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1 A primer on the Heckscher-Ohlin-Samuelson (HOS) model

These notes provide basic information about the HOS model. For more detailed information, consult any textbook on international trade, or my online notes at

<http://are.berkeley.edu/courses/ARE201/Notes2004/Chapter4.pdf>

http://are.berkeley.edu/courses/ARE201/figures/FigureNotes4_ARE201.pdf

There are four important theorems associated with the HOS model: (i) the Factor Price Equalization theorem, (ii) the Stolper Samuelson theorem, (iii) the Rybczynski theorem, and (iv) the Heckscher-Ohlin theorem. The factor price equalization theorem gives conditions under which trade in commodities is a perfect substitute for the international mobility of factors. The Stolper-Samuelson theorem gives conditions under which a change in relative commodity prices has an unambiguous effect on real factor returns (i.e., the welfare of owners of capital and owners of labor). The Rybczynski theorem shows how a change in factor supply alters production, holding fixed all prices. Finally, the Heckscher-Ohlin theorem shows the relation between relative factor endowments and comparative advantage.

Comparative advantage, and thus the direction of trade, depends on a comparison of relative prices in autarky. In the Ricardian model, autarky prices are completely determined by technology; there a difference in technology between two countries is the basis for trade. The HOS model provides an explanation for trade based on different factor endowments (the amount of capital, labor, and any other factors of production that the economy is "endowed" with). In particular, trade depends on differences of countries' *relative factor endowments* (e.g., the number of units of capital for each unit of labor in the economy), rather than different technology or different tastes, or something else.

There is a single factor of production in the Ricardian model. In the HOS setting there are two or more mobile factors. We will consider the special case of this model where there are two commodities and two mobile factors of production, capital and labor. These factors move freely across sectors, so the wage rate and the rental rate must be the same in both sectors, within a country.

The HOS model assumes that each sector has constant returns to scale. This assumption implies that *the optimal capital/labor ratio* $k = \frac{K}{L}$ *depends only on relative factor prices, $\frac{w}{r}$, not on the scale of output, Q .*

1.1 Factor price equalization

Here we consider the relation between relative commodity prices, $p = \frac{p_2}{p_1}$ and relative factor prices $\frac{w}{r}$ (the wage rate divided by the rental rate for capital). This relation is interesting because it determines whether real factor returns are equal in the two countries. If, for example, the real return to labor is the same in the two countries, there is no incentive for international migration of labor. If real factor returns are not equal in the two countries, there is an incentive for factors to move from one country to another. Recall that in the Ricardian model, free trade typically does not imply the equality of real returns to labor. Thus, in general there is no reason to suppose that trade in commodities causes real factor returns to be equal across countries.

The "factor price equalization theorem" states that (under certain technical conditions in the HOS model), the equality of relative commodity prices (amongst countries that trade) implies that relative factor prices, and also real factor returns, are the same in the different countries. Thus, under the conditions of this theorem, trade in commodities is a perfect substitute for trade in factors.

1.1.1 Some implications

The theorem is usually interpreted as providing an indication that as trade moves relative commodity prices in different countries closer together, it also causes real factor returns to move closer together. In other words, trade tends to increase equality between owners of capital in two countries, and between workers in two countries. This tendency is precisely what many people fear from increased globalization: that wages in the rich countries will be moved towards poor country wages. This movement would entail

a decrease in rich country wages and an increase in poor country wages. It would presumably decrease the pressure for migration from poor to rich countries.

Copeland and Taylor (in a forthcoming article) note that the theorem has implications for the Kyoto Agreement. To the extent that the theorem holds, it implies that trade in commodities renders trade in factors unnecessary. Many economists stress the importance of allowing international trade in greenhouse gas (GHG) emissions permits, in order to make the cost of achieving a given level of emissions efficient. For example, if marginal abatement costs are higher in the U.S. than in Europe, then Europe could sell to the US its right to emit one unit of emission. The resulting increase in Europe's abatement costs (resulting from the need to reduce emissions by one unit) is less than the decrease in the U.S. abatement costs (resulting from the right to increase emissions by one unit). Therefore, by choosing a price between the two countries' marginal emissions cost, both countries benefit from the sale; trade reduces worldwide abatement cost without changing the overall level of emissions.

However, if the factor price equalization theorem holds, then trade in commodities makes it unnecessary to trade emissions permits. Output adjusts so that the marginal cost of achieving a particular level of emissions is equal in all countries, even without international emissions trade.

1.2 The Stolper-Samuelson theorem

The Stolper-Samuelson theorem provides conditions under which *an increase the relative price of a good increases the real return to the factor that is used intensively in the production of that good, and decreases the real return to the other factor*. For example, in a two-commodity (beer and soyburgers) and two-factor (capital and labor) model, suppose that soyburgers are relatively capital intensive. This assumption means that the capital/labor ratio used in the production of soyburgers is greater than the capital/labor ratio used in the production of beer. The theorem provides conditions under which an increase in the relative price of soyburgers increases the "real return" (i.e., the welfare) of owners of capital, and decreases the real return to laober (workers' welfare).

1.2.1 Some implications

The factor price equalization theorem implies that when a rich and poor country begin to trade, relative factor prices move closer together. The Stolper-Samuelson theorem describes the effect on real factor returns following a change in relative commodity prices, regardless of the source of that change. Suppose, for example, that the relaxation of trade restrictions between a rich and a poor country causes a fall (in the rich country) of the relative price of commodities that use intensively low skill labor (relative to other commodities). The theorem states that in this case, the real return to low skilled workers in the rich country falls.

People sometimes claim that trade makes everyone better off. The Stolper-Samuelson theorem shows why that claim is false: a change in relative prices caused by anything (including trade) is likely to make some agents worse off. In the absence of distortions, it is true that trade liberalization increases national income. Therefore, it would be possible to make transfers such that all agents would be better off with liberalized trade. However, in the absence of transfers, there is no reason to think that trade makes all agents better off. The Ricardo model, in which there is a single factor of production, obscures this point. We need two or more factors of production in order to determine the effect of a price change on the distribution of income.

As a second example of the implications of the Stolper Samuelson theorem, consider the effect of a price change on the demand for environmental services. We can view increased emissions as an example of an increased demand for environmental services, since higher emissions make greater use of the environment as a dumping ground. We can also think of increased use of a renewable resource, e.g. increased fishing or increased logging, as representing an increased demand for environmental services.

Up to this point I have taken the supply of factors as exogenous; the equilibrium conditions determine their price. However, it is a short step to view the supply of a factor as endogenous, i.e. responsive to its price. In many (but clearly not all) applications, it is reasonable to assume that an increase in the real return to a factor increases the available supply.

Suppose that a country uses the environment (either as a dumping ground for emissions, or a source of a natural resource) to provide services – i.e. factors – for production. More concretely, let $f(K, L, x)$ be the production function for a sector that creates emissions x . Suppose that these emissions

are taxed at the rate τ ; this tax is chosen (perhaps not optimally) to balance economic and environmental considerations. Given the tax, firms in this sector have a unit cost function $c(w, r, \tau)$. The firm views the tax as the cost per unit of emissions, just as the wage is the cost per unit of labor. Under the assumption of CRTS, the emissions per unit of output is $\frac{\partial c}{\partial t}$. If we take the tax τ as exogenous, the aggregate level of emissions, i.e. the aggregate supply of environmental services, is endogenous.

Now suppose that the country opens up to trade, or relaxes trade restrictions. If this liberalization has any effect, it must change commodity prices. Suppose that the liberalization increases the relative price of the commodity that uses intensively the environmental services. If the level of emissions were to remain constant (just as the aggregate stock of labor and capital remain constant), then the Stolper Samuelson theorem implies that there would have to be a more than proportional increase in the price of emissions. That is, τ would have to increase (more than proportionally to the price increase) in order to maintain the same level of aggregate emissions.

Consider the case where the tax does not increase (or more generally, it does not increase by enough to offset the commodity price increase). Since the firm's value of the marginal product of emissions has increased, it increases its level of emissions. In this case, when a country that has a comparative advantage in the environment-intensive commodity opens up to trade, the supply of environmental services – here, the level of emissions – increases.

If the tax is set optimally before and after the trade liberalization, liberalization increases national income. However, suppose that the tax does not adjust optimally. If the tax is optimal before liberalization, but is not adjusted optimally, then the economy has moved from a situation where there is one distortion (the trade restriction) to a situation where that distortion has been reduced but an environmental distortion has been created. In view of the theory of the second best, trade liberalization may not have increased real income in the economy.

1.3 The Rybczynski Theorem

The theorem states that (under certain technical assumptions): *At constant commodity prices, an increase in a factor of production leads to a more than proportional increase in output that uses that factor intensively, and a decrease in output of the other factor.*

Here is the intuition for the theorem. We know that under the assumptions of the theorem, there is a one-one relation between commodity and factor prices. Since commodity prices are held fixed, factor prices are fixed. Therefore, the unit labor requirements in each sector are fixed. Suppose, for example, that the stock of capital increases. Since unit input requirements are unchanged, it is not possible for output in both sectors to increase, as that would call for more labor than is available.

Capital flows into sector 1 (the capital intensive sector) requiring sector 2 to release labor, in order to maintain a constant capital/labor ratio in sector 1. However, as sector 2 releases labor, it must also release some capital, in order to maintain a constant capital/labor ratio in that sector. Output in sector 2 falls, and the amount of additional capital available to sector 1 is greater than the aggregate increase in the stock of capital. Therefore, the output of sector 1 increases more than proportionally to the increase in the aggregate stock of capital.

1.3.1 An example

Suppose that the economy initially has 100 units of capital and 125 units of labor ($K = 100$, $L = 125$). Suppose that (at the given relative commodity prices) it is optimal to produce a unit of soyburgers using 2 units of capital and 1 unit of labor; the capital/labor ratio in the sector is $2/1 = 2$. Suppose that at the given commodity prices, it is optimal to produce one unit of beer using 0.5 units of capital and 1 unit of labor; the capital/labor ratio in the beer sector is $0.5/1 = 0.5$. Let B equal the equilibrium production of beer and S equal the equilibrium production of soyburgers. Full employment of capital and labor requires

$$\begin{aligned} 2S + 0.5B &= 100 \\ 1S + 1B &= 125. \end{aligned}$$

The solution to these two full-employment conditions implies $S = 25$ and $B = 100$. Now suppose that there is a 10% increase in the amount of capital, so $K = 110$. Now the two full employment conditions are

$$\begin{aligned} 2S + 0.5B &= 110 \\ 1S + 1B &= 125. \end{aligned}$$

These conditions imply that at the new equilibrium $S = 31.67$ and $B = 93.33$. The increase in the amount of capital has caused beer production to fall from

100 to 93.33 units and it has caused soyburger production to increase from 25 to 31.67, an increase in $(\frac{31.67}{25} - 1) 100\% = 26.68\%$

1.3.2 Trade implications

Suppose that the country imports soyburgers, the good that is relatively capital intensive and that the stock of labor increases. For example, the population may become more educated, leading to an increase in “effective units” of labor. Alternatively, migration may increase the stock of labor.

At constant prices, the increase in the amount of labor increases national income. If both goods are normal, increased income leads to an increase in demand for both goods. Since the production of the capital intensive good (soyburgers) has fallen, the increased demand for this good requires increased imports. Since the prices have not changed, the increased imports must be paid for by increasing exports. In other words, for this example, an increase in the stock of labor leads to an increase in imports and exports.

1.4 The Heckscher-Ohlin theorem

This theorem gives conditions under which: *A country has a comparative advantage in (and therefore exports) the commodity that uses intensively the factor in which the country is relatively well endowed.*

For example, Home is relatively well endowed in labor if its capital/labor ratio $(\frac{K}{L})$ is less than its trading partner’s capital/labor ratio $(\frac{K^*}{L^*})$. In this example (and assuming that the conditions of the theorem hold), home has a comparative advantage in beer, the commodity that uses labor relatively intensively.