Fishery Management Under Stock Uncertainty

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Abstract

Clark and Kirkwood [2] derive the optimal policy for harvesting fish when stocks are unknown in the special case where the density of present stocks is uniform. This result is important since it embodies the state of the art in the stochastic resource management literature. The objective of this paper is twofold. First, certain important properties of this result are developed. The optimal management of a fishery with stock uncertainty calls for non-monotonic and non-intuitive expected escapement. Clark and Kirkwood attribute this to the exclusion of existence value from their maximization exercise. The following exercise incorporates existence value into the manager's maximization problem and shows that this leads to, in expectation, lower catch quotas. It is also shown that incorporating existence value into the optimization program only increases the probability of obtaining a non-monotonic result for certain stock levels. For other stocks, it's inclusion has no effect whatsoever and the optimal level of escapement remains negatively related to both expected recruitment and the degree of uncertainty.

1 Introduction

Fishery collapse is widely seen as an increasingly common phenomenon. Among other fisheries, concern has been raised in the context of Georges Bank fishery, the Grand Bank fishery off Newfoundland, the Northwest Atlantic fishery and several others in as diverse spots on the planet as the Indian Ocean, off the coast of Peru and the Mediterranean and the Black seas. Environmentalists and marine biologists have cited several causes for the phenomenon, including poaching, reduced recruitment levels in the face of environmental variability and a lack of political will to impose quotas that will ensure sustainability. As Alan F. Sinclair and Steven A. Murawski write, "Environmental change likely affected stock production, especially in northern waters, and may be influencing stock recovery, but no environmental factor alone can explain either the general decline in groundfish productivity since the 1950s or the precipitous decline in the 1990s. Species interactions such as predation and competition were negligible contributors to these declines. Almost without exception, fishing mortality rates have exceeded sustainable levels. Early warning signs of stress on groundfish populations included truncated age structures, altered growth rates, earlier sexual maturation, and increased variability in catches as fisheries became ever more dependent on the strength of incoming year-classes. Stock rebuilding may require a decade or more of harvest rates at or very near zero." [1]

Many scientists attribute widespread fisheries collapse to the increased uncertainty in marine environments, a phenomenon also noted in the quote above. There is increasing evidence that fisheries will be subject to greater variability in the near future. For example, the latest IPCC report states that global warming will result in increased variability of precipitation; more frequent, and more intense precipitation events, and perhaps more severe droughts. Further, a shift in the precipitation mix, to more rain and less snow, coupled with earlier runoff of snowmelt, may amplify the effect of the increase in variability of precipitation, by increasing the variability, over the course of a year, in runoff to streams and reservoirs [4].

This paper focuses on the implications of imperfect information for the management of fisheries. Even without the threat of increased variability, fishery planning is wrought with uncertainty. This may stem from imperfect knowledge of fish stocks, uncertainty about future growth, recruitment and the carrying capacity of reservoir or from imperfect observation of fish catch on the part of the planner. Thus, it is important that catch quotas announced by fishery planners are based on the cognizance of limited and imperfect information.

2 Background

The simplest economic model in the fisheries literature is based on full knowledge of fishery parameters and involves no future uncertainty. This model, presented in Conrad and Clark [3], provides a simple optimal harvesting decision rule: if the fishery is 'small' (implying it cannot affect the market price of fish through output variation), the optimal harvest is given by a "most rapid approach path (MRAP)". Under this rule, the regulator computes the optimal stock of fish. If the present stock happens to be higher than the optimal level, the regulator lets the harvest be the difference between actual stock and desired stock. If the present stock is less than optimal, fishing is disallowed in the present period and stock is allowed to build up. Thus, the movement toward the optimal stock level is quickest - hence the name MRAP. This policy implies that once fish stocks reach the target level, future harvest and escapement will remain constant, such that the stock remains at the optimal steady-state level.

2.1 Imperfect Knowledge of Future Recruitment

Clearly, the deterministic model is unrealistic. Economists have developed essentially two kinds of models that relate to optimal fishery policy under uncertainty. Reed [7] assumes that the regulator knows the present stock with accuracy but is faced with uncertain recruitment (presumably due to an uncertain environment) and hence uncertain future stocks. He shows that the optimal policy in this case is constant escapement: a fixed threshold number of fish is allowed to 'escape' every period.

This simple constant-escapement rule vanishes in the formulation of Clark and Kirkwood.

2.2 Imperfect Information on Current Stocks

Clark and Kirkwood [2] develop a variant to Reed's model. In their model, the regulator only knows the *previous* period's escapement level but *current* stocks are unknown due to uncertain recruitment. The implication of this model is that optimal (planned) escapement is a non-linear function of expected recruitment.[1] In other words, no simple rule can be formulated to describe optimal harvest or escapement based on past escapement or expected recruitment. The following section derives the optimal policy and explores its properties.

3 Optimal Policy Under Imperfect Stock Information

It is assumed that there is only one species of fish in a managed fishery. The price of each unit of fish is one and there is no cost of harvesting. It is also assumed that there is no utility or value from the fish stock *perse* and that the level of escapement can be measured accurately each period. Thus, the manager

[1] Planned escapement is simply the difference between expected recruitment and planned harvest.

does not know the level of recruitment at the beginning of the period but has full information on the level of escapement. The model is as follows:

$$\begin{aligned} x_{t+1} &= z_t G(s_t) \\ s_t &= x_t - h_t \\ h_t &= \min(q_t, x_t) \end{aligned} \tag{1}$$

where z is a random shock with mean 1, s is escapement, $G(.)$ is the stock-recruitment function, x is current stock, h is the catch or harvest and q is the quota announced by the planner. The objective of the planner is to maximize the expected discounted value of future harvests:

$$\max_{q_t \geq 0} E_x \left\{ \sum_0^{\infty} \alpha^t h_t \right\} \tag{2}$$

The dynamic programming equation of this problem is as follows.

$$J_t(s_{t-1}) = \max_{q \geq 0} E_x \left\{ \min(q_t, x_t) + \alpha J_{t+1}(x_t - \min(q_t, x_t)) | s_{t-1} \right\} \tag{3}$$

Since this problem is hard to solve analytically for a general density function, let us consider the case where z is uniformly distributed.

$$f(z) = \begin{cases} 1/(2\varepsilon) & \text{for } 1 - \varepsilon \leq z \leq 1 + \varepsilon \\ 0 & \text{elsewhere} \end{cases}$$

3.1 Derivation of Optimal Policy

Given the density of z, the expectation part of the RHS of (3) can be written as[2]

$$1/(2\varepsilon g) \int_{glo}^{ghi} h + \alpha J(x - h) dx$$

or

$$V(q, x|s) \equiv 1/(2\varepsilon g) \int_{glo}^{ghi} \min(q, x) + \alpha J(x - min(q, x)) dx \tag{4}$$

where $g(= G(s))$ is the mean of current stock x, and $glo(= (1 - \varepsilon)g)$ and $ghi(= (1 + \varepsilon)g)$ are its lower and upper bounds respectively. The planner's problem is to maximize the above expression by choosing the fishing quota q. It is assumed that V is concave, and under certain conditions (see section A.1 of the appendix) this implies that J is concave.

To solve the problem, the domain of x can be partitioned as $[0 \; glo]$ and $[glo \; ghi]$, each of which can be analyzed separately. This division is based on the observation that if the quota is chosen from the former interval, it will be smaller than or equal to the recruitment (or stock) with probability 1.

[2] The time subscripts have been dropped for expositional ease.

3.1.1 Choosing the Quota in the Safe Range

The problem in the interval $[0\ glo]$reduces to

$$1/(2\varepsilon g)\int_{glo}^{ghi} q + \alpha J(x-q)dx$$

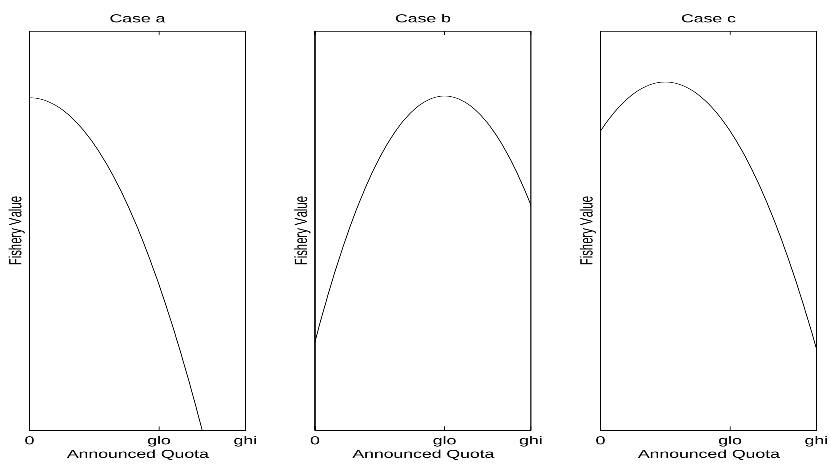
Using Leibniz's rule to take the derivative of this expression,

$$1/(2\varepsilon g)\int_{glo}^{ghi} 1 - \alpha J'(x-q)dx$$

which is

$$1/(2\varepsilon g)\left\{(ghi - glo) - \alpha[J(ghi-q) - J(glo-q)]\right\} \tag{5}$$

Given that the optimal quota lies in this partition, there are three possibilities:



a. The value function reaches its maximum at a quota q less than or equal to zero. In this case, (5) is non-positive in the neighbourhood of $q^* = 0$ which implies

$$2\varepsilon g/\alpha \leq J(ghi) - J(glo) \tag{6}$$

b. The value function reaches its maximum at glo. In this case, (5) = 0 in the neighbourhood $q^* = glo$ which implies

$$2\varepsilon g/\alpha = J(ghi - glo) \tag{7}$$

c. The value function reaches its maximum at a quota $q \epsilon\ (0\ glo)$. In this case,

$$J(ghi - q^*) - J(glo - q^*) = (ghi - glo)/\alpha = 2\varepsilon g/\alpha \tag{8}$$

holds.

3.1.2 Choosing the Quota in the Unsafe Range

If q lies in the sub-domain $[glo\ ghi]$, then the RHS of (3) is

$$1/(2\varepsilon g)\left\{\int_{glo}^{q} x dx + \int_{q}^{ghi} q + \alpha J(x-q)dx\right\}$$

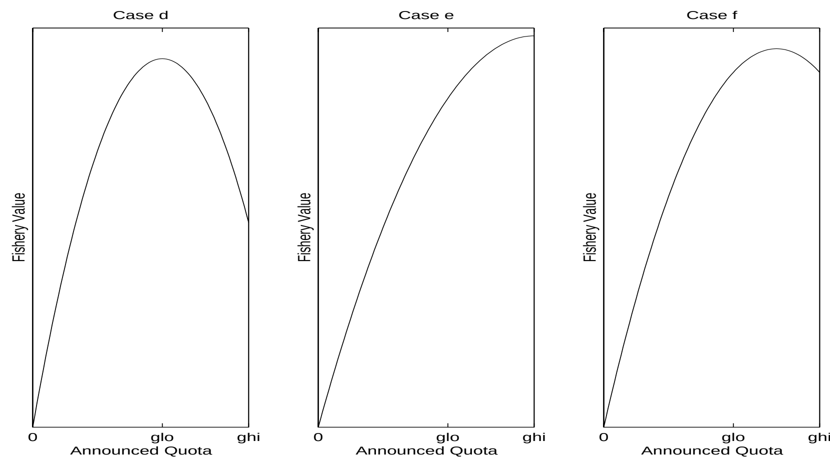
Using Leibniz's rule again to take the derivative of the above,

$$1/(2\varepsilon g)\left\{\int_{q}^{ghi} 1 - \alpha J'(x-q)dx\right\}$$

which is equal to

$$1/(2\varepsilon g)\left\{(ghi-q) - \alpha J(ghi-q)\right\} \tag{9}$$

Again, there are three possibilities.



d. The optimal quota q^* is equal to glo. This implies that in the neighbourhood of glo, (9) is zero

$$2\varepsilon g/\alpha = J(ghi - glo)$$

which, as expected, is identical to (7).
e. The optimal quota q^* is equal to ghi. This implies that in the neighbourhood of ghi, (9) is non-negative which, in turn, implies

$$J'(0) \leq 1/\alpha$$

f. The optimal quota q^* lies in $(glo\ ghi)$. In this case,

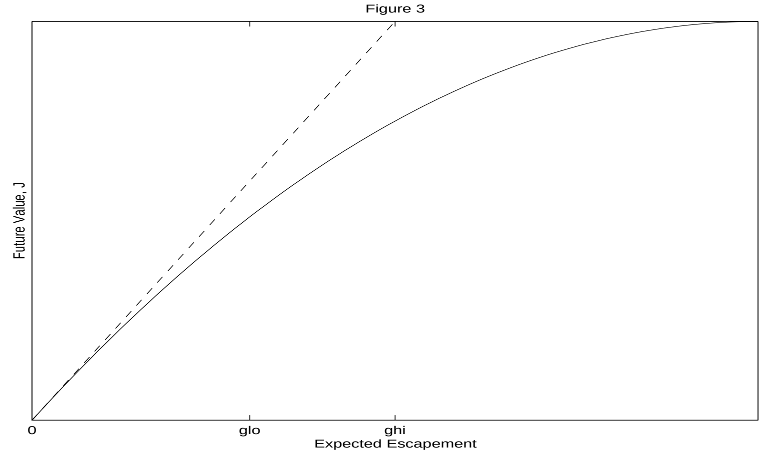
$$J(ghi - q^*) = (ghi - q^*)/\alpha \tag{10}$$

holds exactly. Summarizing the search algorithm for the optimal quota q^*: [3]

$$q^* = \begin{cases} 0 & \text{if } 2\varepsilon g/\alpha \leq J(ghi) - J(glo) \\ \text{solution to (8)} & \text{if } J(ghi) - J(glo) < 2\varepsilon g/\alpha < J(ghi - glo) \\ glo & \text{if } J(ghi - glo) = 2\varepsilon g/\alpha \\ \text{solution to (10)} & \text{if } J(ghi - glo) < 2\varepsilon g/\alpha \ \text{ and } \alpha J'(0) > 1 \\ ghi & \text{if } J(ghi - glo) < 2\varepsilon g/\alpha \ \text{ and } \alpha J'(0) \leq 1 \end{cases} \tag{11}$$

According to the algorithm above, if future stocks are not discounted or are discounted at an infinitesimally small rate (i.e. α is high), the optimal catch quota should zero. At the other extreme, if the future marginal value of the fishery is smaller than the gross discount rate $1+r$ even at very low stock levels, it is economically optimum to harvest the entire stock.

Figure 3



Graphically, if the slope of the dashed line in Figure 3 is less than $1 + r$, it is optimum to harvest the entire stock. When neither of these extreme conditions is true, the manager should locate $2\varepsilon g/\alpha$ relative to $J(ghi) - J(glo)$ and $J(ghi - glo)$ to announce the optimal quota.

3.2 Numerical Computation of the Optimal Policy

Note that in the algorithm above, the optimal quota is conditioned on the level of escapement in the previous period as well as the value function. Since the value function is unknown, how does one figure out the optimal quota for a known escapement? The trick is to employ an iterative process to search for both the optimal policy as well as the value function. The iteration can be described as follows:

- Make an arbitrary guess about the value function.

[3]Section A.2 of the appendix shows that $J(ghi) - J(glo) < J(ghi - glo)$ when $J(.)$ is concave.

- For each level of escapement, use the algorithm above to figure out the optimal quota.
- Substitute the optimal quota in the dynamic programming equation to arrive at a new estimate of the value function.
- If the new value function is 'close' to the original one, the iterative process ends; else, use the new value function and repeat the second and third steps.
- Iterate until convergence. Given the properties of the value function, the contraction mapping theorem guarantees convergence (see section A.3 of the appendix for a statement of this theorem).

4 Properties of the Optimal Policy

Given the roadmap provided by (11), the properties of the optimal quota q^* and correspondingly, of optimal expected escapement $e* \equiv g - q^*$ can be explored.

4.1 The Safe Range

In this section it is established that the optimal expected escapement is not monotonic in both expected recruitment and the level of uncertainty. The necessary and sufficient condition for optimal expected escapement to be monotonically rising in these two parameters are also derived. Finally, it is shown that optimal expected escapement is monotonically decreasing in the discount rate.

4.1.1 The Optimal Quota and Expected Recruitment

We need to compute $sgn\,\{(d(e^*)/dg\}$ over$(0\; glo)$ and $(glo\; ghi)$ where $e^* = g - q^*$ is optimal planned escapement. Totally differentiating (8) with respect to g and q^* and using the implicit function theorem,

$$J'(+)(1+\varepsilon)dg - J'(+)dq^* - J'(+)(1-\varepsilon)dg + J'(-)dq^* = 2\varepsilon/\alpha dg$$

where $(+)$ and $(-)$ refer to $(ghi - q^*)$ and $(glo - q^*)$ respectively. Rearranging terms

$$2\varepsilon dg - \alpha\left\{J'(+)(1+\varepsilon) - J'(+)(-1)dq^* + J'(-)(1-\varepsilon)dg + J'(-)(-1)dq^*\right\} = 0$$

Collecting terms,

$$dq^*/dg = 1 + \frac{\varepsilon}{\alpha\left(J'(-) - J'(+)\right)}\left(2 - \alpha J'(+) - \alpha J'(-)\right)$$

or that

$$dq^*/dg = 1 + \frac{2\varepsilon}{J'(-) - J'(+)}\left\{\frac{1}{\alpha} - \frac{J'(+) + J'(-)}{2}\right\}$$

This implies

$$de^*/dg = \frac{-2\varepsilon}{J'(-) - J'(+)} \left\{ \frac{1}{\alpha} - \frac{J'(+) + J'(-)}{2} \right\}$$

The denominator is positive by virtue of concavity of J. However, the sign of the term in the parenthesis is ambiguous. If that term is negative, one can assert that optimal expected escapement is increasing in expected recruitment. In other words,

$$\frac{J'(+) + J'(-)}{2} > \frac{1}{\alpha} = 1 + r$$

is both necessary and sufficient to claim $de^*/dg > 0$.

4.1.2 The Optimal Quota and the Degree of Variability

Totally differentiating (8) with respect to ε and q^*,

$$2gd\varepsilon - \alpha \{J'(+)gd\varepsilon - J'(+)dq^* + J'(-)gd\varepsilon + J'(-)dq^*\} = 0$$

which implies

$$dq^*/d\varepsilon = \frac{g}{\alpha\,(J'(-) - J'(+))} \{2 - \alpha J'(+) - \alpha J'(-)\}$$

or

$$dq^*/d\varepsilon = \frac{2g}{J'(-) - J'(+)} \left\{ \frac{1}{\alpha} - \frac{J'(+) + J'(-)}{2} \right\}$$

Again, the denominator is positive by concavity of J, but the term in the parenthesis is ambiguous. The ambiguity disappears if, as before,it is assumed that

$$\frac{J'(+) + J'(-)}{2} > \frac{1}{\alpha} = 1 + r$$

which makes $dq^*/d\varepsilon$ negative and $de^*/d\varepsilon$ positive (since $de^*/d\varepsilon = -dq^*/d\varepsilon$).

4.1.3 The Optimal Quota and the Discount Rate

Totally differentiating (8) with respect to q^* and α,

$$-(J(+) - J(-))d\alpha - \alpha(J'(+)(-1)dq^* - J'(-)(-1)dq^*) = 0$$

which implies

$$dq^*/d\alpha = -\frac{J(+) - J(-)}{\alpha(J'(-) - J'(+))}$$

The numerator of the RHS is positive (since J is increasing) and the denominator is also positive (since J is concave). Therefore, the LHS is negative. By implication, $de^*/d\alpha > 0$. Since α is inversely related to the discount rate r, the above inequality implies that the optimal expected escapement is decreasing in the discount rate.

4.2 The Unsafe Range

This section explores the properties of the optimal program in the unsafe range.

4.2.1 The Optimal Quota and Expected Recruitment

Computing the derivative of q^* with respect to g for $(glo\ ghi)$

$$((1+\varepsilon)dg - dq^*) - \alpha J'(+)((1+\varepsilon)dg - dq^*) = 0$$

Totally differentiating (10)

$$((1+\varepsilon)dg - dq^*) - \alpha J'(+)((1+\varepsilon)dg - dq^*) = 0$$

which implies

$$((1+\varepsilon)dg - dq^*)(1 - \alpha J'(+)) = 0 \tag{12}$$

Since (12) holds across the entire sub-domain $(glo\ ghi)$,

$$(1+\varepsilon)dg - dq^* = 0$$

which implies

$$dq^*/dg = (1+\varepsilon)$$

Thus,

$$de^*/dg = -\varepsilon < 0$$

Thus when the optimal quota is between glo and ghi, optimal expected escapement is *decreasing* in expected recruitment.

4.2.2 The Optimal Quota and the Degree of Variability

Totally differentiating (10) with respect to ε and q^*,

$$gd\varepsilon - dq^* + \alpha\left\{J'(+)gd\varepsilon - J'(+)dq^*\right\} = 0$$

This implies

$$(gd\varepsilon - dq^*)(1 + \alpha J'(+)) = 0$$

or that

$$dq^*/d\varepsilon = g$$

Therefore

$$de^*/d\varepsilon = -g < 0$$

The above result shows that in the unsafe range the optimal expected escapement is *decreasing* in the level of uncertainty: the higher the uncertainty, the smaller the optimal expected escapement.

4.2.3 The Optimal Quota and the Discount Rate

Lastly, the sign of $dq^*/d\alpha$ in the range $(glo\ ghi)$ needs to be computed. Totally differentiating (10) with respect to q^* and α,

$$-dq^* - J(+)d\alpha - \alpha J'(+)(-1)dq^* = 0$$

Rearranging terms,

$$dq^*/d\alpha = -\frac{J(+)}{1 - \alpha J'(+)}$$

To sign the above, one can differentiate V twice with respect to q in the unsafe range.

$$V_{qq} = 1/2\varepsilon g[-1 + \alpha J'(+)] < 0$$

by concavity of $V(.)$. This implies $dq^*/d\alpha < 0$. Therefore, optimal expected escapement is decreasing in the discount rate.

4.3 Summary of Properties

As the results of this section show, optimal expected escapement is not monotonic in expected recruitment and the level of uncertainty in the safe range. These properties, as well as the others derived above, can be summarized as follows.

	Safe Range	Unsafe Range
Expected Recruitment	?	-
Level of Uncertainty	?	-
Discount Rate	-	-

However, the condition

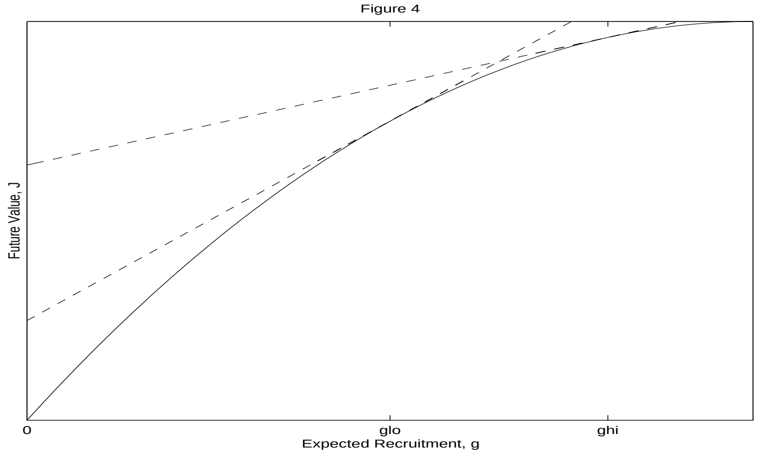
$$\frac{J'(+) + J'(-)}{2} > \frac{1}{\alpha} = 1 + r$$

is necessary and sufficient to resolve the ambiguity. Also, if this condition is assumed, the results are consistent with one's intuition. How may one interpret the above condition graphically? Since q can be infinitesimally small, the condition

$$\frac{J'(ghi) + J'(glo)}{2} > \frac{1}{\alpha} = 1 + r$$

is sufficient to guarantee monotonicity.

Figure 4



Graphically, this means that the *average* of the slopes at glo and ghi of the $J(.)$ function must be more than the gross discount rate. In other words, if the fishery is sufficiently productive in terms of its marginal future value at stock levels of glo and ghi, intuitive results follow and monotonicity is assured.

In the unsafe range, the comparative statics results with respect to both expected recruitment and the level of uncertainty are counter-intuitive. These may be justified as follows. When either of these parameters is high, the potential future loss in terms of stock lost due to a bad shock is correspondingly high. Therefore in such a situation, it makes sense to harvest a disproportionately greater amount of the stock in the present period. In their paper, Clark and Kirkwood surmise that this non-intuitive result is caused by the exclusion of existence value. In the next section it is shown that adding existence value to the fishery manager's problem increases the probability of monotonicity (and intuitive results) in the safe range, but has no effect at all in the unsafe range.

5 Incorporating Existence Value

It is assumed that the existence of fish stock every period has a positive preservation value given by a function $E(.)$ where $E(0) = 0$, $E'(.) > 0$ and $E''(.) < 0$.

5.1 Optimal Quota in the Safe Range

With the inclusion of the function $E(.)$, the maximization problem in the safe range becomes:

$$1/(2\varepsilon g) \int_{glo}^{ghi} q + \alpha E(x-q) + \alpha J(x-q) dx$$

Using Leibniz's rule to take the derivative of this expression,

$$1/(2\varepsilon g)\int_{glo}^{ghi} 1-\alpha E'(x-q)-\alpha J'(x-q)dx$$

or

$$V_q = 1/(2\varepsilon g)\left\{(ghi-glo)-\alpha[E(ghi-q)-E(glo-q)]-\alpha[J(ghi-q)-J(glo-q)]\right\}$$

Let q^{**} solve $V_q = 0$. By 8,

$$V_q(q^*) = -\alpha/(2\varepsilon g).(E(ghi-q)-E(glo-q)) < 0$$

since $E'(.) > 0$. Also, $V_{qq} < 0$ by the concavity of V. Thus $q^{**} < q^*$. In other words, the optimal quota is smaller when existence value is positive.

5.1.1 Effect of Existence Value on Comparative Statics

Defining $e^{**} \equiv g - q^{**}$, one can use the implicit function theorem on the first order condition implied by $V_q = 0$ to get the following results.

$$de^{**}/dg = \frac{-2\varepsilon}{\underbrace{J'(-)-J'(+)}+\underbrace{E'(-)-E'(+)}}\left\{\frac{1}{\alpha}-\frac{\underbrace{J'(+)+J'(-)}+\underbrace{E'(+)+E'(-)}}{2}\right\}$$

$$de^{**}/d\varepsilon = \frac{-2g}{\underbrace{J'(-)-J'(+)}+\underbrace{E'(-)-E'(+)}}\left\{\frac{1}{\alpha}-\frac{\underbrace{J'(+)+J'(-)}+\underbrace{E'(+)+E'(-)}}{2}\right\}$$

The necessary and sufficient condition needed for the above two expressions to be positive is

$$\frac{J'(+)+J'(-)}{2}+\frac{E'(+)+E'(-)}{2} > 1+r$$

Since $E'(.) > 0$, the addition of the last two terms on the LHS increases the likelihood of monotonicity but does not guarantee it. Also,

$$de^{**}/d\alpha = \frac{\underbrace{J(+)-J(-)}+\underbrace{E(+)-E(-)}}{\alpha(\underbrace{J'(-)-J'(+)}+\underbrace{E'(-)-E'(+)})}$$

Since all the underbraced terms are positive by the properties of $J(.)$ and $E(.)$, optimal expected escapement increases with the discount factor, α.

5.2 Optimal Quota in the Unsafe Range

In the unsafe range, the problem of the fishery manager is to maximize

$$1/(2\varepsilon g)\left\{\int_{glo}^{q} xdx+\int_{q}^{ghi} q+\alpha E(x-q)+\alpha J(x-q)dx\right\}$$

Using Leibniz's rule again to take the derivative of the above,

$$1/(2\varepsilon g)\left\{\int_{q}^{ghi} 1-\alpha E'(x-q)-\alpha J'(x-q)dx\right\}$$

or that

$$V_q = 1/(2\varepsilon g)\left\{(ghi-q)-\alpha E(ghi-q)-\alpha J(ghi-q)\right\}$$

Let q^{**} solve $V_q = 0$. By 10,

$$V_q(q^*) = -\alpha/(2\varepsilon g).E(ghi-q) < 0$$

Again, $V_{qq} < 0$ by the concavity of V. Thus $q^{**} < q^*$.

5.2.1 Effect of Existence Value on Comparative Statics

Again, defining $e^{**} \equiv g - q^{**}$, and using the implicit function theorem on the first order condition implied by $V_q = 0$,

$$de^{**}/dg = -\varepsilon < 0$$

and

$$de^{**}/d\varepsilon = -g < 0$$

These results show that the inclusion of existence value has no impact on the comparative static results obtained earlier. Even if fish stock is valued in itself, the optimal expected escapement is decreasing in both expected recruitment and the level of uncertainty. Finally,

$$de^{**}/d\alpha = \frac{J(+)+E(+)}{1-\alpha J'(+)-\alpha E'(+)} > 0$$

which implies that if future stocks are discounted at higher rates, it is optimal to leave fewer fish in the present period.

6 Concluding Remarks

In this paper the properties of the optimal policy for the special case where the density of present stocks is uniform is explored. The results show that for certain model parameters, these properties are not intuitive. The necessary and sufficient conditions for the intuitive result in the safe range have also been derived. However, the comparative statics results are unambiguously non-intuitive in the safe range except with respect to the discount rate. Incorporating existence value into the manager's maximization problem leads to, in expectation, lower catch quotas. The results imply that incorporating existence value into the optimization program only increases the probability of obtaining a non-monotonic result in the safe range; in the unsafe range, the addition of existence value has no effect whatsoever and the optimal level of escapement remains negatively related to both expected recruitment and the degree of uncertainty.

In order to derive the results, the above model makes a number of simplifying assumptions. Like Reed's model, it is characterized by a single source of stochasticity. Real world fisheries, on the other hand, are subject to multiple shocks. It seems natural, therefore, to incorporate at least some of the sources of randomness in an economic model and solve for its optimum. Second, the results above are consistent only with uniformly distributed shocks. This was done for analytical convenience. It may be of interest to derive the optimal policy and study its properties for shocks stemming from other distributions, especially those from normal and log normal densities.

A Appendix

A.1 Concavity of J

It is useful to investigate whether concavity of $V(.)$implies concavity of $J(.)$. First assume that the quota q is in the "safe" range. Define

$$M \equiv q + \alpha/(2\varepsilon g)\int_{glo}^{ghi} J(x-q)dx$$

This implies

$$M_q = 1 - \alpha/(2\varepsilon g)\int_{glo}^{ghi} J'(x-q)dx$$

or

$$M_q = 1 - \alpha/(2\varepsilon g)(J(ghi-q) - J(glo-q))$$

Differentiating M w.r.t. q once again,

$$M_{qq} = \alpha/(2\varepsilon g)(J'(ghi-q) - J'(glo-q)) < 0$$

since M is assumed to be concave in q. This implies

$$J'(ghi-q) - J'(glo-q) < 0$$

or that J(.) is concave in q since ghi is greater than glo.

Now assume that the quota is in the "unsafe" range. Define

$$M \equiv 1/(2\varepsilon g)[\int_{glo}^{q} xdx + \int_{q}^{ghi} q + \alpha J(x-q)dx]$$

This implies

$$M_q = 1/(2\varepsilon g)[(ghi-q) - \alpha J(ghi-q)]$$

Differentiating M w.r.t. q twice,

$$M_{qqq} = -\alpha J''(ghi-q)$$

For $J(.)$ to be concave, the third derivative of M with respect to q is needed to be positive.

A.2 Another Property of J

In this section, another property of J is explored viz. the relationship between $J(ghi) - J(glo)$ and $J(ghi-glo)$. By concavity of J,

$$\forall t > 0 : \int_{0}^{ghi-glo} J'(s+t)ds < \int_{0}^{ghi-glo} J'(s)ds$$

Substituting the variable of integration $s + t$ by z,

$$\int_{glo}^{ghi} J'(z)dz < \int_{0}^{ghi-glo} J'(s)ds$$

This implies

$$J(ghi) - J(glo) < J(ghi - glo) - J(0)$$

Since $J(0)$ equals zero by definition,

$$J(ghi) - J(glo) < J(ghi - glo)$$

A.3 The Contraction Mapping theorem

The following statement is adapted from [5].

Definition: A map $T : Y \rightarrow Z$ on ordered spaces Y and Z is monotone if and only if $y_1 \geq y_2$ implies $Ty_1 \geq Ty_2$.

Definition: A map $T : Y \rightarrow Y$ on a metric space Y is a contraction with modulus $\beta < 1$ if and only if $\| Ty_1 - Ty_2 \| \leq \beta \| y_1 - y_2 \|$.

Contraction Mapping theorem: If X is compact, $\beta < 1$, and Π is bounded above and below, then the map

$$V(x_0) \equiv \max_{u \epsilon U(x)} E\{\Pi(x_0, u_0)\} + E\{\beta V(x_t|x_0, u_0)\}$$

is monotone in V, and is a contraction mapping with modulus β in the space of bounded functions and has a unique fixed point.

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