x(t,l) - u_1(t,l) && (9)
\end{aligned}$$

where $E(l)$ is defined by equation (3). For an interior solution the first necessary condition, equation (5), states that along the optimal path the marginal net margin of the timber sale should equal the shadow price of the forest stock for every t and l. In contrast to lumped optimal control, distributed optimal control requires that this equation holds along the optimal path not only with respect to time, but also with respect to diameter. Thus, the private owner maximizes his/her benefits not only over time but also over diameter at every instant of time. In other words, the private owner practices selective logging. Equation (6) states that the marginal cost of planting trees with diameter l_b should equal at every moment of time the marginal benefits of planting this tree, e.g. the marginal net benefits that accrue from time t to t_1. Hence, in correspondence with the first necessary condition the private owner also practices to some extent selective planting by choosing the time and the number of trees to be planted, however not their diameter. Equation (7) reproduces the constraint associated with $\lambda_b(t)$ and reflects the fact that the diameter of planted trees has to coincide with the stock variable at diameter l_b. Necessary condition (8) shows that the marginal change in the overall net benefits of selective logging due to a decrease in the stock, captured by $-\dfrac{\partial \mathcal{H}_1}{\partial x}$, has to equal the marginal change in the shadow price with respect to time plus the marginal change of the product of the growth rate and the shadow price with respect to diameter. The last necessary condition is just a restatement of the law of motion, and therefore, it will not be discussed here. Finally, since there are no exogenous restrictions on $x(t, l_b)$, $x(t, l_m)$ and $x(t_1, l)$ the following transversality conditions have to be taken into account.

$$\frac{\partial \mathcal{H}_2}{\partial x} - \left.\frac{\partial \mathcal{H}_2}{\partial(\frac{\partial x(t,l)}{\partial l})}\right|_{l=l_b} = 0, \qquad x(t, l_b) \quad \text{free} \tag{10}$$

$$\left.\frac{\partial \mathcal{H}_1}{\partial(\frac{\partial x(t,l)}{\partial l})}\right|_{l=l_m} = 0, \qquad x(t, l_m) \quad \text{free} \tag{11}$$

$$\left.\frac{\partial \mathcal{H}_1}{\partial(\frac{\partial x(t,l)}{\partial t})}\right|_{t=t_1} = 0, \qquad x(t_1, l) \quad \text{free.} \tag{12}$$

Since the term $\partial x(t, l)/\partial l$ enters linearly in the function $f(\cdot)$, the first transversality condition, eqn. (10), requires that $\lambda_b(t) = \lambda(t, l_b)$. In words, in every moment of time the shadow cost for planting trees has to equal the shadow price of the stock at the diameter size of planting. This transversality condition is a result of the link between the distributed and the boundary control by their common stock variable. Transversality condition eqn. (11) states that $\lambda(t, l_m) = 0$. Hence, the shadow price of a tree with the maximum diameter size l_m has to be equal to zero. Finally, transversality condition eqn. (12) yields $\lambda(t_1, l) = 0$, requiring that the shadow price of the trees at the terminal point of time has to be equal to zero.

3 The numerical approach

In practice the necessary conditions, two equations and a system of partial integrodifferential equations, can only be solved analytically under severe restrictions with respect to the specification of the mathematical problem (Muzicant, 1980). Thus, one has to resort to numerical techniques in order to solve the distributed control problem. Available techniques such as the method of finite differences, the method of Galerkin or of finite elements may be appropriate choices (Calvo and Goetz, 2001). However, all of them require the programming of algorithms mostly unknown to economists. Therefore we propose a different method named the Escalator Boxcar Train (de Roos, 1988). In contrast to the other available methods, the Escalator Boxcar Train, EBT, can be implemented with standard computer software utilized for solving mathematical programming problems.

The partial integrodifferential equation 4 describes the time evolution of the population density-function over the domain Ω of the individual structural variable l. Assume for now that E is constant such that the partial integrodifferential equation is now a partial differential equation. Moreover, let Ω be subdivided into n arbitrary and non overlapping domains $\Omega_i(t = 0), i = 1, 2, \cdots, n$ at the initial point of time of the planning horizon and define $\Omega_i(t)$ as

$$\Omega_i(t) = \{l(t, t = 0, l_b) | l_b \in \Omega_i(0)\},$$

i.e. $\Omega_i(t)$ describes the trajectory of the diameter over time. The definition of $\Omega_i(t)$, is such that the domain is transported along the characteristics of the partial differential equation in time.[2] Exploiting the biological interpretation of this equation, the density-function is represented by a set of moments over a collection of n subdomains in Ω, for instance the total number of trees, X(t), and the mean diameter of the trees, L(t) in a particular subdomain $\Omega_i, i = 1, ..., n$. In contrast to other numerical techniques the Escalator Boxcar Train approximates these moments over subdomains of the structuring variable that move along the characteristics and does not approximate the density function at nodal points (de Roos, 1988).[3]

[2] A partial differential equation can be solved analytically by deriving a system of ordinary differential equations. This system consists of the equations of characteristics. Its solution is called the characteristics and coincides with the solution of the partial differential equation. Along the characteristics the changes in the diameter are described by an ordinary differential equation. Thus the characteristics define the biological trajectories of the trees in the time diameter plane.

[3] The presentation of the EBT method follows to a great extent de Roos (1988) and de Roos (1997). In

To describe the evolution of $x(t,l)$, the total number of trees, $X_i(t)$, and the average diameter of the trees, $L_i(t)$ within the subdomain Ω_i are defined as

$$X_i(t) = \int_{\Omega_i} x(t,s)ds, \quad \text{and} \quad L_i(t) = \frac{1}{X_i(t)} \int_{\Omega_i} sx(t,s)ds. \tag{13}$$

To take account of the control variable, let U_{1i} denote the amount of cut trees in the subdomain Ω_i given by

$$U_{1i} = \int_{\Omega_i} u_1(t,l). \tag{14}$$

Within the subdomain or cohort i the population is fully characterized by the total number of trees and their average diameter. The total population is thus a collection of cohorts. Mathematically, the density function $x(t,l)$ is approximated by a set of delta functions of size $X_i(t)$ at diameter $L_i(t)$. For the dynamics of the quantities $X_i(t)$ and $L_i(t)$ the Escalator Boxcar Train method assumes that, except in the case of death, cohorts of individuals stay together indefinitely and do not switch from one cohort to another. The change in time of these quantities is approximately given by[4]

$$\frac{dX_i(t)}{dt} = -\delta(E,L_i)X_i(t) - U_{1i}(t) \quad \text{and} \quad \frac{dL_i(t)}{dt} = g(E,L_i). \tag{15}$$

Equation 15 describes the dynamics of the cohorts that are already present in the population but it does not account for the plantation of new trees. Since the value of the lower interval bound of subdomain Ω_1, denoted by l_1 changes over time, all newly planted trees have a length widening interval $[l_b, l_1]$. This cohort is the boundary cohort characterized by

$$X_0(t) = \int_{l_b}^{l_1} x(t,l)\ dl.$$

Since the number of individuals within the boundary cohort, is initially zero, the average diameter of the boundary cohort is according to equation 13 not defined. Therefore, a slightly

contrast to these references the partial differential equation presented in this paper contains additionally two control variables. Thus, we derive the appropriate set of ordinary differential equations for this case.

[4] The ordinary differential equations for $X_i(t)$ and $L_i(t)$ do not form a solvable system because they involve weighted integrals over the density function $x(t,l)$. To obtain a closed solvable system as presented in equation 15 the functions $\delta(E,l)$ and $g(E,l)$ are approximated by their first order Taylor expansion around $l = L_i(t)$.

different quantity is employed to measure the average diameter of the boundary cohort given by

$$\hat{L}_0(t) = \int_{l_b}^{l_1} (l - l_b) x(t,l) \; dl.$$

Differentiation of $X_0(t)$ and $\hat{L}_0(t)$ with respect to time, and employing first order Taylor approximations of the functions $g(E,l)$ and $\delta(E,l)$ leads to a set of ordinary differential equations. These equations describe the dynamics of the boundary cohort and are given by

$$\begin{aligned} \frac{dX_0}{dt} &= -\delta(E,l_b)X_0(t) - \frac{d}{dl}\delta(E,l_b)X_0(t) + U_2(t) \\ \frac{d\hat{L}_0}{dt} &= g(E,l_b)X_0(t) + \frac{d}{dl}g(E,l_b)\hat{L}_0(t) - \delta(E,l_b)\hat{L}_0(t), \end{aligned} \tag{16}$$

where $u_2(t,l_b)$ is now written as $U_2(t)$ in order to unify notation. Moreover, the resulting term $g(E,l_b)x(t,l_b)$ in the derivation of $\frac{dX_0}{dt}$ is replaced by U_2 according to the boundary condition of equation 4. The boundary cohort cannot be continued infinitely, because the range would become larger and larger and the approximation would break down. Therefore, the cohorts have to be renumbered at regular time intervals $\triangle t$. this renumbering operation transform the current boundary cohort into an internal cohort and initializes a new, empty boundary cohort realizing the following operations

$$\begin{aligned} X_i(k\triangle t) &= X_{i-1}(k\triangle t) \\ L_i(k\triangle t) &= L_{i-1}(k\triangle t) \\ X_1(k\triangle t) &= X_0(k\triangle t) \\ L_1(k\triangle t) &= l_b + \frac{\hat{L}_0(k\triangle t)}{X_0(k\triangle t)} \\ X_0(k\triangle t) &= 0 \\ \hat{L}_0(k\triangle t) &= 0. \end{aligned} \qquad k = 1, 2, 3 \cdots \tag{17}$$

The dynamics described by the equation 15 and 16 for the n internal cohorts and the single boundary cohort approximate the dynamics described by the partial differential equation and its boundary condition 4 for a constant environment E. These ordinary differential equations together with equation 17 form part of the formulation of the mathematical optimization problem (P) such that it can be solved numerically.[5] However, this would require programming skills

[5]Integration methods to numerically solve systems of ordinary differential equations such as the Runge-Kutta

and knowledge of numerical techniques. Alternatively, the system of differential equations may be written as a system of difference equations that can be employed in standard software such as GAMS in order to solve mathematical optimization problems. In this way the numerical solution of distributed optimal control problems is not limited to a small number of economists with advanced programming skills and profound knowledge of numerical techniques.

For the derivation of the EBT method we assumed that the environment is constant. However, this is a simplification which does not take into account that the individual trees compete for space, light and nutrients among each other. To consider this aspect we revoke the validity of equation 3 such that the dynamics are now given by the partial integrodifferenetial equation and the boundary condition 4. As de Roos (1988) shows the EBT method is also applicable in this case. Therefore, equation (3) is approximated yielding

$$E(L_i) = \sum_i^n \pi \Big(\frac{L_i}{2}\Big)^2 X_i \tag{18}$$

Thus, in the case of competition between individuals the decision problem is now given by:

$$\begin{aligned} \max_{\bar{U}_1(t), U_2(t)} \quad & \int_0^{t_1} V_1(\bar{X}(t), \bar{U}_1(t), \bar{L}) e^{-rt}\, dt \\ & - \int_0^{t_1} V_2(\bar{X}(t), U_2(t)) e^{-rt}\, dt \\ & + \quad V_3(\bar{X}(t_1)) e^{-rt_1} \\ & + \int_0^{t_1} V_4(X_n(t)) e^{-rt}\, dt, \end{aligned} \tag{P'}$$

subject to the constraints

$$\begin{aligned} \frac{dX_i(t)}{dt} &= -\delta(E(L_i), L_i) X_i(t) - U_{1i}(t) \quad \text{and} \quad \frac{dL_i(t)}{dt} = g(E(L_i), L_i) \\ \frac{dX_0}{dt} &= -\delta(E(L_i), l_b) X_0(t) - \frac{d}{dl}\delta(E(L_i), l_b) X_0(t) + U_2(t) \\ \frac{d\hat{L}_0}{dt} &= g(E(L_i), l_b) X_0(t) + \frac{d}{dl} g(E(L_i), l_b) \hat{L}_0(t) - \delta(E(L_i), l_b) \hat{L}_0(t) \\ X_i(0) &= \bar{X}^0, \quad g(E(l_b); l_b) x(t, l_b) = U_2(t), \quad U_{1i}, U_2 \geq 0 \quad U_{1i}(t) \leq X_i(t), \end{aligned}$$

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method may be employed in the formulation of the problem.

where $\bar{X}, \bar{U}_1$ and $\bar{L}$ denote the vectors $\bar{X} = (X_1, \cdots, X_n), \bar{U}_1 = (U_{1i}, \cdots, U_{1n}), \bar{L} = (L_1, \cdots, L_n)$ respectively, $\bar{X}_0$ the initial density of each cohort, and $E(L_i)$ is given by equation 18. Like before the utilization of the EBT method requires to incorporate the equation 17 for the numerical formulation of the mathematical optimization problem to take account of the required renumbering operations.

4 Empirical study

The purpose of the empirical analysis is to illustrate the applicability of the EBT method and to determine the optimal selective-logging regime of a diameter-distributed forest, that is the selective logging regime that maximizes the discounted private net benefits from timber production of a stand of *pinus sylvestris* (Scots pine), over time horizon of 300 years. The specie *pinus sylvestris* has been chosen since it occupies most of the catalan forest. Thereafter the optimal selective cutting regime is compared with the optimal clear-cutting regime based on the Faustmann formula.

4.1 Data and Specification of Functions

For given specifications of the economic and biophysical functions of the model, and a given initial diameter distribution of the trees, $\bar{X}_0$, a numerical solution of the decision problem (P') can be obtained. To proceed with the empirical study, different initial diameter distributions of a forest have been chosen, specified on the base of a transformed beta density function $\theta(l)$ with shape parameters γ and ϕ (Mendenhall, Wackerly and Scheaffer, 1990). The initial forest consists of a population of trees which diameter lies within the interval 5 cm $\leq l \leq$ 30 cm. The distribution of the diameter of the trees is given by:

$$\theta(l;\,\gamma,\,\phi) = \begin{cases} \dfrac{1}{25}\dfrac{\Gamma(\gamma+\phi)}{\Gamma(\gamma)\Gamma(\phi)}\Big(\dfrac{l}{25}\Big)^{\gamma-1}\Big(1-\dfrac{l}{25}\Big)^{\phi-1}, & \gamma, \phi > 0;\; 5 \leq l \leq 30, \\ 0, & \text{elsewhere}, \end{cases} \tag{19}$$

where $\theta(l;\,\gamma,\,\phi)$ denotes the density function of the diameter of trees. Thus, the integral $\int_{l_i}^{l_{i+1}} \theta(l;\gamma,\,\phi)\,dl$ gives the proportion of trees lying within the range $[l_i, l_{i+1})$. The beta density function is utilized because it is defined over a closed interval and allows to define a great variety of distinct shapes of the initial distributions of the diameter of the trees. The interval [5, 30] is divided into 10 initial subintervals of identical length. In this way, each cohort comprises trees that differ in their diameter by a maximum of 2.5 cm, and thus, they can be considered as homogeneous. The initial number of trees in each cohort, $X_i(0)$, $i = 1, \cdots, n$, is calculated in such a way that the basal area of the stand is constant (25 m^2/ha) in order to allow for comparisons between the results of the different optimization outcomes. Figure 2, cases a-d) of the appendix presents four different initial distributions obtained by varying the parameters γ and ϕ of the beta density function.

The function $V_1(\bar{X}(t), \bar{U}_1(t),)$ accounts for the net margin of the timber at time t, and is defined as: $[\sum_{i=0}^{n}(p(L_i) - C_0)\, VT(L_i)\, VM(L_i)\, U_{1i}(t) - C_1] - [C_2(X_i(t))]$. The first term in square brackets denotes the sum of the revenue of the timber sale minus the cutting costs of each cohort i, and second term, $C_2(X(t))$, accounts for the maintenance costs. The parameter $p(L_i)$ denotes timber price per cubic meter of wood as a function of the diameter, $VT(L_i)$ the total volume of a tree as a function of its diameter, $VM(L_i)$ the part of the total volume of the tree that is marketable, C_0 variable cutting costs and C_1 fixed cutting costs.

Data about costs and prices was provided by the consulting firm Tecnosylva which elaborates forest management plans throughout Spain. The data shows that timber price per cubic meter is an increasing function of the diameter of the tree, but its second derivative is negative. Thus, a quadratic price function was estimated, given by $p(L) = -23.02 + 4.35L - 0.049L^2$. The cost C_0 comprises logging, pruning, and the collection and removal of the residues. It is given by E23.4 per cubic meter of timber, and C_1 by E3.6 per hectare. Maintenance costs are an increasing function of the number of stems per hectare, and are given by $C_2(X) = 0.07X + 1.18\,10^{-5}X^2$. The planting costs are linear in the amount of planted trees and are given by $V_2(U_2) = E0.60U_2$.

The value of the parameters of tree volume, $VT(L_i)$, and the marketable part of the tree volume, $VM(L_i)$, are estimated using information provided by a study from Cañellas, Martinez García and Montero (2000). The tree volume follows the allometric relation $VT(L) = 0.0002949L^{2.167563}$, and the marketable part of the volume of timber of each tree is an increasing function of the diameter, given by $VM(L) = 0.699 + 0.000411L$. The thinning and planting period, $\triangle t$, is set equal to 10 years, which is a common practice for a *pinus sylvestris* forest (Cañellas et al., 2000).

To determine the dynamics of the forest, the growth of a diameter-distributed stand of *pinus sylvestris* without thinning was simulated with the bio-physical simulation model GOTILWA (Growth Of Trees Is Limited by Water).[6] The model simulates growth and mortality and allows to explore how the life cycle of an individual tree is influenced by the climate, characteristics of the tree itself and environmental conditions given by the total basal area of the trees with a greater diameter than the individual tree. The model is defined by 11 input files specifying more than 90 parameters related to site, soil composition, tree species, photosynthesis, stomatal conductance, composition of the forest, canopy hydrology, and to climate. To generate a wide variety of possible initial distributions, 103 pairs of γ and ϕ were used, where the values of γ and ϕ are taken from the set M, M={ 0, 0.2, 0.4, 0.6, 0.8, 1, 2, 4, 6, 8, 10}.[7] Fourteen of the 103 simulations were rejected because the resulting density of the stand (stems per ha) was biologically not viable.

The results of the simulation were utilized to estimate the function $g(E, L_i)$, which describes the rate of change of the diameter. It was specified as a von Bertalanffy growth curve (von Bertalanffy, 1957), generalized by Millar and Myers (1990) which allows the rate of growth of the diameter to vary with environmental conditions. Thus, the function $g(E, L_i) = (l_m - L_i)(\beta_0 - \beta_1 \, BA_i)$ is estimated by OLS, where β_0 and β_1 are proportionality constants, and BA_i is the sum of the basal area of all trees with a diameter larger than L_i. The estimation yielded the growth function: $g(E, L_i) = (80 - L_i)(0.0068993 - 0.00003107 \, BA_i)$. Other functional forms of $g(E, L_i)$ were evaluated as well, but explained the observed variables to a lesser degree.

As GOTILWA only allows to simulate the survival or death of an entire cohort but not of an individual tree, it was not possible to obtain an adequate estimation of the function $\delta(E, L_i)$ describing the mortality of the forest. Nevertheless, the information provided by Tecnosylva suggests that in a managed forest, the mortality rate can be considered almost constant over time and independent of the diameter. Thus, according to the data supplied by Tecnosylva, $\delta(E, L_i)$ was chosen to be constant over time and equal to 0.01 for each cohort.

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[6]This program has been developed by C. Gracia and S. Sabaté, University of Barcelona, Department of Ecology and CREAF (Centre de Recerca Ecològica i Aplicacions Forestals), Autonomous University of Barcelona respectively.

[7]The set M allows to generate 121 possible pairs of γ and ϕ, but some of them give rise to equivalent initial distributions.

4.2 Optimization Results

The mathematical optimization problem (P') was programmed in GAMS (General Algebraic Modelling System) (Brooke, Kendrick and Meeraus, 1992). For the numerical solution of this problem the Conopt2 solver, available within GAMS, was employed. For a given initial distribution, the numerical solution of the problem determines for every 10 year period the optimal logging, U_{1i}, and planting, U_2, the optimal values of the state variables, X_i and L_i, and consequently economic variables, such as the revenue from timber sale, cutting costs, planting costs, and maintenance costs. Optimizations with different random initializations of the control variables were carried to assure that the numerical method provides solutions that are independent of the initially chosen values for the numerical optimization technique. All optimizations were carried out on a per-hectare basis.

Selective Logging Regime

Given the initial diameter distribution of the trees in Figure 2, case a) of the appendix, Table 1 summarizes the results of the optimization where a discount rate of 2% was assumed. It is shown that the first logging is delayed until the end of the first 10 year period. Consequently, the forest owner has to wait 10 years to obtain the first benefits from the forest. During this period the current value of the total maintenance cost per hectare sum up to E 78.82. The optimal forest management requires to plant a small amount of trees at the initial time periods of the time horizon.[8] It can be observed that all economic and biophysical variables show a cyclical pattern over time. While the phase of this cyclical pattern is maintained over time, the amplitude decreases. In the long-run, the forest consists of approximately 955 trees, and approximately 110 of these trees are logged each 10 year period. The volume of the logged trees is 64 m^3, of which 46 m^3 is marketable timber. The current value revenue from the sale of this amount of timber minus the logging cost is approximately E2000 per hectare. The current value net benefits of the forest in the long-run are nearly E 1900. The total sum of discounted net benefits of the forest over 300 years are E8654 per hectare.

Figure 1 a-f), depicts the number of trees in each cohort at different years of the planning horizon together with their corresponding average diameter denoted below the bar, to illustrate the optimal evolution of the forest.[9] Each bar present the number of trees in one cohort. The

[8]The process of plantation starts after the end of the first 10 year period.

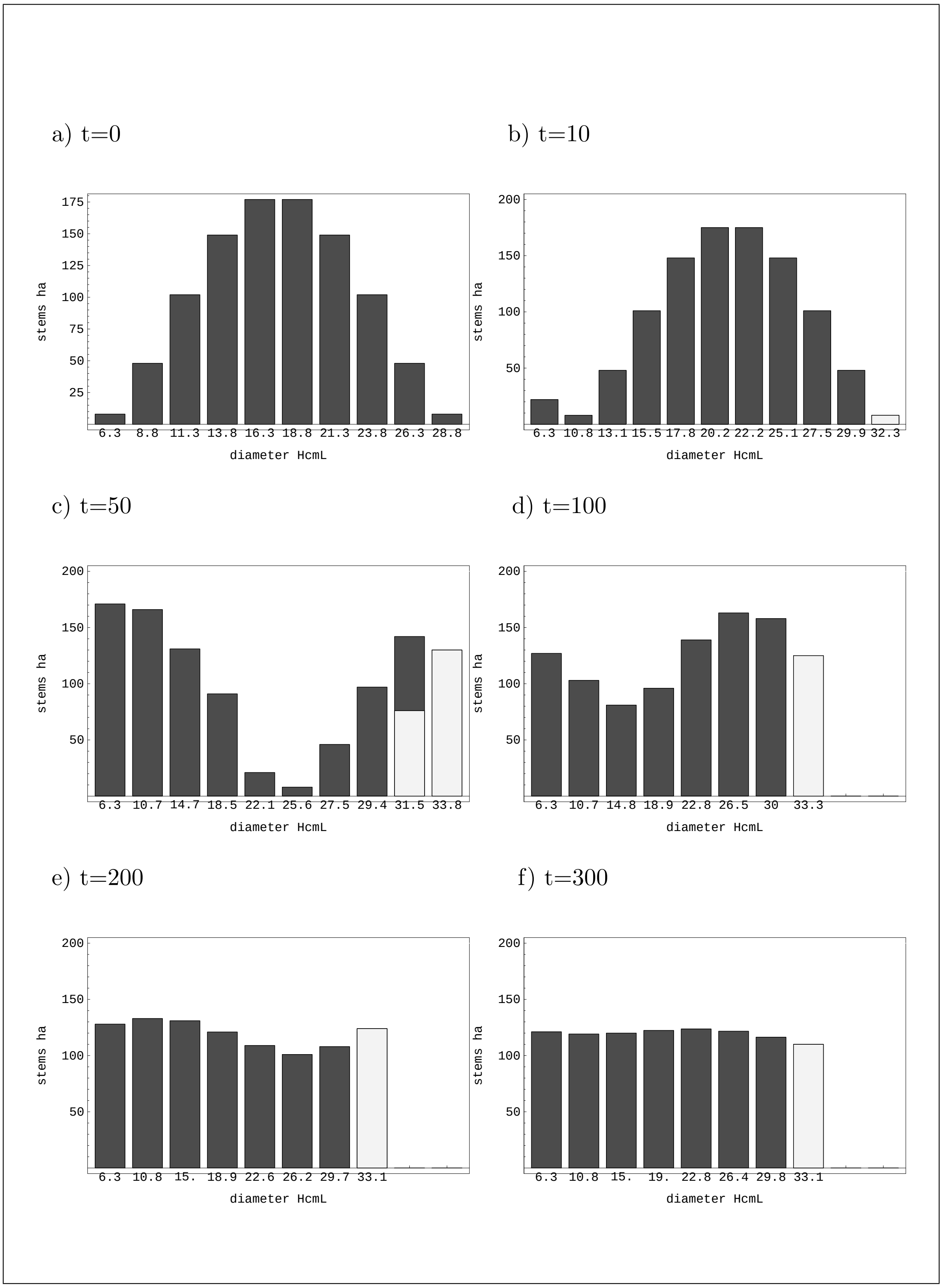
[9]The simulated data does not show a significant difference between the simple and weighted average of the

Table 1: Optimal Selective-Logging Regime Given an Initial Diameter Distribution Determined by $\gamma = \phi = 3$

Year	Number of trees[a]	Planted trees	Logging					Maintenance costs (E/ha)	Planting costs (E/ha)	Net benefit (E/ha)	Discounted net benefit (E/ha)
			Logged trees	BA (m^2/ha)	Volume (m^3/ha)	Timber (m^3/ha)	Revenue - logging costs (E/ha)[b]				
0	968	8	0	0	0.00	0.00	0.00	78.82	0.00	-78.82	-78.82
10	980	22	8	0.65	4.36	3.10	129.60	79.94	6.41	43.25	35.48
20	1057	94	147	11.63	77.95	55.51	2344.11	87.13	26.15	2230.83	1501.28
30	1034	134	145	11.61	77.88	55.46	2357.25	85.00	37.01	2235.24	1234.01
40	1049	168	208	17.22	115.85	82.53	3557.13	86.39	46.58	3424.16	1550.77
50	1003	171	207	17.67	119.20	84.94	3710.54	82.13	49.10	3579.31	1329.81
60	933	145	160	14.20	96.11	68.51	3038.15	75.62	43.21	2919.32	889.76
70	865	99	52	4.74	32.12	22.90	1023.21	69.33	30.34	923.54	230.91
80	886	82	20	1.73	11.66	8.31	359.22	71.32	24.66	263.24	53.99
90	961	104	88	7.58	51.15	36.45	1598.45	78.18	29.92	1490.36	250.77
100	992	127	125	10.84	73.20	52.17	2296.08	81.03	36.10	2178.96	300.77
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
200	954	128	124	10.69	72.13	51.40	2251.02	77.55	37.66	2135.80	40.69
300	955	121	110	9.51	64.21	45.75	2005.13	77.65	35.39	1892.10	4.98

[a] The number of trees in the forest is calculated just after the trees are planted, and before the thinning takes place.
[b] All monetary values apart from the discounted net benefit in the last column of the table are expressed as current values.

Figure 1: Evolution of the Optimal Diameter Distribution with $r = 0.02$



slightly shaded bars indicate the number of trees that should be logged within each 10 year period, while the heavily shaded bars stand for the number of trees that should remain in the stand. Figure 1 shows that it takes more than 100 years to reach a diameter distribution of the trees which is relatively stable over time. Further numerical analysis beyond the 4 initial diameter distribution of the trees presented in Figure 2 of the appendix showed that the obtained uniform long-run distribution of the diameter of the forest is optimal independent of the initial distribution.[10] Hence, our results confirm the supremacy of the normal forest as the optimal forest management objective.

Clear Cutting Versus Selective Logging Regime

Alternatively to adopting a selective-logging regime, the forest owner could continue with the clear-cutting regime, that is, logging the entire stand at regular time periods. In this case, the optimal rotation period can be calculated directly via the Faustmann formula. However, in order to allow for a comparison of the different management regimes, the Faustmann model needs to be specified such that both regimes are comparable. Contrary to the original Faustmann model it is necessary to assume that no trees are planted at time 0, planted trees have a diameter of 6.25 cm, i.e. trees are approximately 15 years old, and the initial diameter distribution of the trees is valid for both regimes, i.e., 1138 trees, with a diameter of 6.25 cm. The Faustmann model maximizes the present value of the perpetual net benefits from the forest. The optimal clear cutting regime is given by the solution of the following optimization problem:

$$\max_{T} \quad \frac{F(T)}{1 - e^{(-r\,T)}}, \tag{FP}$$

where $F(T)$ denotes the discounted net benefits obtained from cutting the entire stand at time T, i.e., the rotation period. In this model, $F(T)$ accounts for the revenue obtained from the timber sale minus logging, maintenance[11] and planting costs, that is:

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diameter of each cohort. Thus, the diameter of each cohort was initialized with the simple average in order to simplify the process of the numerical solution.

[10] In order to keep the length of the paper short we opted for not presenting these results. However, they can be obtained from the authors upon request.

[11] Forest owners following a selective logging regime incur in maintenance costs every 10 year period. In order to account for these costs adequately in the Faustmann model, the maintenance costs cannot be added simply at time T since they incur every 10 years. Thus, the correct maintenance costs are calculated as the sum of discounted periodic payments of the maintenance costs and they are added as a single payment to the Faustmann model.

$$F(T) = \Big((p(L(T)) - C_0)\,VT(L(T))\,VM(L(T))\,X \; - C_1\Big)e^{-r\,T}$$
$$- \frac{C_2(X)e^{-rT} - C_2(X)}{e^{-10r} - 1} - V_2 X\, e^{-r\,T}$$

The resulting optimal rotation length is 62 years. The total discounted net benefits obtained from a clear-cutting regime are about E 1800/ha, and about E 2941/ha from the selective-logging regime. Therefore, using the same parameter values for the clear-cutting and the selective-logging regime, the results show that discounted net benefits of the clear-cutting regime are approximateley E1141 lower than those of the selective-logging regime. That is, the rigidity of clear felling, given by the requirement of logging the entire stand instead of logging a part of the stand at different time periods, together with the unfeasibility of choosing the number of planted trees, causes a loss of the clear-cutting regime of approximately 38% compared to the selective-logging regime.

The benefits of clear felling could only be superior to selective logging if timber prices increase with the amount of timber offered. A clear-cutting regime allows to offer a large amount of timber planks of a particular size. Consequently, the obtained timber prices per m^3 may be higher than with selective logging. While this is true for the case of a small forest area, it may not be true for the case of a large forest area, where the volume obtained from selective logging may be sufficiently large to achieve high timber prices per m^3.

5 Conclusions

This paper presents a theoretical model that allows determining the optimal management of a diameter-distributed forest where the growth process of the trees depend not only on individual characteristics but also on environmental characteristics by considering the distribution of the individual characteristics over the entire population. This modelling allows to account of the fact that the life cycle of each individual tree is affected by the other trees since they compete for light, nutrients and space. The density dependent formulation of the biological growth process leads to a partial integrodifferential equation. The corresponding economic decision problem to determine the optimal management of the forest can be formulated as a distributed optimal control problem where the control variables and the state variable depend on the arguments time and diameter of the tree.

The resulting necessary conditions of this problem include a system of partial integrodifferential equation that usually cannot be solved analytically. For this reason, the utilization of a numerical method (Escalator Boxcar Train) is proposed. The Escalator Boxcar Train method allows to transform the partial integrodifferential equation into a set of ordinary differential equation and thereby to approximate the distributed optimal control problem by a standard optimal control problem. In contrast to the existing literature the resulting optimization problem can be solved numerically utilizing standard mathematical programming techniques and does not require the programming of complex numerical algorithms.

To determine the optimal selective logging regime of a diameter-distributed and privately owned forest where individual trees compete for scarce resources, an empirical analysis is conducted. The empirical analysis shows that the clear-cutting regime, given by the Faustmann solution, leads to lower private benefits than the selective-logging regime. This is due to the fact that the selective logging regime permits the possibility of logging part of the forest after 40 years, while the optimal clear-cutting requires that the forest owner waits until the 62^{st} year before the entire stand is cut. As a result, the clear-cutting regime leads to a loss of approximately 38% of the private benefits of the selective-logging regime.

The presented approach, however is not only helpful for determining the optimal management of natural renewable resources but also for taking optimal economic decisions in fields where network effects are present and the state variable of the dynamic system is structured. Thus, we think that this approach will not only be useful in the field of natural resource management but also in research areas such a optimal economic growth, capital replacement or technology adoption.

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Figure 2: Types of Initial Distributions of the Diameter of the Trees