# Measurement of the benefits of environmental 

## education

Tomoki FUJII *

September 2, 2003

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## 1 Background and Organization

In this paper, we present a way to quantify the benefits of environmental education. Education has been often treated in economics as a form of human capital investment that increases the productivity of individuals. However, this approach is insufficient when environmental education is at issue. The reason is simple. Environmental education is defined as the process of recognizing values and clarifying concepts in order to develop skills and attitudes necessary to understand and appreciate the interrelatedness of humans, their culture and their biophysical surroundings.(Dooms, 1995) This definition does not imply increase in productivity.

One of the major roles of environmental education is to educate people to think more on the interaction between the human society and the environment, and its long-term consequences. It would be easy to say that we should invest more in environmental education programs to educate people to think more "environmentally". However, education does not come for free. There are other socioeconomic problems to be addressed in the society that would come in conflict with environmental education programs for resources. Hence, measurement of the benefits of environmental education programs is critical to make the allocation of scarce resources in a more efficient and transparent manner.

The strategy to measure the benefits of education in this paper is as follows. The trade-off that arise between present behavior and long-term consequences is usually formulated in economics as a problem of discounting. We also follow this approach, but we depart from the standard assumption of
rationality. Based on the psychological evidence ${ }^{1}$, we assume individuals with a self-control problem, where individuals have present-biased preference and behave in a time-inconsistent manner. More precise meaning of a self-control problem will be given in Section 7.

Two types of the benefits of environmental education can be identified in this setting. One type of benefit, which we call sophistication, is to let the individuals recognize their self-control problem and educate them to take it into account when they make decisions. For example, Skanavis (1999) emphasizes the importance of environmental education to raise the awareness of the groundwater pollution the people of Puerto Rico are facing. The other type, which we call rationalization, is to make them use such a discount rate that would give them higher utility when they make choices. We shall develop a theoretical framework to evaluate these two types of benefits of environmental education.

Admittedly, there are a number of other aspects of environmental education that would not be captured in this framework. For example, in the framework we present, preference of individuals is taken as given. But, "slough-off from materialism", which must entail drastic change in the preference of individuals, is sometimes identified as one of the major objectives of environmental education. Though changing the human values to the "right" direction would be important, what the "right" direction means and whether education can play any role would be controversial. Another important ex-

[^1]ample is the informational aspect of environmental education. It is often the case that the polluter has better access to information and ability to process the information, whether the information is private or public. Knowledge and experience acquired through environmental education would help narrow the informational gap.

The types of benefits of environmental education we deal with in this thesis are especially appropriate in a developing country context. People in developing countries, especially poor people, often have very high rate of time preference (RTP). Pender (1996), for example, could not reject the hypothesis that poverty reduction lowers RTP. Hence, we may suspect that people in poor areas tend to have high RTP, which in turn causes rapid depletion of natural resources and the environment.

Holden et al. (1998) measured the RTPs of rural household in Indonesia, Zambia and Ethiopia using hypothetical questions about preferences for current versus future consumption. They found that the rates of time preference were generally high in those countries, and the rates were higher in the case studies in Indonesia and Zambia, where the average levels of income were lower, than in the case study in Ethiopia. The rates varied systematically in each case-study area. Poorer households and/or households with severe immediate cash needs had higher RTPs. They argue that the high average RTPs indicate complementary policies may be needed to ensure sufficient levels of investment in the conservation of natural resources and the environment.

Since the argument provided in this paper is not standard and there are a very limited application of behavioral economic approach to environment and
development, we included a rather lengthy review on the literature to motivate our main findings in Sections 7 to 9. Hence, Sections 2 to 6 are devoted to literature review. In what follows, we start from the standard setting in economics. We go over the existing literature on discounting and its problems in Section 2. In Section 3, we overview the standard arguments on the choice of the discount rate in the exponential discounting setting. Then, we shall present several alternatives to the exponential discounting, which include both non-discounting inter-temporal comparison and time-variant discounting in Section 4. We shall in particular discuss the implications of hyperbolic and quasi-hyperbolic discounting that have recently gained popularity among behavioral economists. In Section 5, we summarize the main findings of the few existing applications of hyperbolic and quasi-hyperbolic discounting to the environmental issues. Based on the literature review, Section 6 further motivates the analysis carried out in the subsequent sections.

Section 7 presents the core results of this paper. It develops the measurement of the benefits of environmental education. Section 7 is followed by two examples in Section 8 and 9. Section 8 carries out rigorous analysis in the simplest setting of non-renewable resource extraction. We analyze both the fixed end problem and the free end problem. In Section 9, we present the results of one-time timber harvest problem. In these examples, we elucidate the interesting trait of sophistication. Finally, Section 10 explores the implications and provides conclusion.

## 2 Discounting and Its Problems

When we make choices involving consequences at different points in time, we consciously or unconsciously compare the benefits and costs. Suppose a consumer has a cake that lasts for two days. Her consumption decision for today determines her consumption tomorrow. If she consumes more now, then she has to consume less in the future. In other words, she is making a decision that involves an inter-temporal trade-off. Such an inter-temporal trade-off is central to the issue of environmental problems as it takes a long time until the adverse effects become apparent.

It is a standard practice in economics to discount future costs and benefits when inter-temporal trade-off is at issue. There are at least three reasons to do this (Clayton and Radcliffe, 1996). Firstly, people are impatient. So, they weigh more the present consumption. Secondly, assuming declining marginal utility and exogenously growing wealth, consequentialist equality is achieved by discounting. Thirdly, since the future benefit is uncertain, the benefit in the present value should be lower than the expected future benefit because there is a risk. We shall discuss this in a more precise terms in the next section.

It makes perfect sense for an individual, or an individual business, to discount. If the consumer in the cake-eating example above has a logarithmic utility function and she has a discount factor of $\delta \in(0,1)$, she will consume $u_{0}=1 /(1+\delta)$ today and $u_{1}=\delta /(1+\delta)\left(<u_{0}\right)$. Hence, if she is very impatient so that $\delta$ is close to zero, she is consuming almost zero in tomorrow. But this is as a consequence of her rational decision. Thus, if no one else is
paternalistic enough to care about her consumption of cake tomorrow, there is nothing wrong with almost zero consumption tomorrow.

Now, let us replace the consumption today and tomorrow by the consumption by the present generation and the next generation. Let us assume the present generation does not overlap with the next generation. Suppose also that the present generation consider the next generation as if it were its future self. Then the present generation makes the decision based on its present-discounted utility. But in this case, it would not be ethically justifiable to give almost zero consumption just because the present generation is impatient. This illustrates the problem inherent in discounting that involves different generations.

This moral intuition was already clear in the seminal work on the intertemporal allocation of resources by Ramsey (1928). He states, "it is assumed that we do not discount later enjoyments in comparison with earlier ones, a practice which is ethically indefensible and arises merely from the weakness of the imagination." Harrod (1948) remarked that discounting is "a polite expression for rapacity and conquest of reason by passion."

Many writers seem to argue that the only ethically defensible discount rate for projects having effects spread over several generations is zero (Perman et al., 1996). Weitzman $(1998,1999)$ gives additional reason for using a low, if not zero, discount rate. He argues with a generic model that the interest rate for discounting among events within the far distant future should be its lowest possible limiting value. The key insight is that what should be averaged over states of the world is not discount rates at various times, but discount factors. In the limit, the properly-averaged certainty-equivalent
discount factor corresponds to the minimum discount rate.
However, a low or zero discount rate may cause problems in other dimensions. Pearce et al. (1990) warn that a low discount rate can also lead to further environmental damage through increased investment and faster economic growth. The reason is that, if the interest rate set by the authority is related to the low social discount rate, it also means that society as a whole discounts further consumption at that rate.

Moreover, using zero discount rate has a consequence that seems to be morally unacceptable. Arrow (1999) points out, in line with the argument in Koopmans' classic papers (Koopmans, 1960, 1965), that, since small decrease in present consumption to invest for the future cause only finite loss of utility for the present generation but results in the infinite sequence of positive gain to an infinity of generations, any sacrifice by the first generation is good. Hence he argues that, even though the strong ethical requirement that all generations be treated alike, itself reasonable, contradicts a very strong intuition that it is not morally acceptable to demand excessively high savings rates of any one generation, or even of every generation.

Discounting has also been criticized from the perspective of psychological reality. Loewenstein (1992) says, "Discounted Utility model is not an explanatory theory; it cannot explain why objects lose or gain in value when delayed. It is simply a way of summarizing and encoding inter-temporal preference. But as a method of encoding preferences, it is also deficient. Its behavioral implications are contradicted by empirical research and common experience."

Another line of criticism against discounting comes from the viewpoint
of intergenerational democracy. For example, discounting will not reject a project that has a short-run benefit but also indefinitely prolonged cost. However, the majority of an intergenerational constituency would oppose the project in the absence of compensation (Page, 1977). Dobbs (1982) objects that an hypothetical majority of unborn voters would not alter the government's decision when it is not prepared to compensate the future generations. This brings us back to the question of whether intergenerational compensation is credible; it is the politics of present generation that makes the decision. Discounting may be based on the assumption of (potential) intergenerational compensation, but there is no guarantee that such compensation is actually carried out.

Despite these problems inherent in discounting and concerned with the choice of discount rate, and despite the fact that they are well-recognized by both academics and practitioners, discounting has been widely used in cost-benefit analysis. Lines (1995) rather bluntly says, "the fact is that, in practice, future well-being is discounted. The issue becomes: which discount rate should be applied?" Implicit in this argument is that the discount rate $r$ is assumed to be constant over the time and thus the present-equivalent payoff of $p(t)$ realized at time $t$ is $p(t) /(1+r)^{t}$ in the discrete time setting and $p(t) e^{-r t}$ in the continuous time setting. The discounting scheme like this is called the exponential discounting.

## 3 Choice of Discount Rate

The standard argument about how to set the discount rate starts from the concept of the rate of pure time preference $\rho$, which is a reflection of the impatience of consumers. The impatience of consumers may be because of a belief that the circumstances of the present are more favorable for consumption than those of the future. Or, it may be a function of the brevity of human life. But even when these factors are allows for, there often remains a pure time preference, that is, a preference due solely to earlier position in time, without reference to circumstances (Price, 1993). In other words, pure time preference is equivalent to pure impatience: being a grasshopper instead of an ant (Cline, 1999).

As we have argued above, discounting on the basis of impatience in an intergenerational context is usually deemed to be inappropriate. Even if $\rho=0$ is accepted, however, there is another basis for discounting as we expect people to be better off in the future. If they are, then the marginal utility of consumption in the future is unlikely to be as much as today's marginal utility of consumption. This leaves us the equation for the social rate of time preference (SRTP) $S R T P=\rho+\theta g$, where $\theta$ and $g$ are the elasticity of marginal utility and the growth rate of per capita consumption respectively. ${ }^{2}$ Opponents of the SRTP approach argue that the opportunity cost of capital should be used as the discount rate. Conceptually, the SRTP and the rate of return on capital are related; letting $w$ be the wedge caused by tax and other obstacles to complete clearing of the market for capital between

[^2]users in physical investment and suppliers (savers), we have $r=S R T P+w$. The fundamental problem with discounting for a very long period of time at today's rate on return on capital is that to do so makes an intergenerational comparison that promises something that cannot be delivered. It is simply not credible that today's generation and all intervening generations will keep intact an investment fund that is capable of continued returns at today's level, to generate a distant-future payment that will compensate a future generation for damage inflicted (Cline, 1999).

Even though there are contentions over the choice of discount rate, the social decision-makers using cost-benefit analysis have been setting a discount rate in one way or another. Arbitrary choice of discount rate should cause no problem if it does not alter the results of analysis much. However, it turns out that cost-benefit analysis is often very sensitive to the choice of discount rate. For example, a dollar thirty years from now is worth 41 cents and 23 cents in the present value at the discount rate of $3 \%$ and $5 \%$ respectively. Within the century from now, a dollar discounted at $3 \%$ is worth almost seven times a dollar discounted at $5 \%$ in the present value. For example, Azar and Sterner (1996) argues that the main conclusion of the Nordhaus' DICE model (Nordhaus, 1993, 1994) may be substantially altered by different discount rates.

Even though exponential discounting is amenable to the standard assumptions in economics such as utilitarianism and rationality, and is a convenient way to explicitly take into account the trade-off between the benefits and costs of present and future generations, there are so many problems with it that its use is questioned. Hence, we shall seek for alternatives to expo-
nential discounting and discuss their merits and limits in the next section.

## 4 Alternatives to Exponential Discounting

### 4.1 Quasi-discounting

Price Price (1993) suggests several modifications to discounting. According to him, quasi-discounting is similar to discounting but more flexible process: it may take account of changing quantities, qualities and probabilities over time, and need not use the straightforward arithmetic of discounting. Though the precise definition of quasi-discounting does not seem to be clear in his argument, he makes a point worth mentioning.

He proposes to have more than one numeraire. In the conventional approach, there is only one discount rate. However, he proposes to use different (quasi-)discount rates applicable to different commodities. In this view, there is in effect a numeraire for each good or income stream is discounted at whatever rate is appropriate to relate its future value to the numeraire of its current value. His justification is that qualitative and probabilistic changes relates to the expected value of a single product or experience, and as such have close affinities with the meaning of discounting. However, since the nature of qualitative and probabilistic change depends on the commodity, using quasi-discount rate as opposed to a general discount rate makes sense.

### 4.2 Reconciliation of intergenerational equity and discounting

To address the intergenerational equity issues explained in Section 2, a number of attempts have been made to rectify discounting ${ }^{3}$. Nijkamp and Rouwendal (1988) uses the aggregate net generation benefit, which is the weighted mean of net present values (NPVs) from all periods; its magnitude depends sensitively upon how NPVs are weighted. This means that late costs and benefits are counted multiple times as they enter the calculation of the NPV for each period. Their method increases the importance of future generations' view point but the means of defining weights is unclear, and the multiple-counting of late revenues and costs favors delayed benefit, which has no obvious justification.

Kula's modified discounting method (Kula, 1981, 1988) is designed for public projects whose benefits are necessarily distributed across generations. The essence of Kula's philosophy is that individuals have a right, under the terms of consumers' sovereignty, to discount within their own lifetimes, but no right to discount utility according to future generations. The method appears to achieve the difficult compromise between intergenerational equity, and efficiency as defined by consumers' sovereignty. It also avoids intuitively offensive conclusions, as that a perpetual source of utility has finite value.

However, the method has inconsistency of three kinds. Firstly, there is a time-inconsistency problem because the discount rate is time-variant. Secondly, the basis of discounting does not match the structure of the discount

[^3]factors. Mortality risk does not remain constant through life, and yet it appears as a constant in individuals' discount functions. Thirdly, the claim of equitability is inconsistent with the factors actually derived. The present generation can justify making only small sacrifices for the good of the next generation, and this lack of required present action is, in practice, a norm that will be handed down the generations, each generation postponing the obligation to make sacrifices.

### 4.3 Chichilnisky's criterion

In the context of global environmental problems, and global warming in particular, the intergenerational concern is one of the most important issues. Literature on sustainable development provides alternative ways to take into account the welfare of future generations. In this subsection and the next, we shall review the alternatives to discounting that have emerged from sustainable development concerns.

Chichilnisky (1996) has proposed a practical way of taking into account the efficiency and equity. ${ }^{4}$ This criterion has a certain intuitive appeal, and the models of optimal economic development using this criterion have been investigated by Beltratti et al. (1994a,b) and Heal (1993). To understand Chichilnisky's criterion, let us begin with the standard resource extraction model:

$$
\begin{equation*}
\max \int_{0}^{\infty} f\left(u_{t}\right) e^{-\delta t} \mathrm{~d} t \quad \text { s.t. } \quad x_{t} \geq 0 \quad \text { and } \quad \dot{x}_{t}=-u_{t} \tag{1}
\end{equation*}
$$

[^4]where $f(\cdot)$ is the instantaneous utility function of the consumption $u_{t}$ and the stock of natural resources $x_{t}$ at time $t$. We obtain the well-known Hotelling rule, which tells us that the present value of the shadow price of the resource has to be the same at all dates at which a positive amount is consumed. In general, the solution can be extremely unequal across generations as the weight on the future generation approaches zero as one goes into more distant future.

In the environmental context, it may be the case that both the flow and the stock are important. For example, people gain utility from the consumption and stock of forest. In other words, people benefit from the use of wood, which is a form of consumption of forest. They also derive pleasure from walking in the forest, which is a direct use of stock. To incorporate this idea, we can add the stock component in our utility function in Equation (1) to arrive at the following model.

$$
\begin{equation*}
\max \int_{0}^{\infty} f\left(u_{t}, x_{t}\right) e^{-\delta t} \mathrm{~d} t \quad \text { s.t. } \quad x_{t} \geq 0 \quad \text { and } \quad \dot{x}_{t}=-u_{t} \tag{2}
\end{equation*}
$$

This give rise to a different rule from the Hotelling rule; if the decreased stock results in higher marginal utility of consumption, there can exist incentives to conserve the stock. However, the fact that the future generation is taken lightly remains. The green golden rule, which happens to be optimal in Rawlasian sense (Heal, 1998b), is a form of solution that treats all the generations equally. The green golden rule can be formalized as follows:

$$
\begin{equation*}
\max _{\text {feasible paths }} \lim _{t \rightarrow \infty} f\left(u_{t}, x_{t}\right) \tag{3}
\end{equation*}
$$

The Chichilnisky criterion is a generalized version of the weighted sum of Equations (2) and (3). Formally,

$$
\begin{equation*}
\max \left(\alpha \int_{0}^{\infty} f\left(u_{t}, x_{t}\right) \Delta(t) \mathrm{d} t+(1-\alpha) \lim _{t \rightarrow \infty} f\left(u_{t}, x_{t}\right)\right) \tag{4}
\end{equation*}
$$

where $\alpha \in(0,1)$ and $\Delta(t)$ satisfies $\int_{0}^{\infty} \Delta(t) \mathrm{d} t=1$. In particular, it could be a conventional exponential discounting factor (i.e. $\Delta(t)=e^{-\delta t}$ ). Although we have derived Equation (4) in a rather arbitrary way, this result actually comes from a very precise rationalization; If we accept certain rather reasonable axioms about the ranking of alternative utility paths, then they must be ranked according to Equation (4).

Chichilnisky's criterion makes an intuitive appeal as it incorporates the conventional utilitarian discounting to address the efficiency, while it also puts weights on the future generation by incorporating the second term. Though it may also appear to be eclectic and the choice of $\alpha$ and $\Delta(t)$ may be arbitrary, it can be considered as a possible and, perhaps, practical departure point from the conventional discounting scheme.

### 4.4 Gamma discounting

So far, the argument been has based upon one principle or another. Now, let us consider discounting from a different angle. If we consider what the experts and non-experts really think, we have a different picture. In this section and the next, we shall discuss gamma discounting proposed by Weitzman (2001), and hyperbolic and quasi-hyperbolic discounting respectively.

Weitzman (2001) takes a radically different approach from the aforementioned approaches; he starts with the recognition that the discount-rate problem is rooted in fundamental differences of opinion, which are unlikely to go away. Hence he contends that we should be operating from within a framework that incorporates the irreducible uncertainty about discount rates directly into our benefit-cost methodology.

He proposes an operational framework to resolve the discount-rate dilemma, which is centered on what he called the gamma discounting. He first develops a model of variable effective interest rates taking the discount rate as a random variable. In essence, his model is:

$$
\begin{equation*}
A(t)=\int_{0}^{t} e^{-x t} f(x) \mathrm{d} x \tag{5}
\end{equation*}
$$

where $x$ is a random variable and $f(x)$ is the associated probability density function. $x$ can be thought of random discount rate and $A(t)$ is the effective discount function for time $t$. In his model, $x$ is a gamma random variable. After several lines of algebra, he arrives at the following effective discount rate $R(t)$.

$$
\begin{equation*}
R(t)=\frac{\mu}{1+t \sigma^{2} / \mu} \tag{6}
\end{equation*}
$$

where $\mu$ and $\sigma^{2}$ are the mean and variance of the gamma distribution. ${ }^{5}$ The effective discount rate is initially $\mu$ but declines monotonically towards

[^5]zero over time.
He then tried to estimate the empirical distribution of the discount rate. He surveyed over 2,000 professional Ph.D.-level economists including 50 representative blue-ribbon "leading economists". He asked the subjects to reply with their "professionally considered gut feeling" about what discount rate should be used to the projects being proposed to mitigate the possible effects of global climate change. What he found is that the aggregate response for the experts and "leading-experts" did not differ much and the $\mu$ and $\sigma$ were estimated at about 4\% per annum and 3\% per annum respectively. He then calculated the "as-if-constant" discount rate $\bar{\gamma}$ of $1.75 \%$ per annum, which would be applied to the infinite horizon of time from the following equation.
\[

$$
\begin{equation*}
\bar{\gamma}=\frac{1}{\int_{0}^{\infty} A(t) \mathrm{d} t}=\frac{\mu^{2}-\sigma^{2}}{\mu} \tag{7}
\end{equation*}
$$

\]

His paper is consistent with the intuition that the discount rate should be lower for a long period of time as slight increase in discount rate makes the benefits and costs virtually zero. Also, his paper is connected to the next section of hyperbolic discounting via Equation (6). From psychological perspectives, hyperbolic discounting seems to approximate how people actually discount. Gamma discounting gives a rationale for hyperbolic discounting as a result of probabilistic treatment of discount rate.

### 4.5 Hyperbolic and quasi-hyperbolic discounting

Many psychological experiments support the use of hyperbolic discounting as opposed to exponential discounting. Let us consider, for example, a trade-off
between 8 hours of work 100 days from now and 12 hours of work 101 days from now. Certainly most of the people would choose the former. However, after 100 days, people may choose differently. 8 hours of work today may look worse than 12 hours of work tomorrow. This kind of inconsistency, called time-inconsistency, is widely observed in human behavior. However, once we allow the assumption of nonexponentiality, we can account for "unwanted" behaviors ranging from character flaws to addictions to mannerisms to pain itself (Ainslie and Haslam, 1992).

The most important observation here is that people tend to have presentbiased preference. In oher words, stronger relative weight is placed to the earlier as the inter-temporal trade-offs at stake get closer to the present. There exist several formulations to incorporate this effect (O'Donoghue and Rabin, 1999a). In this section, we shall present the formula of hyperbolic and quasi-hyperbolic discounting based on Cropper and Laibson (1999), and its extension. We develop an analogous model in a continuous time setting in Section 7.

Loewenstein and Prelec (1992) present an axiomatic analysis of presentbiased preference, which implies a generalized hyperbolic discount function as follows:

$$
\begin{equation*}
\phi(t)=(1+\alpha \gamma t)^{-\gamma / \alpha} \quad, \alpha, \gamma>0 \tag{8}
\end{equation*}
$$

As $\alpha \rightarrow 0, \phi(t)$ approaches the exponential function. When $\alpha$ is large, $\phi(t)$ approximates a step function, implying that all periods after the first receive approximately equal weight. For $\alpha>0, \phi(t)$ lies below the exponential
function at low $t$ and above it at high $t$.
An convenient approximation have been first developed by Phelps and Pollak (1968) for intergenerational analysis and used for intrapersonal analysis by Laibson (1997). Specifically, we examine a representative consumer who live $T$ periods and whose period $t$ self receives utility $V_{t}$ from the consumption sequence $\left(u_{0}, u_{1}, \ldots, u_{T}\right)$ according to

$$
\begin{equation*}
V_{t}\left(u_{0}, u_{1}, \cdots, u_{T}\right)=f\left(u_{t}\right)+\beta \sum_{i=1}^{T-t} \delta^{i} f\left(u_{t+i}\right), \quad 0<\beta, \delta<1 \tag{9}
\end{equation*}
$$

where $f\left(u_{t}\right)$ is the instantaneous hedonic utility from consumption $u_{t}$. When $0<\beta<1$, the discount structure in Equation (9) mimics the qualitative properties of the hyperbolic function, while maintaining most of the analytical tractability of the exponential discount function. We shall refer to the discount factors $\left\{1, \beta \delta, \beta \delta^{2}, \beta \delta^{3}, \ldots\right\}$ as quasi-hyperbolic and we shall call the preference based upon the quasi-hyperbolic discount factor as $\beta-\delta$ preference. ${ }^{6}$

A number of studies have been done on the behavior of quasi-hyperbolic discounters. In particular, studies have been done to analyze the behavior of so called naive and sophisticated selves. A naive self does not recognize that she has time-inconsistency problem. On the other hand, a sophisticated self recognize that she has potentially time-inconsistency problem and takes into account that she will not behave in the future as the way she really wants to right now. To illustrate this point, let us consider the maximization problem as follows where the utility $V_{t}$ in (9).

[^6]\[

$$
\begin{array}{r}
\max _{u_{t}, \ldots, u_{T}} V_{t}\left(u_{0}, u_{1}, \ldots, u_{T}\right) \quad \text { s.t. } \forall \tau \in\{t, \ldots, T\}, \\
0 \leq u_{\tau} \leq W_{\tau}, W_{\tau+1}=R \cdot\left(W_{\tau}-u_{\tau}\right) \tag{10}
\end{array}
$$
\]

where $W_{t}$ is period $t$ wealth and $R$ is the gross return on capital. Naive self just solves (10). The solution to this problem gives the consumption schedule. Let us denote this consumption schedule as $\left(u_{t}^{t}, u_{t+1}^{t}, \ldots, u_{T}^{t}\right)$. However, when the next period comes, naive self re-calculates the consumption schedule. But, there is no guarantee for $u_{t+1}^{t}=u_{t+1}^{t+1}$. In other words, she may actually consume a different amount in period $t+1$ from the amount of period $t+1$ consumption she scheduled in period $t$.

A sophisticated self, on the other hand, resolve this problem by predicting correctly the sequence of future actions. So, the problem she faces is, with a sequence of correct prediction of her future behavior $\left(\hat{u}_{t+1}^{t}, \ldots, \hat{u}_{T}^{t}\right)$,

$$
\begin{array}{r}
\max _{u} V_{t}\left(u_{0}, u_{1}, \ldots, u_{t-1}, c, \hat{u}_{t+1}^{t}, \ldots, \hat{u}_{T}^{t}\right) \quad \text { s.t. } \quad \forall \tau \in\{t, \ldots, T\}, \\
0 \leq u_{\tau} \leq W_{\tau}, W_{\tau+1}=R \cdot\left(W_{\tau}-u_{\tau}\right) \tag{11}
\end{array}
$$

Let us denote the solution to Equation (11) as $u_{t}$. Then we must have $u_{t}=\hat{u}_{s}^{t}$ for $\forall s<t$. This suggests a solution procedure. We can start find a solution to Equation (11) in the last period $T$ without any prediction. Then we have $u_{T}=\hat{u}_{T}^{t}$ for $\forall t<T$. In particular, we have $\hat{u}_{T}^{T-1}$. Hence using (11) for period $T-1$, we have $u_{T-1}$. Repeating this procedure backwards, we
can completely solve the problem. Sophisticated self is different from naive self in that, though she is prone to present-biased utility, she can behave in a time consistent manner because she takes into account the effect of the action taken today on the action taken in the future.

The naive and sophisticate selves are two extremes. Many people know that they have self-control problem in that they do not behave as they scheduled. However, they sometimes make costly commitment because they know that they have self-control problem. This sort of commitment sometimes works and sometimes does not work as they are not necessarily fully aware of the self-control problem they have. Hence, it would be fair to say that the real life human being is usually between naive and sophisticated selves.

Recently, O'Donoghue and Rabin (2001) developed a model to explain the behavior of partially sophisticated and partially naive self. They introduced the projected $\beta$, which they denote $\hat{\beta}$. Usually, we have $\hat{\beta} \in(\beta, 1)$. In the two limiting cases where $\hat{\beta}=\beta$ and $\hat{\beta}=1$, we have the sophisticated self and naive self respectively.

Like the sophisticated self, the partially sophisticated and partially naive self takes into account the consequences of the present action in the future action. However, the prediction is not perfect; they use $\hat{\beta}$ to derive $\hat{u}_{t}^{\tau}$ for $t<\tau$. Assuming appropriate constraints apply, the problem the sophisticate self at time $t$ is solving is:

$$
\begin{equation*}
\max _{u} U_{t}\left(u_{0}, u_{1}, \ldots, u_{t-1}, c, \hat{u}_{t+1}^{t}, \ldots, \hat{u}_{T}^{t}\right) \tag{12}
\end{equation*}
$$

, where we have

$$
\begin{align*}
\hat{u}_{t}^{\tau}= & \arg \max _{u} \hat{V}_{\tau}\left(u_{0}, u_{1}, \ldots, u_{\tau-1}, u, \hat{u}_{\tau+1}^{\tau}, \ldots, \hat{u}_{T}^{\tau}\right) \\
& \text { for } \forall \tau \in\{t+1, \ldots, T\} \text { and } \hat{V}_{t}=f\left(u_{t}\right)+\hat{\beta} \sum_{i=1}^{T-t} \delta^{i} f\left(u_{t+i}\right) \tag{13}
\end{align*}
$$

we shall call this preference based on Equations (12)-(13) as $\beta-\hat{\beta}-\delta$ preference. $\beta-\hat{\beta}-\delta$ formulation allows us to predict the different modes of behavior when there is a gap between the predicted $\beta$ and the actual $\beta$, and the individual fails to fully take into account his/her self-control problem. Most of the studies are focused on the individual behavior. However, from the perspective of environmental and resource economics, we are also interested in the relationship to the social choice. In the next section, we shall go over the existing literature on the application of quasi-hyperbolic discounting in environmental and resource economics.

## 5 Applying Hyperbolic Discounting to Envi-

 ronmental IssuesCompared with the analysis of individual behavior, there have been a limited number of applications of hyperbolic or quasi-hyperbolic discounting to the environment. Henderson and Bateman (1995) argues that the social discount rate may be a hyperbolic function rather than a exponential function. Public choice has resulted lower discretionary exponential discount rates for many intergenerational projects in Britain and the USA. Using a household survey
data on hypothetical pollution control programs that include inter-temporal substitution of human lives, the estimated discount factor fits well to a hyperbolic function. They claim that, for intergenerational time frames, hyperbolic discount rates should be employed together with exponential discount rates in cost-benefit sensitivity analysis.

Fischer (1999b) develops a model of time-consistent procrastination to assess the extent to which the observed behavior is compatible with rational behavior. Key qualitative findings of psychological studies of academic procrastination are consistent with the standard natural resource management principles implied by the model, when suitably adapted to task aversiveness, uncertainty, and multiple deadlines. However, quantitatively, the fully rational model requires an extremely high rate of time preference or elasticity of inter-temporal substitution to generate serious procrastination; furthermore, it cannot explain undesired procrastination.

Fischer (1999a) also considers dynamically inconsistent preference and investigates the extent to which dynamically inconsistent preferences can better explain impatience and address the issue of self-control failures. She presents two types of discount functions, hyperbolic and differential discounting functions, which is actually a variant form of quasi-hyperbolic discounting. It is also found that they have different implications for policies to induce work, reduce procrastination and improve welfare.

Karp (2002) applied hyperbolic discounting to the context of global warming. He point out two disadvantages of using a constant discount rate. First, the prescribed policy is sensitive to the discount rate. Second, with moderate discount rates, large future damages have almost no effect on current deci-
sions. He argues that time-consistent quasi-hyperbolic discounting alleviates both of these modeling problems, and is a plausible description of how people think about the future.

Although the number of applications of hyperbolic discounting is yet small, hyperbolic discounting has been increasingly attracting the attention of environmental and resource economists. The study presented in this paper should also be understood in this context. However, the approach we take in this thesis is considerably different from the above-mentioned research. First, we deal with environmental education. To the best of my knowledge, there have been no study to quantify the effect of environmental education using behavioral economic approach. Second, our approach has strong implications in a developing country context. We shall get back to this point in Section 10. Third, as opposed to the majority of the studies, we use a continuous time setting. This allows us to separate more clearly the misconception of future behavior from the present-biasedness. In the next section, we recapitulate the discussion so far, and discuss the connection between self-control problem and the issues of environment and development.

## 6 Self-Control Problem in Environment and Development

We have overviewed discounting and its associated problems. One of the most important issues is the ethical one. Since discounting with a positive discount rate gives necessarily smaller weight on the future generation, it has
been argued that discounting is not morally right. Psychologists also argued that discounted utility theory does not describe the actual inter-temporal choice of people very well.

Despite those and a number of other problems with discounting, it is still widely used in practice. Even if the use of conventional discounting is taken for granted, the choice of discount rate has always been contentious. This rather classical issue has gained a renewed attention in the context of global warming as the impact of even a slightest change in discount rate can be enormous in the long run.

To address the problems with discounting over a long period of time, a lot of researchers have proposed different discounting criteria and a combination of discounting and other criteria. For example, Chichilnisky has proposed a weighted sum of usual discounting approach and the green golden rule. Price tried to find ways to modify the conventional way of exponential discounting. But such modification tends to add another arbitrary assumptions. Least said, there has been no good alternative to discounting agreed by most economists.

Weitzman has taken a radically different approach. Instead of trying to reach consensus on a discount rate, he took the disagreement over the discount rates as given. Using the experts' opinion, he arrived at the formula for gamma-discounting. His result supports the use of hyperbolic discounting. Recent studies on hyperbolic discounting and its simplified version called quasi-hyperbolic discounting seem to give us a useful insight about intertemporal social choice.

Though there have been relatively few applications of hyperbolic and
quasi-hyperbolic discounting to inter-temporal social choice, including environmental applications, it sheds light on the issues from a different angle. If, for example, the discount factor we observe from a revealed choice reflects $\beta \delta$ instead of $\delta$. we seriously over-predict the relevant discount rate. If we allow for the sub-rationality of a society, we may have a completely different picture.

As we have briefly discussed at the beginning of this paper, discount rates in developing countries are estimated to be very high. The high RTPs seem to have contributed to environmental degradation, which in turn further impoverish already poor people. It seems plausible that they are a timeinconsistent decision makers and have a self-control problem. This motivates the analysis in the next section, where we analyze the behavior of a partially naive and partially sophisticated social planner present in a continuous time setting. We propose a way to measure the benefits of environmental education. We subsequently provide two examples related to resource management in Sections 8 and 9.

## 7 Formulating the Benefits of Environmental Education

Let us first consider a problem of environmental management for a social planner. The indicator of the environmental quality at issue is $x(t)$ with its initial value $x_{0}$ at the beginning of the planning period $t_{0}(\geq 0) .{ }^{7} x(t)$ is affected by the consumption schedule $u(t)$ of the society. $g(\cdot)$ describes the time-dependent relationship between the environmental quality and the consumption as $\dot{x}(t)=g(t, x(t), u(t))$. This equation is called the state equation, and, for the notational convenience, we shall denote it for $t_{2} \geq t_{1}$ in the following way:

$$
\begin{equation*}
\mathcal{S}_{t_{1}}^{t_{2}} \equiv\left\{\dot{x}(t)=g(t, x(t), u(t)) \text { for } \forall t \in\left[t_{1}, t_{2}\right]\right\} \tag{14}
\end{equation*}
$$

The social planner is concerned about her "lifetime utility" $V(u ; \delta)$ for her planning period between $t_{0}$ and $T .{ }^{8}$ The end of her planning period $T$ may be fixed or free, and may be infinite. To avoid unnecessary complication, we assume $T$ is fixed in this section. ${ }^{9}$ In Subsection 8.2, we deal with numerical examples where $T$ is fixed and $T$ is free. As with the previous discussions, The discount factor $\delta(t)$ is the weight attached to each point in time and is assumed to be exogenous. In this study, we assume that the functional

[^7]$V(u ; \delta)$ is independent of time. In other words, given the consumption schedule, her valuation of the lifetime utility does not change over time for given $\delta$. If $\delta(t)$ is interpreted as the discount factor referenced at $t_{0}, V(u ; \delta)$ may be interpreted as the utility she gets if she knows exactly how she will consume in the course of her planning period. ${ }^{10}$ In particular, when the rate of time preference is constant, $\delta(t)$ is the familiar exponential function.

We assume that $V(u ; \delta)$ can be expressed as the following integration of her instantaneous utility function $f(\cdot)$ over the planning period:

$$
\begin{equation*}
V(u ; \delta) \equiv \int_{t_{0}}^{T} f(t, x(t), u(t)) \delta(t) \mathrm{d} t \tag{15}
\end{equation*}
$$

Let us assume she also cares about the environmental quality passed on to the next planning period. The scrap function $\phi(x(T), T)$ gives the instantaneous utility at $T$ from leaving the environment of quality $x(T)$ at $T$. Hence, the maximization problem she faces is as follows:

$$
\begin{equation*}
\max _{u(t), t \in\left[t_{0}, T\right)}(V(u ; \delta)+\phi(x(T), T) \delta(T)) \text { s.t. } x\left(t_{0}\right)=x_{0}, \mathcal{S}_{t_{0}}^{T} \tag{16}
\end{equation*}
$$

Some cautions are in order. The indicator of the environmental quality $x(t)$ is defined generically. It may be the number of fish in the pond or the concentration of carbon dioxide in the atmosphere. Larger $x(t)$ is more desirable in the former case whereas smaller $x(t)$ is more desirable for the latter for the ranges of $x(t)$ in real-life environmental problems. $u(t)$ is typically

[^8]non-negative, but may be negative in some cases. For example, it would be reasonable to define net afforestation as "negative timber consumption." The problem above is a standard optimal control problem, and both theoretical and numerical methods are available to analyze it.

A rational social planner in this non-stochastic problem solves Equation (16) and she is time-consistent. Since there is no uncertainty, she can plan her optimal consumption schedule $\hat{u}_{t_{0}}^{R}\left(t ; x_{0}, \delta\right)$ at $t_{0}$, given $x_{0}$ and $\delta$. The hat is used to denote her planned consumption as opposed to her actual consumption path $u^{R}\left(t ; x_{0}, \delta\right) .{ }^{11}$ She will not change her mind as the time goes by and thus we have $u^{R}\left(t ; x_{0}, \delta\right)=\hat{u}_{t_{0}}^{R}\left(t ; x_{0}, \delta\right)$. In other words, she will consume as she planned. Moreover, she would never regret as she knows she is maximizing her lifetime utility. That is, she does not change her consumption schedule even if she is hypothetically given an opportunity to change her past consumption schedule retroactively at a later point in time.

The society and individuals do, however, make time-inconsistent decisions and regret. In particular, a number of psychological evidence suggests that individuals seem to have present-biased preference, in which too much weight is placed on instantaneous gratification. ${ }^{12}$ To illustrate the present-biased preference, let us consider an everyday situation in a college. A student has her most favorite concert scheduled two days from now, but she also has a pass-or-perish examination the day after the concert. If she knows a good two-day preparation is just about enough to pass the examination, her rational behavior would be to study today and tomorrow so that she can go

[^9]to the concert and still pass the exam. However, she may end up talking over the phone for hours and hours today just to avoid studying, even though she will have to pay the price of missing the concert the day after tomorrow and regret. If she could make the decision three weeks ago what to do today, she would have decided to study today.

Procrastination described in this example is quite common. People often behave on the principle of "don't do today what you can put off until tomorrow," even though they know it is better to behave on the principle of "don't put off until tomorrow what you can do today." This may be formalized by an approach developed by O'Donoghue and Rabin (2001). In this study, we generalize their argument to a continuous time setting.

To develop an argument, we start by defining several terms. Our departure point is a rational individual, who has the discount factor $\delta(t)$. To make it clear that $\delta(t)$ is the discount factor for a rational person, $\delta(t)$ will be hereafter referred to as the rational discount factor. $\delta(t)$ is known to all types of social planners introduced hereafter in the sense that they know what would be the best action to take in the long run. But it is not the basis of their decisions if they are sub-rational.

One of the reasons why time-inconsistency occurs is that individuals and society have a present-biased utility, which too much weight is placed on instantaneous gratification. In the example of college student, she underweighted the benefit of the future concert and over-weighted the disutility from studying. We shall, therefore, need to define the discount factor on which the decision is actually based, or the decision discount factor $\beta(t)$. We assume $\beta(t)$ is continuous in $t$. As with $\delta(t), \beta(t)$ is the weight one gives
to each point in time $t$. When the individual has a (strictly) present-biased utility, present is over-valued and thus $\beta$ and $\delta$ has the following relationship.

$$
\begin{equation*}
\frac{\beta(s)}{\beta(t)}<\frac{\delta(s)}{\delta(t)} \quad \text { for } \quad \forall s, \forall t \text { s.t. } s>t \geq 0 . \tag{17}
\end{equation*}
$$

If the social planner is completely naive, she does not realize at all that her decision discount factor depends on the time at which her decision is made and hence she is time-inconsistent. She believes she will behave like a rational social planner with her rational discount factor $\delta(t)$ in the future.

Now let us consider her decision-making problem at time $\tau\left(\in\left[t_{0}, T\right]\right)$. She finds her perceived optimal consumption schedule $\hat{u}_{\tau}^{N}(t)$ with $t \in[\tau, T]$ at each point in time $\tau$ based on her decision discount factor, where the initial environmental quality $x_{\tau}$ at time $\tau$ is given. ${ }^{13}$ For the notational convenience, we write this initial condition as $\mathcal{I}_{\tau} \equiv\left(x(\tau)=x_{\tau}\right)$ and the current perceived value of the scrap function as $\tilde{\phi}_{\tau} \equiv \phi(x(T), T) \beta(T)$.

$$
\begin{align*}
\hat{u}_{\tau}^{N}\left(t ; x_{\tau}, \beta, \delta\right)= & \underset{u(t), t t \in[\tau, T]}{\operatorname{argmax}}\left(V(u(t) ; \beta)+\tilde{\phi}_{\tau}\right) \\
& \text { s.t. } \mathcal{I}_{\tau}, \mathcal{S}_{\tau}^{T}, u(t)=u^{R}(t ; x(t), \delta) \text { for } t \in(\tau, T] \tag{18}
\end{align*}
$$

There are three points to note. Firstly, in effect, the naive social planner only chooses present (i.e. $t=\tau$ ) level of consumption $u(\tau)$ as she thinks she will behave rationally so that $u(t)=u^{R}(t ; x(t), \delta)$ for $t \in(\tau, T]$, and takes her future behavior as given. Only how she thinks she will behave influences her current decision, and how she actually behaves in the future has no

[^10]relevance here. Secondly, it should be noted that $\hat{u}_{\tau}^{N}(t)$ for $t>\tau$ is a plan that only exists in the head of the naive social planner. As soon as a little amount of time elapses, she changes her consumption schedule. The actual consumption path for the naive social planner is $u^{N}(t)=\hat{u}_{t}^{N}\left(t ; x_{t}, \beta, \delta\right)$. In general, $\hat{u}_{\tau}^{N} \neq \hat{u}_{\tau^{\prime}}^{N}$ for $\tau \neq \tau^{\prime}$, and $u^{N}(t) \neq \hat{u}_{\tau}^{N}\left(t ; x_{\tau}, \beta, \delta\right)$ for $t>\tau$. Thirdly, the rational social planner is a special case of the naive social planner where $\beta=\delta$.

The assumption of a completely naive agent would be too strong as people often know that they tend have a self-control problem. Now let us consider the other extreme and assume the social planner knows exactly how she will behave in the future so that she can act now in accordance with her future behavior and such social planner is called completely sophisticated. The behavioral principle for a completely sophisticated social planner is similar to backward induction in game theory. The planning starts from the end of the planning period. Let us assume that $\Delta$ is very small. The social planner first determines the optimal consumption $u(T-\Delta T)$ where the constant and the environmental quality $x(T-\Delta T)$ is given. Then she goes a little backward and considers the optimal consumption $u(T-2 \Delta)$ when $x(T-2 \Delta)$ is given. This solution procedure implies time-consistency as she behaves in a manner consistent with her perceived optimal future behavior. Therefore, the solution can be described in the following way ${ }^{14}$

$$
\hat{u}_{\tau}^{S}\left(t ; x_{\tau}, \beta\right)=\underset{u(t), t t \in[\tau, T]}{\operatorname{argmax}}\left(V(u(t) ; \beta)+\tilde{\phi}_{\tau}\right)
$$

[^11]\[

$$
\begin{equation*}
\text { s.t. } \mathcal{I}_{\tau}, \mathcal{S}_{\tau}^{T}, u(t)=\hat{u}_{t}^{S}(t ; x(t), \beta) \text { for } t \in(\tau, T] \tag{19}
\end{equation*}
$$

\]

Equations (18) and (19) are similar in the sense that they both take their future behavioral patterns as given. But how they perceive their future behavior is different. The sophisticated social planner know how she will behave in the future, and thus $u(t)=\hat{u}_{t}^{S}(t ; x(t), \beta(t))$ appears in the constraint.

The solution procedure for the sophisticated social planner is essentially the same as the solution procedure for the rational social planner. The only difference is that the sophisticated social planner use her decision discount factor instead of her rational discount factor. Hence, the actual consumption for a sophisticated social planner with her decision discount factor $\beta$ is

$$
u^{S}\left(t ; x_{t}, \beta\right)=\hat{u}_{\tau}^{S}\left(t ; x_{\tau}, \beta\right)=u^{R}\left(t ; x_{t}, \beta\right) \text { for } \forall(t, \tau) \text { with } \tau \leq t \leq T
$$

We assumed that the sophisticated social planner knows her future decision discount factor. But, even if she knows she has a self-control problem and tries to take it into account when making the current decisions, she may not know it precisely. Partially sophisticated and partially naive social planners ${ }^{15}$ have perceived discount factor $\hat{\beta}(t)$, and she thinks her future behavior is decided by $\hat{\beta}$. As the name suggests, partially sophisticated and partially naive have naive social planners and sophisticated social planners as the special case. $\hat{\beta}=\delta$ corresponds to the former, and $\hat{\beta}=\beta$ the latter. Usually, we have $\beta \geq \hat{\beta} \geq \delta$, and we shall assume this holds hereafter.

[^12]\[

$$
\begin{align*}
\hat{u}_{\tau}^{P}\left(t ; x_{\tau}, \beta, \hat{\beta}\right)= & \underset{u(t), \forall t \in[\tau, T]}{\operatorname{argmax}}(V(u(t) ; \beta)+\phi(x(T), T) \beta(T)) \\
& \text { s.t. } \mathcal{I}_{\tau}, \mathcal{S}_{\tau}^{T}, u(t)=u^{S}(t ; x(t), \hat{\beta}) \text { for } t \in(\tau, T] \tag{20}
\end{align*}
$$
\]

The partially sophisticated and partially naive social planner is timeinconsistent in general. As with naive social planners, her actual consumption is $u^{P}(t)=\hat{u}_{t}^{P}\left(t ; x_{t}, \beta, \hat{\beta}\right)$. Now, let us define the value function as follows:

$$
\begin{equation*}
J^{P}\left(\tau, x_{\tau} ; \beta, \hat{\beta}\right)=V\left(u_{\tau}^{P}\left(t ; x_{\tau}, \beta, \hat{\beta}\right) ; \beta\right)+\phi(x(T), T) \beta(T) \tag{21}
\end{equation*}
$$

Then, denoting $f_{\tau} \equiv f(\tau, x(\tau), u(\tau)), \beta_{\tau} \equiv \beta \tau$ and $\phi_{T} \equiv \phi(x(T), T)$, we have

$$
\begin{align*}
& J^{P}\left(\tau, x_{\tau} ; \beta, \hat{\beta}\right) \\
& =\max _{\substack{u(t) \\
t \in[\tau, \tau+\Delta \tau)}}\left(\int_{\tau}^{\tau+\Delta \tau} f_{t} \beta_{t} \mathrm{~d} t+\max _{\substack{u(t) \\
t \in[\tau+\Delta \tau, T]}}\left(\int_{\tau}^{\tau+\Delta \tau} f_{t} \beta_{t} \mathrm{~d} t+\phi_{T} \beta_{T}\right)\right) \\
& \quad \text { s.t. } \mathcal{I}_{\tau}, \mathcal{S}_{\tau}^{T}, u(t)=u^{S}(t ; x(t), \hat{\beta}) \text { for } t \in(\tau, T] \\
& =\max _{\substack{u(t) \\
t \in[\tau, \tau+\Delta \tau)}}\left(\int_{\tau}^{\tau+\Delta \tau} f_{t} \beta_{t} \mathrm{~d} t+J^{P}\left(\tau+\Delta \tau, x_{\tau+\Delta \tau} ; \beta, \hat{\beta}\right)\right) \\
& \quad \text { s.t. } \mathcal{I}_{\tau}, \mathcal{S}_{\tau}^{\tau+\Delta \tau}, u(t)=u^{S}(t ; x(t), \hat{\beta}) \text { for } t \in(\tau, \tau+\Delta \tau] \tag{22}
\end{align*}
$$

When $\Delta \tau$ is small, by Taylor-expanding $J^{P}$, we have

$$
J^{P}\left(\tau, x_{\tau} ; \beta, \hat{\beta}\right)
$$

$$
\begin{align*}
= & \max _{u_{\tau}}\left(f_{\tau} \beta_{\tau} \Delta \tau+J^{P}\left(\tau, x_{\tau} ; \beta, \hat{\beta}\right)+J_{t}^{P}\left(\tau, x_{\tau} ; \beta, \hat{\beta}\right) \Delta \tau\right. \\
& \left.\quad+J_{x}^{P}\left(\tau, x_{\tau} ; \beta, \hat{\beta}\right) \Delta x+o^{2}(\Delta \tau)\right) \text { s.t. } \Delta x=g\left(\tau, x_{\tau}, u_{\tau}\right) \Delta \tau \\
= & J^{P}\left(\tau, x_{\tau} ; \beta, \hat{\beta}\right)+\Delta \tau \cdot\left(\operatorname { m a x } _ { u _ { \tau } } \left(f_{\tau} \beta_{\tau}\right.\right. \\
& \left.\left.\quad+J_{t}^{P}\left(\tau, x_{\tau} ; \beta, \hat{\beta}\right)+J_{x}^{P}\left(\tau, x_{\tau} ; \beta, \hat{\beta}\right) g\left(\tau, x_{\tau}, u_{\tau}\right)+o(\Delta \tau)\right)\right) \tag{23}
\end{align*}
$$

Dividing through by $\Delta \tau$ and letting $\Delta \tau \rightarrow 0$, we have

$$
\begin{equation*}
-J_{t}^{P}\left(\tau, x_{\tau} ; \beta, \hat{\beta}\right)=\max _{u_{\tau}}\left(f\left(\tau, x_{\tau}, u_{\tau}\right) \beta_{\tau}+J_{x}^{P}\left(\tau, x_{\tau} ; \beta, \hat{\beta}\right) g\left(\tau, x_{\tau}, u_{\tau}\right)\right) \tag{24}
\end{equation*}
$$

By taking the first order condition, we have

$$
\begin{equation*}
0=\beta_{\tau} \frac{\partial f\left(\tau, x_{\tau}, u_{\tau}\right)}{\partial u}+J_{x}^{P}\left(\tau, x_{\tau} ; \beta, \hat{\beta}\right) \frac{\partial g\left(\tau, x_{\tau}, u_{\tau}\right)}{\partial u} \tag{25}
\end{equation*}
$$

This is the dynamic programming equation (DPE) for partially sophisticated and partially naive social planners. It looks very similar to the Hamilton-Jacobi-Bellman in the standard setting ${ }^{16}$, but it should be pointed out that this is different from the completely sophisticated social planners' DPE, unless $\hat{\beta}=\beta$.

The interpretation of Equation (25) is straightforward. The first term means the marginal lifetime utility the social planner thinks she will get from the marginal extraction of the resource now. The second term is the marginal opportunity cost she thinks she will incur from the marginal extraction of the resource now. It should be emphasized that $J_{x}^{P}$ has $\hat{\beta}$ in its parameters, and

[^13]this is the parameter that let us formulate her perceived, as opposed to her actual, future behavior.

The argument above is precisely how we solve the problem in a general setting as will be shown in Section 8 using a numerical example. Firstly, we find the rational and sophisticated social planner's solution. Then, we can find the value function $J^{P}$ for the partially sophisticated and partially naive social planners. By Equation (25), we can find their control rule $u^{P}$.

We are now in a position to measure the benefits of environmental education. As we have discussed in the beginning of this paper, there are two types of benefits we can think of in this setting. The first type of benefit is to make the social planner's perceived discount factor $\hat{\beta}$ closer to her decision discount factor $\beta$, which will be referred to as sophistication. The natural measurement of the benefit of sophistication $B^{S}$ is the additional lifetime utility she acquires from sophistication, which can be expressed follows:

$$
\begin{equation*}
B^{S}\left(t, x_{t}, \delta, \beta, \hat{\beta}\right) \equiv V\left(u^{S} ; \delta\right)-V\left(u^{P} ; \delta\right) \tag{26}
\end{equation*}
$$

The second type of benefit is to make the social planner's decision discount factor closer to her rational discount factor, which will be referred to as rationalization. The benefit of rationalization $B^{R}$ can be written in a way similar to Equation (26).

$$
\begin{equation*}
B^{R}\left(t, x_{t}, \delta, \beta\right) \equiv V\left(u^{R} ; \delta\right)-V\left(u^{S} ; \delta\right) \tag{27}
\end{equation*}
$$

The total benefit of environmental education $B^{T}$ is, therefore,

$$
\begin{equation*}
B^{T}\left(t, x_{t}, \delta, \beta, \hat{\beta}\right)=B^{S}\left(t, x_{t}, \delta, \beta, \hat{\beta}\right)+B^{R}\left(t, x_{t}, \delta, \beta\right) \tag{28}
\end{equation*}
$$

It should be noted that $B^{T}$ and $B^{R}$ are always non-negative by construction, but $B^{S}$ may not be positive. Let us provide some intuition of nonpositive $B^{S}$. When the future looks grim because of one's own self-control problem, there is less incentive to do well now. Let us think about fish in the pond as an example. Let's say the number of fish goes up as the time goes by. Let us assume that the fish increases faster when there are more fish in the pond. If the fisherman is naive, he thinks that he will not over-harvest in the future. On the other hand, if he is sophisticated, he knows he will overharvest. When we think about the benefits of keeping additional fish in the pond, the perceived marginal utility may be higher for the naive fisherman. We shall show this with a more concrete example in Sections 8 and 9.

## 8 Example 1: Non-renewable Resource Extraction

### 8.1 Analytics

As an illustration, let us consider a simple problem of non-renewable resource extraction. Non-renewable resources are such resources that does not grow or regenerate and include such as resources as oil and iron ore. The social planner supplies the quantity $u(t)(\geq 0)$ to the market. She faces a linear demand curve $d(t) \equiv a-b \cdot p(t)$, where $p(t)$ denotes the price of the unit
quantity of the resource, and $a$ and $b$ are strictly positive. The social planner derives her instantaneous utility $f(u(t))$ from the area defined by the demand curve, marginal cost curve and the vertical axis. To simplify the problem, we assume the marginal cost of extraction is zero. The quantity $x(t)$ of nonrenewable resource has the initial reserve $x(0)=r$ and follows the differential equation $\dot{x}(t)=-u(t)$. This means $g(t, x, u)=-u$ in Section 7. The social planner's instantaneous utility satisfies Equation (29).

$$
\begin{equation*}
f(u)=\int_{0}^{u} p(z) d z=\int_{0}^{u} \frac{a-z}{b} d z=\frac{2 a u-u^{2}}{2 b} \tag{29}
\end{equation*}
$$

Let us first consider a rational social planner with her rational discount factor of $\delta(t) \equiv e^{-\gamma t}$ as a benchmark case, where $\gamma$ is a constant. Her planning period starts at time $t=0$ and ends at time $t=T$. We first assume $T$ is fixed, and then we allow $T$ to change. Her problem can be solved by the standard optimal control techniques. The optimal control problem she is facing is to find the value function $J^{R}(r ; \gamma)$, the corresponding optimal control rule $u^{R}(t ; \gamma)$, and the time $T$ to deplete the resource. It should be emphasized that the value function is her perceived lifetime utility she thinks she will obtain when she obeys her perceived optimal control rule. Perceived lifetime utility is equal to her actual lifetime utility when she is rational (i.e. $J^{R}=V^{R}$ ). We assume the social planner doesn't care about the world after her planning period so that $\phi(x(T), T)=0$. In this example, first we assume $T$ is fixed and we then allow $T$ to be free to illustrate how the discussion in Section 7 is altered.

$$
\begin{array}{r}
J^{R}(r ; \gamma) \equiv \max _{u(t)} \int_{0}^{T} f(u(t)) e^{-\gamma t} \mathrm{~d} t \quad \text { s.t. } \quad \dot{x}=-u \\
u(t) \geq 0, x(t) \geq 0 \text { for } t \in[0, T], x(0)=r \tag{30}
\end{array}
$$

Letting the costate variable be $\lambda$, the Hamiltonian $H(t)$ is given in Equation (31).

$$
\begin{equation*}
H=\frac{2 a u-u^{2}}{2 b} e^{-\gamma t}+\lambda \cdot(-u) \tag{31}
\end{equation*}
$$

The necessary conditions for optimality gives

$$
\begin{align*}
\frac{\partial H}{\partial u} & =\frac{a-u}{b} e^{-\gamma t}-\lambda=0  \tag{32}\\
\dot{\lambda} & =\frac{-\partial H}{\partial x}=0  \tag{33}\\
\dot{x} & =\frac{\partial H}{\partial \lambda}=-u \tag{34}
\end{align*}
$$

By the end-point condition, we have

$$
\begin{equation*}
x(T)=0 \tag{35}
\end{equation*}
$$

Solving Equations (32)-(35), we have

$$
\begin{align*}
u^{R}(t ; r, \gamma) & =a-\frac{\gamma(a T-r)}{e^{\gamma T}-1} e^{\gamma t}  \tag{36}\\
J^{R}(r ; \gamma) & =\int_{0}^{T}\left(\frac{2 a u^{R}-\left(u^{R}\right)^{2}}{2 b} e^{-\gamma t}\right) \mathrm{d} t \\
& =\frac{1}{2 b \gamma}\left(a^{2}\left(1-e^{-\gamma T}\right)-\frac{\gamma^{2}(a T-r)^{2}}{e^{\gamma T}-1}\right) \tag{37}
\end{align*}
$$

Now, let us consider the sophisticated social planner's problem. She knows that she has a self-control problem with her decision discount factor $\beta(t) \equiv e^{-\rho t}$ where $\rho(\leq \gamma)$ is a constant. As noted in Section 7, she acts as if she is a rational social planner with her discount factor $e^{-\rho t}$. Hence, we have the following equations corresponding to Equations (36) and (37)

$$
\begin{align*}
u^{S}(t ; r, \rho) & =a-\frac{\rho(a T-r)}{e^{\rho T}-1} e^{\rho t}  \tag{38}\\
J^{S}(r, T ; \rho) & =\frac{1}{2 b \rho}\left(a^{2}\left(1-e^{-\rho T}\right)-\frac{\rho^{2}(a T-r)^{2}}{e^{\rho T}-1}\right) \tag{39}
\end{align*}
$$

We can now analyze how the partially sophisticated and partially naive person behaves with her perceived discount factor $\hat{\beta}(t) \equiv e^{-\hat{\rho} t}$, where $\hat{\rho}$ is constant and $\gamma \leq \hat{\rho} \leq \rho .{ }^{17}$ Let us consider the value function $J^{P}$ at $T=0$. Since her control rule $u^{P}(t ; r, \rho, \hat{\rho})$ is equal to $u^{S}(t ; r, \hat{\rho})$ for $t(>0)$,

$$
J^{P}(r, T ; \rho, \hat{\rho})
$$

[^14]\[

$$
\begin{align*}
& =\int_{0}^{T} f\left(u^{s}(t ; r, \hat{\rho})\right) e^{-\rho T} \mathrm{~d} t \\
& =\frac{1}{2 b}\left(\frac{a^{2}\left(1-e^{-\rho T}\right)}{\rho}+\frac{\hat{\rho}^{2}(a T-r)^{2}\left(1-e^{(2 \hat{\rho}-\rho) T}\right)}{\left(e^{\hat{\rho} T}-1\right)^{2}(2 \hat{\rho}-\rho)}\right) \tag{40}
\end{align*}
$$
\]

It should be noted that $J^{P}$ is her perceived lifetime utility. From Equation (40), we can derive

$$
\begin{align*}
J_{r}^{P}(r, T ; \rho, \hat{\rho}) & =\frac{\partial J^{P}(r, T ; \rho, \hat{\rho})}{\partial r} \\
& =\frac{\hat{\rho}^{2}\left(1-e^{(2 \hat{\rho}-\rho) T}\right)(r-a T)}{b(2 \hat{\rho}-\rho)\left(e^{\hat{\rho} T}-1\right)^{2}} \tag{41}
\end{align*}
$$

Therefore, using Equation (25), we have

$$
\begin{equation*}
0=\frac{a-u}{b}-J_{r}^{P}(r, T ; \rho, \hat{\rho}) \tag{42}
\end{equation*}
$$

This equation gives the control rule for the partially sophisticated and partially naive social planner $u^{P}(r, T ; \rho, \hat{\rho})$.

$$
\begin{align*}
u^{P}(r, T ; \rho, \hat{\rho}) & =a-b J_{r}^{P}(r, T ; \rho, \hat{\rho}) \\
& =a-\frac{\hat{\rho}^{2}\left(1-e^{(2 \hat{\rho}-\rho) T}\right)(r-a T)}{(2 \hat{\rho}-\rho)\left(e^{\hat{\rho} T}-1\right)^{2}} \tag{43}
\end{align*}
$$

$u^{P}$ is the amount of extraction when the remaining time until the end of the planning period is $T$ and the remaining resource is $r$. Hence, the time evolution of $x^{P}$ for partially sophisticated and partially naive social planners
can be found as:

$$
\begin{align*}
x^{P}(x, t ; \rho, \hat{\rho})= & r-\int_{0}^{t} u^{P}(x(\tau), T-\tau ; \rho, \hat{\rho}) \mathrm{d} \tau \\
& \text { s.t. } x(0)=r, \dot{x}(\tau)=-u^{P}(x(\tau), T-\tau ; \rho, \hat{\rho}) \tag{44}
\end{align*}
$$

Equations (43) and (44) are analytically intractable. Hence, we need to use numerical methods to solve for them. In Subsection 8.2, we provide a numerical example together with the results for the case in which $T$ is free. Before moving on to the numerical results, let us find the equations corresponding to Equations (43) and (44) when $T$ is free. The maximization problem the rational social planner is facing is modified to the following equation:

$$
\begin{array}{r}
J^{R}(r ; \gamma) \equiv \max _{u(t), T} \int_{0}^{T} f(u(t)) e^{-\gamma t} \mathrm{~d} t \quad \text { s.t. } \quad \dot{x}=-u \\
u(t) \geq 0, x(t) \geq 0 \text { for } t \in[0, T], x(0)=r \tag{45}
\end{array}
$$

By the first order conditions, Equations (32)-(35) need to hold. Since $T$ is free, we have the following additional end-point condition.

$$
\begin{equation*}
\left.H\right|_{t=T}=0 \tag{46}
\end{equation*}
$$

Solving Equations (32)-(35) and (46), we have the control rule and value function for the rational social planner.

$$
\begin{align*}
u^{R}(t ; r, \gamma) & =a\left(1-e^{t-T(r ; \gamma)}\right)  \tag{47}\\
J^{R}(r ; \gamma) & =\int_{0}^{T(r ; \gamma)}\left(\frac{2 a u-u^{2}}{2 b} e^{-\gamma t}\right) \mathrm{d} t \\
& =\frac{a^{2}}{2 b \gamma}\left(1-e^{-\gamma T(r ; \gamma)}\right)^{2} \tag{48}
\end{align*}
$$

,where $T(r ; \gamma)$ satisfies

$$
\begin{equation*}
r=a T(r ; \gamma)-\frac{a}{\gamma}\left(1-e^{-\gamma T(r ; \gamma)}\right) \tag{49}
\end{equation*}
$$

Since $T$ is free, $T$ now depends on $r$. Equation (49) shows this clearly. As with the fixed $T$ case, the sophisticated social planner's control rule and value function can be found easily using $\rho$ instead of $\gamma$ in Equations (47) and (48). Thus,

$$
\begin{align*}
u^{S}(t ; r, \rho) & =a\left(1-e^{t-T(r ; \rho)}\right)  \tag{50}\\
J^{S}(r ; \rho) & =\int_{0}^{T(r ; \rho)}\left(\frac{2 a u-u^{2}}{2 b} e^{-\rho t}\right) \mathrm{d} t \\
& =\frac{a^{2}}{2 b \rho}\left(1-e^{-\rho T(r ; \rho)}\right)^{2} \tag{51}
\end{align*}
$$

,where $T(r ; \rho)$ satisfies Equation (49). Let us consider how a partially sophisticated and partially naive social planner behaves with her perceived discount factor $\hat{\beta}(t) \equiv e^{-\hat{\rho} t}$, where $\hat{\rho}$ is constant. To do so, we need to find the value function $J^{P}$ first. Since her control rule $u^{P}(t ; r, \rho, \hat{\rho})$ is equal to
$u^{S}(t ; r, \hat{\rho})$ at $t(>0)$,

$$
\begin{align*}
& J^{P}(r ; \rho, \hat{\rho}) \\
& =\int_{0}^{T(r ; \hat{\rho})}\left(\frac{u^{S}(t ; r, \hat{\rho})\left(a-u^{S}(t ; r, \hat{\rho})\right)}{2 b} e^{-\rho t}\right) \mathrm{d} t \\
& \quad=\frac{a^{2}}{2 b} \cdot\left(\frac{1-e^{-\rho T(r ; \hat{\rho})}}{\rho}+\frac{e^{-2 \hat{\rho} T(r ; \hat{\rho})}-e^{-\rho T(r ; \hat{\rho})}}{2 \hat{\rho}-\rho}\right) \tag{52}
\end{align*}
$$

Hence,

$$
\begin{align*}
\frac{\partial J^{P}(r ; \rho, \hat{\rho})}{\partial r} & =\frac{\partial J^{P}}{\partial T} \frac{1}{\partial r} \\
& =\left(\frac{a^{2}}{2 b} \cdot \frac{2 \hat{\rho}}{2 \hat{\rho}-\rho}\left(e^{\rho T(r ; \hat{\rho})}-e^{-2 \hat{\rho} T(r ; \hat{\rho})}\right)\right) \cdot\left(\frac{1}{a\left(1-e^{-\hat{\rho} T(r ; \hat{\rho})}\right)}\right) \\
& =\frac{a \hat{\rho}\left(e^{-\rho T(r ; \hat{\rho})}-e^{-2 \hat{\rho} T(r ; \hat{\rho})}\right)}{b(2 \hat{\rho}-\rho)\left(1-e^{-\hat{\rho} T(r ; \hat{\rho})}\right)} \tag{53}
\end{align*}
$$

Using Equation (49), her perceived optimal control rule $u^{P}$ at $t=0$ is derived.

$$
\begin{align*}
u^{P}(r ; \rho, \hat{\rho}) & =a-b \frac{\partial J^{P}}{\partial r} \\
& =a\left(1-\frac{\hat{\rho}\left(e^{-\rho T(r ; \hat{\rho})}-e^{-2 \hat{\rho} T(r ; \hat{\rho})}\right)}{(2 \hat{\rho}-\rho)\left(1-e^{-\hat{\rho} T(r ; \hat{\rho})}\right)}\right) \tag{54}
\end{align*}
$$

The time evolution of $x(t)$ is

$$
\begin{align*}
x^{P}(t ; \rho, \hat{\rho})= & r-\int_{0}^{t} u^{P}(x(\tau) ; \rho, \hat{\rho}) \mathrm{d} \tau \\
& \text { s.t. } x(0)=r, \dot{x}(\tau)=-u^{P}(x(\tau) ; \rho, \hat{\rho}) \tag{55}
\end{align*}
$$

It should be noted that $T(r ; \hat{\rho})$ is the perceived duration of extraction of resources when a sophisticated social planner has the remaining resource of $r$. The actual time $T_{P}(\rho, \hat{\rho})$ at which the social planner finish the resource $R$ can be derived by solving for $T$ in the following equation.

$$
\begin{equation*}
x^{P}(T ; \rho, \hat{\rho})=0 \tag{56}
\end{equation*}
$$

### 8.2 Numerics

Equations (43) and (44) describes the time evolution of resource stock and consumption when the social planner is partially naive and partially sophisticated for fixed $T$. Equations (54) and (55) do the same for the free $T$ case. However, the equations we have in Subsection 8.1 are not analytically tractable. In this subsection, we take a look at different types of social planners using numerical integration.

Let us first calibrate the parameters. Some of the parameters may be arbitrarily set without the loss of generality. Let us start with $b$. It should be pointed out that $u(t)$ is independent of $b$, which is the slope of the demand curve. This may be counter-intuitive, but it makes sense if one think about the currency in which $p$ is expressed. Let's say $p$ is expressed in dollars and $\tilde{p}$
is the corresponding price in yen. Since the two demand curves $d \equiv a-b \cdot p$ and $d \equiv a-\tilde{b} \cdot \tilde{p}$ should express the same thing, it should be in dependent of $b$. In this subsection, we assume $b=1$. We can also set, without loss of generality, $a=1$, too, by using an appropriate unit. $a$ is the demand for the unit of the resource when the resource is free. We can change the unit in which the quantity is measured so that $a=1$.

Once $a$ is set, $r$ carries a meaning as $\frac{r}{a}$ is the number of units of time that resource lasts without the social planner. For our convenience, we assume $T$ is expressed in years. We set $r=9.00$ as this is often about the amount of time it takes for rapid environmental degradation to manifest itself. For the rational social planner, we choose a relatively low, but non-zero, discount rate $\gamma=0.01$. This may be around the acceptable discount rate for a long-term environmental and resource problems.

Let us now introduce all the social planners that appear in this subsection. Their traits are summarized in Table 1. We shall come back to the interpretation of the numbers later, and let us pay attention to $\rho$ and $\hat{\rho}$ for now. For example, P3 social planner has $\rho=0.50$ and $\hat{\rho}=0.20$. $\gamma=0.01$ for everyone. In this example, we have one rational, three sophisticated, three naive and three partially sophisticated and partially naive social planners. P1, P2 and P3 social planners sometimes do not appear on graphs as they are somewhere between a naive social planner and a sophisticated social planner.

We have chosen $0.07,0.20$ and 0.50 as values of $\rho$ and $\hat{\rho} .0 .07$ is about the growth rate which many east and southeast Asian counties have experienced. It may be understood as something close to a fair discount rate for a relatively

Table 1: Lifetime utility for different values of $\rho$ and $\hat{\rho}$. Planning Period is fixed. $\gamma=0.01, r=9.00, T=10.00, a=1.00$ and $b=1.00$. The letters in the parentheses will be used to refer to each type of social planner.

|  |  | $\hat{\rho}$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 0.01 |  | 0.07 |  | 0.20 |  | 0.50 |  |
| $\rho$ | 0.01 | 4.7106 | (R) | - |  | - |  |  |  |
|  | 0.07 | 4.7094 | (N1) | 4.7092 | (S1) | - |  |  |  |
|  | 0.20 | 4.7012 | (N2) | 4.6992 | (P1) | 4.6973 | (S2) | - |  |
|  | 0.50 | 4.6720 | (N3) | 4.6647 | (P2) | 4.6523 | (P3) | 4.6423 | (S3) |

short-term projects with close link to the financial market. 0.50 is an example of extremely high discount rate, though it could be higher in countries like Cambodia. 0.20 is probably about the discount rate for myopic agents in developed countries as many institutions in the consumer credit sector charge interest rates around this number.

Figure 1 shows the time evolution of $x(t)$ for different types of social planners. As the graph shows, the difference in the path $x(t)$ takes is small. This is not very surprising given that the planning period is relatively short, even though the discount rate used in each type of the agents are quite different. When we look at the control rule $u(t)$, which is shown in Figure 2 , there are substantial differences among the social planners. Note how the control rule differ between a sophisticated social planner and a naive social planner with the same $\rho$. When $\rho$ is close to $\delta, \mathrm{S} 1$ social planner and N1 social planner behave in a similar manner. However, when $\rho$ gets larger, their behavioral patterns differ significantly. N3 social planners starts with less resource extraction as $u(t)$ does not decline as fast as S3.

Let us now go back to Table 1, and consider the benefit of education.


Figure 1: Graph of $x(t)$ for different types of social planners. $T$ is fixed.


Figure 2: Graph of $u(t)$ for different types of social planners. $T$ is fixed.

Table 2: Lifetime utility for different values of $\rho$ and $\hat{\rho}$. Planning period is free. $\gamma=0.01, r=9.00, a=1.00$ and $b=1.00$. The numbers in the parentheses at the top and bottom are the initial planning period and the actual duration of resource extraction respectively.

|  |  | $\hat{\rho}$ |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 0.01 |  | 0.07 |  | 0.20 |  | 0.50 |  |
| $\rho$ | 0.01 | 6.7164 | $\begin{aligned} & (45.651) \\ & (45.651) \end{aligned}$ | - |  | - |  | - |  |
|  | 0.07 | 5.7250 | $\begin{aligned} & (45.651) \\ & (16.127) \end{aligned}$ | 6.0195 | $\begin{aligned} & (19.684) \\ & (19.684) \end{aligned}$ | - |  | - |  |
|  | 0.20 | 4.9283 | $\frac{(10.651)}{(11.652)}$ | 5.2344 | (19.684) | 5.2780 | $\begin{aligned} & (13.676) \\ & (13.676) \end{aligned}$ | - |  |
|  | 0.50 | 4.5676 | $(45.651)$ | 4.7167 | (19.684) (10.497) | 4.7485 | (13.676) (10.870) | 4.7521 | (10.992) |

Notice that, for a given $\hat{\rho}$, the social planner's lifetime utility declines as $\hat{\rho}$ gets closer to $\rho$. This corresponds to what we saw in the previous paragraph. The naive starts with less extraction, because she thinks she behaves rationally in the future, and that rational future self says not to consume too much now. Hence, knowing that she will have a self-control problem does not help. It makes her more pessimistic about her future.

Now, let us consider a case where $T$ is free. As shown in Table 2, we shall use the same ten types of social planners in this setting to allow comparison between the $T$ fixed and $T$ free cases. The values of $a, b, r$ and $\gamma$ are the same as the $T$ fixed case. Since $T$ is free, the planning periods are different. Moreover, the initial planning period and the actual planning period are different if the social planner is time-inconsistent. The numbers in the brackets denote the initial planning period and actual planning period.

The time-inconsistency issue may be more clearly shown in Figure 3. This illustrate the scheduled time of the termination of extraction at each given point in time. More precisely, the horizontal axis measures the actual time $t$
elapsed whereas the scheduled time $\hat{T}(x(t) ; \hat{\rho}) \equiv t+T(x(t) ; \hat{\rho})$ depends on the actual time. When the graph hits the diagonal 45-degree line, the extraction is terminated. It should be noted that the time-consistent social planners, or the rational and sophisticated social planners, have a horizontal graph, whereas the naive social planners $\hat{T}$ declines as the time elapses.

Now, let us look at the time evolution of $x(t)$ and $u(x)$. The graphs for $x(t)$ are shown in Figures 4 and 5. The two figures differ only in their domains of $t$ shown in the graph. The graph shows that, in comparison with the fixed $T$ case, the graphs are quite different. Once again, the difference is clearer in the graph of $u(t)$. Figures 6 and 7 represent the extraction path for each type of social planners.

The naive individuals tend to change their consumption much more rapidly. This is because they tend to overconsume initially for the given $\rho$. The most typical example is N3 social planner. N3 social planner's $u(t)$ start just below 1.0 and maintain over 0.9 until the last minute. It would be interesting to note that the graphs S3 and N3 cross twice whereas S1 and N1 cross only once.

The measures of the benefits of environmental education $B^{S}, B^{R}$ and $B^{T}$ can be derived from Tables 1 and 2. The results are summarized in Table 3. Comparison of graphs and tables between the fixed $T$ case and the free $T$ case seems to tell us the importance of commitment and future prospects. When $T$ is fixed, the benefit of sophistication was negative as bad future prospect realizes itself. The naive social planners do better than the sophisticated ones, because the optimism about the future unintentionally made them to commit to conserve the resource.


Figure 3: Scheduled time $\hat{T}(x(t) ; \hat{\rho})$ of the end of extraction and actual time $t$ elapsed for different types of social planners. The horizontal axis is the calendar time $t$ and the vertical axis is the scheduled time of completion at $t$, which is constant for time-consistent (i.e. rational and sophisticated) social planners.


Figure 4: Graph of $x(t)$ for different types of social planners. $T$ is free.


Figure 5: Graph of $x(t)$ for different types of social planners. $T$ is free. This is the same graph as Figure 4, but the domain is more focused.


Figure 6: Graph of $u(t)$ for different types of social planners. $T$ is free.


Figure 7: Graph of $u(t)$ for different types of social planners. $T$ is free. This is the same graph as Figure 6, but the domain is more focused.

Table 3: Measures of the Benefits of Environmental Education when $T$ is fixed and $T$ is free.

| Type of <br> Social Planner | $T$ Fixed |  |  | $T$ Free |  |  |
| :--- | ---: | :---: | :---: | :---: | :---: | :---: |
|  | $B^{S}$ | $B^{R}$ | $B^{T}$ | $B^{S}$ | $B^{R}$ | $B^{T}$ |
| R | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| N1 | -0.0002 | 0.0014 | 0.0012 | 0.2945 | 0.6969 | 0.9914 |
| N2 | -0.0039 | 0.0133 | 0.0094 | 0.3498 | 1.4384 | 1.7881 |
| N3 | -0.0297 | 0.0683 | 0.0385 | 0.1846 | 1.9643 | 2.1489 |
| S1 | 0.0000 | 0.0014 | 0.0014 | 0.0000 | 0.6969 | 0.6969 |
| S2 | 0.0000 | 0.0133 | 0.0133 | 0.0000 | 1.4384 | 1.4384 |
| S3 | 0.0000 | 0.0683 | 0.0683 | 0.0000 | 1.9643 | 1.9643 |
| P1 | -0.0019 | 0.0133 | 0.0114 | 0.0436 | 1.4384 | 1.4820 |
| P2 | -0.0224 | 0.0683 | 0.0458 | 0.0354 | 1.9643 | 1.9997 |
| P3 | -0.0100 | 0.0683 | 0.0583 | 0.0037 | 1.9643 | 1.9679 |

When there is enough freedom in the part of social planners, gnothiseauton (know thyself) is a good motto. However, it is not so when the current action binds the future action, as was the case in the fixed $T$ case of Section 8. We shall see a similar case in Section 9. The fact that the benefits of sophistication is negative when $T$ is fixed and positive when $T$ is fixed further motivates the exploration of the property of $B^{S}$. In the next section, we present an example in which the end of the planning period is free and $B^{S}$ is negative for some range of $\hat{\rho}$.

## 9 Example 2: One-time Timber Harvest

Let us consider another example of one-time timber harvest problem. The resource is renewable this time. This section provides another example in which sophistication can harm the individual. We assume a forester makes
an investment of $k(\geq 0)$ at and only at $t=0$. Once the investment is made, the timber grows as the time elapses. At time $t$, it will be worth $e^{2 \sqrt{k t}-k^{2}}$. Though the choice of this functional form is for the convenience of illustration, it captures an important feature often found in ecological systems. The resource increases faster than the exponential function initially, but the speed of growth gets slower than the exponential function.

The forester can harvest the timber any time after the investment is made. To simplify the problem, the forester only has the choice between all or nothing. In other words, he can either keep or clear the forest, but he cannot, for example, harvest half the forest and keep the other half. He has the rational discount factor of $e^{-\gamma t}$. As with previous examples, let us start with a rational forester.

The lifetime utility maximization problem he faces is:

$$
\begin{equation*}
\max _{k, t}\left(\exp \left(2 \sqrt{k t}-\gamma t-k^{2}\right)\right) \tag{57}
\end{equation*}
$$

Now suppose that the initial investment $k$ is made. Then, the optimal timing at which the forest is harvested is, by maximizing the expression in Equation (57), $t_{0}(k ; \gamma)=\frac{k}{\gamma^{2}}$. Hence a rational forester chooses $k$ by solving the following equation:

$$
\begin{equation*}
\max _{k}\left(\exp \left(2 \sqrt{k t_{0}(k ; \gamma)}-\gamma t_{0}(k ; \gamma)-k^{2}\right)\right) \tag{58}
\end{equation*}
$$

By solving Equation (58), we have the rational forester's initial investment $k^{R}(\gamma)$ and the time of harvest $t^{R}(\gamma)$.

$$
\begin{align*}
k^{R}(\gamma) & =\frac{1}{2 \gamma} \\
t^{R}(\gamma) & =\frac{1}{2 \gamma^{3}} \tag{59}
\end{align*}
$$

Therefore, the rational forester's lifetime utility is

$$
\begin{equation*}
V^{R}=\exp \left(-\frac{1}{4 \gamma^{3}}\right) \tag{60}
\end{equation*}
$$

The solution for a sophisticated forester with his decision discount factor of $e^{-\rho t}$ can be obtained by replacing $\gamma$ by $\rho$ in Equation (59). We assume $\gamma \leq \rho$. The sophisticated forester's lifetime utility is, therefore,

$$
\begin{equation*}
V^{S}=\exp \left(\frac{3 \rho-4 \gamma}{4 \rho^{3}}\right) \tag{61}
\end{equation*}
$$

Now, let us consider a partially sophisticated and partially naive forester with his decision discount factor of $e^{-\rho t}$ and his perceived discount factor of $e^{-\hat{\rho t}}$. We assume $\gamma \leq \hat{\rho} \leq \rho$. At $t=0$, he thinks he will behave like a completely sophisticated harvester with the decision discount factor $e^{-\hat{\rho} t}$. Hence, he thinks he will harvest at $\hat{t}_{0}^{P}(k)=k / \hat{\rho}^{2}$ if he invests $k$ now. The problem he is solving is, therefore,

$$
\begin{align*}
& \max _{k}\left(\exp \left(2 \sqrt{k \hat{t}_{0}^{P}(k)}-\rho \hat{t}_{0}^{P}(k)-k^{2}\right)\right) \\
= & \max _{k}\left(\exp \left(\frac{2 k}{\hat{\rho}}-\frac{k \rho}{\hat{\rho}^{2}}-k^{2}\right)\right) \tag{62}
\end{align*}
$$

Solving this, we obtain his initial investment $k^{P}(\rho, \hat{\rho})$.

$$
\begin{equation*}
k^{P}=\frac{2 \hat{\rho}-\rho}{2 \hat{\rho}^{2}} \cdot \operatorname{Ind}(2 \hat{\rho}-\rho>0) \tag{63}
\end{equation*}
$$

If $2 \hat{\rho}-\rho \leq 0, k^{P}=0$ and he clears the forest at $t=0$. and he gains the lifetime utility of 1 . Hereafter, we shall assume $2 \hat{\rho}-\rho>0$. Once investment is made, he decide at each point in time whether to harvest the forest. When he keeps the forest, he thinks he will clear the forest at $\hat{t}^{P}=\hat{t}_{0}\left(k^{P} ; \hat{\rho}\right)$. Hence, he clears the forest when his perceived utility from keeping the forest gets smaller than his present-biased utility from clearing forest immediately. Hence the following equation must be satisfied.

$$
\begin{align*}
& \exp \left(2 \sqrt{k^{P} t}-\left(k^{P}\right)^{2}\right)>\exp \left(2 \sqrt{k \hat{t}}-\rho\left(\hat{t}^{P}-t\right)-k^{P}\right) \\
\Longleftrightarrow & 0>\left(\sqrt{t}-\left(-\sqrt{\hat{t}^{P}}+\frac{2 \sqrt{k^{P}}}{\rho}\right)\right) \cdot\left(\sqrt{t}-\sqrt{\hat{t}^{P}}\right) \\
\Longleftrightarrow & (0<)-\sqrt{\hat{t}^{P}}+\frac{2 \sqrt{k^{P}}}{\rho}<t<\sqrt{\hat{t}^{P}} \tag{64}
\end{align*}
$$

Therefore, he harvests at

$$
\begin{align*}
t^{P} & =-\sqrt{\hat{t}^{P}}+\frac{2 \sqrt{k^{P}}}{\rho} \\
& =\frac{1}{\hat{\rho}}\left(\frac{2 \hat{\rho}-\rho}{\rho \hat{\rho}}\right) \sqrt{\frac{2 \hat{\rho}-\rho}{2}} \tag{65}
\end{align*}
$$

Hence, his lifetime utility $V^{P}$ is

$$
\begin{align*}
V^{P} & =\exp \left(2 \sqrt{k^{P} t^{P}}-\gamma t^{P}-\left(k^{P}\right)^{2}\right) \\
& =\exp \left(\frac{(2 \hat{\rho}-\rho)^{2}}{\hat{\rho}^{3}} \cdot \frac{4 \hat{\rho}^{2}-4 \gamma(2 \hat{\rho}-\rho)-\rho \hat{\rho}}{4 \rho \hat{\rho}}\right) \tag{66}
\end{align*}
$$

In particular, the lifetime utility for naive individuals are

$$
\begin{equation*}
V^{N}=\exp \left(\frac{(2 \gamma-\rho)^{2}}{\gamma^{3}} \cdot \frac{3 \rho-4 \gamma}{4 \rho \gamma}\right) \tag{67}
\end{equation*}
$$

We are in a position to derive the benefits of environmental education. Using Equation (26), we plotted the benefit of sophistication against $\gamma$ for a given value of $\rho$. The graph shows that there is some range of $\gamma$ for while $B^{S}$ takes negative (Figures 8-10). Hence, sophistication does harm the forester under some circumstances. However, it should also be noted that the harm sophistication can cause is quite small in magnitude.

## 10 Summary and Discussion

As the examples in Sections 8 and 9 show, we can measure the benefits of environmental education using the concepts of decision and perceived discount factor. We derived Equation (25), which is the Hamilton-Jacobi-Bellman equation equivalent in the sub-rational settings. We identified two kinds of benefits of environmental education, namely one from sophistication and one from rationalization. The total benefits and benefit from rationalization are always positive, but we constructed examples where sophistication actually harms. Though we emphasized the importance of the fact that sophistication


Figure 8: The benefit of sophistication $B^{S}$ versus $\gamma$ for $\rho=2.0$. Full domain of $\gamma \in[0,2]$ is shown.


Figure 9: The benefit of sophistication $B^{S}$ versus $\gamma$ for $\rho=2.0$. This graph is the same as Figure 8 but the scale and the domain are different.


Figure 10: The benefit of sophistication $B^{S}$ versus $\gamma$ for $\rho=0.5$.
can harm, our intention is not to downplay the importance of sophistication. Rather, it is that the future prospect does matter a lot in the determination of the current behavior.

The analysis provided in this paper opens up a new horizon of research. There are a number of research topics that may derive from Equation (25). For example, including stochastic component is one of the natural ways of extension. As we have seen, $\gamma$ discounting rests crucially on the uncertainty associated with the discount factor. It would also be plausible that $\beta$ and/or $\hat{\beta}$ are stochastic.

From the perspective of environmental management in developing countries, it would be also important to design mechanisms that would enhance the lifetime utility of the social planner. As examples suggest, when a subrational social planner can make a commitment for the future, it may help her to do better . In the context of global environmental negotiations, the rise and fall of Kyoto protocol is understood as a commitment device for the future, which failed as there was no enforcement mechanism. O'Donoghue and Rabin (1999b) provides some useful insights to the incentive problems, but more research will be needed to come up with a better mechanism to tackle the issues of environment and development.

Finallny, we can extend the argument in Section 7 to a game-theoretic framework. It would be interesting to look at how the implications for the tragedy of commons may be altered when there are a number of sub-rational agents and the resource has the common, or open access, property.

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[^0]:    *Ph D Candidate, Department of Agricultural and Resource Economics. University of California at Berkeley / John L. Simpson Memorial Research Fellow. Email: fujii@are.berkeley.edu.

[^1]:    ${ }^{1}$ Rabin (2002) provides a perspective on the recent development in behavioral economics, which is based on psychological evidence. An excellent book edited by Kahneman and Tversky, eds (2000) offers a good introduction to self-control problems and other related issues. See also O'Donoghue and Rabin (2001).

[^2]:    ${ }^{2}$ See Perman et al. (1996) for derivation

[^3]:    ${ }^{3}$ This section is based on Price (1993).

[^4]:    ${ }^{4}$ This section is based on Heal (1998a,b).

[^5]:    ${ }^{5}$ Usually two parameters $\alpha$ and $\beta$ are used to describe the gamma distribution function. But the mean and variance give the same information as those two parameters. Since the mean and variance have more economically intuitive meaning the mean and variance are mainly used in his paper

[^6]:    ${ }^{6}$ There have been a number of applications of $\beta-\delta$ preference. See the cited references in O'Donoghue and Rabin (1999a).

[^7]:    ${ }^{7}$ As long as the indicator of the environmental quality is expressed by a continuous variable, the subsequent argument holds. We shall come back to the interpretation of $x(t)$ later after introducing Equation (16).
    ${ }^{8} V$ is a functional. It depends on the whole consumption schedule over the planning period, and not the consumption at one point in time.
    ${ }^{9}$ However, the main point of the argument holds for $T$ free. In particular, Equation (49) and subsequent arguments in this section are valid when $T$ is free. In Section 8, we compare the cases when $T$ is fixed and $T$ is free.

[^8]:    ${ }^{10} \mathrm{We}$ shall hereafter interpret in this way. because it is closest to the standard economic assumption. However, it is reasonable to assume that the lifetime utility is dependent on the past levels of consumption. No uncertainty is assumed in this study.

[^9]:    ${ }^{11}$ The superscript $R$ denotes the rational social planner.
    ${ }^{12}$ For further discussion of the definitions of present-biased preference, see O'Donoghue and Rabin (1999a) and citations therein.

[^10]:    ${ }^{13}$ The superscript $N$ denotes the native social planner.

[^11]:    ${ }^{14}$ The superscript $S$ denotes the sophisticated social planner.

[^12]:    ${ }^{15}$ The superscript $P$ is used to denote partially sophisticated and partially naive social planners

[^13]:    ${ }^{16}$ See, for example, Kaminen and Schwartz (1991, pp.259-261) for the argument in the standard setting.

[^14]:    ${ }^{17}$ We shall implicitly assume $\rho \neq 2 \hat{\rho}$. But all the results hereafter hold as a limiting case even when $\rho=2 \hat{\rho}$.

