

EEP100 - Lanjouw  
Fall 2004

Problem Set 1

1. Find the first derivative for the following functions:

a)  $g(x) = 5x + 6$

b)  $f(x) = x(x^2)(1 - x)$

c)  $f(x) = \frac{x+2}{3x-6}$

d)  $f(x) = \ln(2x^2)$

e)  $f(t) = te^{5t+t^2}$

2. Find all the partial derivatives for the following functions:

a)  $f(x, y) = -x^2y + y$

b)  $f(x, y) = \ln(2xy)$

c)  $g(x, y, z) = z^2 + (1/[x^2 + y^2])$

3. Find all of the second-order partial derivatives for the functions in (2)

4. Describe the level sets of the following functions (e.g., solve for  $y$  in terms of  $x$  along paths on which the function is constant):

a)  $h(x, y) = 7x + 5y - 4.$

b)  $f(x, y) = 3 \ln(x^2) - 2y$

c)  $g(x, y) = \alpha x^\beta y^{1-\beta}$ , where  $\alpha, \beta$ , are constants.

5. In each of the cases above, calculate the slope of the level sets at an arbitrary point  $(x, y)$ . Hint: for b and c use the implicit function theorem.

6. Find all solutions (if any exist) to the following systems of equations:

a)

$$\begin{aligned} 5x + 12y &= 10 \\ 2x - 5y &= 3 \end{aligned}$$

b)

$$\begin{aligned} x^3 - 3y &= 0 \\ y &= x^2 \end{aligned}$$

c)

$$\begin{aligned} y &= x^2 + 4 \\ y + 4x &= 0 \end{aligned}$$

7. Determine any points that satisfy the first order condition for a local maximum or minimum. For each point, indicate whether it is a maximum, a minimum, or a point of inflection (ie, where the second derivative is zero). Graph each function.

- a)  $f(x) = x^3 - x$
- b)  $f(x) = -5x + x^2$
- c)  $f(x) = x^5$
- d)  $f(x) = x^3 + x^2 - x$
- e)  $f(x) = -1/x$

8. If we cut four equal-sized squares out of the corner of a square of cardboard 12 inches on a side, and fold up the remaining flaps, we get a tray shape without a top. What sized square should be cut out to maximize the volume of the tray?

9. Find the total derivative  $df/dt$  for the following functions. **Do it two ways:** using substitution and using the chain rule. Show your work:

- a)  $f(x, y) = x + y^3$ , where  $x(t) = t$ ,  $y(t) = \ln(t)$ .
- b)  $f(x, y) = \sqrt{xy} + e^{-x}$ , where  $x(t) = t + 1$ ,  $y(t) = 5$

10. In Chapter 1 of Varian he describes a market for apartments. There he argues that, in the short run, a \$50 tax placed on landlords by the city would not change the rent paid by consumers - that is, the full tax would be paid by the landlords. What would happen to total payment by renters if, instead, the city levied the tax on the renters? Explain your answer.