Elasticity Handout

EEP 162-Spring 2006

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1. Conditional Input Demand: $x_k = g^k(w_1, ..., w_N, y)$

   (a) Conditional own price elasticity:

   $$_c\varepsilon_i^i = \frac{w_i}{x_i} \cdot \frac{dg_i(\cdot)}{dw_i}$$

   - $_c\varepsilon_i^i < 0 \rightarrow$ Must be non-positive.
   - $|_c\ \varepsilon_i^i| \leq 1 \rightarrow$ inelastic
   - $|_c\ \varepsilon_i^i| > 1 \rightarrow$ elastic

   (b) Conditional Cross price elasticity:

   $$_c\varepsilon_j^i = \frac{w_j}{x_i} \cdot \frac{dg_i(\cdot)}{dw_j}$$

   - $_c\varepsilon_j^i < 0 \rightarrow$ complements
   - $_c\varepsilon_j^i > 0 \rightarrow$ substitutes

   (c) Conditional Output elasticity:

   $$_c\varepsilon_y^i = \frac{y}{x_i} \cdot \frac{dg_i(\cdot)}{dy}$$

   - $_c\varepsilon_y^i > 0 \rightarrow$ normal input
   - $_c\varepsilon_y^i < 0 \rightarrow$ inferior input

2. Unconditional Input Demand $x_k = h^k(w_1, ..., w_N, p)$

   (a) Unconditional Own Price Elasticity

$$\varepsilon_i^i = \frac{w_i}{x_i} \cdot \frac{dh_i(\cdot)}{dw_i}$$

   - ${}_c\varepsilon_i^i \leq 0 \Rightarrow$ Must be non-positive.
   - $\mid \varepsilon_i^i \mid < 1 \Rightarrow$ inelastic
   - $\mid \varepsilon_i^i \mid > 1 \Rightarrow$ elastic

   (b) Unconditional Cross Price Elasticity

$$\varepsilon_j^i = \frac{w_j}{x_i} \cdot \frac{dh_i(\cdot)}{dw_j}$$

   - $\varepsilon_j^i < 0 \rightarrow$ complements
   - $\varepsilon_j^i > 0 \rightarrow$ substitutes

   (c) Unconditional Input Demand Output Price Elasticity

$$\varepsilon_p^i = \frac{p}{x_i} \cdot \frac{dh_i(\cdot)}{dp}$$

   - $\varepsilon_p^i > 0 \rightarrow$ normal input
   - $\varepsilon_p^i < 0 \rightarrow$ inferior input

3. Unconditional supply $y* = y(w_1, ..., w_N, p)$

   (a) Output price elasticity of supply

$$\varepsilon_p^y = \frac{p}{y} \cdot \frac{dy(w_1, ..., w_N, p)}{dp}$$

   - $\varepsilon_p^y \geq 0 \rightarrow$ Typically positive

   (b) Input price elasticity of supply

$$\varepsilon_i^y = \frac{w_i}{y} \cdot \frac{dy(w_1, ..., w_N)}{dw_i}$$

   - $\varepsilon_i^y <$ input is normal
   - $\varepsilon_i^y >$ input is inferior

4. Short Run Conditional Input Demand:

$$x_k = g^k(w1, ..., w_{N-1}, \bar{X_N}, y)k = 1, ...N - 1$$

5. Short Run Unconditional Input Demand:

$$x_k = h^k(w_1, ..., w_{N-1}, \bar{X_N}, p), k = 1, ..., N - 1$$

6. Short Run Unconditional Optimal Output:

$$y = y(w_1, ..., w_{N-1}, \bar{X_N}, p)$$

7. Unconditional vs Conditional

   (a)
$$x_k = h^k(w_1, ..., w_N, p) \equiv g^k(w_1, ..., w_N, y(w_1, ..., w_N, p))$$

   (b) Output price derivative

   - In derivatives:
$$\frac{dh^i}{dp} = \frac{dg^i}{dy} \cdot \frac{dy}{dp}$$
     Elasticity:
$$\varepsilon_p^i =_c \varepsilon_y^i \cdot \varepsilon_p^y$$
   - $\varepsilon_p^y \geq 0$
   - ${}_c\varepsilon_y^i \geq 0$ if normal good, ${}_c\varepsilon_y^i < 0$ if inferior good
   - Therefore, $\varepsilon_p^i \geq 0$ if input normal, $\varepsilon_p^i < 0$ if input inferior

   (c) Own Price Derivative

   - In derivatives:
$$\frac{dh^i}{dw_i} = \frac{dg^i}{dw_i} + \frac{dg^i}{dy} \cdot \frac{dy}{dw_i}$$
   - Elasticities:
$$\varepsilon_i^i =_c \varepsilon_i^i +_c \varepsilon_y^i \cdot \varepsilon_i^y$$
   - We know ${}_c\varepsilon_i^i \leq 0$ and ${}_c\varepsilon_y^i \cdot \varepsilon_i^y \leq 0 \rightarrow \mid \varepsilon_i^i \mid \geq \mid_c \varepsilon_i^i \mid$

   (d) Cross Price Derivative:

- In Derivatives:

$$\frac{dh^i}{dw_j} = \frac{dg^i}{dw_j} + \frac{dg^i}{dy} \cdot \frac{dy}{dw_j}$$

- In Elasticities

$$\varepsilon_j^i =_c \varepsilon_j^i +_c \varepsilon_y^i \cdot \varepsilon_j^y$$

- ${}_c\varepsilon_j^i \geq 0$ if substitutes, ${}_c\varepsilon_j^i < 0$ if complements
- ${}_c\varepsilon_y^i \geq 0$ if normal, ${}_c\varepsilon_y^i < 0$ if inferior
- $\varepsilon_j^y \geq 0$ if inferior, $\varepsilon_j^y < 0$ if normal
- So whether cross price elasticity of unconditional demand function is larger or smaller than the conditional demand depends on whether the inputs are normal or inferior

(e) Long Run can be derived from short run (show conditional)

- We express the relationship as follows:

$$x^k = g^k(w_1, ..., w_N, y) = g^k(w_1, ..., w_{N-1}, g^N(w_1, ..., w_N, y), y)$$

- All derivatives of long run demand functions are larger in absolute value that the corresponding derivatives of short run demand functions.