Answer Key 1

1. (1) $\mathbb{R}$ = real numbers. $\mathbb{Q}$ - rational numbers.

Denseness of $\mathbb{R}$: $\forall a, b \in \mathbb{Q}, a \neq b$, $\exists r \in \mathbb{R}$ s.t. $\min\{a,b\} < r < \max\{a,b\}$

(2) $\exists x \in X$, $\cancel{\forall}\ \exists \varepsilon > 0$, $\forall \delta > 0$ $\exists y \in B_\delta(x)$, $f(y) \notin B_\varepsilon(f(x))$

3. I will prove this for set-notation version:

$$P \subseteq Q \iff \sim P \cup Q = X$$

(1) $P \subseteq Q \iff P \cap Q = P \iff \sim P = \sim(P \cap Q) = \sim P \cup \sim Q$

(2) $P \subseteq Q \iff \sim P \cap \sim Q = \sim Q \iff Q = \sim(\sim P \cap \sim Q) = P \cup Q$

$$\begin{aligned} \iff \quad \sim P \cup Q &= (\sim P \cup \sim Q) \cup (P \cup Q) \\ &= (\sim P \cup P) \cup (\sim Q \cup Q) \qquad (3) \\ &= X \cup X = X \end{aligned}$$

(3) follows from (1) and (2). And the first equality of (3) implies the last equalities of (1) and (2) so we have both directions.

5.

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 1 \\ 1 & 3 & 6 & 1 & 3 \\ 3 & -1 & -k_1 & 15 & 3 \\ 1 & -5 & -10 & 12 & k_2 \end{array}\right] \longrightarrow \left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 1 \\ & 2 & 4 & -2 & 2 \\ & -4 & -(k_1+6) & 6 & 0 \\ & -6 & -12 & 9 & k_2-1 \end{array}\right]$$

$$\longrightarrow \left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 1 \\ & 2 & 4 & -2 & 2 \\ & & -k_1+2 & 2 & 4 \\ & & & 3 & k_2-5 \end{array}\right]$$

$k_1 \neq 2 \rightarrow$ (2) Unique soln

$k_1 = 2 \longrightarrow$ $\left[\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 1 \\ & 2 & 4 & -2 & 2 \\ & & & 2 & 4 \\ & & & 0 & k_2-11 \end{array}\right] \Rightarrow$ $k_1 = 2$, $k_2 \neq 11$ - no soln. (1)

$k_1 = 2$, $k_2 = 11$ ∞ soln. (3)