

# Fishery Management Under Multiple Uncertainty

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## Abstract

Explanations of worldwide fishery collapse are numerous. There is general consensus that variability and uncertainty play important roles. Roughgarden and Smith (1996) argue that three source of error are important for fisheries management: variability in fish dynamics, inaccurate stock size estimates, and inaccurate implementation of harvest quotas. We develop a bioeconomic model with these three sources of error, and solve for optimal harvest “announcements” based on a measurement of fish stock in a discrete-time model. Among other results we find: (1) when errors are large, we generally reject the “constant escapement” rule advocated by much of the existing literature, (2) Inaccurate stock estimation affects policy in a fundamentally different way than the other error sources, and (3) When existence value is included, management is much more conservative, substantially increasing stock viability through time.

## 1 Introduction

Fishery collapse is an increasingly common phenomenon worldwide. Though the Gordon-Schaeffer type models suggest management can overcome the economic (and indeed biological) deficiencies of unregulated fisheries, many managed fisheries have collapsed. Suggested causes include habitat destruction, reduced recruitment levels in the face of environmental variability, fishery overcapitalization, and a lack of political will to impose quotas that will ensure sustainability (Johnson and Libecap 1982; Dupont 1990; Homans and Wilen 1997; Wilen 2000). Despite differences in fisheries spatially and temporally, excessive harvest rates are nearly universally accepted as a major contributor to declines.

Others have noted that the inherent problem of over-fishing is exacerbated by uncertainty in fish stock size and dynamics (Roughgarden and Smith 1996). This combination is thought to be largely responsible for the recent decline (and closure) of the pacific ground-fish fishery (cite). In the past, some have attributed collapses to uncertainty in marine environments; correctly suggesting that ignoring uncertainty can lead to excessive harvest.

This paper focuses on the implications of environmental and managerial uncertainty for the management of fisheries. Even without the threat of increased variability from phenomena like climate change, fishery planning is hard. The theories put forth by the biological community suggests that three sources of uncertainty are important in determination of fishery policy. Environmental variability shocks fish growth, subsequent stock measurement is inaccurate, and once a total harvest is announced, it is implemented inaccurately. This paper frames and solves this fishery management problem under uncertainty. Among other implications for fishery policy, we find that measurement error has the greatest potential to affect policy. We also find a general rejection (with sufficiently large uncertainty) of the “constant escapement” policy suggested by deterministic models (Gordon 1954), models of only growth uncertainty (Reed 1979), and other more heuristic approaches to fishery modeling from the biological community (Roughgarden and Smith 1996).

The outline is as follows. In section 2 we provide a background of economic and biological models of fishery management under uncertainty. We then turn to our approach of integrating the two types of models, discussing its relevance in section 3. The formal characterization is presented in section 4, and results in section 5. We then extend results to include the possibility of existence value in section 6 and conclude in section 7.

## 2 Background

There exists a large economic and biological modeling literature addressing management of renewable resources under uncertainty. Economists pose allocation questions, pushing biological realism only to the extent to which it permits clean analytical solutions. Biological modelers often criticize such economic models for their inadequate treatment of biological uncertainties. Although they incorporate more biological realism, biological models are often criticized by economists because they either ignore economic principles (harvest costs, prices, or time preference), or they fail to solve for an optimal allocation. We briefly describe the literature in each discipline, and then present our approach of integrating the important elements of each into a stochastic dynamic decision-making framework.

### 2.1 Economic Models

All analytical models in the economics literature use one source of uncertainty for tractability. The two important results that relate to fishery management under uncertainty are due to Reed (1979) and Clark and Kirkwood (1986).

### 2.1.1 Future Stock Uncertainty

Reed (1979) notes that although previous deterministic models of fish growth are easily understood and solved, they ignore the important reality that the environment is subject to stochastic fluctuations. Reed assumes the stock of fish from one period to the next is governed by a deterministic, “compensatory” growth function, but that a random multiplicative shock disturbs this growth every period. Despite perfect knowledge of the size of the fish stock left to spawn after harvesting (“escapement”), future stocks are unknown due to the random disturbance. Reed assumes a constant price per unit harvest, and a marginal cost function which decreases in stock. Letting  $\alpha$  be the discount factor and  $E_x$  be the expectation operator over stock,  $x$ , Reed solves:

$$\max_{\{h_t\} \geq 0} E_x \left\{ \sum_0^{\infty} \alpha^t (ph_t - h_t C(x_t)) \right\}$$

subject to

$$\begin{aligned} x_{t+1} &= z_{t+1}^g G(s_t) \\ s_t &= x_t - h_t \end{aligned}$$

where  $p$  is the price,  $h$  is the harvest,  $s$  is the escapement,  $C$  is the cost function and  $z^g$  is the growth shock.

Under the above model, Reed concludes that despite uncertainty about future stocks, the optimal policy is to allow a constant escapement every period, regardless of stock at the beginning of the period (provided the initial stock is larger than the desired escapement). Thus if the manager finds herself in the beginning of a period with very high stock (because of a large positive shock to previous period’s growth), she should enjoy a large current harvest down to the optimal escapement level. Should she find herself with a very small stock (caused by a deleterious previous period shock), she will only harvest a small amount. The intuition behind this policy is straightforward. The manager wishes to harvest to the point where the value of the last fish harvested is equal (in expectation) to the value of leaving it in the ocean, and harvesting the fish at a later time. As a consequence of the marginal profit in any period (price minus marginal cost) being independent of initial stock in that period, the value of the last fish harvested should always be the same, regardless of initial stock in the period. Thus, the manager should leave constant escapement every period. Since the stock of fish is known every period, management policy can never lead to accidental extinction in this model.

Costello, Adams, and Polasky (1998) extend Reed’s results by asking how management should respond to predictions of future environmental variability. They find that with a forecast of beneficial future conditions, harvest should be reduced in the current period, thus rejecting the “constant escapement” rule. Their result is largely driven by the fact

that fish stocks cannot accidentally go extinct.

### 2.1.2 Present Stock Uncertainty

Clark and Kirkwood (1986) modify Reed's model by noting that in addition to considerable uncertainty about future growth, fishery managers can rarely measure stock levels accurately. In fact, they note that stock assessments typically have confidence intervals of  $\pm 50\%$ . With this practical motivation in mind, they alter Reed's model by changing the timing of the shock. In their model, the manager knows the escapement in the previous period but is unsure about recruitment in the current period. In addition, Clark and Kirkwood eliminate fishing costs from the profit expression, noting that costs have no qualitative effect on results in which they are interested.

$$\max_{\{h_t\} \geq 0} E_x \left\{ \sum_0^{\infty} \alpha^t h_t \right\}$$

subject to

$$\begin{aligned} x_t &= z_t^g G(s_{t-1}) \\ s_t &= x_t - h_t \end{aligned}$$

Using their model, Clark and Kirkwood obtain results very different from those of Reed. When managers cannot perfectly measure current stock, the optimal policy is no longer one of constant escapement. Perhaps somewhat surprisingly, the optimal policy is less cautious (for some levels of expected initial stock) than the constant escapement policy (see figure 2). Furthermore, in some cases it turns out to be optimal to harvest the population to extinction. This result is in direct contradiction to the constant escapement policy which systematically guards the population from extinction every period.

### 2.1.3 Models Incorporating Existence Value

There is evidence of existence value (cite). Often biologist make an implicit assumption about this, but have not made it explicit. Some economists have included into models. Has the effect of increasing stocks.

## 2.2 Biological Models

Biologists have developed growth simulation models, which typically either do not solve for optimal management strategies or do not include multiple uncertainty sources. In other words, the biology literature has posed a realistic problem of fishery management problem under uncertainty but has not developed an optimal solution to this problem.

### 2.2.1 Solutions for Management With Multiple Uncertainty

The striking difference between the results of the above two models has sparked interest in the field about the importance of different types of uncertainty fishery policy. Recently, Roughgarden and Smith (1996) tackle this problem from a biological perspective. Their model is motivated by the danger facing many of the world's fisheries, and the belief that harvesting *as if* the resource growth and measurement is deterministic, when in fact it is stochastic, can quickly lead to unintended extinction. With this observation in mind, Roughgarden and Smith extend Reed's and Clark and Kirkwood's models by introducing two additional sources of uncertainty, stock measurement error and harvest implementation error. In their model the fishery manager enters the period and measures the stock with some error. She must then make a harvest announcement *knowing* that the true harvest will be imprecisely implemented.

The significant increase in complexity of the Roughgarden and Smith model as compared to the Clark and Kirkwood model precluded them from obtaining analytical results. They turn to numerical simulation to solve the problem, and conduct several experiments altering the levels of the three sources of uncertainty in order to provide policy recommendations. Although they frame their problem as a bio-economic problem, they solve it by "rule of thumb", locating the optimal "target stock" (constant escapement) rule given three sources of uncertainty. They recommend  $\frac{3}{4}$  of the carrying capacity,  $K$ , as the optimal target stock. The authors appropriately point out that the target stock may be set elsewhere (depending on the assumptions about the level of uncertainty), but that it should definitely be above the  $\frac{K}{2}$ , or maximum sustainable yield, level. Roughgarden and Smith recommend an escapement level significantly higher than the deterministic optimum (which lies slightly below  $\frac{K}{2}$ ) because the high stock creates a buffer away from extinction in the presence of multiple uncertainty.

An altogether different approach to fishery management under uncertainty is taken by Lauck et.al. (Lauck, Clark, Mangel, and Munro 1998). They propose the establishment of a protected marine reserve to deal with fluctuations in recruitment, imprecise stock surveys and imperfect implementation of seasonal quotas. Their results show that even small changes in catch variability has a significant impact on stock survival.

## 3 This Research

Both biology and economics are affected by uncertainty, and they combine to complicate optimal decision-making by the manager. Biologically, the importance of different forms of uncertainty has to do largely with extinction risk. Under the Reed model, the population

cannot go extinct, except by human design, because the manager always chooses escapement precisely. In the Clark and Kirkwood model, extinction is only possible in the unlikely case when there is such extreme miscalculation in stock measurement that the manager sets the harvest too high and drives them to extinction. Under assumptions in the Roughgarden and Smith model, extinction is much more likely because of the implementation error (now including the possibility of over-estimating the stock size, *and* over-harvesting in the same period). Economically, the importance of uncertainty enters in the optimal decision-making by the manager. As the number of sources of uncertainty increase, the manager must base her expectations on less precise information.

We solve for optimal fishery management under the multiple forms of uncertainty posed by Roughgarden and Smith (growth, measurement, and implementation). In the spirit of the questions posed by fishery modelers and managers, we address the following questions within the context of our multiple uncertainty model:

1. Given three sources of uncertainty (growth, measurement, implementation), possibly in addition to an intrinsic value on stock viability, how should the total allowable catch announcement depend on the stock measurement in any given period?
2. What are the implications of optimal management for stock survival through time?
3. How important are the individual sources of uncertainty in determining optimal management?
4. How should management optimally respond to changes in the magnitude of each source of uncertainty?
5. What is the effect (on all questions above) when existence value of stock *in situ* is included in the optimal stochastic dynamic decision framework?

The next section describes the model developed to answer these questions.

## 4 The Model

In this section we state our assumptions, formalize the optimization problem, and describe the solution technique. The general model and method below is useful for setting up and solving any stochastic dynamic programming problem with Markovian transitions<sup>1</sup>.

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<sup>1</sup>The Markovian property is satisfied when the density of the state next period depends only on the the current value of the state variable. In other words, realizations of the state in the past have no bearing on next period's state.

## 4.1 Assumptions

We make the following assumptions:

1. Let  $x_t$  be single fish population that is managed <sup>2</sup>. Harvest is  $h_t$  and escapement is  $s_t \equiv x_t - h_t$ .
2. There are three random shocks every period:  $z_t^g$ ,  $z_t^m$ , and  $z_t^i$ , which stand for growth, measurement, and implementation shocks, respectively. These shocks are independent. <sup>3</sup>
3. In each period, the manager knows escapement in the previous period with certainty. Fish stock growth is as follows:

$$x_t = z_t^g G(s_{t-1}) \quad (1)$$

where  $G(s_{t-1})$  gives the stock  $x_t$  in the deterministic case<sup>4</sup>.

4. Stock measurement,  $m_t$  is as follows:

$$m_t = z_t^m x_t \quad (2)$$

5. Given the announced quota,  $q_t$ , the harvest is given by:

$$h_t = z_t^i q_t \quad (3)$$

6. The price of fish is  $p$  and the unit harvest cost is constant,  $c$ . We normalize  $p - c$  to unity for the remainder of the analysis.

## 4.2 The Manager's Problem

Letting  $\alpha$  be the discount factor and  $E_x$  be the expectation operator over stock,  $x$ , the manager's infinite horizon problem can be stated as follows:

$$\max_{\{q_t\} \geq 0} E_x \left\{ \sum_0^{\infty} \alpha^t h_t \right\} \quad (4)$$

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<sup>2</sup>While this technique can handle multiple stocks, the focus of this research is centered on the impact of multiple uncertainty on fisheries management.

<sup>3</sup>In principle, one could include some covariance between shocks of different type. We do not explore shocks of that type here.

<sup>4</sup>This is identical to the way Reed (1979) shocks growth. It differs from Roughgarden and Smith, who shock the *parameters* of the growth function, which, in their formulation, amounts to a uniform shock to the carrying capacity each period.

subject to

$$\begin{aligned}
x_t &= z_t^g G(s_{t-1}) \\
s_t &= x_t - h_t \\
h_t &= \min(z_t^i q_t, x_t) \\
m_t &= z_t^m x_t
\end{aligned} \tag{5}$$

We use the numerical technique of value function iteration to solve 4. The fishery manager measures the stock each period (state variable) and announces the seasonal quota (control variable) on this basis. The dynamic programming equation for this problem is as follows:

$$J_t(m_t) = \max_{q_t \geq 0} E_x \{h_t + \alpha J_{t+1}(z_{t+1}^m z_{t+1}^g G(x_t - h_t) | m_t)\} \tag{6}$$

To numerically solve for the value function, we transform the model into a discrete state space and discrete control space dynamic programming problem. The support of the state and the control is the set of integers from zero to  $K(1 + z_g)$ . We then compute the fishery value for each announcement and each measurement. For a given measurement, the optimal announcement is the one that maximizes the fishery value.

The maximand in 6 has two terms. The first of these represents the value of the expected harvest in the current period. The second term is simply the expected value of the state the following period. We discuss below how each of the two terms above was numerically computed. First, the expected harvest in the present period can be expressed as

$$E_x h(q_k | m_l) = \sum_{j=1}^n f^i(h_j | q_k) \sum_{v=1}^j \sum_{p=v}^n b^m(x_p | m_l)$$

where  $f^i$  is the density of harvest  $j$  conditioned on the catch quota  $k$ ,  $b^m$  represents the density of recruitment  $p$ , conditioned on the measurement  $l$  and  $n$  is the number of possible states. The density  $b^m$  can be obtained by applying Bayes' Law to  $f^m$ , the conditional density of measured recruitment. The rightmost summand is simply the probability that the recruitment is at least  $v$  when the measurement is  $l$ . Summing over all  $v$  from 1 to  $j$  yields the expected catch when the attempted catch is  $j$ . Finally, multiplying this expression by  $f^i(h_j | q_k)$  and summing over all  $j$  yields the expected harvest, conditioned on the measurement  $l$  and the quota  $k$ .

Given that the conditional densities of recruitment  $f^g$  and harvest  $f^i$  are known, it is straightforward to compute the density of escapement, which is conditioned both on the measurement and announced quota. Let  $y$  represent that density. Then the second term in the maximand of (6) can be expressed as

$$E_x J(q_k | m_l) = \sum_c J \sum_k y(s_k | q_k, m_l) \sum_u f^m(m_c | x_u) f^g(x_u | s_k)$$

The rightmost summand represents the density of measuring  $c$  fish when the escapement in the previous period is  $k$ . Multiplying this by the conditional density of escapement  $k$  and summing over all  $k$  yields the density of measuring  $c$  fish in the next period, conditioned on measuring  $l$  fish in the present period and announcing a quota of  $k$ . Finally, multiplying this term by the value function  $J$  and summing over all  $c$ , one gets the expected future value of announcing a quota of  $k$  when the measurement is  $l$ .

The manager's problem can be solved by adding the current period value and the future value of making an announcement for each stock measurement. The optimal announcement maximizes this value for each measurement.

### 4.3 Markovian Analysis

We can obtain numerous results of interest by exploiting the special dynamic structure of this problem. Most renewable resource problems under uncertainty are Markov processes. Loosely defined, a Markov process is a stochastic process where the current state completely defines the probability distribution of the next state. In particular, this means future transitions are independent of the past. In this section we outline how exploiting the special structure of a Markov process can help us obtain very useful results about the fishery in question. Examples of such statistics include the distribution of stock through time, the probability of extinction through time, and the mean time to extinction.

For most fishery problems, the state variable is the stock level,  $N$  (we assume the relevant state space is  $\{0, 1, \dots, S\}$ ), and future stock depends only on current stock, thus satisfying the Markov property. Because the fish stock in one period completely determines the distribution of stock the next period, we can construct a matrix,  $P$ , which defines the probabilities of transitioning to every state given any current state in the absence of exploitation, as follows:

$$P_{ij} = Pr\{N_{t+1} = j | N_t = i\} \quad \forall t, \text{ and } \forall i, j \in \{0, 1, \dots, S\} \quad (7)$$

That is, an element  $P_{ij}$  of the matrix  $P$  gives the probability of transitioning from state  $i$  to state  $j$  in one step. Thus, the rows of  $P$  must sum to 1. Transition probability matrices (TMP) provide a useful way to describe biological systems. However, they do not, in general, include information about human exploitation.

It is possible to construct a matrix,  $Q$ , which describes the transitions between states *taking into account the effect of human harvest on the stock in question*. To do so, we first calculate a feedback control rule, call it  $\hat{h}(N)$  (it need not be the "optimal" rule) which defines a control given a state. In this example, the control is an announcement of harvest given information about the system, such as the stock size. By applying  $\hat{h}(N)$  to  $P$  above,

we calculate  $Q$ , the “policy embedded transition probability matrix” (PETPM). All analysis of the system must now be in the context of  $Q$ , and not  $P$ , since it is assumed that human harvest is part of the system.

Since  $Q_{ij}$  describes the probability of transitioning from state  $i$  to state  $j$  in one step, it can be shown that the  $ij^{th}$  element of  $Q^t$  gives the probability of transitioning from state  $i$  to state  $j$  in  $t$  steps. This extremely useful observation (derived from the Chapman Komolgorov equations) allows us to calculate the probability distribution of the stock of fish at any time in the future given an initial starting distribution,  $v_0$ , as  $v_0Q^t$ . For example, suppose the initial stock level is known to be 600. Then the  $v_0$  vector would have a “1” in the position corresponding to at stock level of 600, and zeros elsewhere. The distribution of stock in period 7, given that we start with exactly 600 fish is given by the vector  $v_0Q^7$ .

To further the analysis, we must first classify all states according to their potential transitions. In the case of fisheries, when extinction is a real possibility (with sufficient uncertainty), the state 0 is said to be an “absorbing” state because if the process ever reaches 0, it remains there forever. Furthermore, it is likely that all other states,  $\{1,2,\dots,S\}$ , are “transient” states, because they could, in theory, eventually be absorbed (i.e. become extinct). In fact, if there is any probability that the system will *ever* move from state  $k$  to state 0 in any number of steps, then state  $k$  is a transient state. This feature of the system implies that if left running long enough, the system will always go extinct, though it may take arbitrarily long to do so. For fisheries with this property (where extinction is possible, and where all other states could eventually reach 0), it does not make sense to discuss the limiting distribution of fish stocks, because in the limit, all probability mass is at zero.

We now describe how to determine (1) The probability of extinction through time, (2) the expected number of visits to each state prior to extinction, and (3) the expected time to extinction. The first item of interest to biologists, managers, and economists, is the probability of extinction through time. Analytical representation of such a time series allows us to make direct comparisons between management practices, and allows us to more systematically address the risks of extinction to a particular population. The probability of extinction after  $t$  years,  $Z_t$ , given initial population distribution  $v_0$ , is given by the first element of  $v_0Q^t$ . That is,

$$Z_t = v_0Q^t(1) \tag{8}$$

where (1) refers to the first element of the vector  $v_0Q^t$ . This element gives the desired probability (since the first element corresponds to the state 0). The second item of interest is the mean number of visits,  $M$ , to each state prior to extinction. It is calculated as follows. First, we exclude all absorbing states from the  $Q$  matrix. In this case, the only absorbing state is 0, so we eliminate the first row and column from  $Q$ . Call the new matrix  $\tilde{Q}$ , and

let  $w_0$  be  $v_0$  with the first element eliminated. The mean number of visits  $M$  is calculated as follows:

$$M = w_0(I - \tilde{Q})^{-1} \tag{9}$$

where  $I$  is the identity matrix.  $M$  is a vector whose  $i^{th}$  element is the expected number of visits to state  $i$ , given the initial state distribution,  $v_0$ , prior to extinction. Of course the expected time to extinction is just the sum of the elements in the vector  $M$ .

#### 4.4 Numerical Computation of the Optimal Policy

How does one simultaneously figure out the optimal quota for a known escapement? We use value function iteration, which is based on an important result called the contraction-mapping theorem. <sup>5</sup>

Definition: A map  $T : Y \rightarrow Z$  on ordered spaces  $Y$  and  $Z$  is monotone if and only if  $y_1 \geq y_2$  implies  $Ty_1 \geq Ty_2$ .

Definition: A map  $T : Y \rightarrow Y$  on a metric space  $Y$  is a contraction with modulus  $\beta < 1$  if and only if  $\|Ty_1 - Ty_2\| \leq \beta \|y_1 - y_2\|$ .

Contraction-mapping theorem: If  $X$  is compact,  $\beta < 1$ , and  $\Pi$  is bounded above and below, then the map

$$V(x_0) \equiv \max_{u \in U(x)} E\{\Pi(x_0, u_0)\} + E\{\beta V(x_t|x_0, u_0)\} \tag{10}$$

is monotone in  $V$ , and is a contraction mapping with modulus  $\beta$  in the space of bounded functions and has a unique fixed point.

Operationally, this means one can use the following recipe to solve a dynamic programming problem.

1. Make an arbitrary guess about the value function.
2. For each measurement, use the algorithm above to figure out the optimal quota, assuming the guess above is correct.
3. Substitute the optimal quota in the dynamic programming equation to arrive at a new estimate of the value function.

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<sup>5</sup>For details, see ??.

4. If the new value function is ‘close enough’ to the previous one, the iterative process ends; else, use the new value function and repeat the second and third steps.
5. Iterate until convergence. Given the properties of the value function, the contraction mapping theorem guarantees convergence (see section of the appendix for a statement of this theorem).

The iterative scheme described above converges to the solution only asymptotically. In practice, one iterates until the difference between successive  $V$ 's is smaller than a pre-determined tolerance level. Since there are only a finite number of control rules and the contraction theorem applies, the process will converge in finite time.

#### 4.5 Parameter Values and Functional Forms

The drawback of relying on numerical results is that one must make specific assumptions about the function forms and parameters. We adopt the following specific assumptions:

1. Expected fish growth follows the logistic curve:

$$G(s_t) = \frac{As_t(K - s_t)}{K} + s_t \quad (11)$$

where  $A$  ( $=1$ ) is the growth parameter of the logistic function and  $K$  ( $=100$ ) is the carrying capacity of the environment.

2.  $z_t^g$ ,  $z_t^m$ , and  $z_t^i$  are independent, stationary, uniform distributions of the following form:

$$z^\xi = 1 + (2\tilde{u} - 1)\varepsilon^\xi \quad \xi = \{g, m, i\} \quad (12)$$

where  $\tilde{u}$  is drawn from a uniform distribution lying between zero and one. For example, if  $\varepsilon^m = 0.4$ ,  $z^m$  is distributed uniformly between 0.6 and 1.4. Furthermore, if recruitment is 100, this implies measured stock is a uniform random variable with support  $[60, 140]$ . The corresponding coefficient of variation is 0.2309.

## 5 Results

In the results that follow, we explore both ‘‘small’’ and ‘‘large’’ shocks for each random disturbance term (and in various combinations). These refer to uniform random changes within a range of 20% to 80%, respectively. These shocks translate into a coefficient of variation of 0.1155 and 0.4619 respectively.

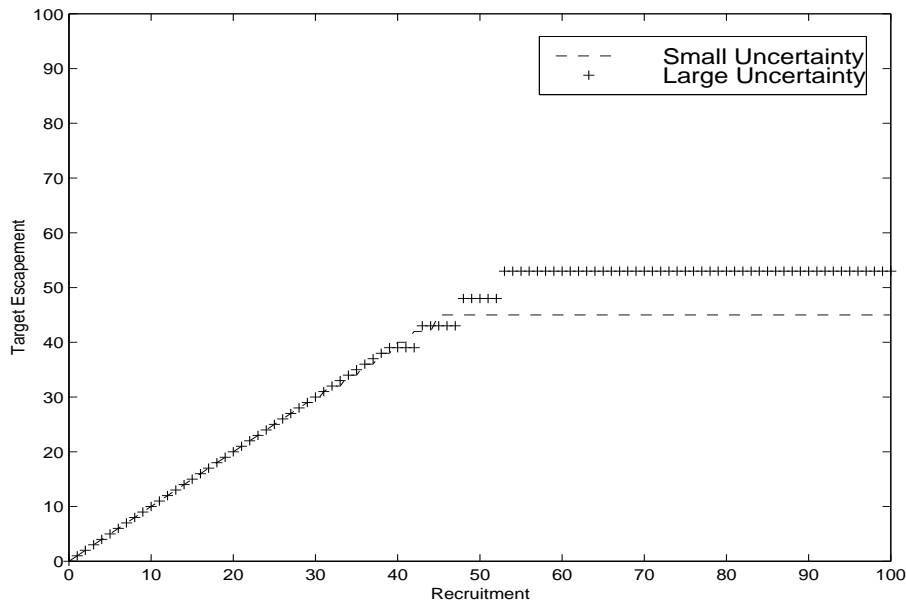


Figure 1: Optimal Fishing Policy Under Future Growth Uncertainty

### 5.1 Previous Results

We start by presenting the results obtained by authors of previous work. In the presence of only growth uncertainty and when present stocks can be measured accurately, the optimal policy is one of constant escapement. This result is due to Reed (1979) and is presented in Figure 1.

Clark and Kirkwood (1986) note that in many situations it may be hard to measure the stock after the fish have spawned, but relatively easier to do so at the end of each fishing season. Therefore, if escapement can be measured with precision (but not stock), then the policy function entails non-constant escapement (see Figure 2).

Roughgarden and Smith (1996) consider only constant-escapement policies in the presence of multiple uncertainty. The “optimal” constant-escapement policies for the small and large shocks are shown in Figure 3.

### 5.2 Our Results

We explore several combinations of shocks. The results are presented as a graph of the optimal policy function. Recall, the optimal policy function indicates optimal announced escapement (measured recruitment minus announced quota) as a function of measured recruitment. In particular, we explore the following cases: (1) all errors small, (2) one error large, (3) multiple errors large. In all cases the optimal policy function is a line of slope 1 for sufficiently low measured recruitment. This line indicates a fishery closure for very low

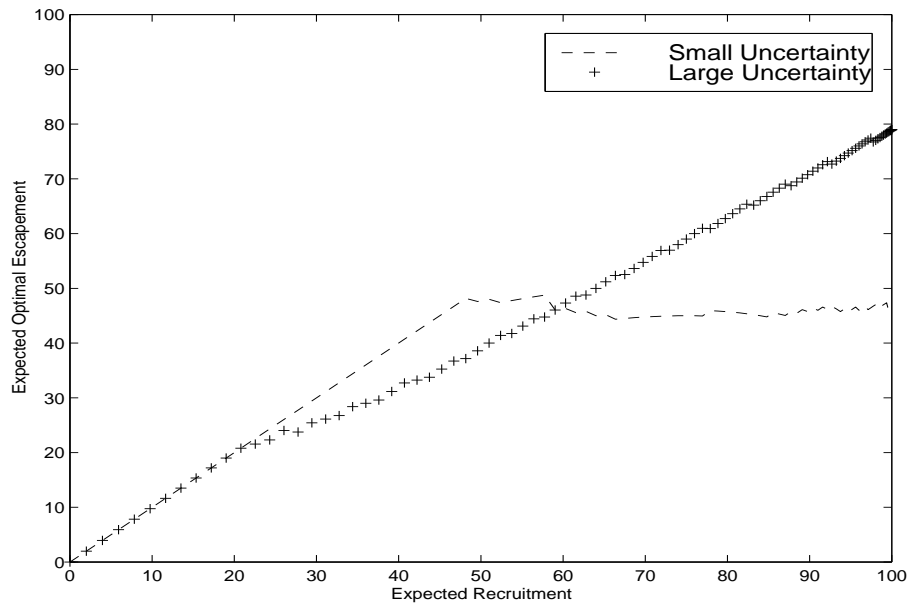


Figure 2: Optimal Fishing Policy Under Current Stock Uncertainty

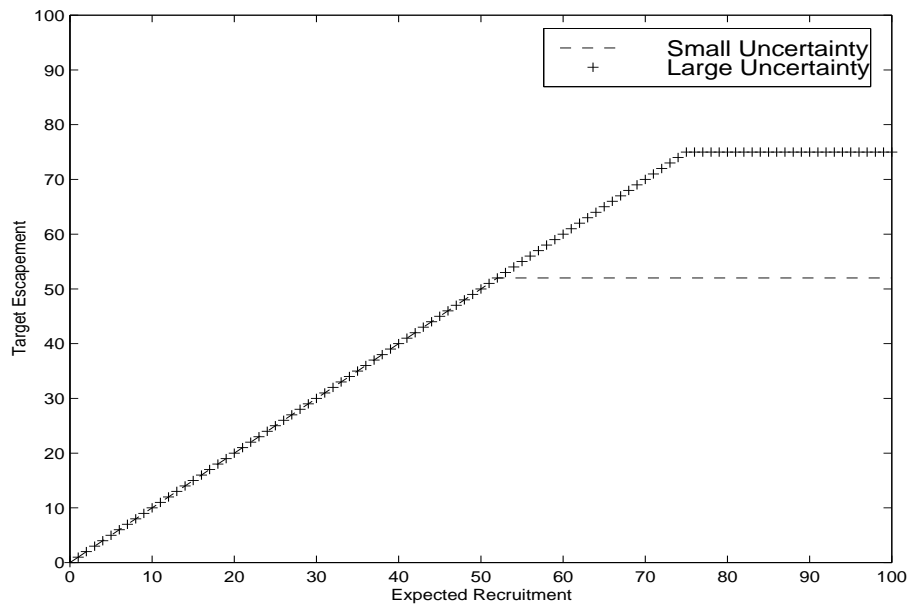


Figure 3: Policy Recommendations of Roughgarden and Smith

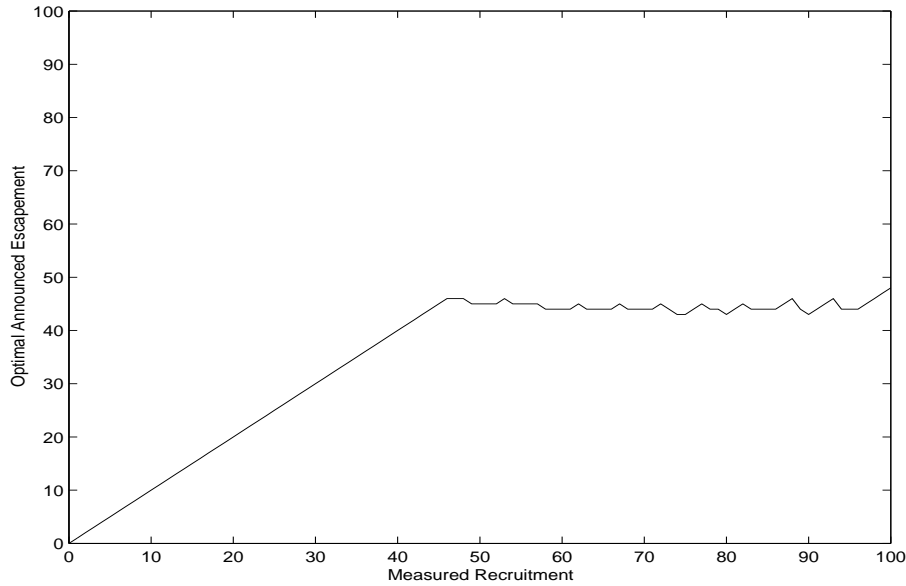


Figure 4: Optimal Fishing Policy Under Small Multiple Uncertainty

stocks.<sup>6</sup>

### 5.3 Small errors

When all shocks are zero we have the familiar deterministic model, where the optimal policy is a “bang-bang” solution with constant escapement level at the point where the discount rate equals the slope of the logistic growth curve. As may be expected, small shocks - whether considered individually or all together - lead to a policy which is not significantly different from the deterministic rule. Figure 4 shows the graph of the optimal policy function for values of  $\varepsilon$  of .2 for each shock type. [Note that although we consider  $\varepsilon = .2$  a “small” shock, it is actually fairly large, as it indicates a uniform deviation of 20% on either side of the mean.]

This policy suggests that for low levels of uncertainty, not only is the deterministic rule qualitatively appropriate (i.e. it suggests a constant escapement policy), but it is quantitatively appropriate (the escapement target is approximately  $K/2$ ).

### 5.4 One source with large error

How does the optimal policy change when one of the shocks is large, holding the others at a small level? The answer depends on which variable has a high shock. Figure 5 confirms

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<sup>6</sup>The small “wiggles” in the graph of the optimal policy function are a consequence of partitioning the state and control spaces into discrete units and are not important. See appendix.

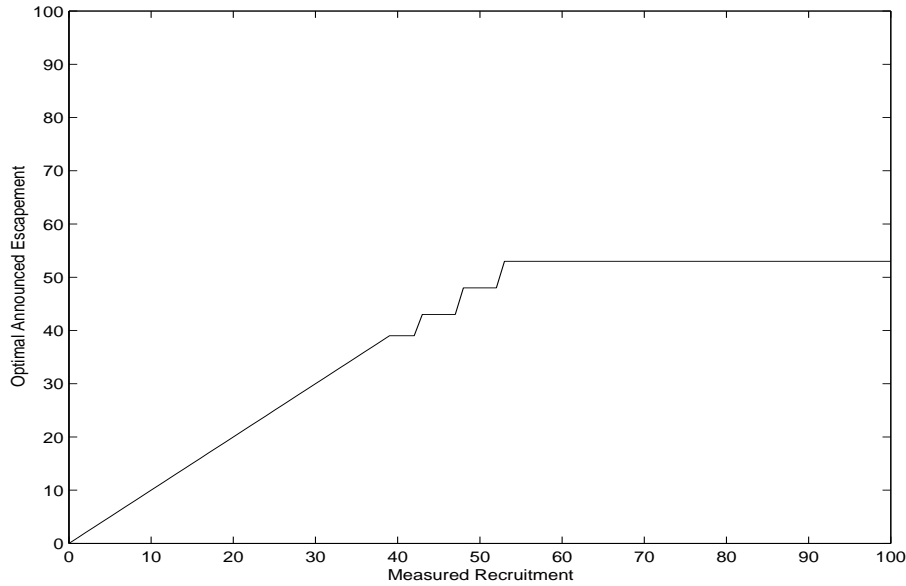


Figure 5: Optimal Fishing Policy With Large Growth Uncertainty

Reed’s result that even a large growth shock does not alter the optimality of a constant escapement policy. If the fishery is subject to a large implementation shock, the optimal expected escapement level rises, though not significantly. Figure 6 shows this case.

These figures suggest that if only growth or implementation errors are large, even if the shocks are highly variable, a constant escapement policy may be qualitatively appropriate. There is a qualitative change in the policy, however, if a large measurement error is introduced. In this case, escapement levels *fall* for small recruitment levels and rise for higher ones. The increased level of escapement makes good intuitive sense; since the error is multiplicative, the variance of the density of true stock increases and this leads the manager to call for lower quotas as a guard against risk.

But what accounts for the lower levels of escapement for small measurements of stock around the kink of the deterministic rule? Consider a manager who faces no uncertainty at all and makes a stock measurement just under the kink. In a deterministic world, she would harvest nothing. Now assume that she makes the same measurement but she believes her measurements are flawed. In this case, either the true stock is smaller than she measured or it is larger. If it is smaller, she should harvest nothing at all. However, if it is larger, she wants a harvest a positive number of fish. Thus, in expectation, her harvest for this measurement is non-zero, and *more* than it was under no uncertainty.

Interestingly, the policy implied by the graph of the optimal policy for high measurement

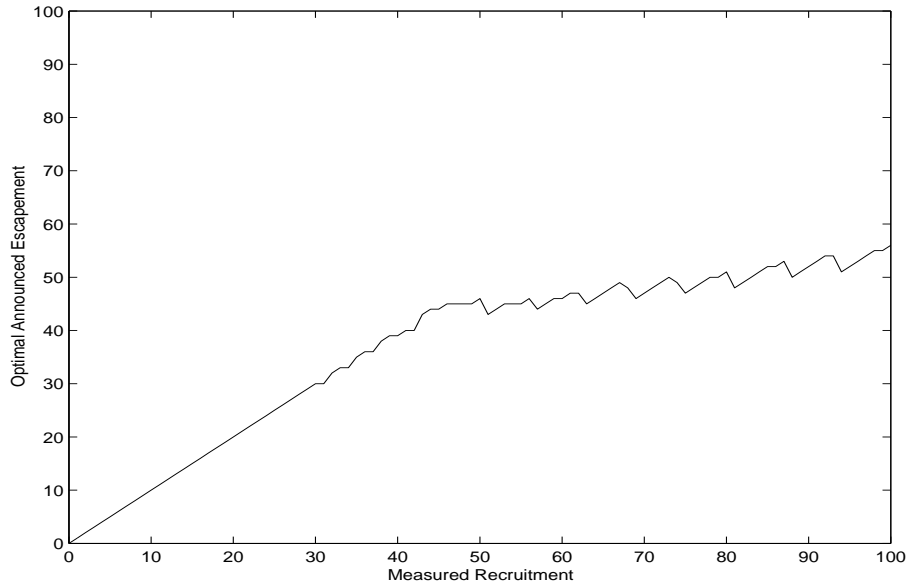


Figure 6: Optimal Fishing Policy Under Large Implementation Error

error (figure 7) is nearly linear beyond the kink; its slope is roughly 0.5, which implies an escapement rate of approximately 50% of the measured stock.

### 5.5 Multiple sources with large error

Now consider a large measurement error combined with a large implementation error. While this does not lead to a significant change in the shape of the policy function, it does imply an increased escapement rate of approximately 66% of the measured stock that period (see figure 8).

Finally, adding a large growth shock to the previous set of shocks (measurement and implementation errors) has no significant impact even on the level of escapement. This suggests that growth uncertainty, in the presence of large measurement and implementation errors, does not change the optimal policy in any meaningful way. The graph of the optimal policy function with large growth, measurement, and implementation errors is shown in figure 9.

## 6 Extinction and Existence Value

The previous section describes optimal management of a fishery under multiple sources of uncertainty, with varying degrees of each type of uncertainty. In the presence of large errors (all three), the optimal policy is associated with a significant extinction probability.

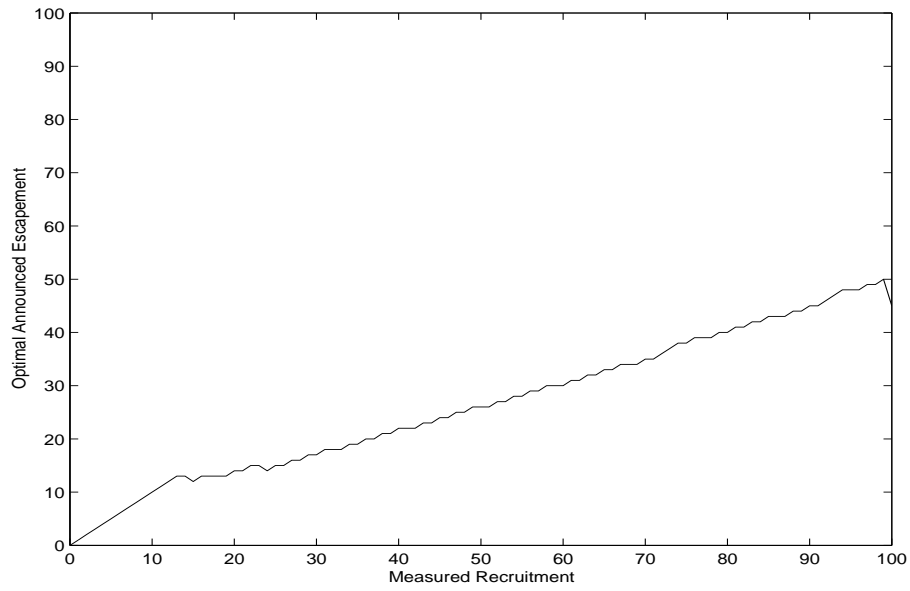


Figure 7: Optimal Fishing Policy Under Large Measurement Error

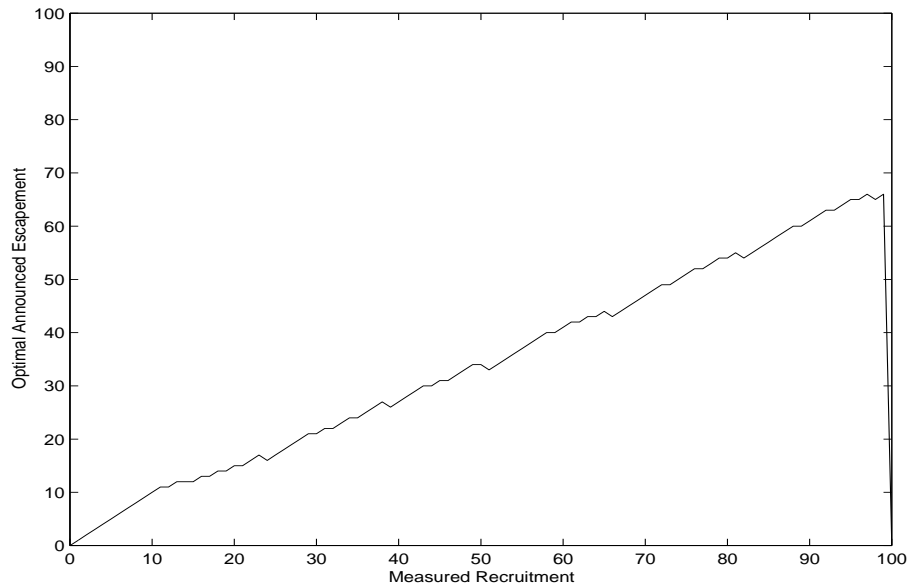


Figure 8: Optimal Fishing Policy Under Large Measurement and Implementation Uncertainty

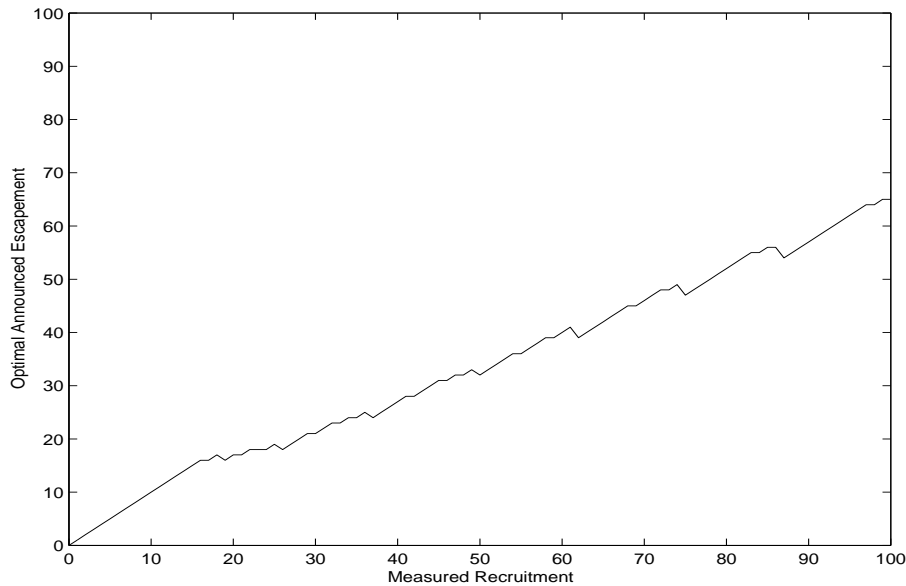


Figure 9: Optimal Fishing Policy Under Multiple Uncertainty

Figure 10 gives confidence intervals for the stock size through time provided it is managed optimally - as defined above.

For example, the 95% line plots the stock size such that the probability that the stock will be below this line at any point of time is 95%. Since the median line (50%) hits zero at about year 9, following the economically optimal policy in the presence of such extreme uncertainty will lead to extinction of the stock in approximately 9 seasons (though there is some non-negligible chance that the stock will survive beyond the fiftieth season). This is probably not a socially desirable result.

The previous exercise has a narrow focus since the goal of the fishery manager is narrowly framed to maximize the present discounted value of profits derived from selling the harvest. While profits are undoubtedly a significant component of the fishery value, they may not capture the entire value that society places on the resource. One motivation for much of the biological literature on this subject is to limit (or even eliminate) extinction risk. Implicit in many of these models is an infinite cost of the stock going extinct.

In economics, the non-market valuation literature has spawned a number of studies suggesting that society may derive benefit from aspects of a resource not captured by markets. In the context of fisheries, people may care not only about catch but may also derive utility from the knowledge that the fishery *exists* and has not gone extinct. Individuals may also value the *option* of going fishing, or simply enjoying the fishery in terms of its natural beauty, at some future date. Furthermore, ecologists and geneticists argue that larger popu-

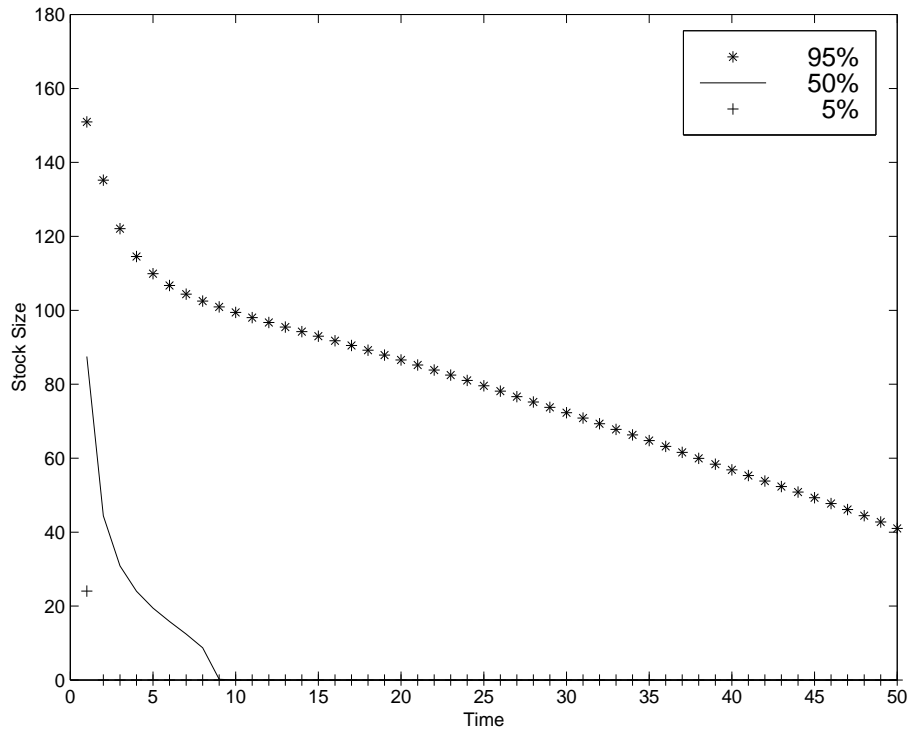


Figure 10: Confidence Region of Stock Under Multiple Uncertainty Without Existence Value

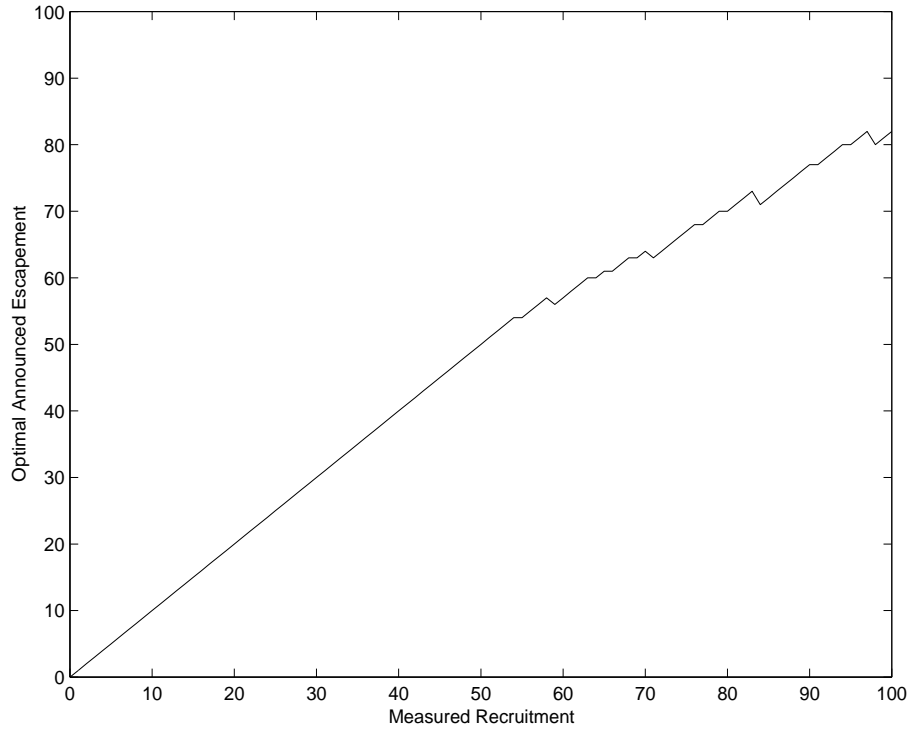


Figure 11: Optimal Fishing Policy Under Large Multiple Uncertainty With Existence Value

lations enhance stock viability and facilitate the use of genetic material for anthropocentric purposes. Thus, to capture the value of the fishery to society, these sources of value need to be included in the maximand. While we realize this is a hard task in practice, we postulate a straightforward function form for use in our exercise. We assume that the marginal value of *in situ* fish ( $W(x)$ ) diminishes in stock ( $W'(x) < 0$ ), as follows:

$$W(x) = \delta e^{-\gamma x} \quad (13)$$

where  $\delta$  and  $\gamma$  are parameters of the function. For the purpose of this exercise, we assume that the marginal existence value for nearly extinct stocks equals price times the carrying capacity (in this case, \$100). When the stock is at the carrying capacity the marginal existence value equals price divided by the carrying capacity (in this case, one penny). This explicit recognition of existence value has a significant effect on the optimal policy and a dramatic one on extinction probability. In this case, the optimal policy calls for a much higher escapement level (as seen in figure 11), providing a buffer against extinction. The analog to figure 10 is provided in figure 12. Under the parameters used, the steady state stock distribution has a median of approximately 75 with upper and lower 95% confidence limits of about 160 and 20, respectively.

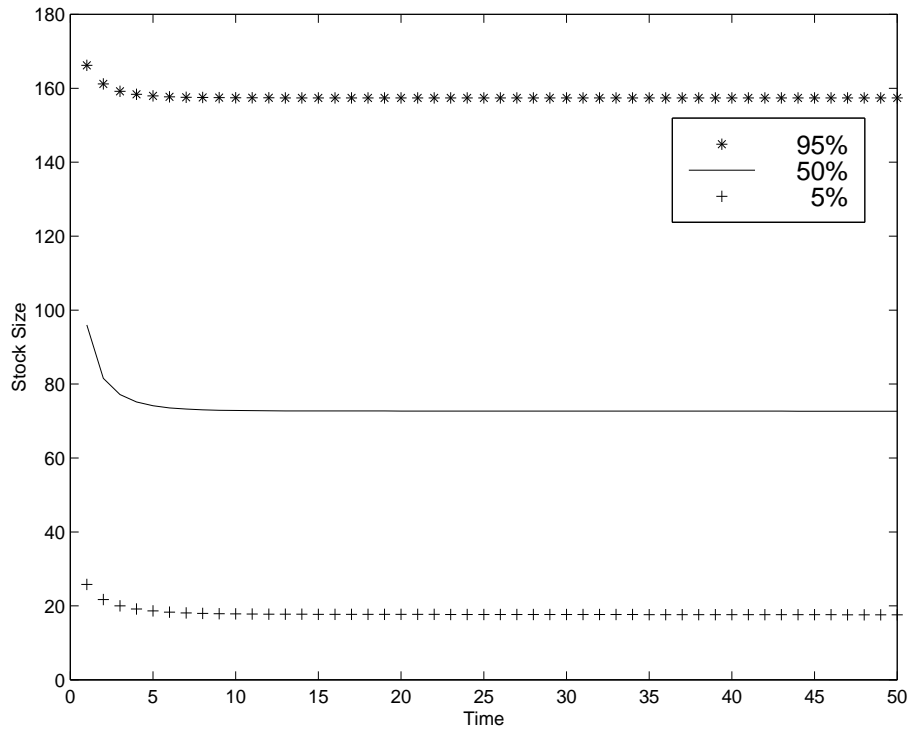


Figure 12: Confidence Region Of Stock Over Time With Existence Value

## 7 Concluding Remarks

The model presented above addresses a global concern of increasing importance. Several economically and culturally important fisheries around the world are threatened with extinction. While there are many causes for this phenomenon, uncertainty in the management of fisheries is one of the central concerns. Previous approaches, in economics and biology, are inadequate. In general, the economic literature has focused on stylized models at the expense of biological, and even economic, realism. Biological models of fishery management often satisfy expectations of biological realism, but are often not solved for allocative efficiency. Instead, “rules of thumb” are calculated, disregarding the inherent tradeoff in any allocation problem.

We frame an economic allocation problem by incorporating sources of uncertainty identified by biologists and fishery managers as important. The model is solved through iteration on the value function from the dynamic programming equation. Results include the following insights about management:

- Small shocks have no significant effect on policy vis-a-vis the deterministic rule.
- Growth shocks and implementation errors have only a small effect on optimal policy when they are large.
- Measurement errors have the largest impact on fishery policy, especially when they combine with implementation errors. This highlights the importance and value of stock surveys.
- Under highly stochastic environments, a simple rule-of-thumb should strive towards maintaining escapement that is at least two-thirds the measured stock. The fact that the optimal policy function happens to be almost linear eases its implementability.

Unlike others who have attempted to solve the problem of fishery management under multiple uncertainty, our model makes no presumptions about the shape of the policy function. Despite all these strengths, the methodology above can be improved upon in one crucial sense. Our optimization method discards the beliefs that the manager might have about previous period’s escapement levels; in each period she relies only on *current* period stock measurements to derive her estimate about the level of the recruitment. Using her beliefs about escapement levels (based on all past measurements) can only improve her decision in terms of increasing the fishery value.

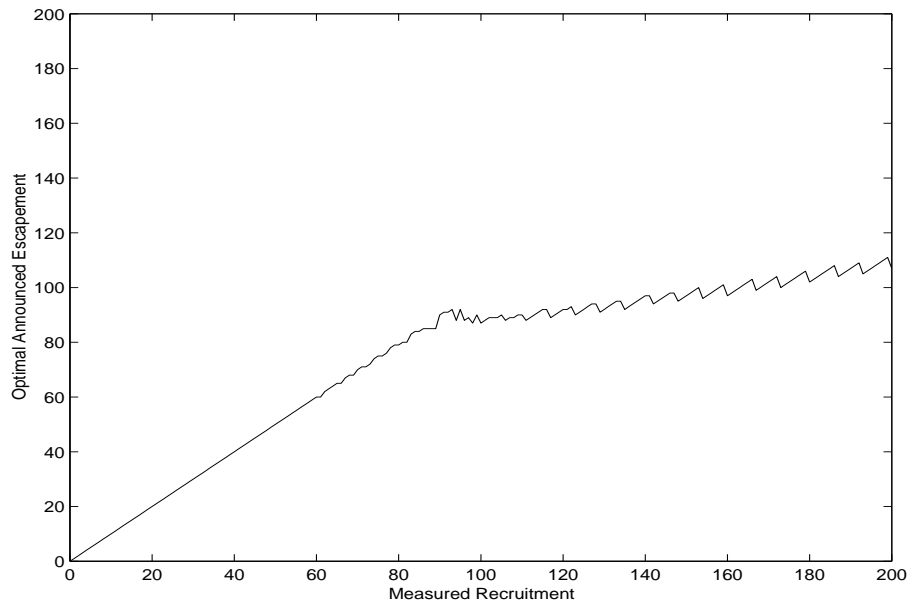


Figure 13: Optimal Fishing Policy Under Large Measurement and Implementation Uncertainty (with finer grid)

## Appendix

### The “Wiggles” Issue

As previously noted, the “wiggles” in our results are the result of the state and control space being coarse. While it is straightforward to make the grids of these finer, the memory requirements to run the code (presented in the next section) increase exponentially. Here is an example of the results one can obtain if the grid size is doubled. The figure corresponds to 8 above.

### The Code

We present below the code we developed (using MATLAB) to solve the problem.

```
% This code solves the stochastic fishery problem with multiple uncertainty.
clear all; close all

% s = 0 if no EV; else s = 1
s = 0;

% Specify carrying capacity and convergence tolerance
K = 100; tol = K/1000; r = 1;
```

```

z = [0.0 0.0 0.8]; zg = z(1); zm = z(2); zh = z(3); L = K*(1+zg);
E = (0:L)'; lE = length(E);

% Specify interest rate, level of uncertainty
int = 0.05; rho = 1/(1+int);

% Generate expected recruitment
EXR = growth(r,K,E);

% Generate growth uncertainty transition matrix
GTM = markov(E,EXR,zg);

% Specify escapement and measurement domain
[R,M,A,H] = deal([0:length(GTM)-1]'); [lA,lM] = deal(length(GTM));

% Generate measurement error transition matrix
MTM = markov(R,M,zm);

% Apply Bayes rule to MTM
MBM = bayes(MTM);

% Generate implementation error transition matrix
HTM = markov(A,H,zh);

% Prob that stock is at least i when meas is j
CGM = flipud(cumsum(flipud(MBM')));

% Announced harvest possibilities
ANH = repmat(A,1,lA);

% Expected harvest | announcement i & meas j
EXH = sparse(HTM*(ANH.*CGM)) + cumsum(ANH.*MBM') - ANH.*MBM';

% Compute measurement(t+1) density | ann(t) i and meas(t) j
MON = sparse(convolute(MBM,HTM)*GTM*MTM);

% Initialize value function and diff

```

```

vnew = zeros(1,1E); diff = 2*tol;

% The marginal existence value function is  $m = ae^{bx}$ . We assume that
%  $m(0) = p*K$ , which implies  $a = K$ .
%  $m(K) = p/K$ , which implies  $b = -(\log(K^2))/K$  or that  $b = -0.0921$ .
% The corresponding (total) existence value function is  $EXV = (a/b)*(e^{bS} - 1)$ .

if s == 0; a = 0; else a = K; end b = -(\log(K^2))/K;

EXV = a/b*(exp(b*A')-1); XSV = repmat(EXV,1A,1);

% Iterate until value function converges
while diff > tol
    vold = vnew;

    F = rho*sparse(reshape(MON*vold',1E,1E));

    % Expected value of announcing i when meas is j
    V = EXH + XSV + F;

    % Build the new value function & harvest
    [vnew index] = max(V);

    diff = max(abs((vnew - vold)));
end

% Compute optimal announcement | measurement
OAN = M(index);

% Compute optimal escapement | measurement
OES = max(0,M - OAN); plot(M(1:K+1),OES(1:K+1)); axis([0 K 0 K])
xlabel('Measured Recruitment')

ylabel('Optimal Announced Escapement')

title(strcat('Optimal Policy, z = ',num2str(z)))

```

```

% Compute the value function of the policy derived above.
VFN = V(sub2ind(size(V),index,1:size(V,2)));

% Compute Policy Embedded Transition Matrix
GTM = full(GTM); OGM = GTM(round(OES)+1,:);

% Compute long-run stock distribution
LRM = unique(OGM^50,'rows');

subfunction g = growth(r,K,E)

% This function calculates the recruitment of fish, g, based on the present
% stock level, s, and the growth function chosen.

g = E + r*E.*(K-E)/K;

subfunction M = markov(X,Y,z)

% Last edited October 24th, 2000.

% M = markov(X,Y,z)
% This function computes the transition matrix from state X to state Y
% when the stochastic shock is z.

glo = round((1-z)*Y); ghi = round((1+z)*Y); g = ghi-glo+1;

prob = min(1,1./(g)); L = length(X);

M = zeros(L,L); for i = 1:L
    M(i,glo(i)+1:ghi(i)+1) = prob(i);
end

if length(M) > L
    [a b] = size(M); M(:,L) = sum(M(:,L:b)',1)'; M(:,L+1:b) = [];
end

```

```

subfunction B = bayes(Q)

% BAYES(Q) finds the bayesian probabilities of Q.

LQ = length(Q); B = (Q./repmat(sum(Q),LQ,1))';

subfunction V = convolute(P,Q)

%CONVOLUTE(P,Q) convolves over rows of P and Q.
% P and Q are square matrices of dimension n where P is the growth
% transition matrix and Q is the implementation error matrix. This
% code generates a matrix V of dimension n^2 by n, each row of
% which is the density of escapement. The density of escapement when the
% state is i and the control is j can be found in row n*(i-1)+j of V.

n = length(P); V = zeros(n,2*n-1);

for i = 1:n
    V(n*(i-1)+1:n*i,end:-1:1) = conv2(Q,P(i,end:-1:1));
end V(:,n) = sum(V(:,1:n),2) ; V(:,1:n-1)= [];

```

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